



**HAL**  
open science

# Reduced Basis method applied to large scale non linear multiphysics problems

Cécile Daversin, Christophe Prud'Homme, Christophe Trophime

► **To cite this version:**

Cécile Daversin, Christophe Prud'Homme, Christophe Trophime. Reduced Basis method applied to large scale non linear multiphysics problems. MoRePaS2015, Oct 2015, Trieste, Italy. hal-01219061

**HAL Id: hal-01219061**

**<https://hal.science/hal-01219061v1>**

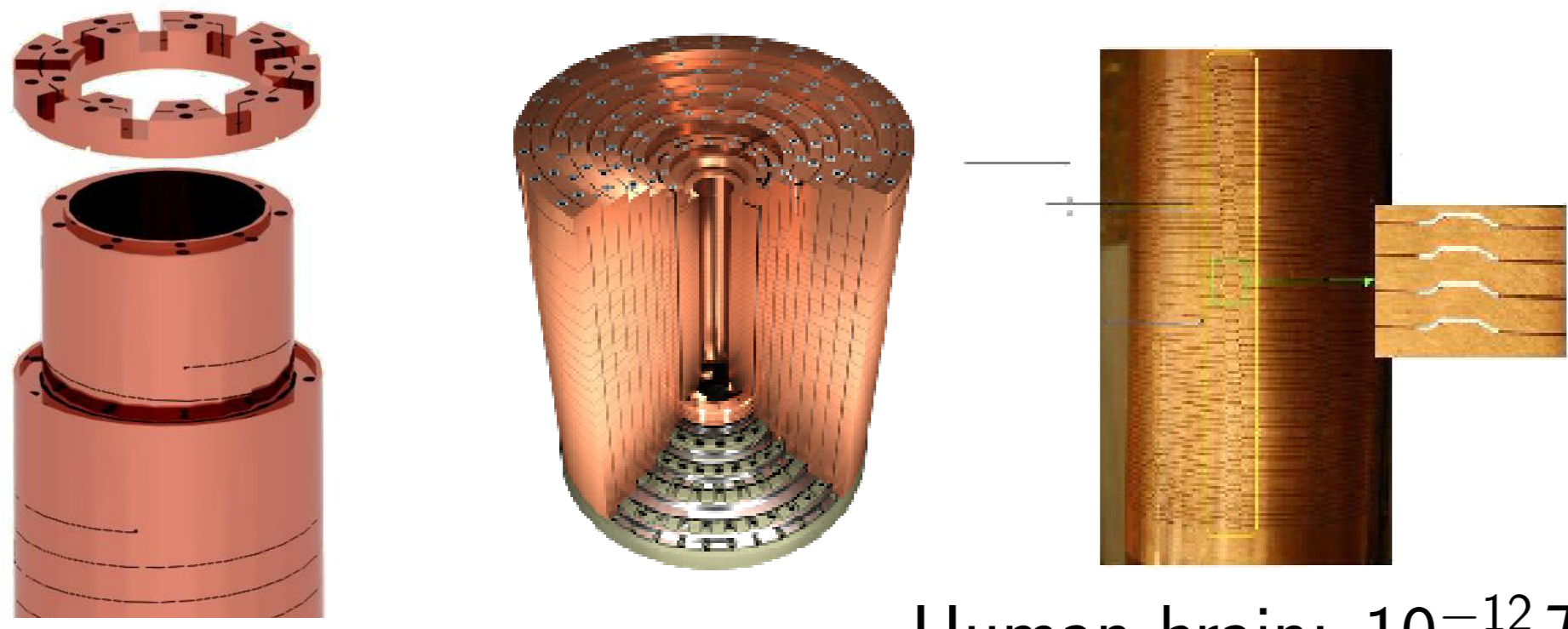
Submitted on 22 Oct 2015

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



## LNCMI : French high magnetic field laboratory



### Large scale facility :

High magnetic field : > superconductors (24 T)  
Grenoble : Continuous fields (→ 36 T)  
Toulouse : Pulsed fields (→ 90 T)

### Applications :

(Bio-)chemistry  
Magnetoscience  
Applied superconductivity

Human brain:  $10^{-12} T$ , Earth :  $5.10^{-5} T$ , Pace-maker :  $10^{-3} T$ , IRM : 1T, Superconductor : 24T

