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Reduced Basis method applied to large scale non linear multiphysics problems

Cécile Daversin, Christophe Prud'Homme, Christophe Trophime

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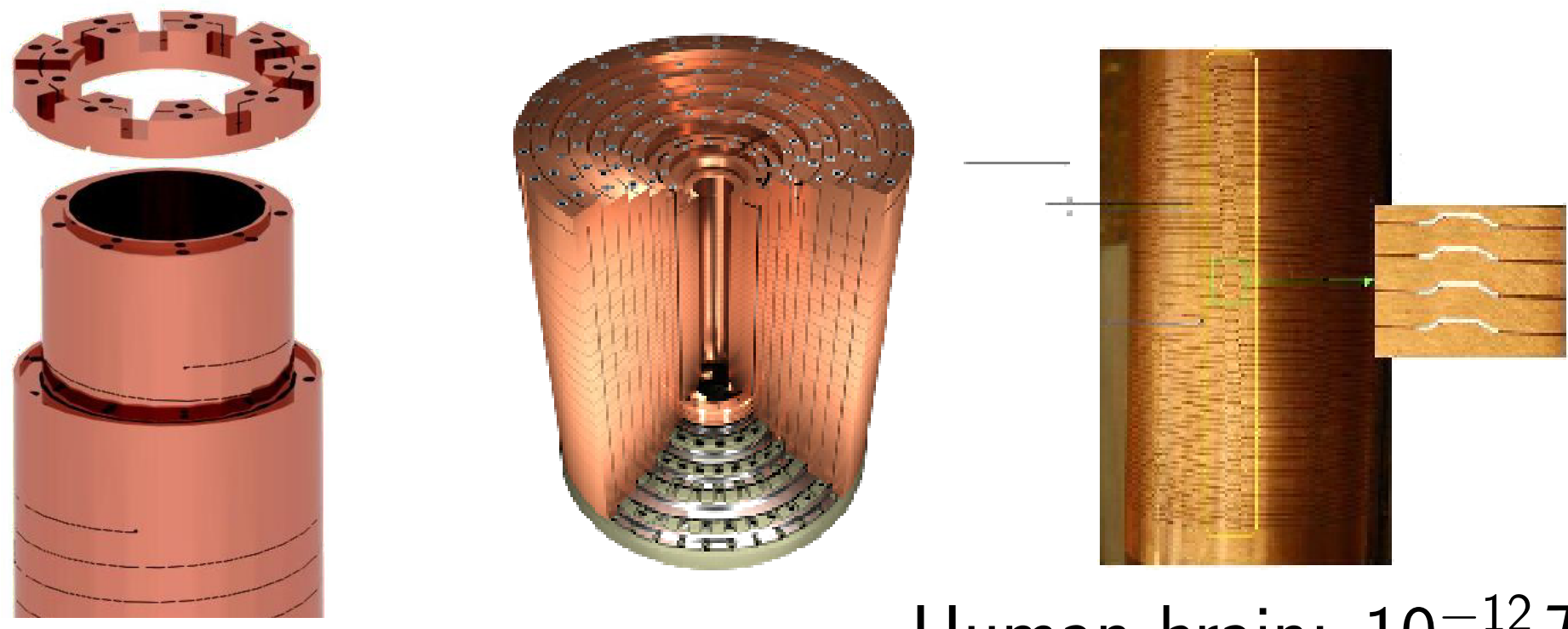
<https://hal.science/hal-01219061>

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LNCMI : French high magnetic field laboratory



Large scale facility :

- High magnetic field : > superconductors (24 T)
- Grenoble : Continuous fields (→ 36 T)
- Toulouse : Pulsed fields (→ 90 T)

Applications :

- (Bio-)chemistry
- Magnetoscience
- Applied superconductivity

Human brain: $10^{-12} T$, Earth : $5.10^{-5} T$, Pace-maker : $10^{-3} T$, IRM : 1T, Superconductor : 24T

Reduced Electro-thermal Model

Electro-thermal model

V : Electrical potential [V] T : Temperature [K]

