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# Wavelets and Binary Coalescences Detection

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**Abstract:** Gravitational waves generated by coalescing binary systems of neutron stars or black holes are expected to behave like *chirps*, *i.e.* amplitude and frequency modulated signals, buried into strongly correlated noise, with very low signal to noise ratio. This note presents a wavelet based algorithm for on line processing and detection of such signals, from interferometric detectors which are currently being constructed, and discusses a few examples. The details of the method and more complete simulations can be found in [6].

The detection of gravitational waves, predicted by general relativity but never observed so far, is a major challenge in today's experimental physics. Several projects are currently being developed, among which one may quote the LIGO [1], GEO [4] and VIRGO [2] laser interferometric detectors.

Among the potential sources for gravitational waves, the most promising are presumably pulsars, and coalescing binary systems of black holes or neutron stars. Such systems are expected to produce "simple" signals, for which available models are considered reliable. We focus here on the case of coalescing binary systems. The corresponding gravitational waves take the form of *chirps*, i.e. *amplitude and frequency modulated signals.*

The detection problem may be formulated as follows. The signal takes the form

$$f(x) = h_\theta(x - x_0) + n(x) \tag{1}$$

where $h_\theta(x)$ is the reference signal (given in good approximation by various post-Newtonian theories [8]), depending on a set of parameters to be estimated, represented generically by $\theta$, $x_0$ is the date of coalescence (to be estimated as well) and $n(x)$ is the experimental detector noise.

The classical method to such a detection problem is provided by *matched filters* [8]. However, the matched filter applies to the case where the parameters $\theta$ of the signal are known. Since the parameters are *not* known, one has to construct filters for sufficiently many choices of the parameters, which leads to a very large number of filters.

This note describes an alternative approach based on wavelet transform. Since the detection problem actually amounts to that of estimating a shift in a time-scale domain, wavelet methods are a natural tool. In addition, it is important for detection purpose to keep scaling and translation covariance properties of the time-scale representation, we focus on continuous wavelet transform. The interested reader is refered to [6] for more details.

## I. SIGNAL AND NOISE MODELS

The signal measured at the detector is a strain, denoted by $h(x)$, and is the superposition of a gravity wave signal, and an additive detector noise.

### A. Signal

Consider a system of two stars of masses $m_1$ and $m_2$ respectively, at distance $R$ from the observer. Following the conventional use, we introduce the *chirp mass* $\mathcal{M} = \left((m_1 m_2)^3/(m_1+m_2)\right)^{1/5}$. In the Newtonian approximation, the expected reference signal is given by $h(x) = A h_F(x_0 - x)$, where

$$h_F(x) = A x^\alpha \Theta(x) \cos\left(\Phi - \frac{2\pi}{\beta+1} F x^{\beta+1}\right) , \tag{2}$$

with $\Theta(x)$ the Heaviside step function, $\alpha = -1/4$ and $\beta = -3/8$. Here, $x_0$ is the coalescence date, $\Phi$ is a global phase, and $A$ and $F$ are related to physical parameters by $F = C_F \mathcal{M}^{-5/8}$ and $A = \epsilon C_A \frac{1}{R} \mathcal{M}^{5/4}$ for some numerical constants $C_A$ and $C_F$, and a constant $0 \le \epsilon \le 1$ depending on the relative orientation between the source and the detector.

The expected signal is then a chirp, to which local amplitude $A(x_0 - x)^\alpha$ and frequencies $\nu(x) = F(x_0 - x)^\beta$ may be associated. An example is displayed in Figure 1.

More general post-Newtonian approximations are described in [8] for example.

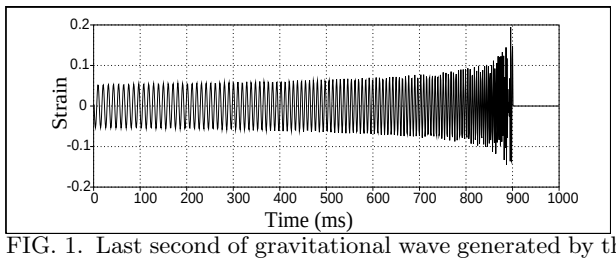


FIG. 1. Last second of gravitational wave generated by the collapse of a binary with $m_1 = 10 M_\odot$, $m_2 = 1 M_\odot$, coalescing at a distance $R = 200 Mpc$. The amplitudes are to be multiplied by $10^{-21}$.

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### B. Noise

The detector noise is the superposition of several contributions, and is generally modeled as a weakly stationary process, with power spectral density $S(\nu) = \mathcal{E}(2\pi\nu)$ of the form

$$\begin{aligned} S(\nu) &= S_1\nu^{-5}+S_2\nu^{-1}+S_3\left(1+\left(\tfrac{\nu}{500Hz}\right)^2\right) \\ &+ \tfrac{1}{\nu}\textstyle\sum_{n=1}^{\infty}\tfrac{K}{(\nu^2-\nu_n^2)^2-\phi^2\nu_n^4} \ . \end{aligned} \tag{3}$$

Here, $S_1, S_2, S_3, K, \phi^2$ are constants depending on the experimental setup. The spectral density used in our simulations is displayed in Figure 2.

