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Marc REBILLAT, Kerem EGE, Maxime GALLO, Jerome ANTONI - Exponential sine sweeps for the autonomous estimation of nonlinearities and errors assessment by bootstrap Application to thin vibrating structures - 2015

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# Exponential sine sweeps for the autonomous estimation of nonlinearities and errors assessment by bootstrap Application to thin vibrating structures

Marc Rébillat<sup>1</sup>, Kerem Ege<sup>2</sup>, Maxime Gallo<sup>2</sup>, Jérôme Antoni<sup>2</sup>

# A linear behavior in vibroacoustics ?

- Geometrically nonlinear vibrations of plates, shells:

when the amplitude of the transverse displacement  $w$   
exceeds the plate/shell thickness  $h$



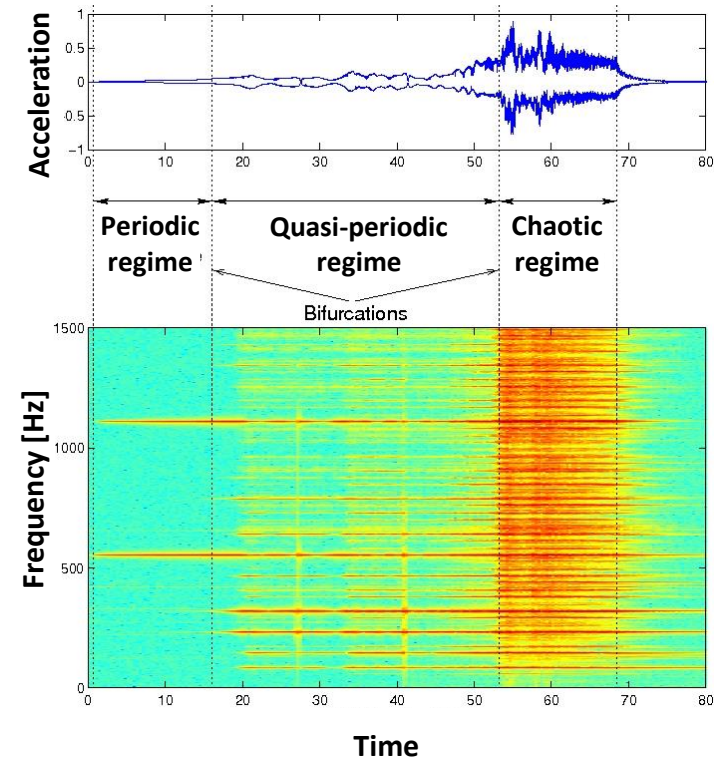
→ *jump phenomenon, hysteresis,  
internal resonance*

- Boundary conditions, joints, contacts...



Example : Gong

Cyril Touzé, Olivier Thomas



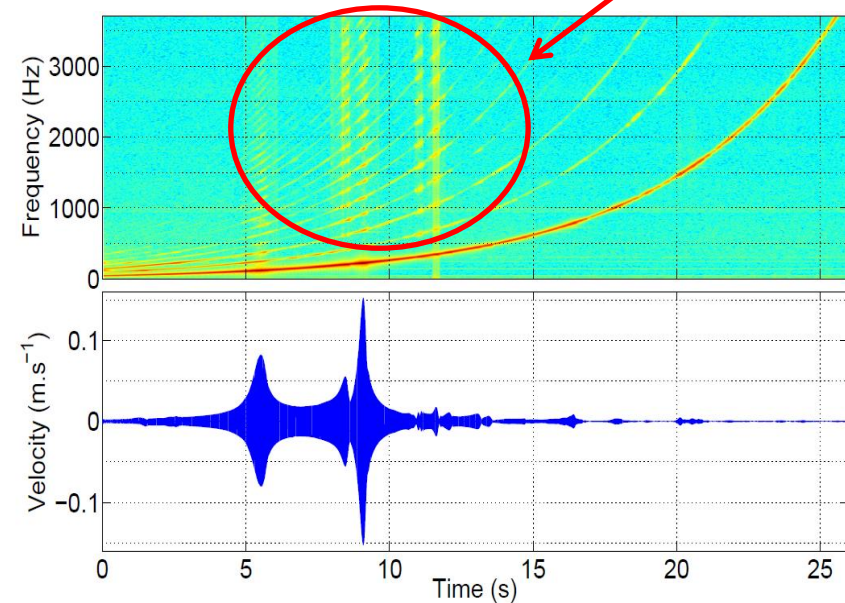


# A linear vibroacoustic measurement ?

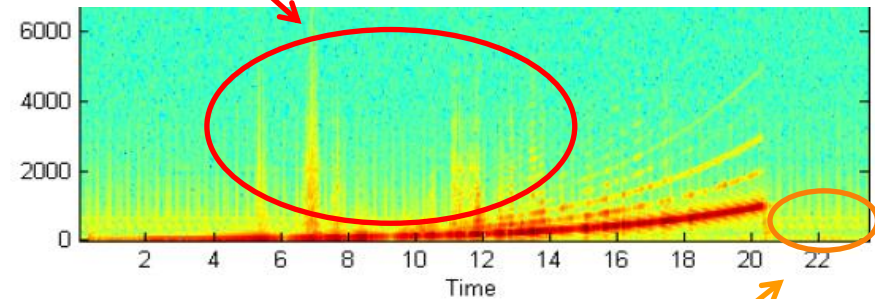
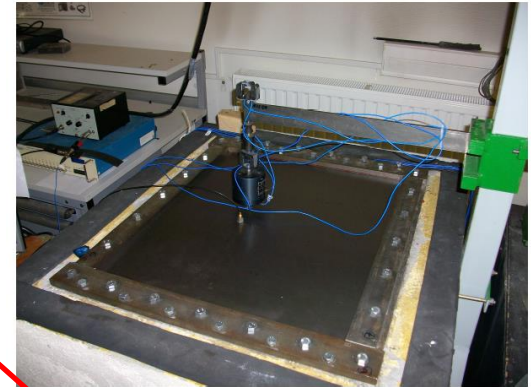
## Acoustical excitation



*How to extract the linear/nonlinear parts of a typical vibroacoustic measurement ?*



## Vibrational excitation

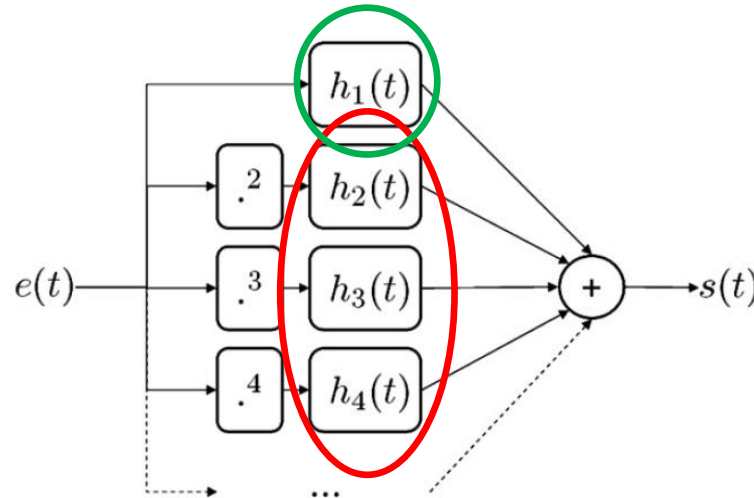


*How to estimate the effects of experimental noise ?*

- Estimation of Parallel Hammerstein models
  - Theory
  - Application to a vibrating plate
  - Separation of the intrinsic nonlinear contributions
  