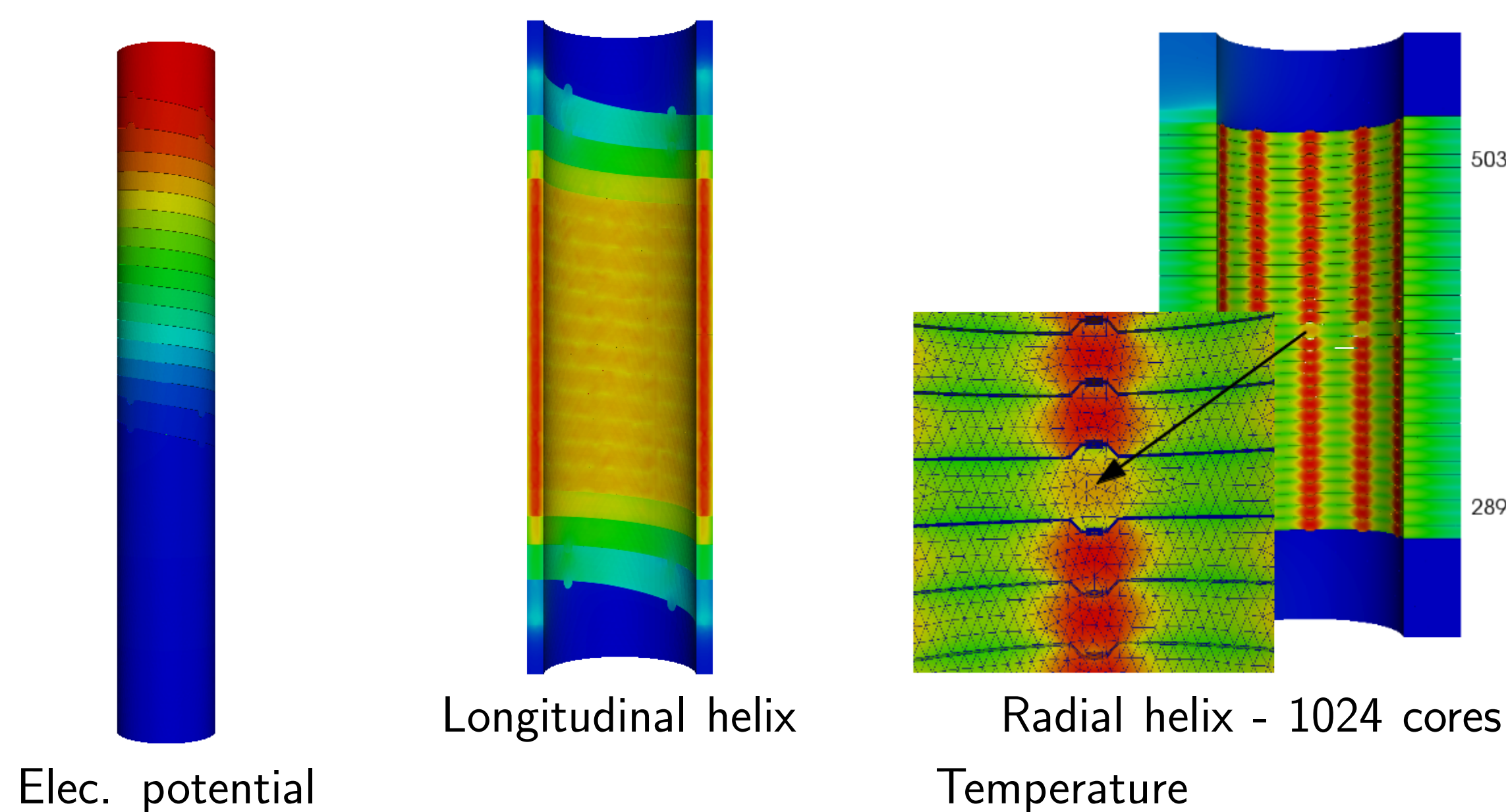
$$\begin{cases} -\nabla \cdot (\sigma(T) \nabla V) = 0 & \text{dans } \Omega \\ -\nabla \cdot (k(T) \nabla T) = \sigma(T) \nabla V \cdot \nabla V & \text{dans } \Omega \end{cases}$$