## Reduced Electro-thermal Model

### Electro-thermal model

$V$  : Electrical potential [V]

$T$  : Temperature [K]

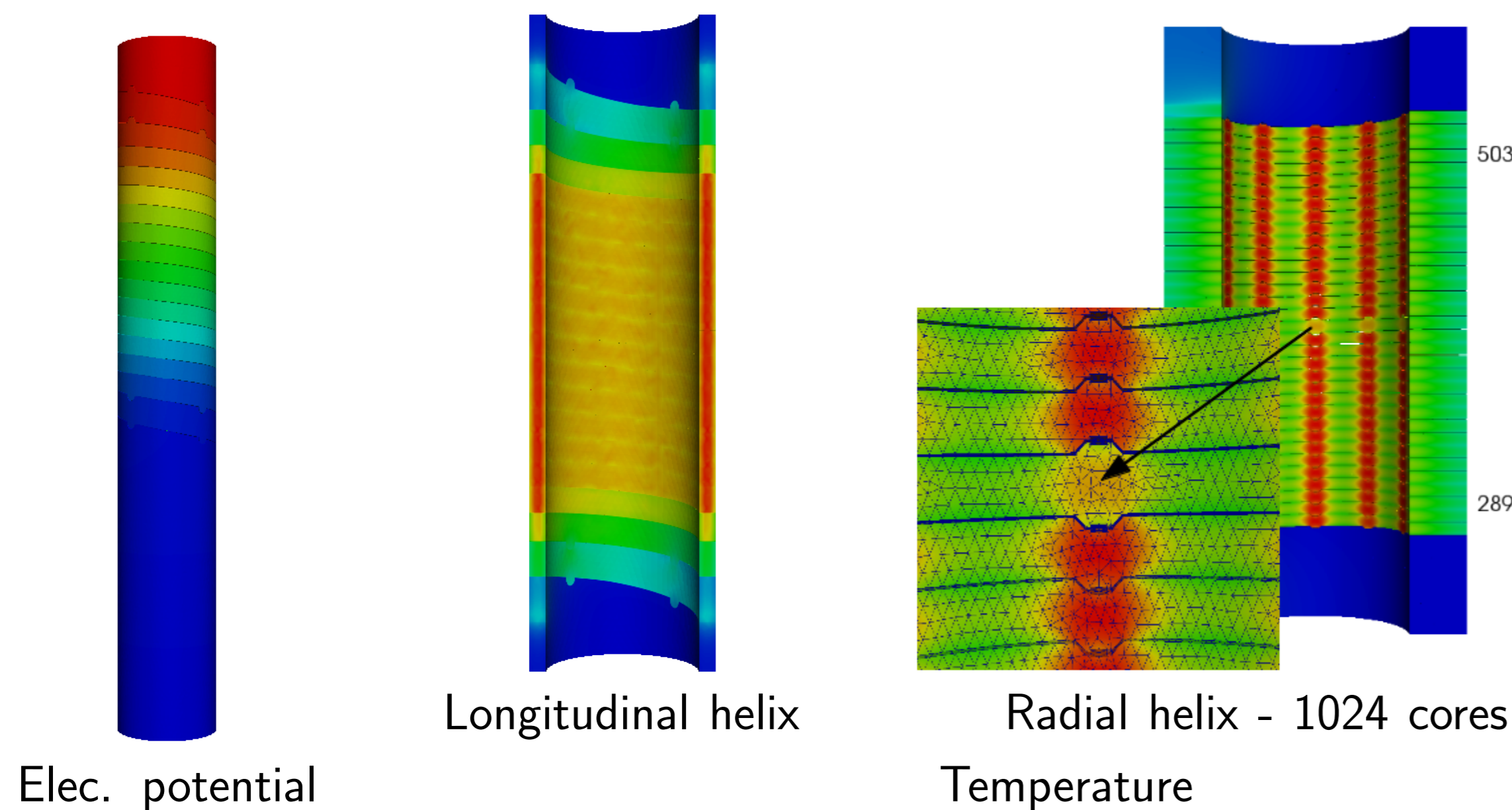
$$\begin{cases} -\nabla \cdot (\sigma(T) \nabla V) = 0 & \text{dans } \Omega \\ -\nabla \cdot (k(T) \nabla T) = \sigma(T) \nabla V \cdot \nabla V & \text{dans } \Omega \end{cases}$$

$$\sigma(T) = \frac{\sigma_0}{1 + \alpha(T - T_0)} \quad \text{Material properties} \quad k(T) = LT\sigma(T)$$