- Improvements of the method (sweep repetition)
  - Theory
  - Noise estimation by synchronous averaging
  - Uncertainty estimation by bootstrap
  - Autonomous kernel order estimation
  
- Conclusion and perspectives

# Parallel Hammerstein models estimation

⇒ Slightly nonlinear system modeled as **parallel Hammerstein models**

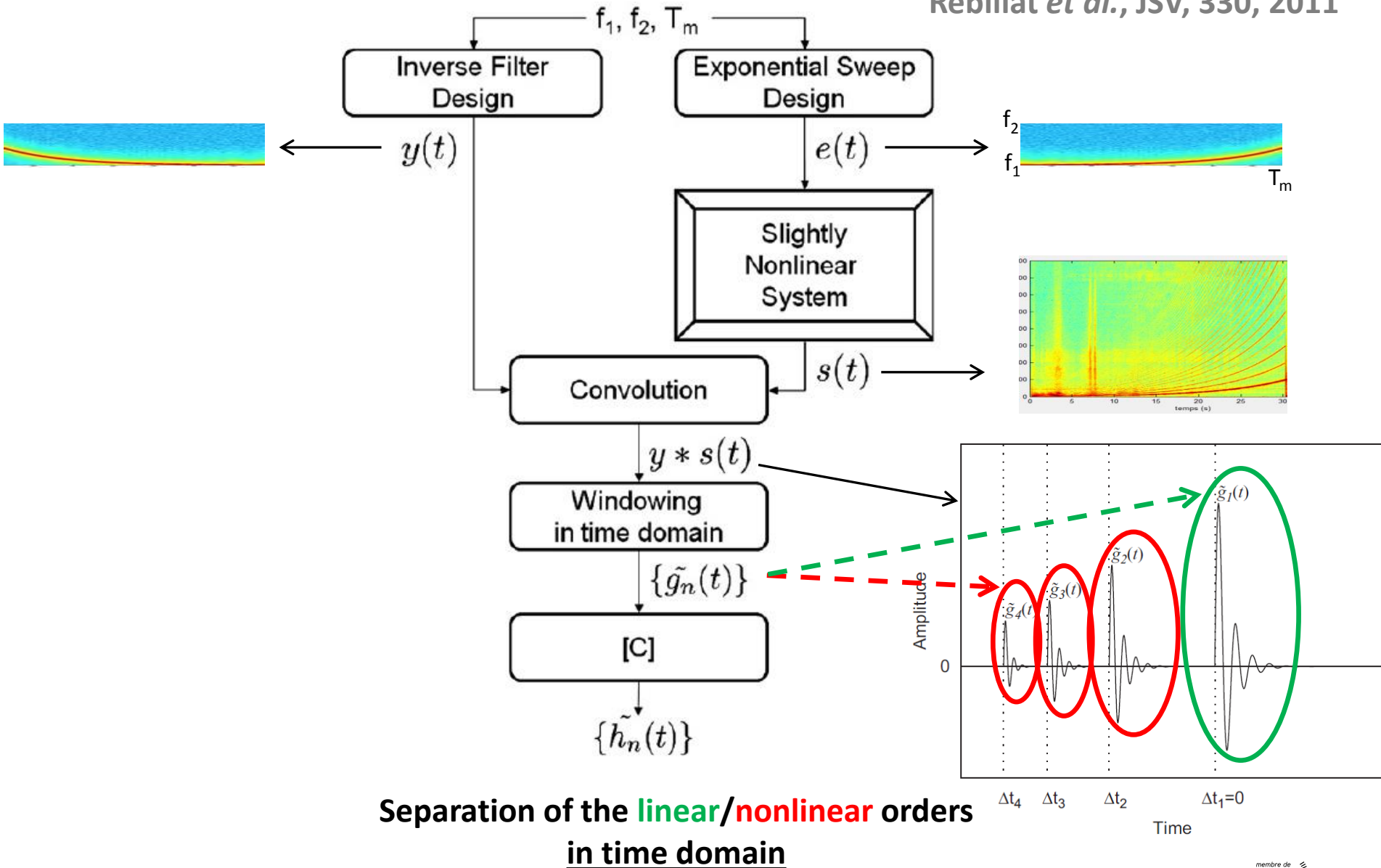


⇒ Kernels easily estimated using **exponential sine sweeps**  
(see [Farina, AES, 2000], [Rébillat, JSV, 2011] or [Novak, IEEE Instrumentation, 2011] for example)

→ Separation of the **linear**/**nonlinear** orders

# Parallel Hammerstein models estimation

Rébillat *et al.*, JSV, 330, 2011

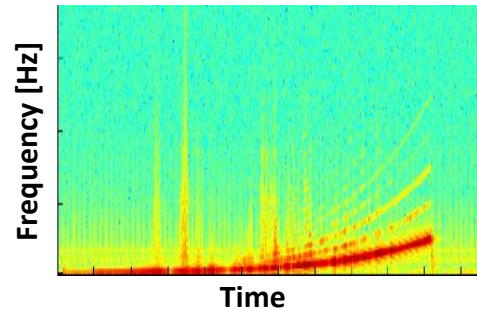


Separation of the **linear/nonlinear** orders  
in time domain

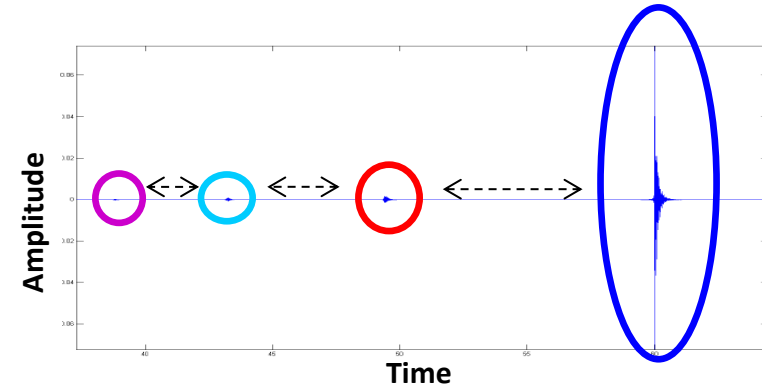
# Application to a vibrating plate

- Clamped Steel plate (1mm)
- Shaker + Accelerometer

One sweep of **20 seconds**  
[20Hz-1kHz]

