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Analyzing and Comparing Basel's III Sensitivity Based Approach for the interest rate risk in the trading book

MABELLE SAYAH

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Abstract

A bank's capital charge computation is a widely discussed topic with new approaches emerging continuously. Each bank is computing this figure using internal methodologies in order to reflect its capital adequacy; however, a more homogeneous model is recommended by the Basel committee to enable judging the situation of these financial institutions and comparing different banks among each other.

In this paper, we compare different numerical and econometric models to the sensitivity based approach (SBA) implemented by BCBS under Basel III in its February 2015 publication in order to compute the capital charge, we study the influence of having several currencies and maturities within the portfolio and try to define the time horizon and confidence level implied by Basel's III approach through an application on bonds portfolios.

By implementing several approaches, we are able to find equivalent VaRs to the one computed by the SBA on a pre-defined confidence level (97.5 %). However, the time horizon differs according to the chosen methodology and ranges from 1 month up to 1 year.

Keywords: *Capital charge, Sensitivity Based approach, Basel III, GARCH, PCA, ICA, Dynamic Nelson Siegel, bonds portfolio, trading book, interest rate risk*

I. INTRODUCTION

Commercial banks are a key component of today's financial and economic system. Banks allocate funds from depositors to borrowers, convert maturities, and provide financial products. These services among others enhances the efficiency of the overall economy. Given this crucial role, adequate regulations should apply to monitor banks risks.

Since its first issuance in 1988, Basel has been the main banking regulation authority initializing with some main credit risk rules. In 1996, the market amendment was issued setting the basic standards regarding trading assets and the value at risk computation methodology. It also divided the risk into market and credit risk; market risk being divided between equity, interest rate, foreign exchange, commodity, and option risk with a standardized capital computation approach treating each asset class separately. In 2006, Basel II came along with more 'personalized' approaches such as internal rating based IRB for credit risk, internal models for OTC derivatives exposure along with the introduction of the operational risk charge.

However, these regulations did not prevent major crisis hitting the international market causing huge losses in different sectors (cf. Baptistab et al. (2012)). As a response to this shortage, Basel 2.5 was created beginning 2011 as a response adding more capital to the trading book (especially on the poorly modeled products). Basel 2.5 additions targets the stressed VaR concept, the incremental risk charge and few new standard rules regarding the banking book.

In 2013, after a thorough observation of the consequences of the crisis and the attempt for correction made by 2.5, Basel III was introduced. The main functions of this issuance are: increasing capital for counterpart exposures, tightening the definition of the bank's capital, adding buffers

for liquidity and introducing a new leverage ratio (cf. BCBS (2014)). In parallel, increasing the trading book capital requirement under Basel 2.5 was required following the crisis and was not well designed: the calculation remains non-risk sensitive and highly conservative, and differences in model approval persist between jurisdictions.

As a result, in May 2012 the Basel Committee published a Consultative Paper on a 'Fundamental Review of the Trading Book' (FRTB) to improve this framework then in February 2015, "Fundamental review of the trading book: outstanding issues", the BCBS exposed the weaknesses of Basel's previous approaches. It suggested some major changes to the trading book to be implemented by 2018: Modeling Issues (stressed expected shortfall, liquidity adjustments, diversification, default and migration risk, and non-modelable risk factors), scope and approval process (boundary of the trading book, desk level model approval, model testing, model independent assessment tool) and new standard rule for capital charges computation.

The standard rule suggested by the FRTB, is based on sensitivities hence, it is called the "Sensitivity based approach" (SBA): it was implemented as the 'homogeneous' method for capital charge computation across all banks. The SBA is based on percentages and correlations between different maturities and currencies (cf. BCBS (2015)). The existing standard rules poorly reflected hedging or diversification thus inflating the trading book capital level. The SBA is simple yet risk sensitive which is already a big improvement. SBA is a standardized method that reflects the risk resulting from: Interest rate, credit spread, equity, commodity, foreign exchange, default, and options risk.

Comparing regulatory approaches, an obvious contrast between Basel and Solvency is noted: Solvency II has a similar three pillar structure as Basel's Accords. The target capital requirements (called SCR) are described under the first pillar and refer to all types of risks an insurance undertaking is exposed to: market risk (interest rate risks, equity risk, property risk, spread risk, concentration risk, and currency risk) and counterparty default risk. Both frameworks take diversification effects into account and use square root formulas. However, these aggregation approaches are applied at different levels: a considerably stronger risk differentiation is shown under Basel III. For example, the SBA equity risk distinguishes 10 risk categories in order to assign the risk weights, in contrast to one single shock for all listed equities under Solvency II. Under the interest rate risk, SBA is Basel's III approach whereas in solvency II a shocked scenarios based computation sets the capital charge (cf. LAAS and Siegel (2015)).

Seemingly, the SBA has several add-ons regarding sensitivity and diversifying considerations however it still has some issues (specifying the coefficients) and details concerning the aggregations that need to be tweaked; for instance the figure under the square root is sometimes negative (cf. ISDA (2015)). In this paper we aim to focus on the suggested standard rule by the FRTB in order to compare it with other econometric models to find an equivalent capital charge computation technique with few additional details such as time horizon and confidence level for a given capital. Among the different risk modulations, we chose to focus on the interpretation of the interest rate calculation.

According to Oxford Dictionary of Economics, interest rate is defined as "The charge made for the loan of financial capital expressed as a proportion of the loan". More formally, Basel Committee on Banking Supervision (cf. BCBS (2004)) indicated that interest rate risk (IRR) is the exposure of a bank's financial condition to adverse movements in interest rate.

The sound IRR management conducted by Basel Committee on Banking Supervision had been the source on which analysts rely to evaluate the activities of bank's risk management of interest rate (cf. BCBS (2004)). In the guideline, the committee offers four basic elements of IRR: appropriate board oversight, comprehensive internal controls, adequate policies and appropriate risk measure. The SBA falls under this forth element in modeling the interest rate risk in the trading book.

Our aim in this paper is to understand the computation of the interest rate risk in banks based on BASEL's III approach. However, the SBA remains relatively vague and the choices of its coefficient and correlation parameters are not robustly detailed and documented. Hence, studying the interest rate risk from an econometric point of view in order to compare and contrast the results is critical and highly important.

Based on different central banks approaches for term structure interest rate, we selected few econometric models. The main idea was to reduce the dimensions of the database, study the dynamics of these 'reduced' factors and then conclude on the wider data range dynamic. We introduce the methods used to derive capital requirement and compare them with SBA's in order to conclude on some equivalence between them.