Non-linearity

$V = 0$  (bottom)  
 $V = V_D$  (top)

$-\sigma(T) \nabla V \cdot \mathbf{n} = 0$  (electrical insulation)  
 $-k(T) \nabla T \cdot \mathbf{n} = h(T - T_w)$  (water cooling)



### Reduced Basis method

$u^N(\mu) \approx u^N(\mu)$  : linear combination of FEM solutions

$$W^N = \text{span}\{\xi_i \equiv u^N(\mu_i)\}_{i=1}^N \rightarrow u^N(\mu) = \sum_{i=1}^N u_i^N(\mu) \xi_i$$

RB approximation space 10 ≤ N ≤ 100

### Efficient Offline/Online strategy

**Non-affinely parametrized functions** → Empirical Interpolation Method

$$w(u, \mathbf{x}; \mu) \approx w_M(u, \mathbf{x}; \mu) = \sum_{m=1}^M \beta_m^M(u; \mu) q_m(\mathbf{x})$$

**Non-linearity** → Iterative fixed-point methods

$$\sum_{i=1}^N \left[ \sum_{q=1}^{Q_a} \sum_{m=1}^{M_a^q} \underbrace{\gamma_{a,m}^q(k, u, \mu)}_{\text{online}} \underbrace{a_m^q(\xi_i, \xi_j)}_{\text{offline}} \right]^{k+1} u_i^N(\mu) = \sum_{q=1}^{Q_f} \sum_{m=1}^{M_f^q} \underbrace{\gamma_{f,m}^q(k, u, \mu)}_{\text{online}} \underbrace{f_m^q(\xi_j)}_{\text{offline}}$$

### Electro-thermal model

$u = (V, T)$  with  $\mu = (\sigma_0, \alpha, L, V_D, h, T_w)$

EIM approx. :  $\sigma_M \approx \sigma(T), k_M \approx k(T), Q_M \approx \sigma(T) \nabla V \cdot \nabla V$

Feel++ [www.feelpp.org](http://www.feelpp.org)  
Reduced Basis Framework

User specifications  
(geometries, inputs, ...)