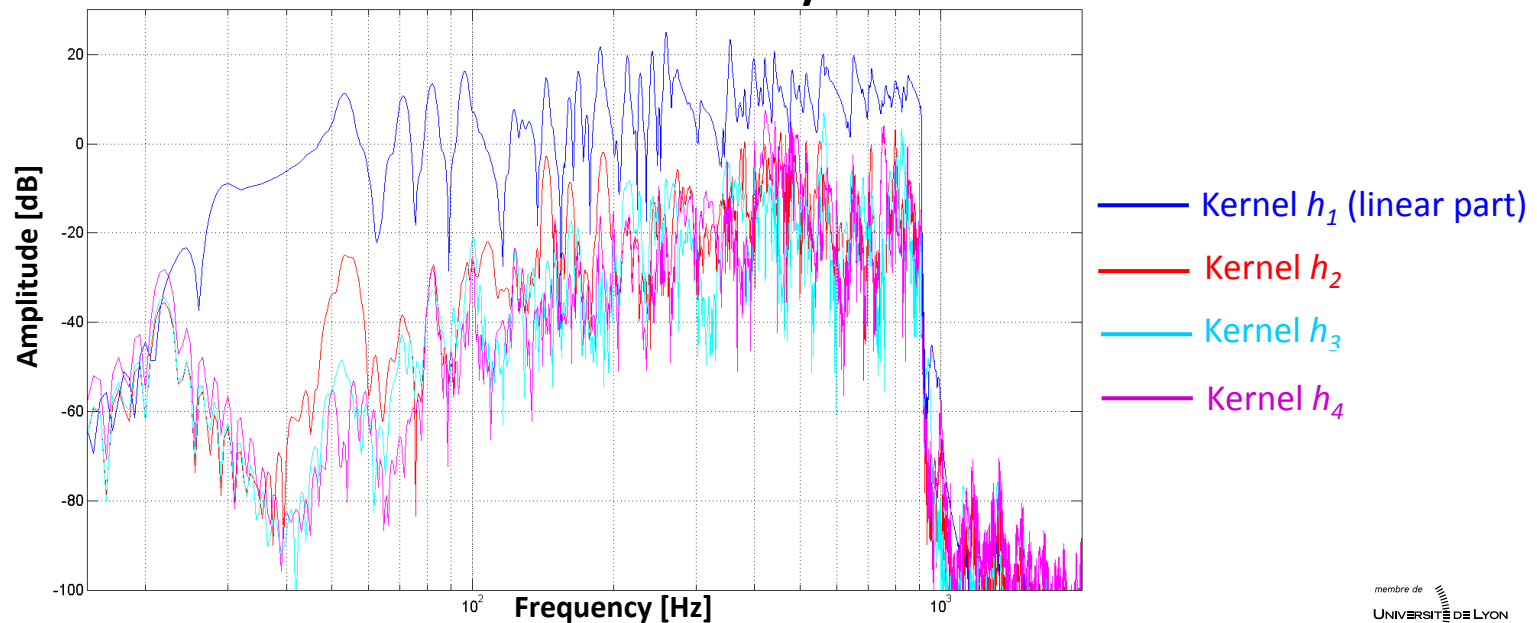


After deconvolution:



Importance of the **length of the sweep** for  
time domain separation

## Identified Kernels of the system

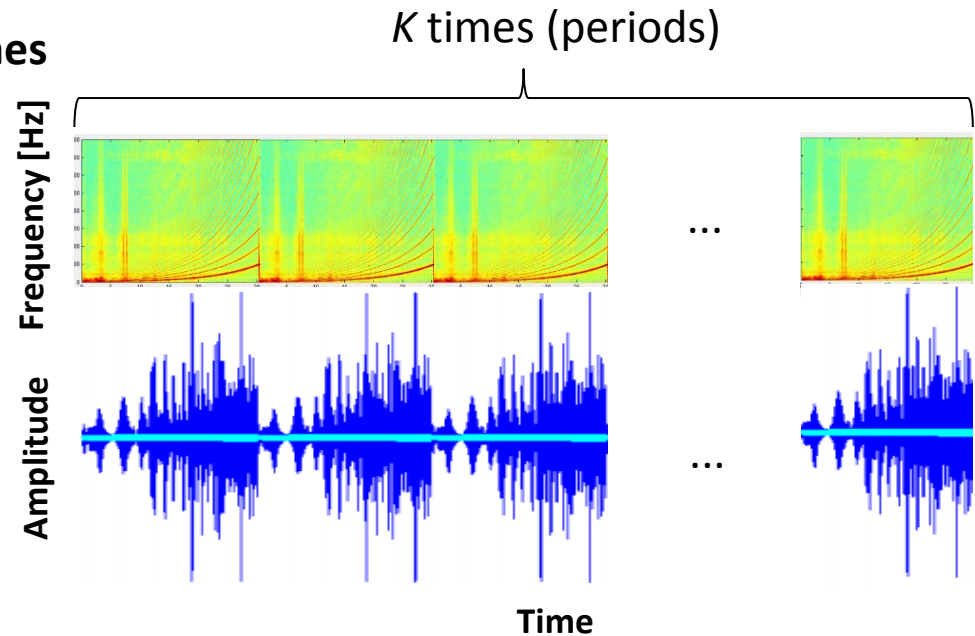




# Improvement of the method: Sweep repetition & Noise estimation

- Measured signal = system response + experimental noise  $x(t) = s(t) + n(t)$

- Repetition of the same sweep  $K$  times



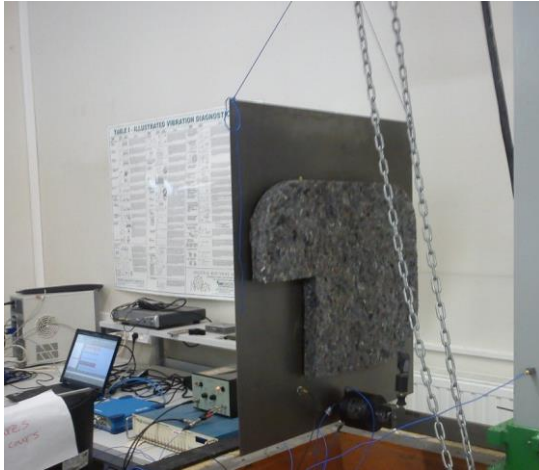
- Estimation of the noise  $\bar{n}(t) = x(t) - \bar{s}(t)$

using the time synchronous averaging  $\bar{s}(t)$   
of the system response  $x(t)$

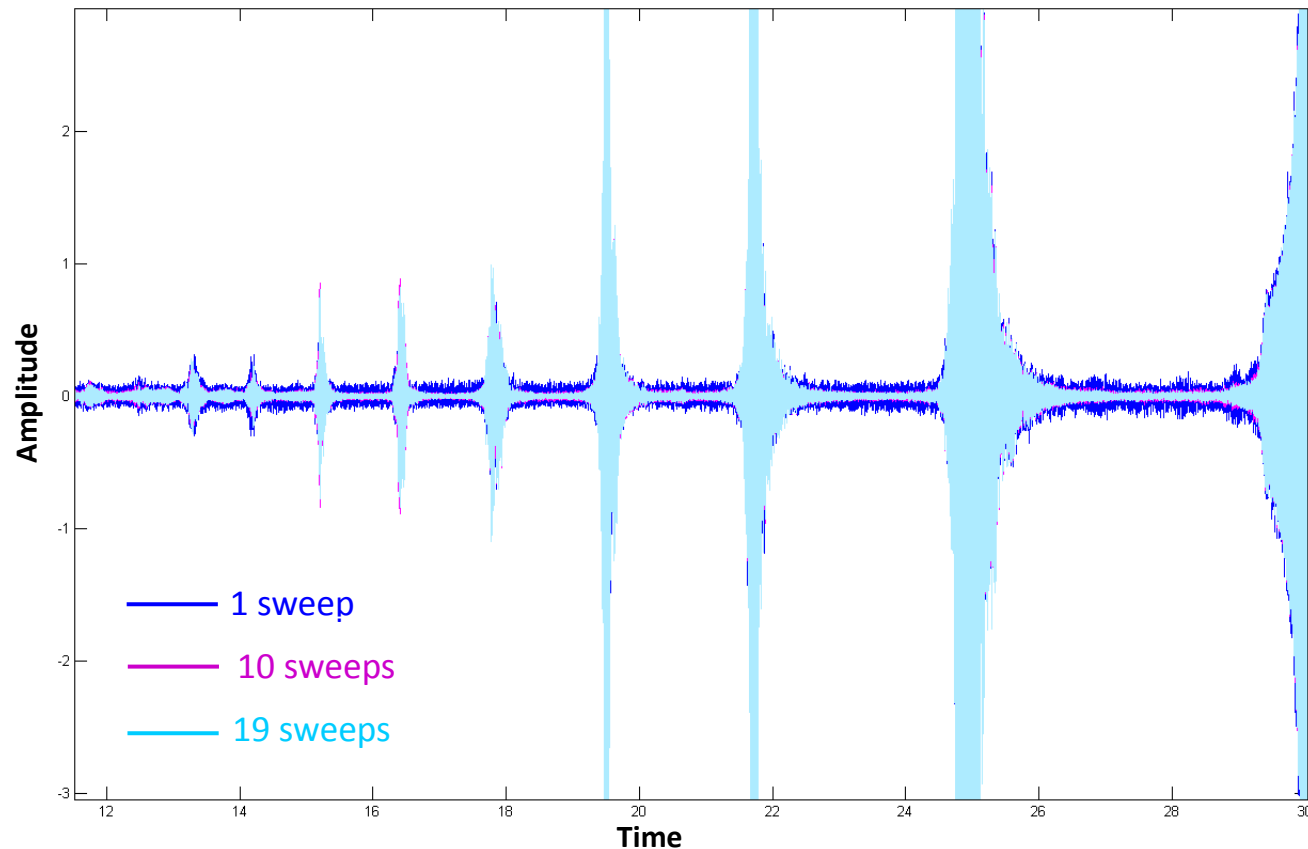
$$\bar{s}(t) = \frac{1}{K} \sum_{k=0}^{K-1} [x(t - kT)]$$