Having this objective in mind, the structure of the work is as follows: We started by modeling each interest rate curve on a stand-alone maturity basis using a GARCH approach. It is worth noting that by doing so, we dropped a crucial information which is the strong correlation between these maturities, however we needed this phase as a starting point and a comparison threshold. Modeling the volatility of term structures using GARCH processes has become a current practice due to its numerous advantages relative to alternative models. GARCH-methods are a way of investigating how a function of past returns, in a specific financial series, should be constructed and mapped onto the second moment (cf. Hull (2000)). Proposed by Engle (1982) and then generalized by Bollerslev (1986), GARCH models explain high frequency financial data series through the autoregressive conditional heteroskedasticity and can model simultaneously conditional mean and conditional variance (cf. Edison and Liang, 1999). Two parameters for orders could be used in order to optimize the results of the tests regarding GARCH coefficients convergence. However, in practice, low orders are more frequently used. The first-order ($p=q=1$) GARCH model (cf. Taylor (1986)) has become the most popular GARCH model.

Secondly, we introduced the component approaches starting with the Principal component analysis (PCA) which is one of the multivariate analysis techniques usually used for correlation studies, data reduction and efficiency assessment (cf. Leveuge et al. 1 (2010)). This method incorporates the interdependence between term structures maturities: it considers the correlated curves and generates new non-correlated variables. Each factor is related to a loading and a cumulative variance defining the variance explained by each one of the new variables. PCA creates the same number of term structures included in the model however, we need to choose the reduced number of factors that we want to handle. In this work, we chose to cover 98% of the variance, by considering two or three factors. Using a GARCH model, we only project the chosen factors and not the loadings; then we re-create the entire data from the projected factors and previously observed loadings.

Thirdly, was the implementation of the Independent component analysis (ICA): it provides a mechanism of decomposing a given signal into statistically independent components. PCA uses only second order statistical information however, ICA uses higher order (kurtosis) for separating the signals which permits more conclusive results in financial data (cf. Comon (1994)). A drawback in the ICA is its inability to indicate the data variance coverage for each factor, therefore the modeler has to define the number of factors to be considered; in this paper we chose to include three ICA factors.

Regarding the last approach, a factor model is suggested: the Dynamic Nelson Siegel. No GARCH processes are used, instead a mix of Nelson Siegel estimation and ARIMA processes projection are put in practice. Factor models for the yield curve, such as Nelson-Siegel (1988), its dynamic version (cf. Diebold and Li (2006)) and its arbitrage-free counterpart proposed by Christensen, Diebold and Rudebusch (2011), have been extensively applied to forecast bond yields. We used the Dynamic Nelson Siegel due to its flexibility in representation especially for the long term projection. By fitting the curves, projecting the factors using Diebold method and the

loadings employing an ARIMA process, we are able to reconstruct the curves from which we concluded the capital requirement.

The capital charge using the previously mentioned methods, except SBA, can be computed on a certain confidence level basis and for a given time horizon; therefore comparing these methods to SBA would determine a common time horizon and confidence level, reaching the purpose of this paper. We start by explaining in details the procedure of the SBA, the correlation between the duration of the portfolios and the capital charge required by this procedure then compare the methods using different approaches. These latter will be based on bonds portfolios denoted in: euros, dollars and Turkish lira from the French, German, US and Turkish governments yields respectively, for maturities between 1 month and 30 years.

In this paper we proceed as follows: in Section 2 we provide a detailed description of the sensitivity based approach through hypothetical portfolios, we also show the link between this capital charge computation and the portfolios duration. In section 3, we proceed with an overview of four different approaches then we explain how we computed the capital charge using these processes. In section 4 we present the empirical analysis, describe the data, estimate the models, compute the VaRs and conclude with analyzing, comparing and back-testing the capital requirement calculations. In section 5 we offer some interpretation and conclusive remarks.

II. SENSITIVITY BASED APPROACH (SBA)

II.1. Introducing the approach

This new method would require banks to use prices and rate sensitivities in order to compute their capital charge. This revised (sensitivity-based) standardized approach would capture more granular or complex risk factors across different asset classes in the trading book (cf. BCBS (2015)). It builds on the standardized framework tested in the trading book QIS conducted in the second half of 2014. (cf. Basel (2014))

The proposed methodology covers the delta and optionality risk: general interest rate risk, credit spread risk of non-securitization and securitization exposures, equity, commodity and FX risk. Vega and curvature risk measurements are under development in order to measure the sensitivity of the value of an option with respect to a modification in volatility and the rate of change of delta.

II.2. Implementation reasons

- The approach must provide a method for calculating capital requirements for banks with a level of trading activity that does not require sophisticated measurement of market risk.
- It provides a fall back in the event that a bank's internal model is deemed inadequate, including the potential use as add-on or floor to an internal model-based charge.
- The approach should facilitate consistent and comparable reporting of market risk across banks and jurisdictions.

II.3. Computational steps

We first compute the net sensitivity of the bond (relative 1 bps change) and multiply it by its corresponding risk weight in order to get the weighted sensitivity. We note that for each maturity a different risk weight is allocated based on a matrix provided by the Basel committee. For each currency, the 'average' is computed as the square root of the sum of squared single weighted sensitivities and double products of these latter weighted by given, maturity based, correlation coefficients. Aggregation on a portfolio level is another sum of the squared capital charges computed for each currency plus the double products weighted by a factor of 0.5 fixed by Basel (between currencies).

1. Get the observed yield and price on the market.
2. Compute the net sensitivity of each instrument and recalculate the price.
3. Based on the matrix imposed by Basel, get the weighted sensitivities.
4. Form buckets by sorting each currency in a separate bucket.
5. For each bucket compute the following:

$$K_{bucket} = \sqrt{\sum_{i=1}^N WS_i^2 + \sum_{i=1}^N \sum_{j \neq i} \rho_{ij} WS_i WS_j}$$

6. Compute the capital charge:

$$\text{Capital Charge} = \frac{\sqrt{\sum_{i=1}^N K_i^2 + 0.5 \sum_{i=1}^N \sum_{j \neq i} S_i * S_j}}{\text{initial portfolio value}} \text{ where } S_i = \sum WS_t$$

II.4. Hypothetical example

Let us consider a hypothetical portfolio composed of only one zero coupon bond. The portfolio is studied on a rolling basis and the bond does not include optionality. In this paper, we do not consider the effect of currency or default risk.

Price	$P = 100e^{-rT}$
Modified price	$P' = 100e^{-rT - 0.0001T}$
Sensitivity	$S = \frac{P' - P}{0.0001}$
Weighed sensitivity	$WS = RW * S$
Capital Charge	$\frac{WS}{P}$

The previous example proves that the capital charge under the SBA is only dependent of the zero bond maturity (therefore duration) and the associated risk weight of this particular maturity.

We analyzed the three following cases: a portfolio consisting of one bond, two same currency bonds and different currencies bond in order to come up with one generalized approach. Step by step computation and numerical examples can be found in **Appendix 1**.