$\sigma(T) = \frac{\sigma_0}{1 + \alpha(T - T_0)}$ Material properties $k(T) = LT\sigma(T)$

Non-linearity

$V = 0$ (bottom) $-\sigma(T) \nabla V \cdot \mathbf{n} = 0$ (electrical insulation)

 $V = V_D$ (top) $-k(T) \nabla T \cdot \mathbf{n} = h(T - T_w)$ (water cooling)



Reduced Basis method

$u^N(\mu) \approx u^N(\mu)$: linear combination of FEM solutions

 $W^N = \text{span}\{\xi_i \equiv u^N(\mu_i)\}_{i=1}^N \rightarrow u^N(\mu) = \sum_{i=1}^N u_i^N(\mu) \xi_i$

RB approximation space $10 \leq N \leq 100$

Efficient Offline/Online strategy

Non-affinely parametrized functions → Empirical Interpolation Method

$$w(u, \mathbf{x}; \mu) \approx w_M(u, \mathbf{x}; \mu) = \sum_{m=1}^M \beta_m^M(u; \mu) q_m(\mathbf{x})$$

Non-linearity → Iterative fixed-point methods

$$\sum_{i=1}^N \left[\sum_{q=1}^{Q_a} \sum_{m=1}^{M_a^q} \underbrace{\gamma_{a,m}^q(k, u, \mu)}_{\text{online}} \underbrace{a_m^q(\xi_i, \xi_j)}_{\text{offline}} \right]^{k+1} u_i^N(\mu) = \sum_{q=1}^{Q_f} \sum_{m=1}^{M_f^q} \underbrace{\gamma_{f,m}^q(k, u, \mu)}_{\text{online}} \underbrace{f_m^q(\xi_j)}_{\text{offline}}$$

Electro-thermal model

$u = (V, T)$ with $\mu = (\sigma_0, \alpha, L, V_D, h, T_w)$

 EIM approx. : $\sigma_M \approx \sigma(T)$, $k_M \approx k(T)$, $Q_M \approx \sigma(T) \nabla V \cdot \nabla V$

Feel++ www.feelpp.org

 Reduced Basis Framework

User specifications (geometries, inputs, ...)