# Application to a vibrating plate – Noise estimation

- Free-free damped steel plate (1mm)
- Shaker + Accelerometer



Influence of sweep repetitions (number of periods K)



Increase of K



Synchronous averaging  
on **more periods**

$$\bar{s}(t) = \frac{1}{K} \sum_{k=0}^{K-1} [x(t - kT)]$$

→ Better extraction  
of the noise

→ More precise estimations of  
the kernels of high orders

# Uncertainty of measurement using Bootstrap

How to study the  
variability of the measurements ?



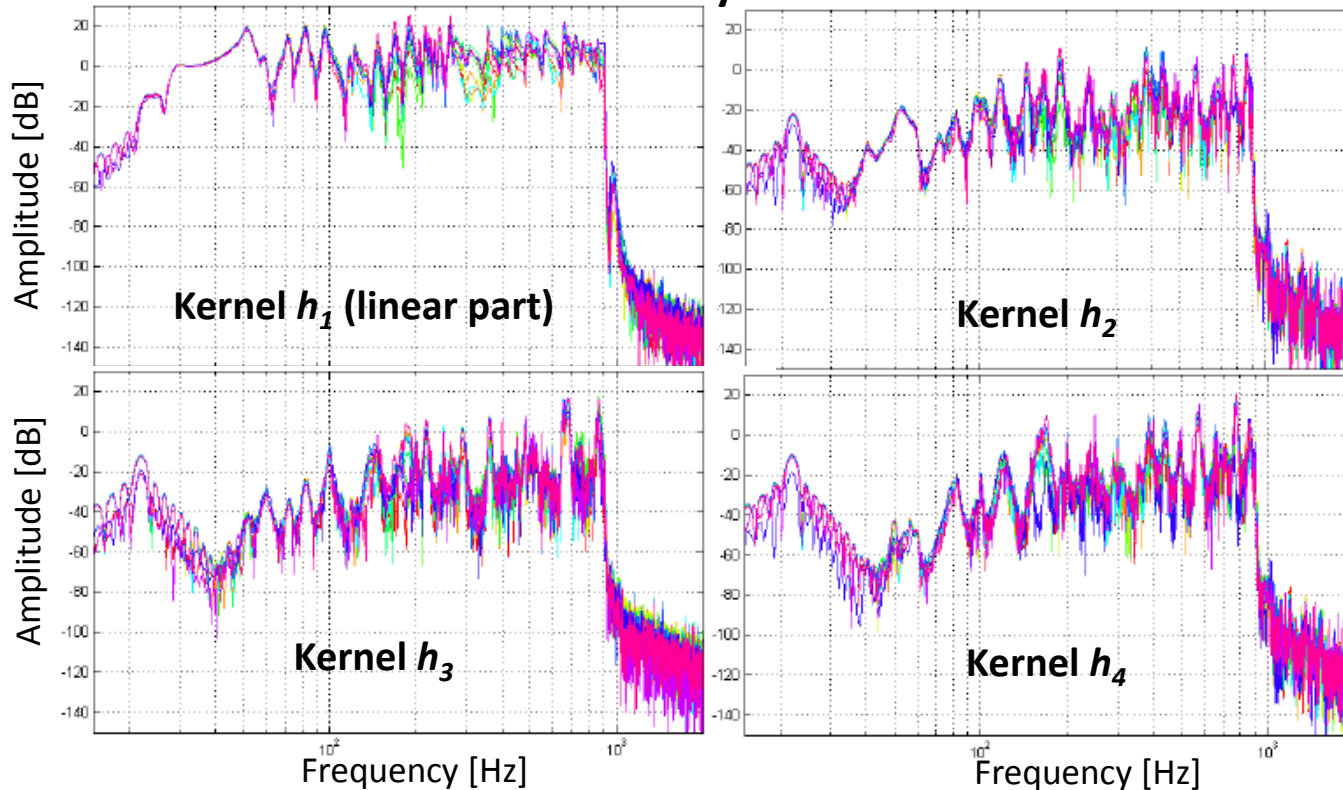
Bootstrap method

**B** random sample with replacement (of K sweeps each time)  
Synchronous averaging on K repetitions



**B** different  
kernel estimations

Identified Kernels of the system for  $B=10$  and  $K=19$



Way to identify and  
quantify variability  
(uncertainty) of plate  
nonlinearities  
estimations

# Uncertainty of quantification

Synchronous averaging on  $K$  random draw with replacements (performed  $B$  times)

Estimation of parallel Hammertein model (performed  $B$  times)

Mean over the  $B$  estimations

$K$  repetitions of exponential sine sweeps (ESS)