Affine  
Decomposition

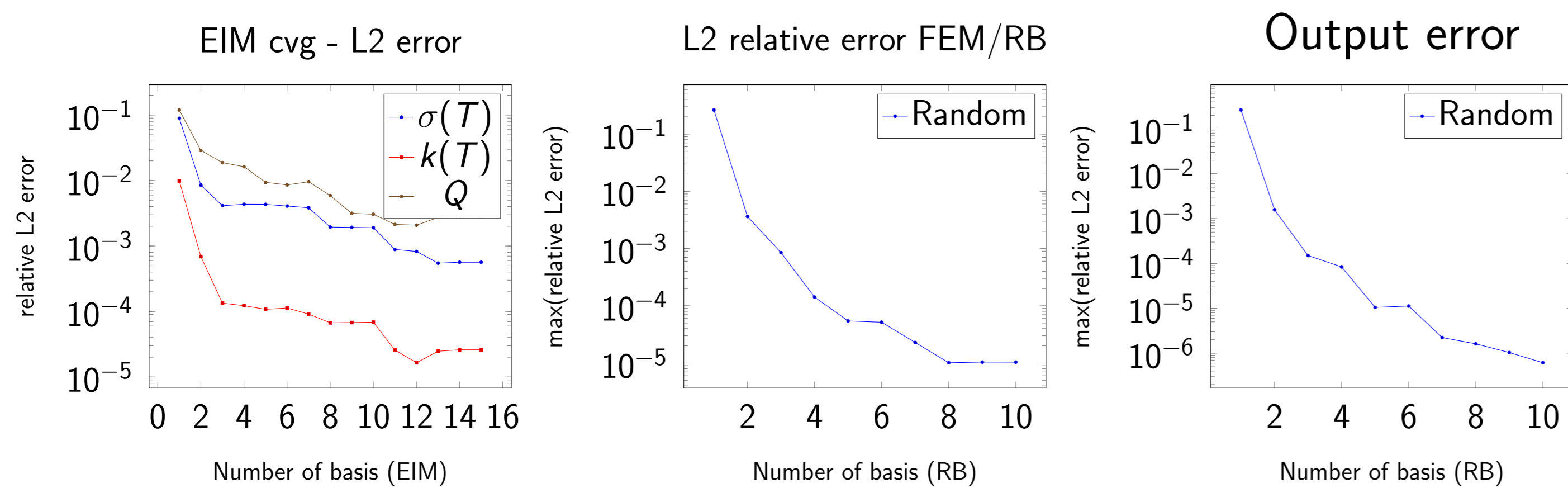
RB  
PFEM

OpenTurns  
Octave

### Electro-thermal model on radial helix

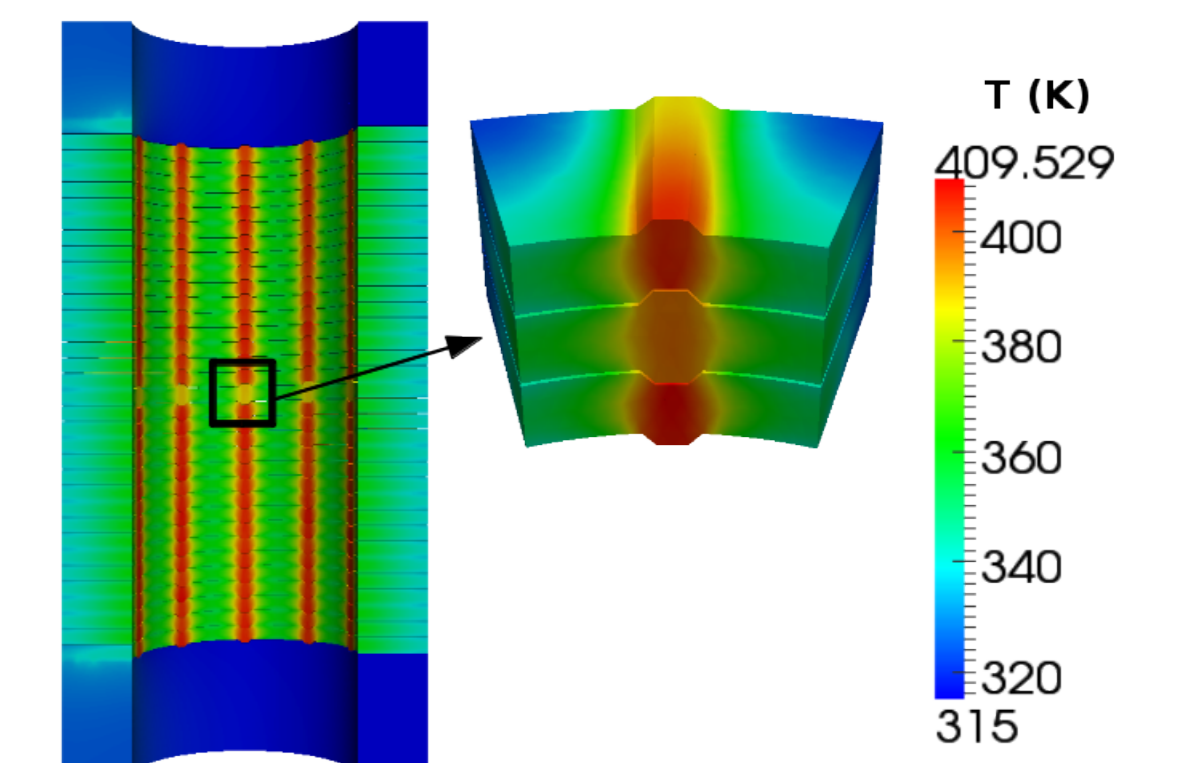
Number of dofs  $\approx 5 \cdot 10^5$   
HPC on 12 procs  
EIM : 15 basis, RB : 10 basis