Hereafter we consider a portfolio combining three bonds: two in a same currency and a third in a different one, we obtain the following:

The price of the three bonds being: $P_i = \sum C_{it} e^{-r_{it}t}$ and the durations $D_i = \frac{\sum t C_{it} e^{-r_{it}t}}{P_i}$ with C_{it} being the cashflows and r_{it} the interest rate.

We compute the net sensitivity NS_i and the weighted sensitivity WS_i :

$$NS_i = \frac{\sum_t C_{it} e^{-r_{it}t} - \sum_t C_{it} e^{-(r_{it} - 0.01\%)t}}{0.0001} \text{ and } WS_i = \frac{RW_i}{0.0001} \sum_t C_{it} e^{-r_{it}t} (1 - e^{-0.0001t}) = RW_i * D_i * P_i.$$

Having two different currencies, two buckets are created and a K_{α_i} is computed for each:

$$K_{\alpha_1} = \sqrt{WS_1^2 + WS_2^2 + 2\rho_{12} WS_1 WS_2} \text{ in the second bucket having only one bond, } K_{\alpha_2} \text{ equals } WS_3.$$

Bringing these together, SBA demands a 0.5 coefficient for the correlation and a sum of square to compute the squared capital charge:

$$CC = \frac{\sum_{i=1}^3 RW_i^2 P_i^2 D_i^2}{\sum_{i=1}^3 P_i} \left(1 + \frac{2\rho_{1,2} RW_1 D_1 P_1 RW_2 D_2 P_2 + RW_1 D_1 P_1 RW_3 D_3 P_3 + RW_2 D_2 P_2 RW_3 D_3 P_3}{2(\sum_{i=1}^3 RW_i^2 D_i^2 P_i^2)} \right).$$

III. EQUIVALENT INTEREST RATE RISK ASSESSMENT METHODS

Computing the capital charge of a given portfolio needs the computation of the value at risk using the traditional approaches. Therefore, we will compute the value at risk combining different known methods in order to compare them with the SBA results.

III.1. ARCH-GARCH

ARCH methods were introduced by Engle in 1982 (cf. Engle (1982)) then generalized by Bollerslev in 1986 (cf. Bollerslev and Tim (2008)) as a conditional variance prediction model, especially useful when the volatility of the financial data is the main issue.

Let X_t denote a real-time stochastic process; the GARCH(p,q) process is given by:

$$X_t | \sigma_t \sim N(0, \sigma_t^2)$$
$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i X_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2$$

The GARCH approach has been used in modeling financial time series, test financial theories and interpret key features of a given data in a time-dependent matter.

In our bonds portfolio we consider government yield curves with maturities ranging from 3 months to 30 years, compute the return and apply GARCH models for each maturity.

The orders of a GARCH process play a major role in determining the results: q can be based on model selection tests such as the autocorrelation function of the squared residuals; however with large q, estimation error might increase. Finding both p and q parameters can be facilitated through time series testing.

In our paper GARCH parameters are determined on the basis of coefficients significance and Jarque Berra test (cf. Bollerslev (2008)): skewness and kurtosis are used for constructing Jarque Berra's test statistic to find whether the coefficients of skewness and excess kurtosis are jointly nul(cf. Jarque and Bera (1981)).

When estimating GARCH models, computer-based softwares have to be used. Different softwares have different functionalities, drawbacks and features. Several works present these GARCH estimating software packages and compare them (cf. Brooks *et al.* (2001)). In our work, the 'tseries' R-package is used to estimate these models.

We start by estimating the GARCH process for each maturity, after having adequately selected the parameters. Since the projection requires initial values for each yield curve, we adopted the traditional way of having the last observed yield as the first data point of projection and the volatility as the historically observed volatility for each curve. We project for one year, fixing a year as 252 days, using Monte Carlo simulations then extract the value at risk for different confidence levels from 95% to 99.9 %. The capital charge percentage was computed on a mean relative figure basis, i.e. the mean of all simulations is extracted of the VaR and divided again by the mean (10,000 simulations were in order).

III.2. PCA-GARCH

Principal component analysis is mainly used to reduce the dimension of the handled data (cf. Jackson (1991)): This is useful in visualizing extraction. PCA can be derived from a number of starting points and optimization criteria. The most important of these are minimization of the mean-square error in data compression, finding mutually orthogonal directions in the data having maximal variances, and decorrelation of the data using orthogonal transformations.

The transformation is defined by a set of p-dimensional vectors of weights or loadings $w^{(k)} = (w_1, \dots, w_p)_{(k)}$ that map each row vector $x_{(i)}$ of X to a new vector of principal component scores $t_{(i)} = (t_1, \dots, t_p)_{(i)}$, given by $t_{k(i)} = \mathbf{x}_{(i)} \cdot \mathbf{w}^{(k)}$

Therefore instead of having 15 yield curves with different maturities, using the PCA process we reduce it to a number of principle factors that represent more than 98% of our data.

Having these factors (in most cases 2 or 3) we project them on a one year basis using GARCH models and choose their parameters based on the previously mentioned details. After rebuilding the 15 maturities using the projected factors and the previously computed loadings, we generate the VaR and capital charge of our portfolios using the same way selected in the previous methodology for the capital charge computation.

This process adds the correlation between different maturities and shows the interdependency between all tenors even when the portfolio does not include the entirety of maturities.

III.3. ICA-GARCH

Independent components analysis, (cf. Comon (1994)), is another method to reduce dimensions that has the same functionality as PCA except for the difference in the determination of the components and the loadings: In PCA, the aim is to find vectors that best explain the variance of the data whereas in ICA the kurtosis is in focus. The latent variables are assumed non-Gaussian and mutually independent.

The data analyzed by ICA could originate from many different kinds of application fields, including digital images, document databases, economic indicators and psychometric measurements. In many cases, the measurements are given as a set of parallel signals or time series; the term blind source separation is used to characterize this problem. Typical examples are mixtures of simultaneous speech signals that have been picked up by several microphones, brain waves recorded by multiple sensors, interfering radio signals arriving at a mobile phone, or parallel time series obtained from some industrial process.

In this ICA method follows the same process as PCA's: ICA to the full data panel, GARCH projection, rebuilding of the data, determining the VaR, capital requirement computation.

Again this model includes correlation as well as GARCH estimations however it is an add-on to the previous method due to the following: ICA does not assume the non-correlation of the factors instead it supposes the independence, such that the normality of the data is not a must; on the contrary, non Gaussian factors have an added-value.

The difference between PCA and ICA is explained in the literature (cf. Bugli (2007)), (cf. Burgos (2013)):

Given a set of multivariate measurements, the purpose is to find a smaller set of variables with less redundancy, which would best represent the data: this is the common goal of PCA and ICA.