Affine Decomposition

RB / PFEM

OpenTurns Octave

Electro-thermal model on radial helix

Number of dofs $\approx 5 \cdot 10^5$

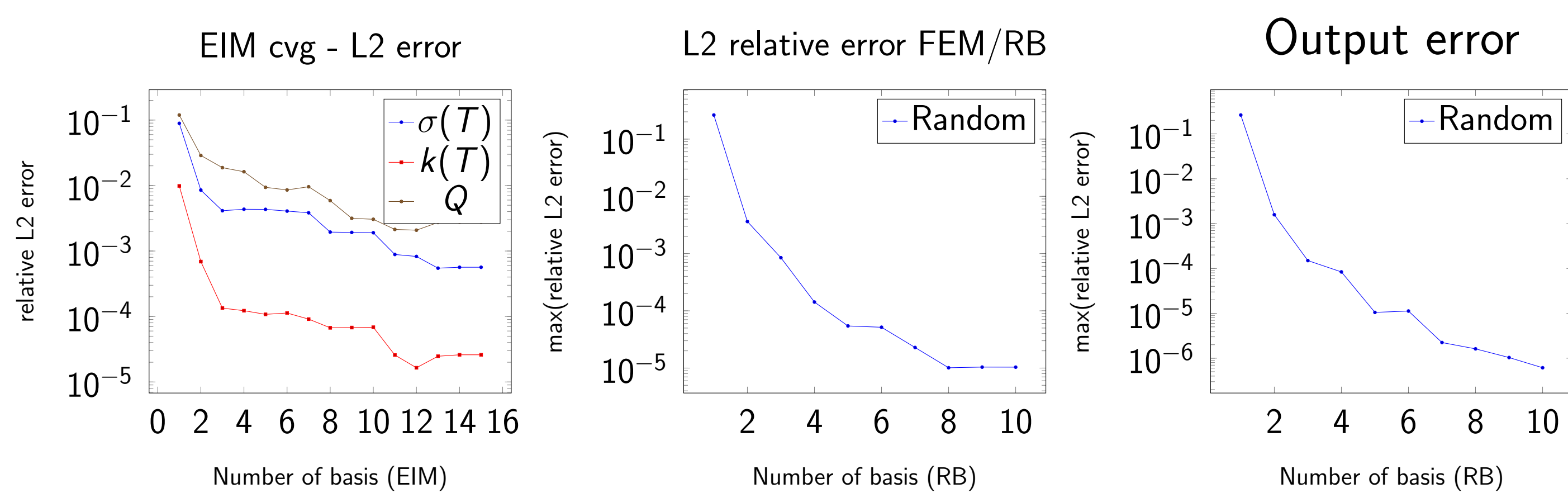
 HPC on 12 procs

 EIM : 15 basis, RB : 10 basis

FEM : 1011 seconds ≈ 16 min

 RB (Online) : 6.7 seconds

Gain factor : 150



Sensitivity Analysis

$\sigma_0 \in [50.10^6; 50, 2.10^6](S.m^{-1})$

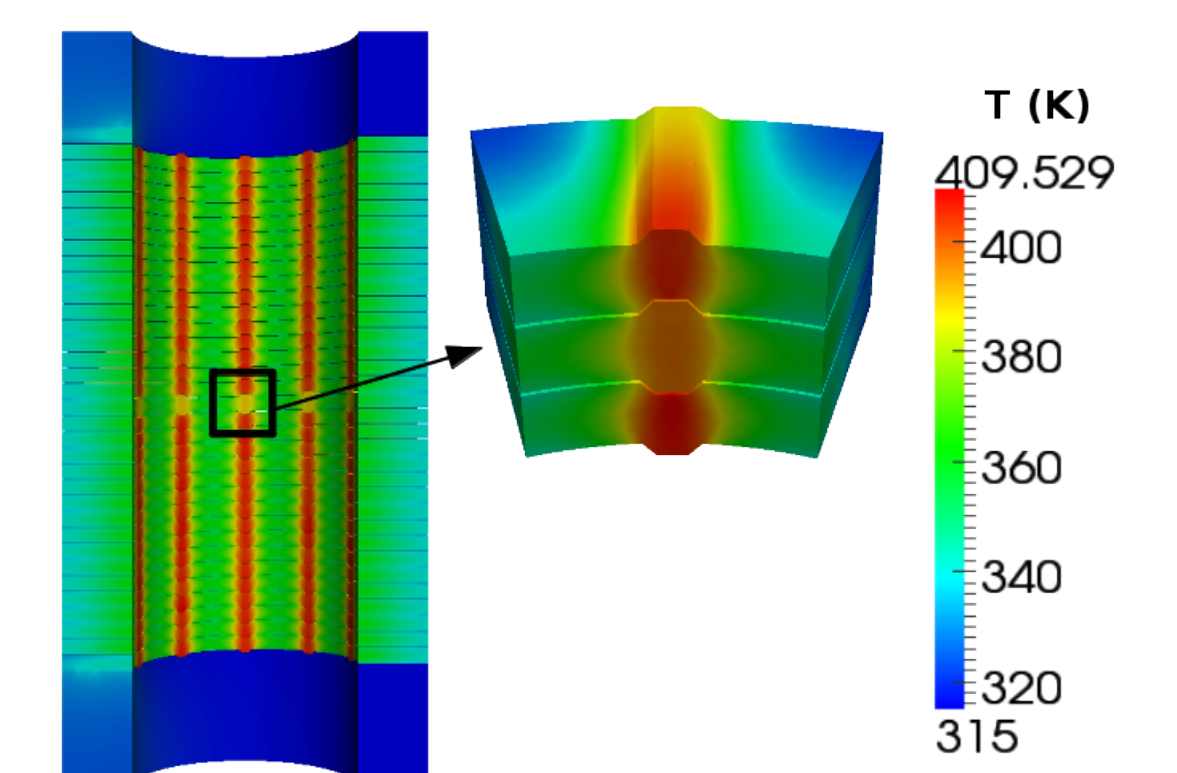
 $\alpha \in [3, 3.10^{-3}; 3, 5.10^{-3}](K^{-1})$