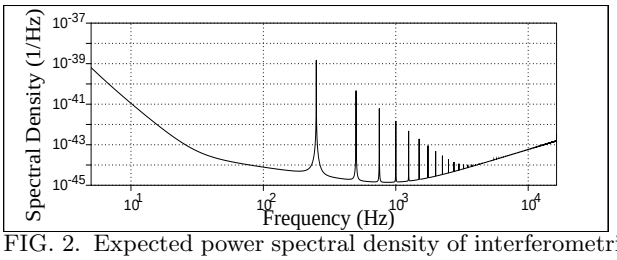


FIG. 2. Expected power spectral density of interferometric detector noise.

## II. A DETECTION ALGORITHM

### A. Prewhitening wavelet transform

Our main tool will be a version of continuous wavelet transform adapted to the problem as follows. Let $\psi(x)$ be an integrable and square-integrable function, such that $\hat{\psi}(k) = 0\ \forall k \leq 0$ and that $0 < c_\psi := \int_0^\infty |\hat{\psi}(k)|^2 \frac{dk}{k} < \infty$ Let $n(x)$ be a weakly stationary process, and let $C$ be the associated covariance operator:

$$Cf(x) = \frac{1}{2\pi}\int e^{ikx}\mathcal{E}(k)\hat{f}(k)dk \ . \tag{4}$$

The usual continuous wavelet transform of $f(x) \in L^2(\mathbb{R})$ is defined by $T_f(b,a) = \langle f, \psi_{(b,a)}\rangle$, where $\psi_{(b,a)}(x) = (1/\sqrt{a})\psi((x-b)/a)$. We define the corresponding *prewhitening wavelet transform* of any $f(x) \in L^2(\mathbb{R})$ as follows

$$\begin{aligned} W_f(b,a) &= \langle C^{-1/2}f, \psi_{(b,a)}\rangle \\ &= \tfrac{\sqrt{a}}{2\pi}\textstyle\int e^{ikb}\tfrac{\hat{f}(k)\overline{\hat{\psi}(ak)}}{\sqrt{\mathcal{E}(k)}}dk \ . \end{aligned} \tag{5}$$

Clearly, given a stationary process $n(x)$ of spectral density $\mathcal{E}(k)$, we have $V(a) := \mathbb{E}\{|W_n(b,a)|^2\} = ||\psi||^2$. In practice, we are given a signal of the form $f(x) = f_0(x) + n(x)$, where $n(x)$ is as before. Introducing the new representation,

$$M(b,a) = |W_f(b,a)|^2 - V(a) \ , \tag{6}$$

we easily obtain that $\mathbb{E}\{M(b,a)\} = |W_{f_0}(b,a)|^2$.

### B. Transform of binary coalescence signals

Let us come back to the detection problem. We stick to the Newtonian situation. If $\psi(x) = A_\psi(x)\exp\{i\phi_\psi(x)\}$ is an analytic wavelet with linear phase $\phi_\psi(x) = \omega_0 x$, it follows from the analysis in [5] that the standard continuous wavelet transform is localized in a neighborhood of a curve in the time-scale plane, called the ridge of the wavelet transform, of equation

$$b = b_{(F,x_0)}(a) = x_0 - \left(\frac{2\pi aF}{\omega_0}\right)^{8/3} \ . \tag{7}$$

We assume that the prewhitening wavelet transform has the same localization properties (see [6] for more details).

### C. Line Integrals

Consider now the realistic situation

$$f(x) = Ah_F(x_0 - x) + n(x) \ , \tag{8}$$

where $n(x)$ is a weakly stationary stochastic process with spectral density $\mathcal{E}(k)$ given in (3), modeling the detector noise. Given a candidate $(\tau,\gamma)$ for the pair $(x_0, F)$, we consider the corresponding candidate $b_{(\tau,\gamma)}$ (see Eq. (7)), and the *line integral*

$$\mathcal{L}_\tau(\gamma) = \int M(b_{(\tau,\gamma)}(a), a)\frac{da}{a} \ , \tag{9}$$

with $M(b,a)$ as in (6). The algorithm we propose is based on the following maximization problem

$$\max_{(\tau,\gamma)} \mathcal{L}_\tau(\gamma) \ . \tag{10}$$

### D. The Maximization Algorithm

The maximization problem (10) amounts to a search in a two-dimensional space, parametrized by the pair $(x_0, F)$. We now describe an algorithm compatible with a possible real time implementation.