However, in PCA the redundancy is measured by correlations between data elements, while in ICA the concept of independence is used, and in ICA the reduction of the number of variables is given less emphasis. (cf. Hyvarinen *et al.* (2001))

The earliest ICA algorithm that we are aware of and one which generated much interest in the field is that proposed by (cf. Herault and Jutten (1986)). Since then, various approaches have been proposed in the literature to implement ICA. These include: minimizing higher order moments (cf. Cardoso (1989)) or higher order cumulants (cf. Cardoso and Souloumiac (1993)), minimization of mutual information of the outputs or maximization of the output entropy (cf. Bell and Sejnowski(1995)), minimization of the Kullback-Leibler divergence between the joint and the product of the marginal distributions of the outputs (cf. Amari *et al.* (1996)).

This work is applied using the fastICA algorithm implemented in a R-package. (cf. Delac *et al.* (2006))

III.4. Dynamic Nelson Siegel

The interest rate curve is essential for pricing, hedging and evaluating a portfolio. Various curve fitting spline methods have been introduced (quadratic and cubic splines (McCulloch (1971, 1975)), exponential splines (Vasicek and Fong (1982)), B-splines (Shea (1984))..., these methods were criticized for not being too representative of the economical situations. Nelson and Siegel (1987) and Svensson (1994,1996) therefore suggested parametric curves that are flexible enough to describe the large frame of the financial conditions.

Nelson Siegel method consists of estimating three parameters using the maximum likelihood process or OLS to rebuild the yield curve (cf. Siegel and Nelson (1988)): it models the slope, curvature and level of the curve with their loadings and movements through time in order to fit an existing curve at each tenor t .

$$y(\tau) = \beta_1 + \beta_2 \frac{1-e^{-\lambda\tau}}{\lambda\tau} + \beta_3 \frac{1-e^{-\lambda\tau}}{\lambda\tau - e^{-\lambda\tau}}$$

The Nelson Siegel model is extensively used by central banks and monetary policy makers (ex: Bank of International Settlements (2005), European Central Bank (2008)).

As a development to the traditional fitting approach, Diebold and Li (2006) introduce the Dynamic Nelson-Siegel (DNS) model by estimating the classical formula with time-varying factors and model them using autoregressive specifications projecting therefore the yield curves by adding dynamism to the parameters: This method shows very encouraging results especially on a long time horizon.

$$y_t(\tau) = \beta_{1t} + \beta_{2t} \frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau} + \beta_{3t} \frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau - e^{-\lambda_t \tau}}$$

Our chosen approach in this paper is to fit the yield curves using the traditional Nelson Siegel and to project it afterwards: estimating the different yields using the Nelson Siegel function in R,(R-package: YieldCurve),projecting the betas computed using adequate ARIMA processes (based on best-fit approach); maintaining the loadings as calculated historically, rebuilding the projected yields based on diebold and Li's dynamic approach.

Having the projected yields, a new portfolio evaluating could be placed, a value at risk and therefore a capital charge is computed.

Lambda:

The lambda factor in the nelson siegel formula, as provided by Nelson and Siegel (1987), does not have any exact economic meaning. In fact, from the econometric point of view, it is a constant assuring a specific slope of yield curve. In practice, higher values produce a faster decay of yield curve and vice versa. In conclusion, the factor plays a key role especially in fitting the longer end of yield curves.

Nelson and Siegel (1987) fixed λ to simplify the computation allowing usage of the simple and linear ordinary least squares. Diebold and Li (2006) follow the same logic of exogenously predetermined λ however they suggested interpretation for the parameters as the slope, curvature and level. Based on such assumptions, and interpreting lambda as the maturity at which the loading on the curvature factor achieves its maximum, maximizing this loading would result in a numeric value for this coefficient: 0.0609.

Alternatively, in the related literature, Diebold et al. (2006) instead use the value of 0.077. Adding to that, other papers suggest different approaches for computing lambda not related to the aforementioned interpretation(cf. Comisef (2010)) and (cf. Gilli *et al.* (2012)).

In this paper, lambda of the projected yield is also considered as a constant chosen as the mean of the previously estimated lambdas for each specific yield.

IV. APPLICATION: BONDS PORTFOLIOS

IV.1. Data used

The data is fetched from Bloomberg: Government yield curves 3 months up till 30 years maturities on a daily basis for: France, Germany, USA and Turkey. For each country, different dates are available, France data starts on 04/30/1998, German on 10/04/1991, US on 11/24/2003 and Turkey on 04/01/2005; all the data ending point is on 05/15/2015.

IV.2. Portfolios's duration and different capital charge requirements

In order to compare these methods, we build portfolios using either one unique currency or multiple currencies.

The following plots represent the different yields for each currency:

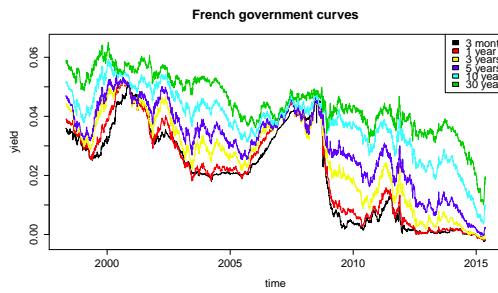


Figure 1: FRANCE government yield curves

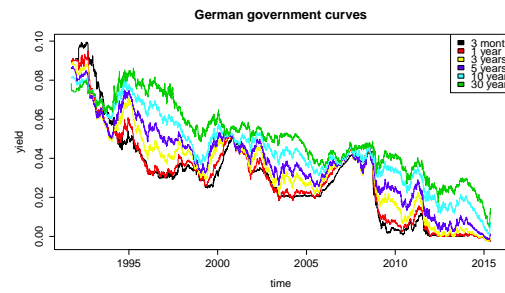


Figure 2: GERMANY government yield curves

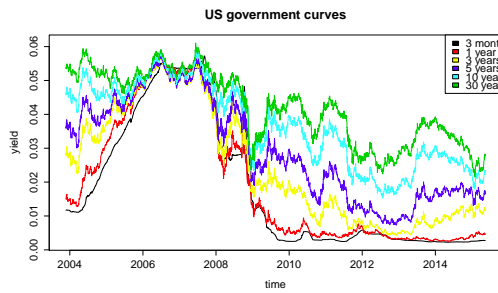


Figure 3: US government yield curves
In these plots we can distinguish four phases:

1. The normal phase up until 2004.
2. The moderation phase as called by Basel between 2004 and 2007, ending with the beginning of the financial crisis (low volatility).
3. The liquidity crisis between 2007 and 2008.
4. The zero bound phase where the volatility decreases going from 2008 up till the end of the sample: low short term volatility, high long term volatility.

Data statics can be found in *Appendix 2*.