$B$  estimations of the response

$B$  estimations of the  $N$  kernels

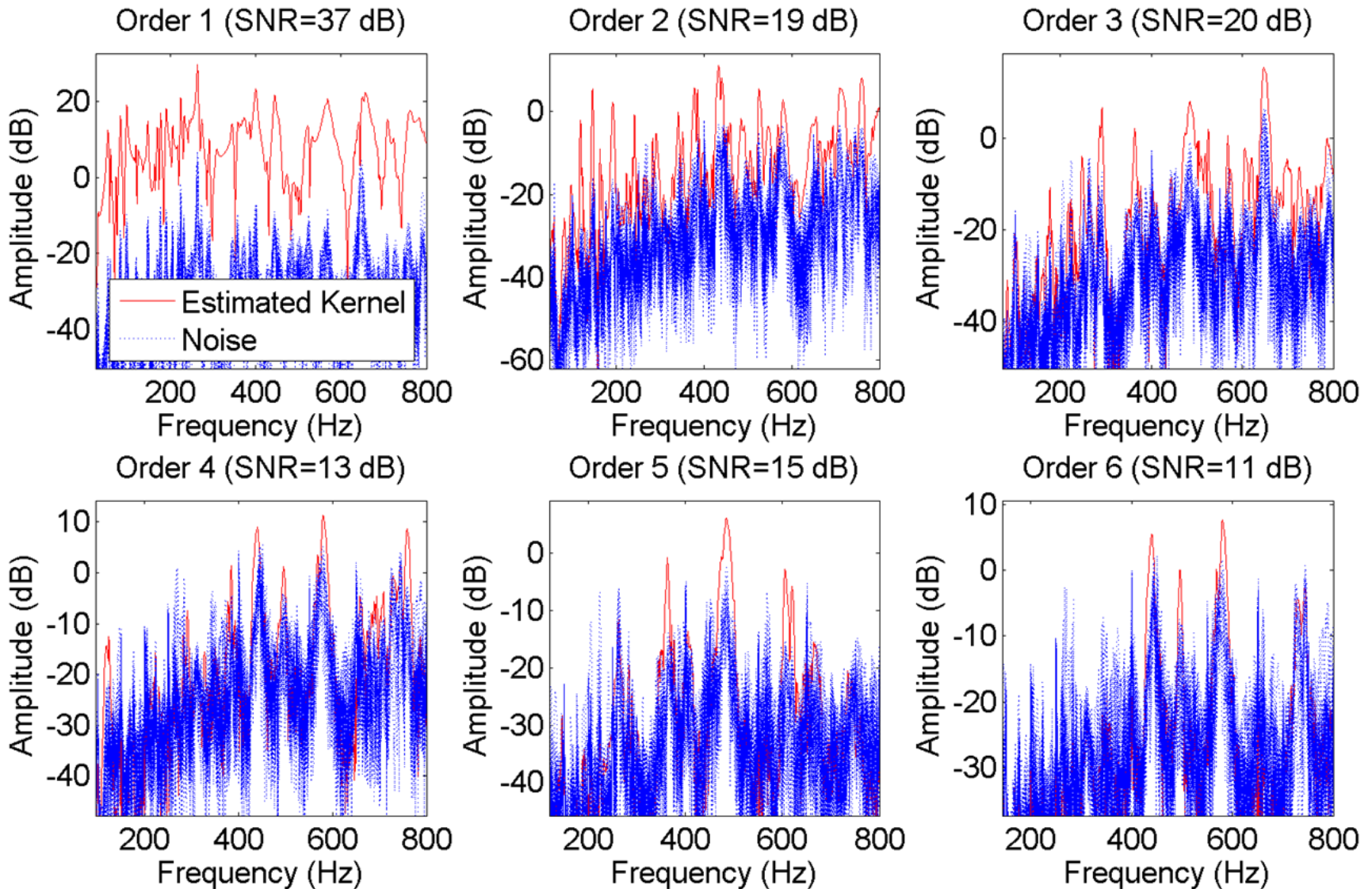
1 mean estimation of the  $N$  Kernels



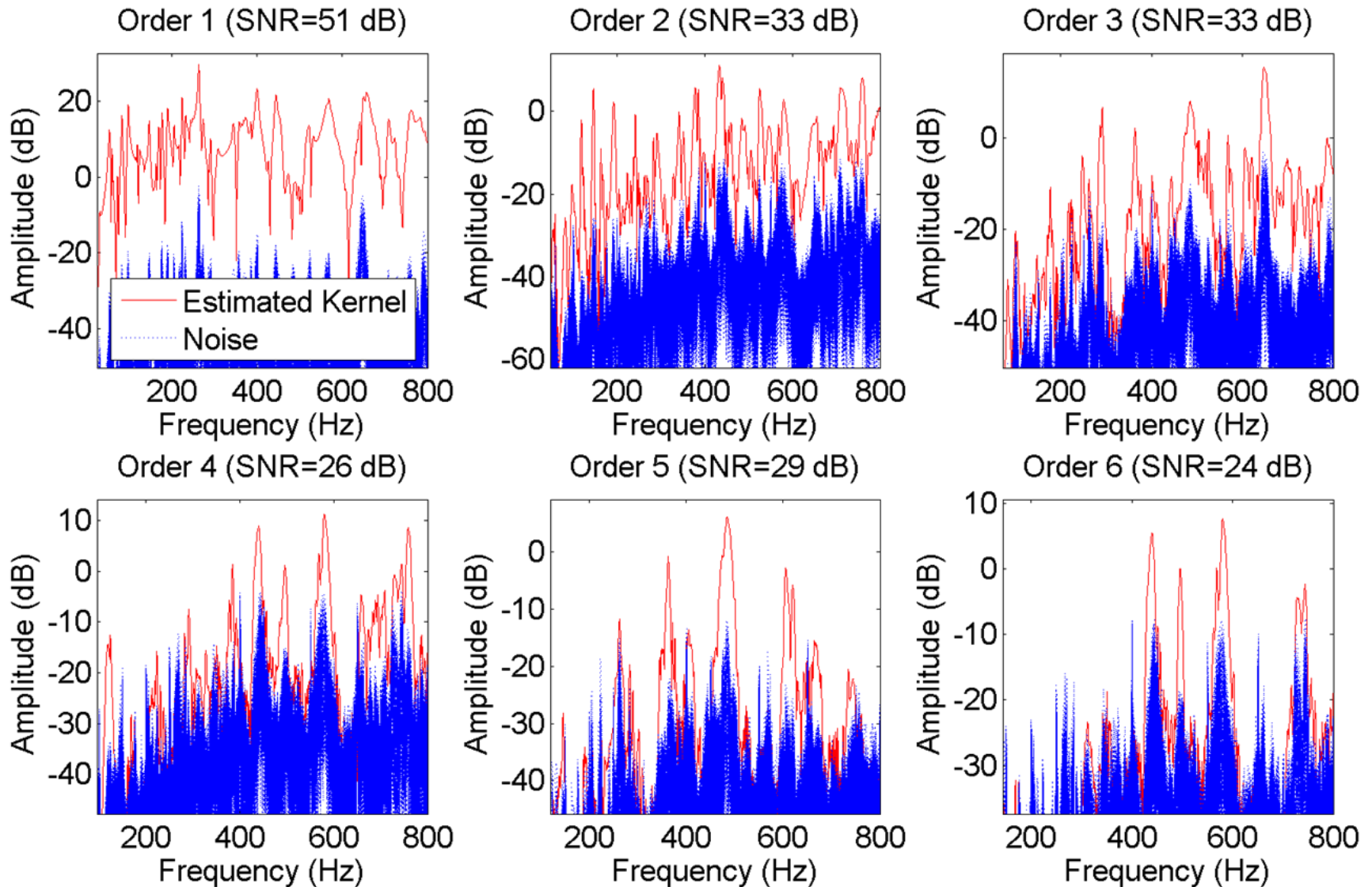
$B$  estimations of the « estimation noise » on the  $N$  kernels