We first fix a threshold, hereafter denoted by `THRESHOLD` for the line integral (9), the choice of which is based upon an a priori knowledge of the noise, and some a priori estimates on the false alarm or non detection probabilities. For a given date $\tau$, set

$$\hat{F}(\tau) = \arg\max_\gamma \mathcal{L}_\tau(\gamma) \ , \tag{11}$$

$$\hat{A}(\tau) = \max_\gamma \mathcal{L}_\tau(\gamma) \ , \tag{12}$$

Our approach relies on a two-step optimization:

- `FOR` $\tau = \tau_{min}$ `TO` $\tau = \tau_{max}$
  1. Solve the problem (12) with respect to $\gamma$.
  2. Store the values $\hat{F}(\tau)$ and $\hat{A}(\tau)$.
- `IF` $\hat{A}(\tau) \geq$ `THRESHOLD`: mark $\tau$ as a possible date for an event.

The maximization is carried out by standard maximization methods (an adapted version of Brent's method in our implementation)

## III. NUMERICAL EXAMPLES

We present here some of the numerical results obtained with the algorithm described above, in three typical situations. A detailed analysis and more examples can be found in [6], together with an analysis of the statistics of detection.

The numerical tests have been performed on signals generated following the lines of Section I. The continuous wavelet transform is discretized in the form $T_f(b_m, a_n)$ with $b_m = m\nu_s$ ($\nu_s$ being the sampling frequency) and $a_n = 2a_0^n$, with $a_0 = 2^{1/8}$ and $n = 0, \ldots 39$ (for $\nu_s = 2\,kHz$, this corresponds to the frequency range $30\,Hz - -500\,Hz$).

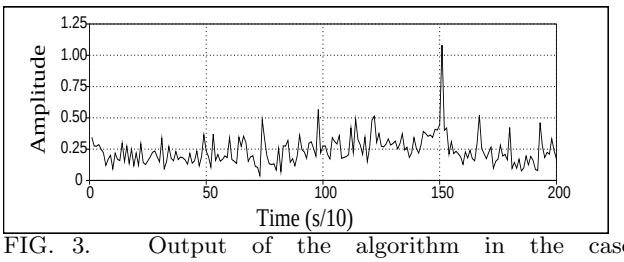


FIG. 3. Output of the algorithm in the case $m_1 = m_2 = 1.4M_\odot$ and $R = 150Mpc$.

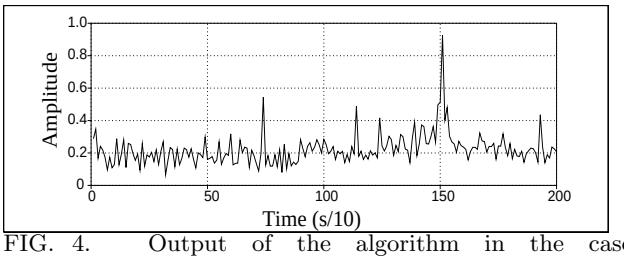


FIG. 4. Output of the algorithm in the case $m_1 = 1M_\odot$, $m_2 = 10M_\odot$ and $R = 250Mpc$.

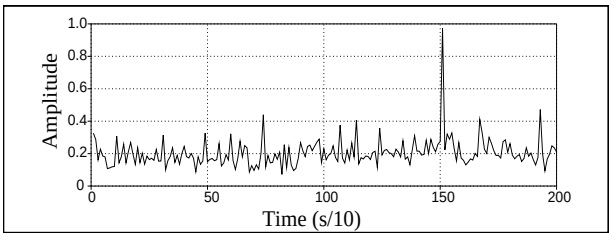


FIG. 5. Output of the algorithm in the case $m_1 = m_2 = 10M_\odot$ and $R = 1.2Gpc$.

We display in Figure 3 (resp. Figure 4, resp. Figure 5) the function $\hat{A}(\tau)$ introduced in Eq (12), for the case of a binary system formed of two neutron stars of 1.4 solar masses each (resp. 1 and 10 solar masses, resp. 10 solar masses each), coalescing at a distance of $150\,Mpc$ (resp. $250Mpc$, resp. $1.2Gpc$). In each case, the coalescence date was set to $x_0 = 15.1sec.$. The sampling frequency was set to $\nu_s = 2kHz$ (resp. $\nu_s = 1kHz$). A significant peak is clearly seen, and in the three considered cases, the peak is located precisely at the correct date $\tau = x_0$.

A more detailed statistical analysis shows that in the three considered situations, the peak value $\hat{A}(x_0)$ is far above the 0.1% critical value of the distribution of $\hat{A}$ in the noise alone reference.

The algorithm can also be used for estimating the frequency parameter $F$ when an event has been detected. In the three considered examples, we obtained $\hat{F} = 134.6$ while the true value was $F = 133.47$ (resp. $\hat{F} = 85.26$ for $F = 85.97$, resp. $\hat{F} = 39.59$ for $F = 39.05$), i.e. a precision of the order of 1%.

## IV. CONCLUSIONS

In this note, we have summarized a wavelet based algorithm for on line processing of interferometric detector signals and binary coalescence detection. The results in [6] and those shown here prove that this approach is a reliable one. In addition, the computational cost is low enough to allow real time processing of data on standard Unix workstations. For these reasons, we believe that this approach should be considered seriously for on line detection of binary coalescence, as well as for the corresponding parameter estimation problems.

The details of the method are given in [6], together with results of simulations and statistical analysis of the performances of the method.

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