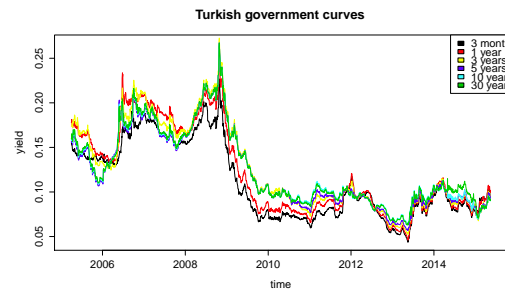


Figure 4: TRY government yield curves

Single currency portfolios

We consider four portfolios, for each governmental yield, with the same composition, consisting of 12 zero coupons each: 3 one year ZC, 3 two years ZC, 2 three years ZC, 1 five years ZC, 1 ten years ZC and 2 fifteen years ZC.

Sensitivity Based Approach

SBA capital requirement is compared below to the portfolio's duration at every date:

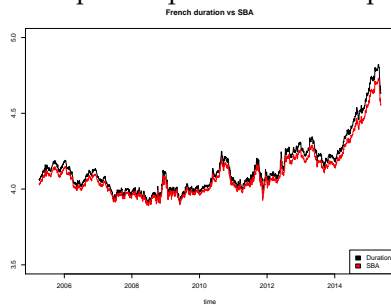


Figure 5: FR portfolio duration vs SBA CC

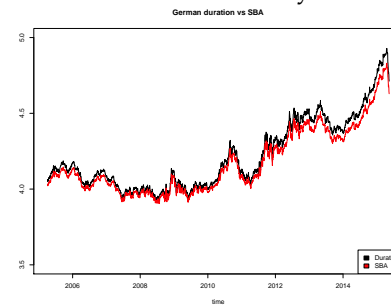


Figure 6: GR portfolio duration vs SBA CC

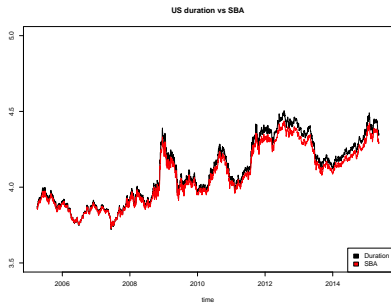


Figure 7: US portfolio duration vs SBA CC

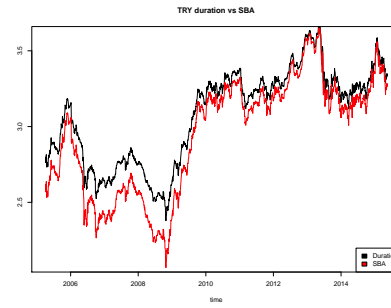


Figure 8: TRY portfolio duration vs SBA CC

As previously shown by the three cases equations, the plots above reflect the correlation between the duration of the four chosen portfolios and the capital requirement computed using the SBA method.

It is clear that the coefficient depends on a certain factor reflected by the risk weight used. Having the same portfolio composition, the factor between the duration and the capital charge is the same. After noting the obvious resemblance between the behavior of the SBA capital requirement and the duration of these single currency portfolios, we proceed in applying the main goal of this paper.

Our aim is to define an equivalent to the SBA capital requirement using a VaR on a given confidence level and time horizon; we proceed as follows: For each chosen methodology, we compute the VaR of the portfolios price (Monte-Carlo simulations basis) and compare these VaRs to the SBA requirement; the interception between these figures will simulate the confidence level and time horizon equivalent to Basel's requirement. We initiate this comparative work by computing the GARCH process value at risk.

GARCH model

For each currency, we have 15 yield curves with different maturities; we compute: $\Delta i_t = i_t - i_{t-1}$. Building on these differences, we estimate them using an ARCH(1) model; projecting this model one year ahead (252 days) we build our 'future' portfolios. Repeating this process 1000 times, based on a Monte Carlo logic, we conclude the VaR and therefore the capital charge. Note that for all methods the SBA capital charge is computed as a percentage of the initial portfolio, in the other methods capital charge is computed as the relative change between the projected mean and values at risk of the initial portfolio value.

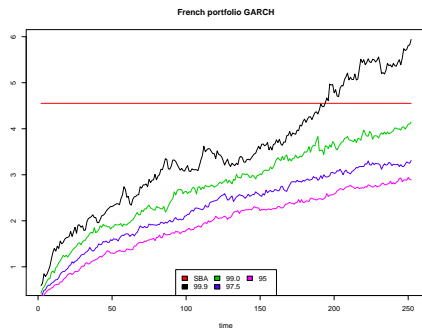


Figure 9: French GARCH capital charge

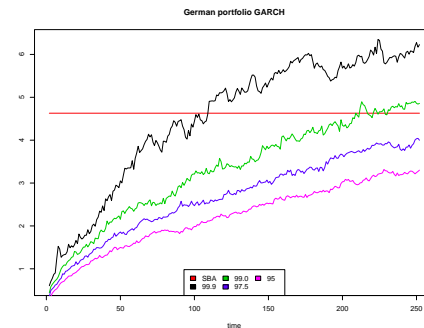


Figure 10: German GARCH capital charge

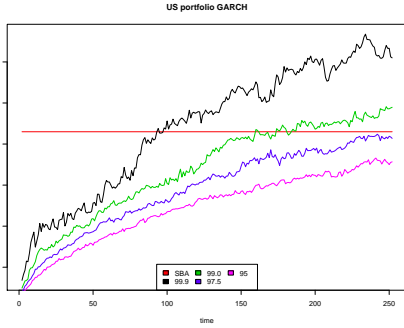


Figure 11: US GARCH capital charge

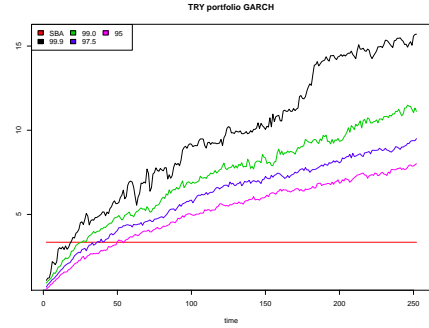


Figure 12: Turkish GARCH capital charge

Not accounting for the inter-correlation in our GARCH approach, the capital charge is expected to fall mostly below the SBA approach. We can observe the resemblance between the French, German and US market, however a very unstable behavior in the Turkish data.

PCA-GARCH model

Applying the PCA on the 15 maturities of each currency, we reduce the data into two components covering at least 98 % of the data. We project the first differential of these components using an adequate GARCH model then rebuild the entire maturities using the projected factors and the previously computed loadings. Monte Carlo simulations permits the extraction of the VaR at different levels and the capital charge.