FEM : 1011 seconds  $\approx 16$  min  
RB (Online) : 6.7 seconds  
**Gain factor : 150**



### Sensitivity Analysis

$\sigma_0 \in [50.10^6; 50, 2.10^6](S.m^{-1})$   
 $\alpha \in [3, 3.10^{-3}; 3, 5.10^{-3}](K^{-1})$   
 $L \in [2, 5.10^{-8}; 2, 9.10^{-8}]$   
 $U \in [0.14; 0.15](V)$  Intensity  $\approx 25$  kA  
 $h \in [70000; 90000](W.m^{-2}.K^{-1})$   
 $T_w \in [293, 313](K)$



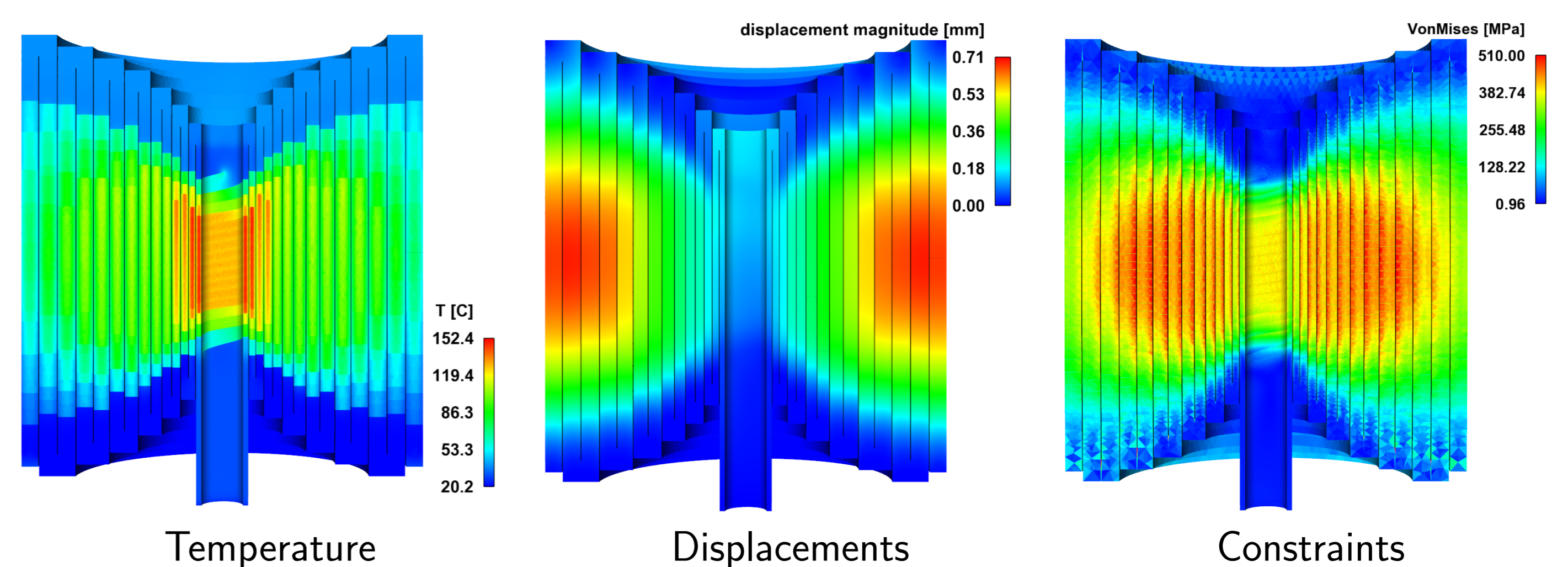
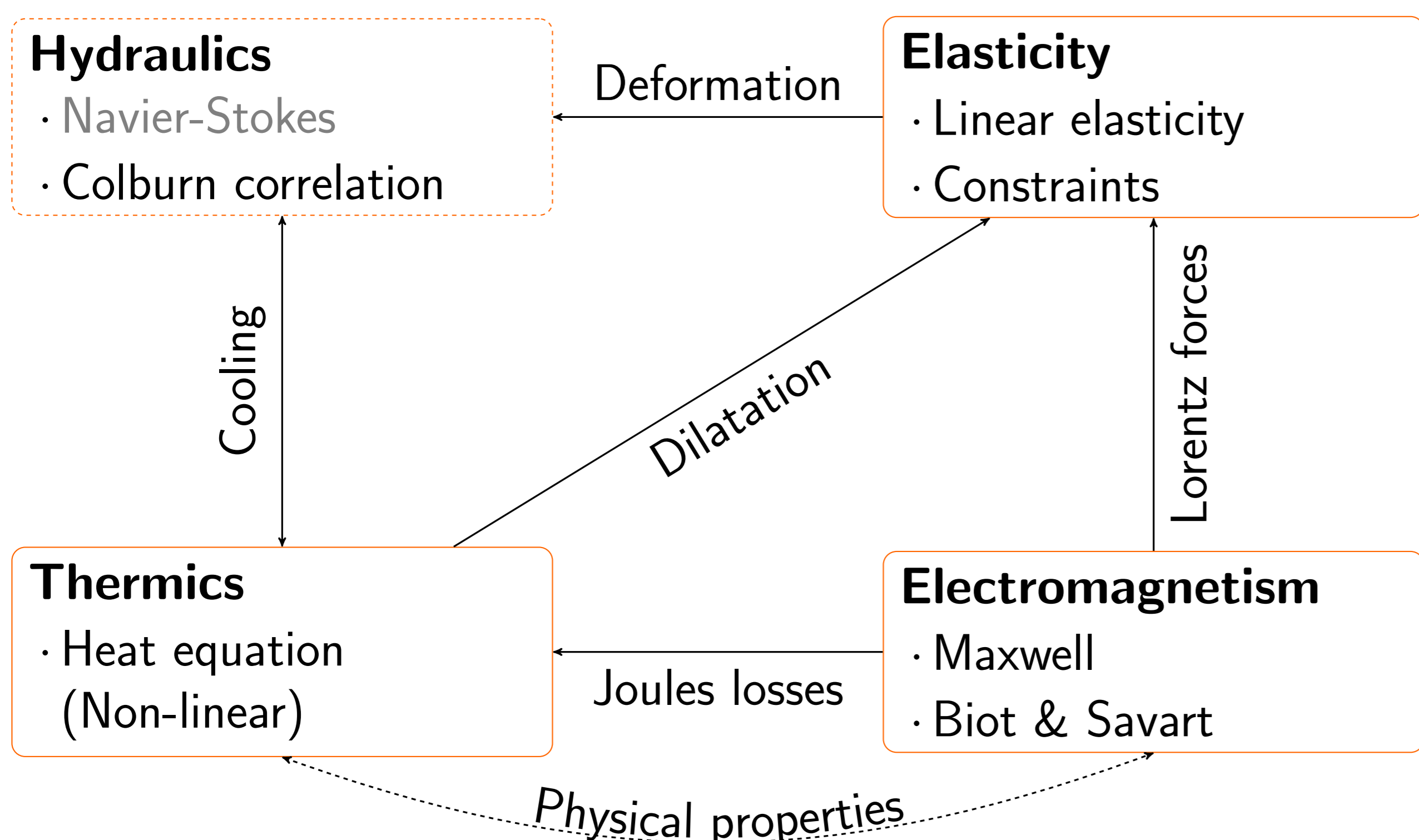
### Sensitivity indices (Sobol) :

—  $\sigma_0$  : 6.85e-05    —  $V_D$  : 1.28e-01  
—  $\alpha$  : 4.54e-04    —  $h$  : 2.40e-01  
—  $L$  : 9.21e-03    —  $T_w$  : 6.21e-01

### Quantiles :

$q(\gamma)$  such that  $P(Y < q(\gamma)) > \gamma$   
99.0 quantile = 380 K = 107 C  
80.0 quantile = 377.5 K = 104.5 C

## Towards a full 3D non-linear multi-physics reduced model



### Quantities of interest

— Mean temperature  
— Magnet power  
— Magnetic field on a point  
— Field homogeneity  
— Mean displacements (x, y, z)  
— Mean constraints