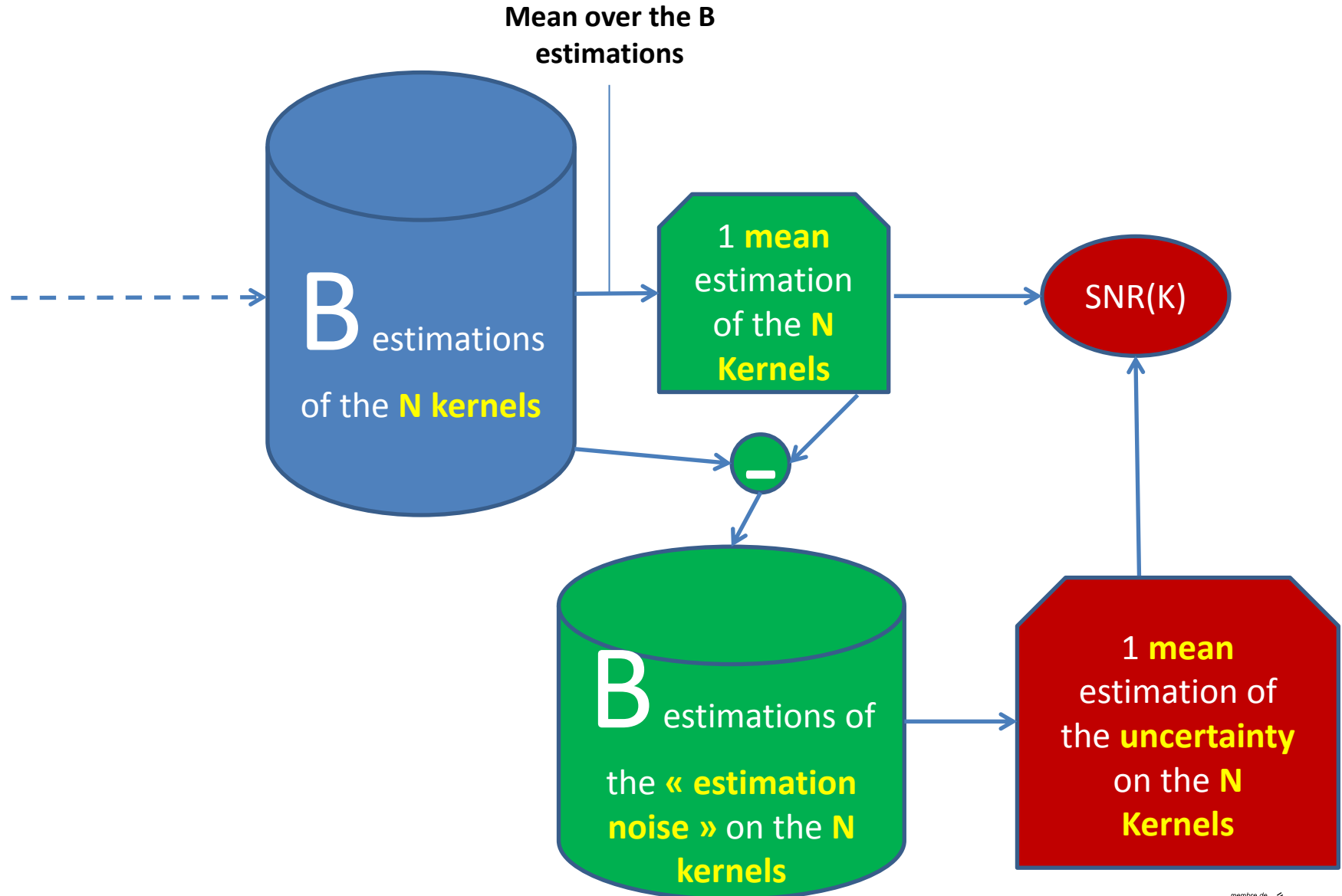
# No averaging (K=1, B=150)



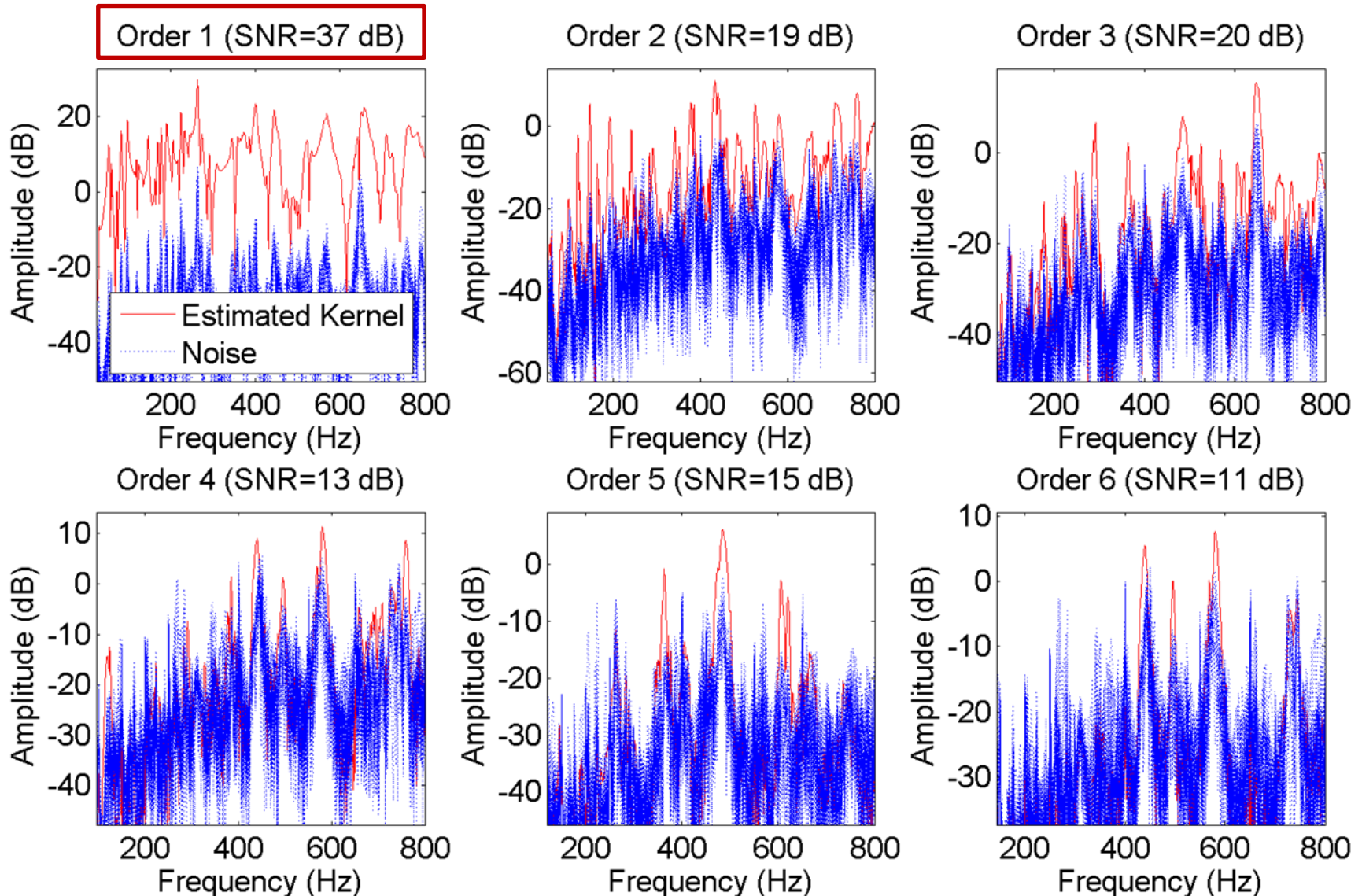
# Averaging (K=19, B=150)