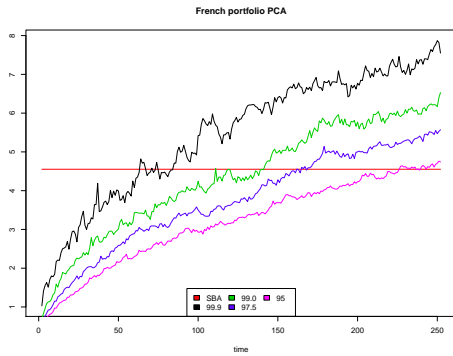


Figure 13: French PCA capital charge

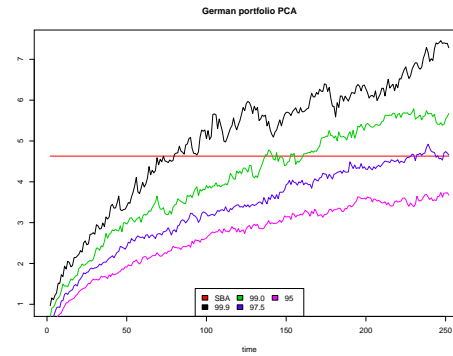


Figure 14: German PCA capital charge

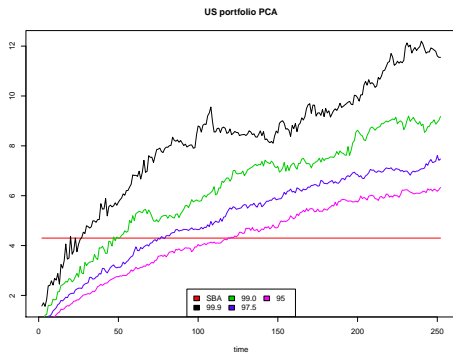


Figure 15: US PCA capital charge

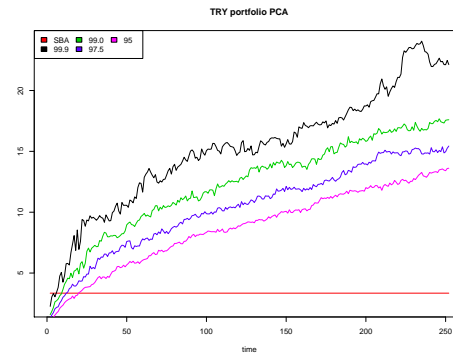


Figure 16: Turkish PCA capital charge

Once again European and US data show the same behavior, whereas the Turkish data being very volatile shows different results. Quickly comparing GARCH and PCA methodologies, a clear increase in the required capital is showing in the PCA figures due to the incorporation of the correlation between different maturities.

ICA-GARCH model

This method is similar to the previous one, however instead of using the principal component approach we used the independent components approach to increase the precision and reduce the assumptions made in the PCA.

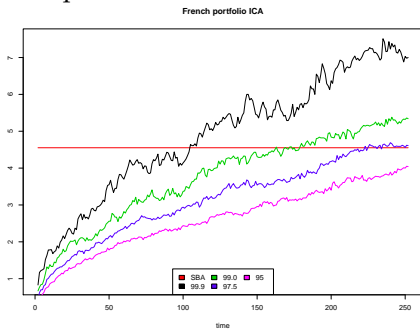


Figure 17: French ICA capital charge

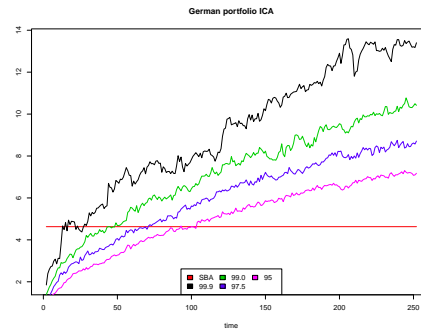


Figure 18: German ICA capital charge

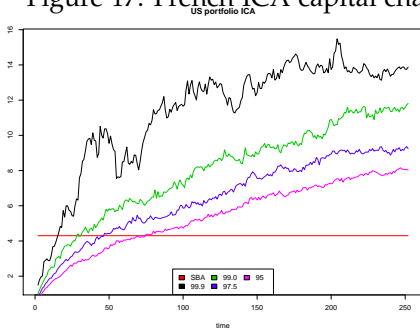


Figure 19: US ICA capital charge

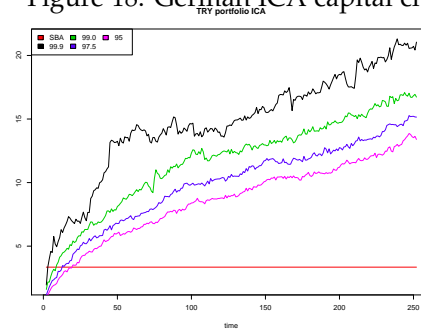


Figure 20: Turkish ICA capital charge

DNS-ARIMA model

After estimating the curves using NS model, we projected the beta parameters using the best fitted ARIMA(p,d,q) process. Along with the mean of the historically observed lambda's and the projected beta's, we rebuild the curves, estimate the VaR and compute the capital charge.

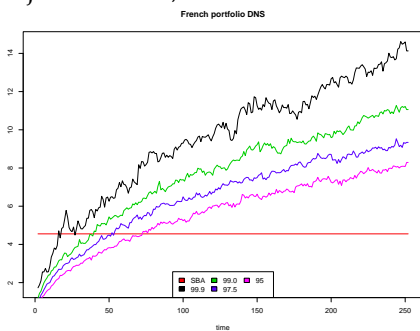


Figure 21: French DNS capital charge

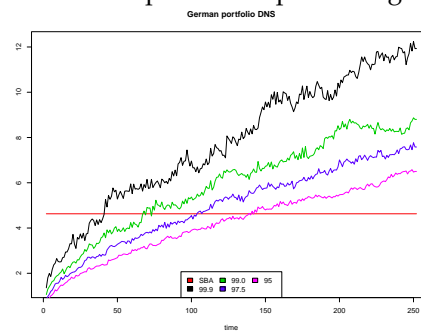


Figure 22: German DNS capital charge

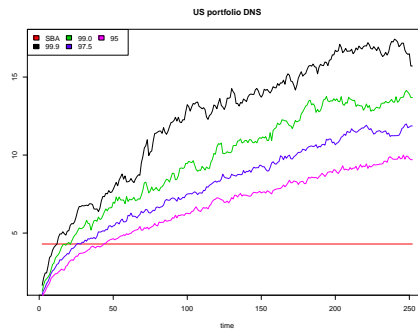


Figure 23: US DNS capital charge

The results show a quicker convergence in this method. However, in an unstable market such as the Turkish case, we do not observe any convergence in the Nelson Siegel parameter, therefore no projection could be applied.

Multiple currencies portfolios

Denoting the previous portfolios by their government yield we have: *port_FRANCE*, *Port_GERMANY*, *Port_US* and *Port_TURKEY*.