 $L \in [2, 5.10^{-8}; 2, 9.10^{-8}]$

 $U \in [0.14; 0.15](V)$ Intensity ≈ 25 kA

 $h \in [70000; 90000](W.m^{-2}.K^{-1})$

 $T_w \in [293, 313](K)$



Sensitivity indices (Sobol) :

- σ_0 : 6.85e-05
- α : 4.54e-04
- L : 9.21e-03
- V_D : 1.28e-01
- h : 2.40e-01
- T_w : 6.21e-01

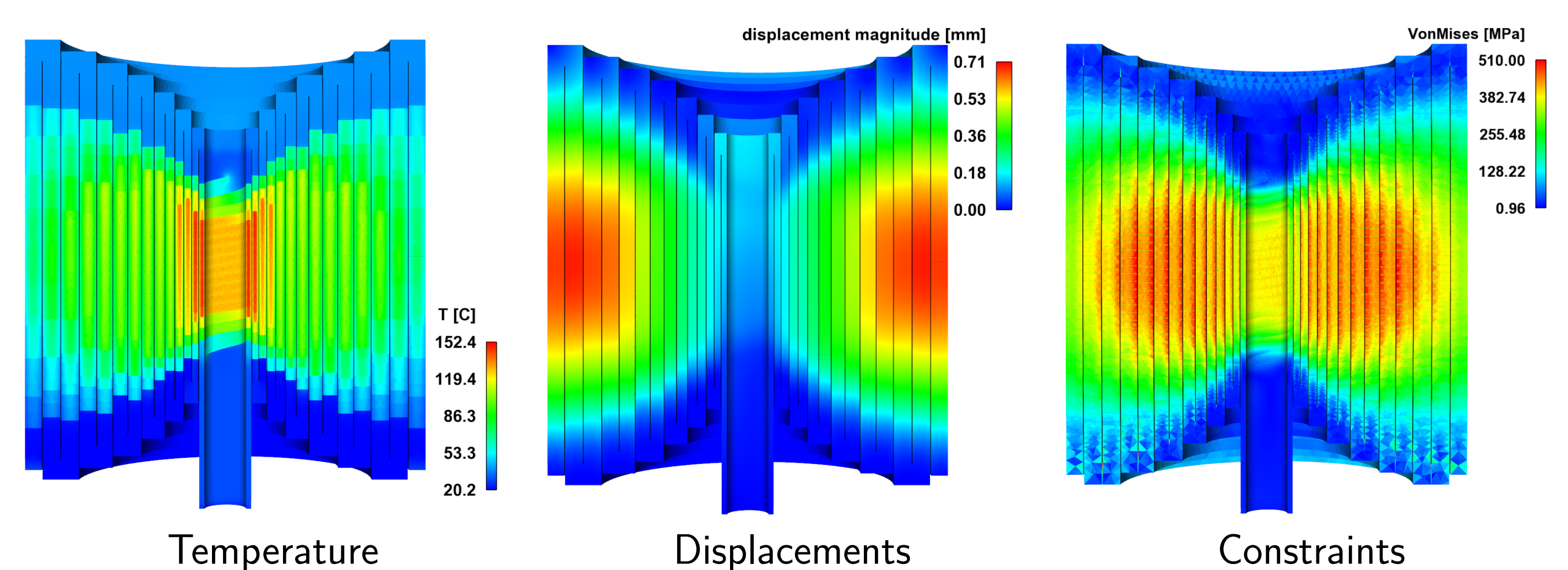
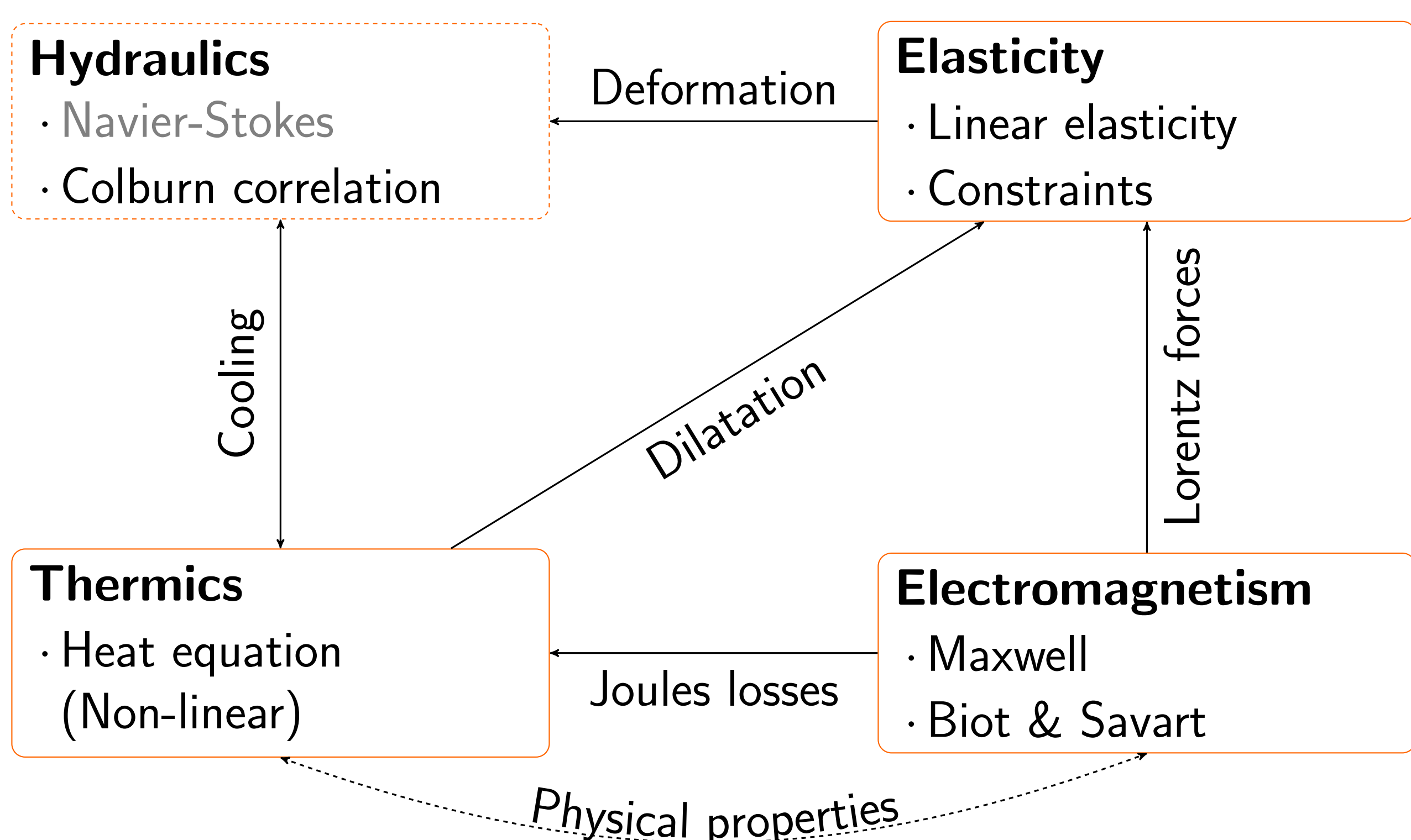
Quantiles :

$q(\gamma)$ such that $P(Y < q(\gamma)) > \gamma$

 99.0 quantile = 380 K = 107 C

 80.0 quantile = 377.5 K = 104.5 C

Towards a full 3D non-linear multi-physics reduced model



Quantities of interest

- Mean temperature
- Magnet power
- Magnetic field on a point
- Field homogeneity
- Mean displacements (x, y, z)
- Mean constraints