# Uncertainty of quantification

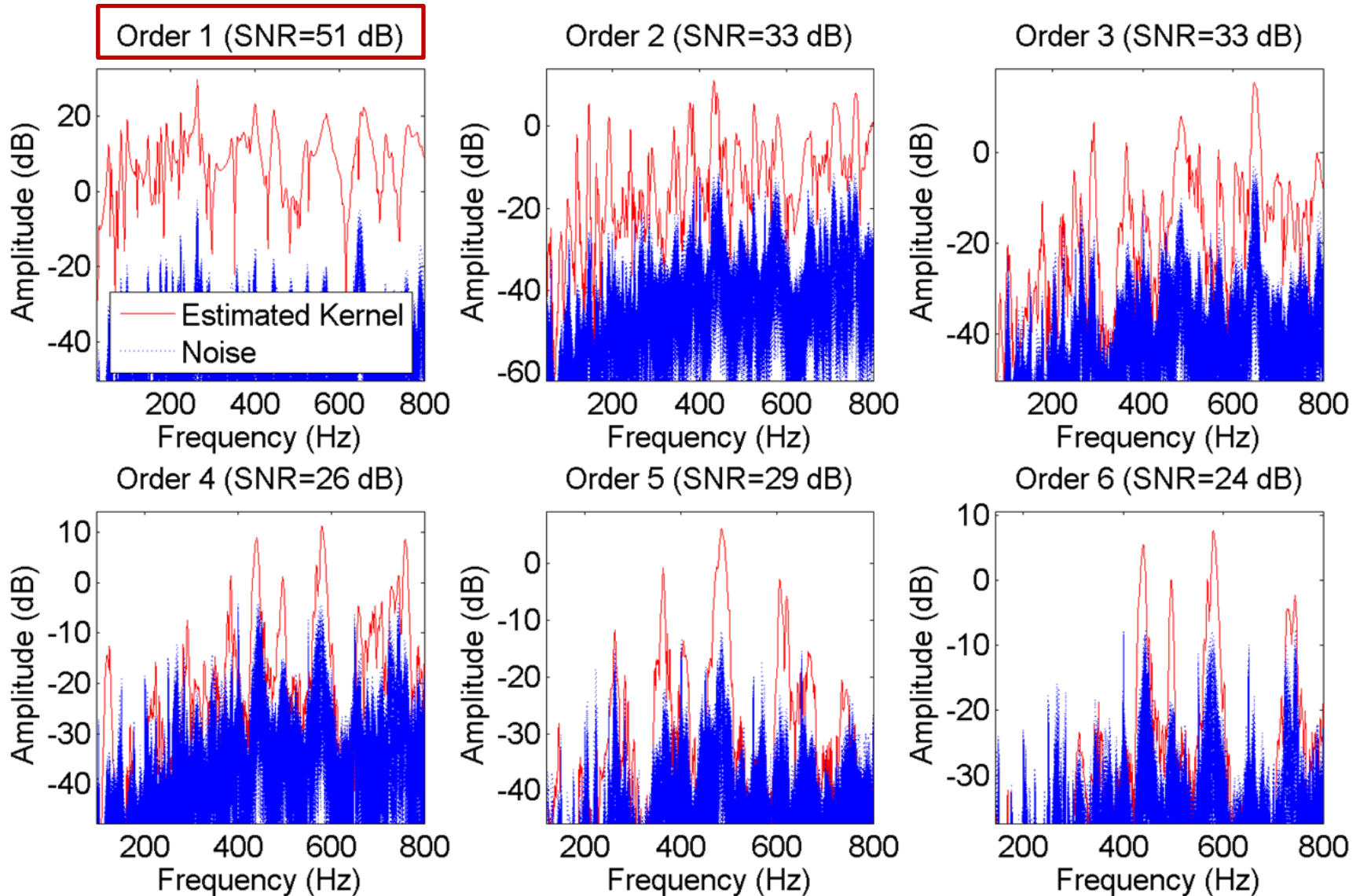


# No averaging (K=1, B=150)

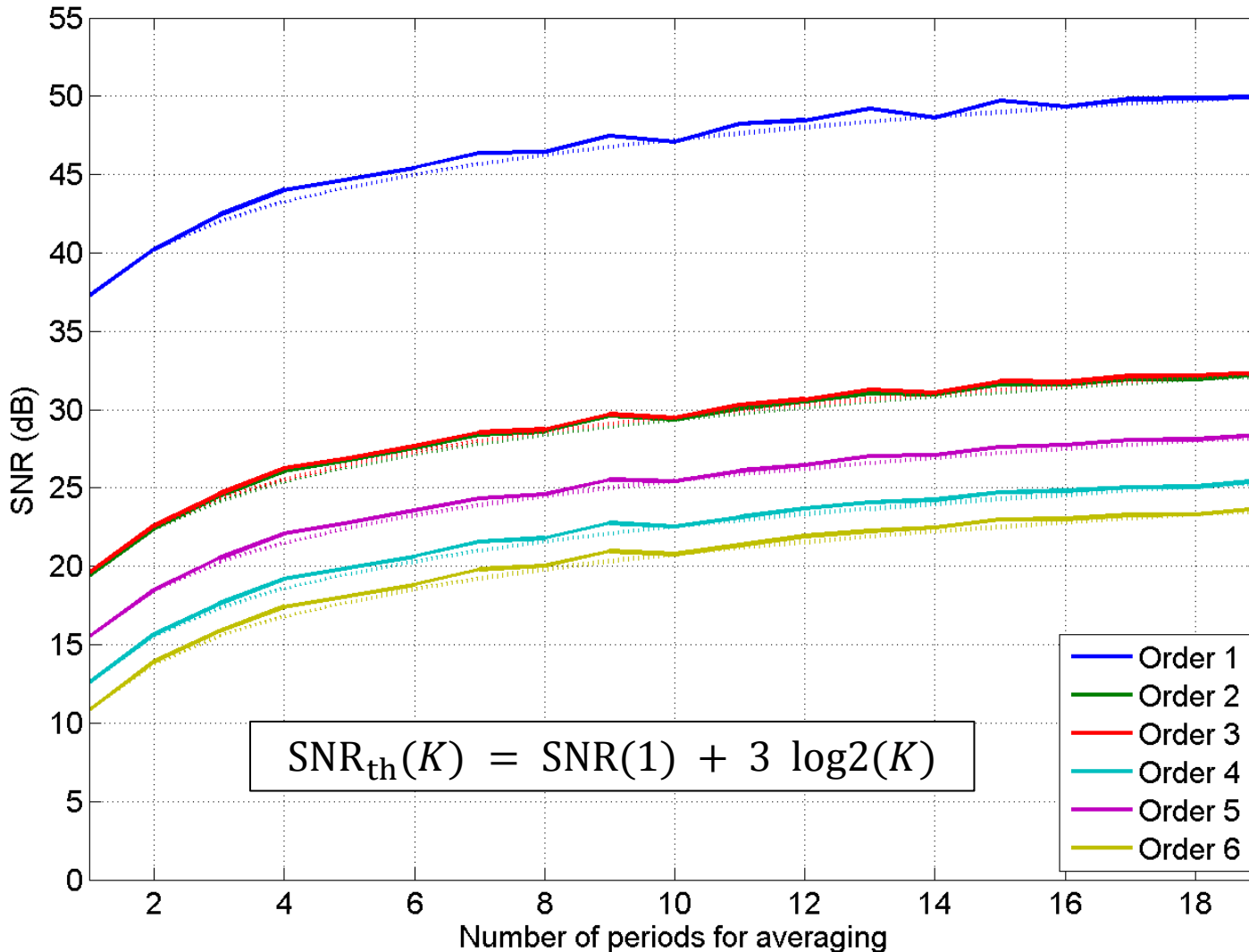




# Averaging (K=19, B=150)

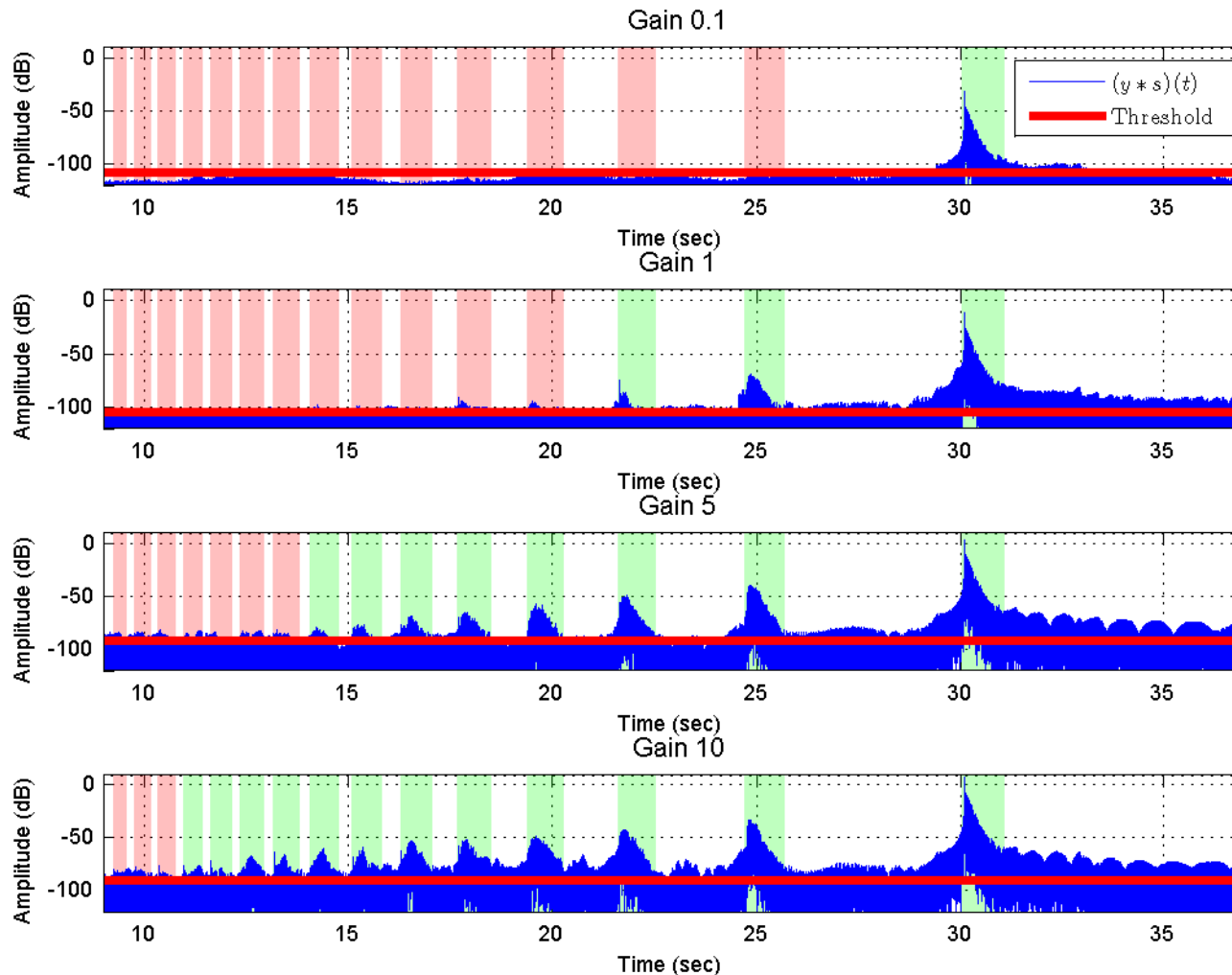


# Effect of repetition number K on SNRs



# Autonomous kernel order estimation

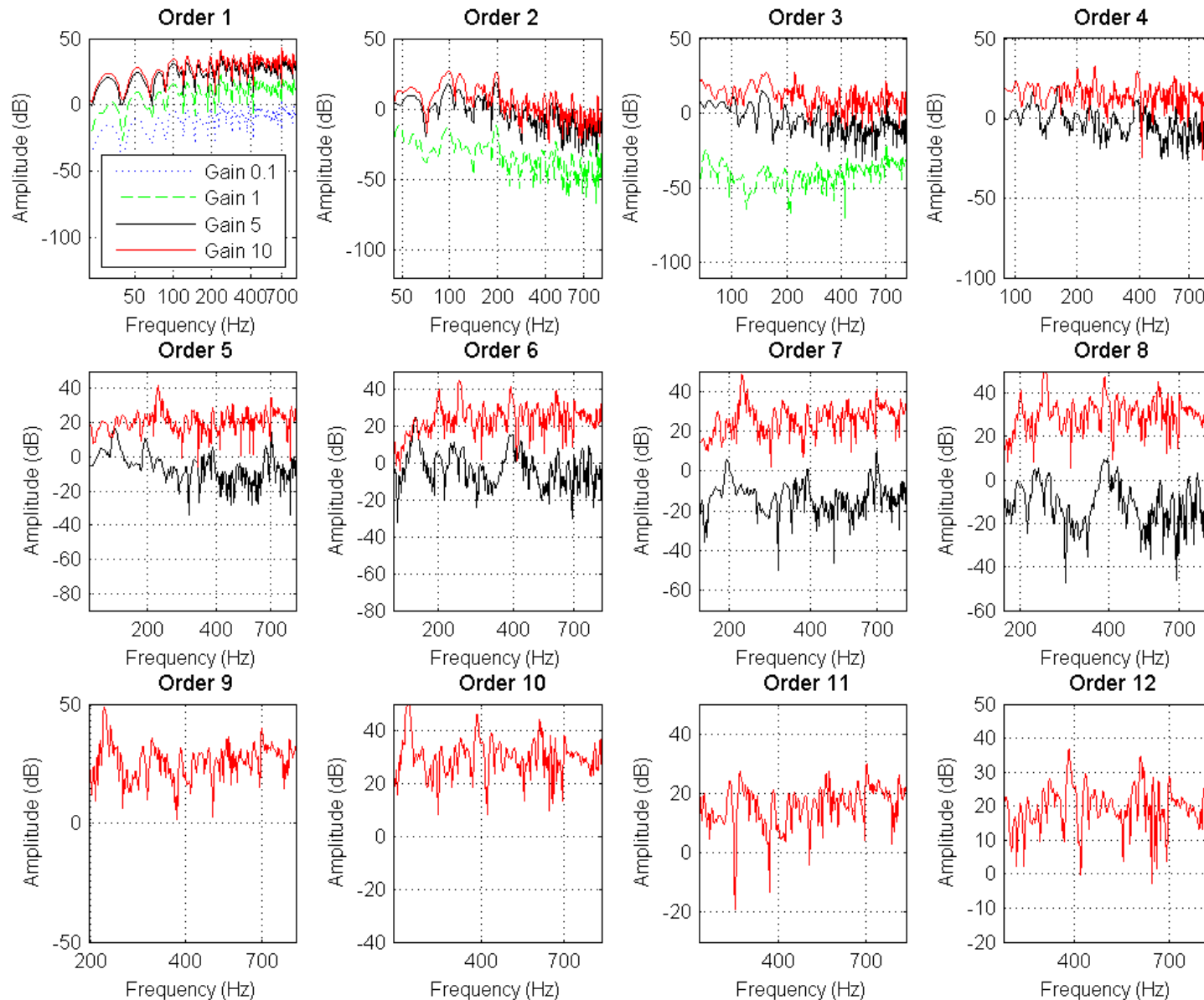
→ Example on the suspended damped plate (5 different gains)



Estimation of the proper kernel order  $N$  to identify

Threshold defined following a statistical F-test (Fisher)

# Example of autonomous Kernel estimation





# Conclusion & Perspectives

- Original method to **estimate the nonlinearities** of a vibro-acoustical structure
  - Slightly nonlinear systems modelled as **parallel Hammerstein models**
  - Kernels easily estimated using **exponential sine sweeps**
- **Improvements** of the sine sweep method
  - **Repetition of the excitation signal** –  $K$  sweeps
  - **Extraction of the noise** through time synchronous averaging
  - **Uncertainty estimation** by bootstrap
  - **Autonomous kernel order estimation**
- **Perspectives**
  - **Comparisons** with other methods (spectral domains): **periodic multisines...**