In this section we consider multiple currencies combining the previously mentioned portfolios:

Port_FR_GR, *Port_FR_US*, *Port_FR_TRY*, *Port_FR_GR_US*, *Port_GR_US_TRY* and *Port_FR_GR_US_TRY*

Sensitivity Based Approach

In this section, having multiple currencies, the correlation parameters between different curves at different tenors will be added. Note that both France and Germany are held in euros therefore they are part of the same bucket. SBA capital requirement vs duration:

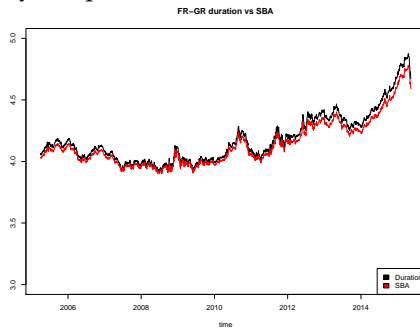


Figure 24: FR GR duration vs SBA CC

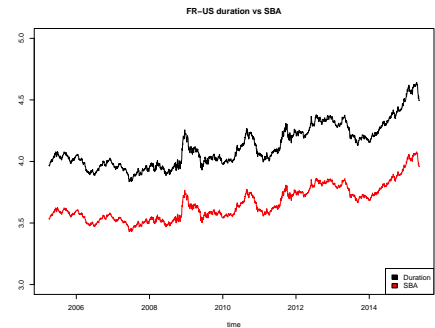


Figure 25: FR US duration vs SBA CC

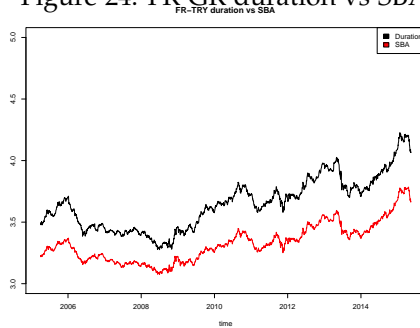


Figure 26: FR TRY duration vs SBA CC

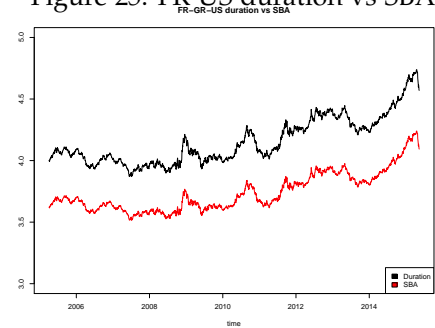


Figure 27: FR GR US duration vs SBA CC

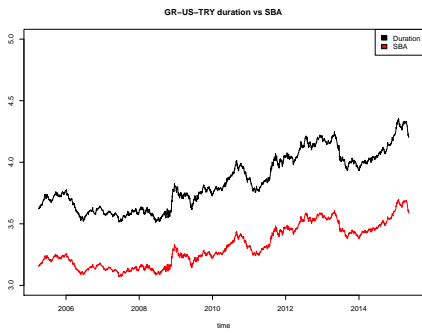


Figure 28: GR US TRY portfolio duration vs SBA capital charge

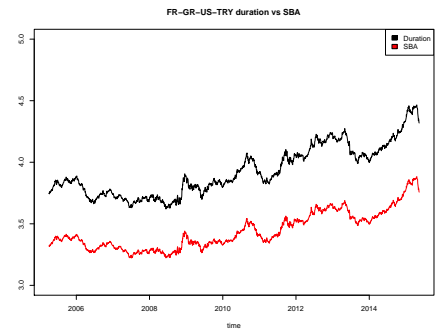


Figure 29: FR GR US TRY portfolio duration vs SBA capital charge

Similar remarks could be presented here regarding the parallel movement of the duration and capital requirement of these portfolios. This could be interpreted using the equations in appendix 1, case 3.

FR-GR portfolio

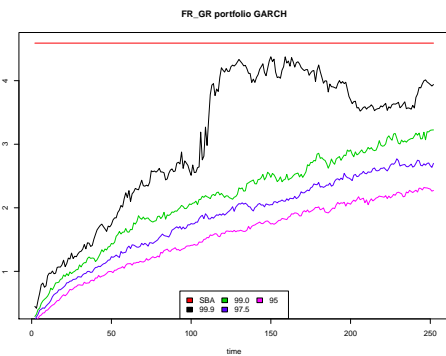


Figure 30: FR-GR GARCH vs SBA CC



Figure 31: FR-GR PCA vs SBA CC

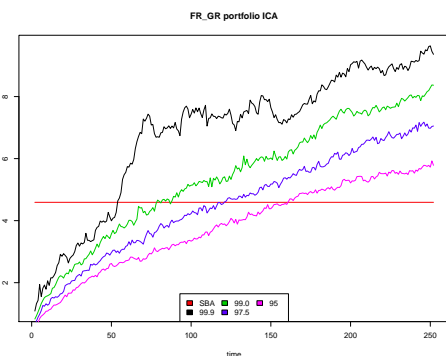


Figure 32: FR-GR ICA vs SBA CC

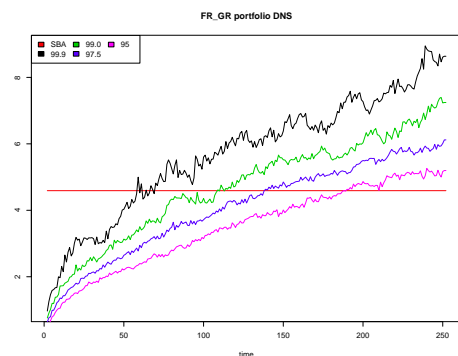


Figure 33: FR-GR DNS vs SBA CC

FR-US portfolio

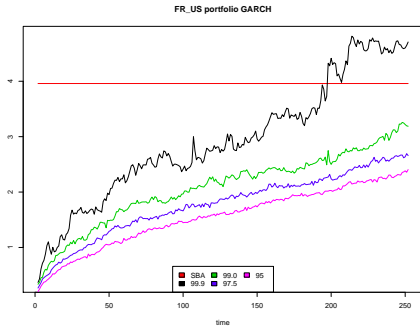


Figure 34: FR-US GARCH vs SBA CC



Figure 35: FR-US PCA vs SBA CC

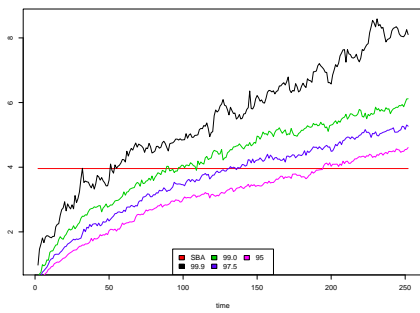


Figure 36: FR-US ICA vs SBA CC

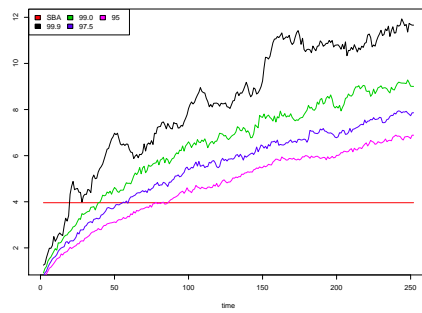


Figure 37: FR-US DNS vs SBA CC

FR-TRY portfolio

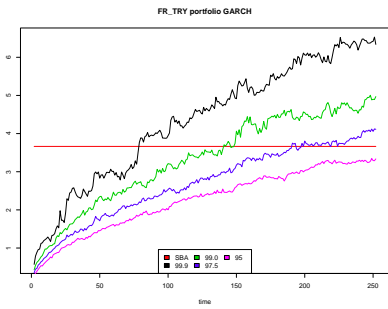


Figure 38: FR-TRY GARCH vs SBA

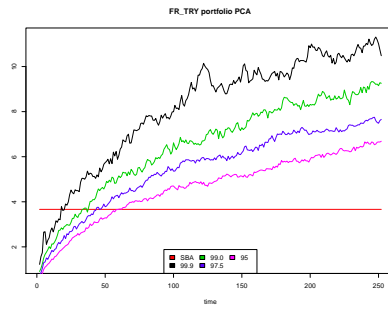


Figure 39: FR-TRY PCA vs SBA

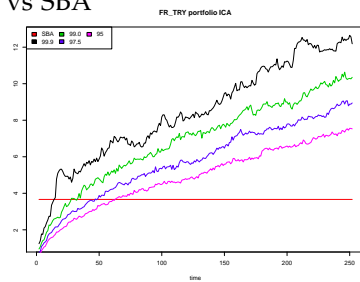


Figure 40: FR-TRY ICA vs SBA

FR-GR-US portfolio

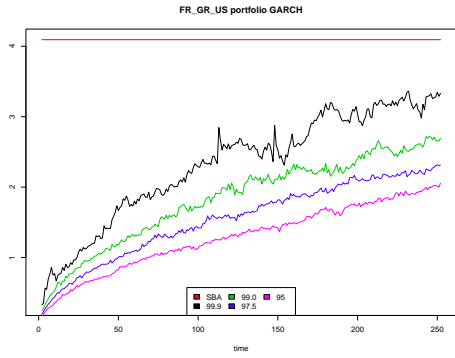


Figure 41: FR-GR-US GARCH vs SBA

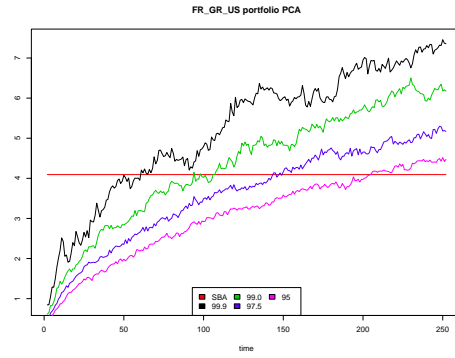


Figure 42: FR-GR-US PCA vs SBA CC

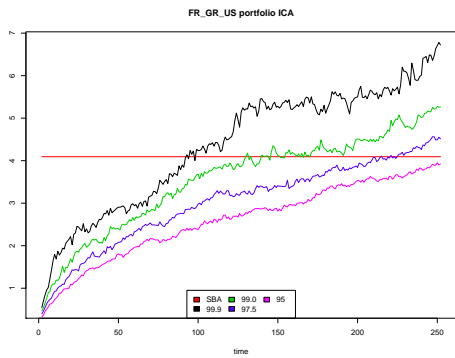


Figure 43: FR-GR-US ICA vs SBA CC

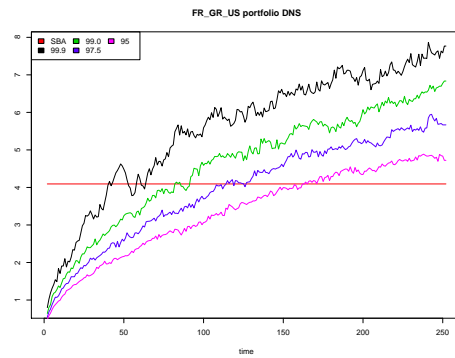


Figure 44: FR-GR-US DNS vs SBA CC

GR-US-TRY portfolio

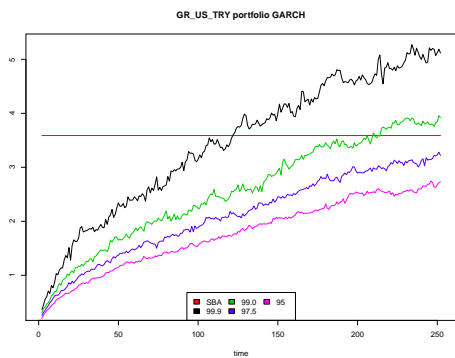


Figure 45: GR-US-TRY GARCH vs SBA

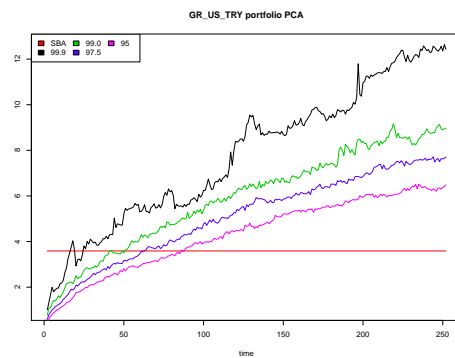


Figure 46: GR-US-TRY PCA vs SBA CC

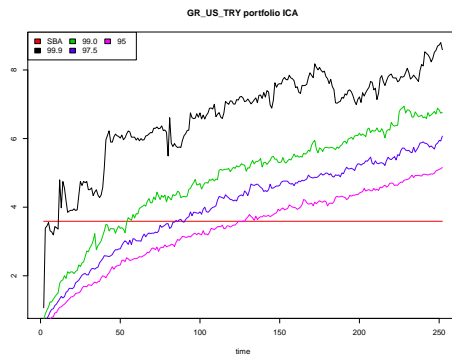


Figure 47: GR-US-TRY ICA vs SBA CC

FR-GR-US-TRY portfolio

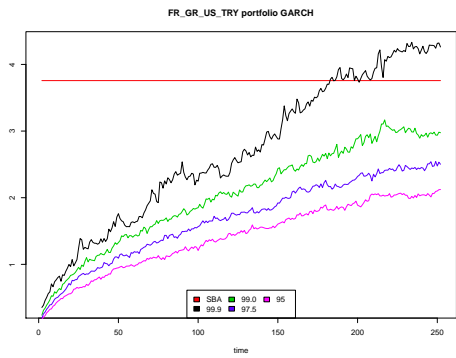


Figure 48: FR-GR-US-TRY GARCH

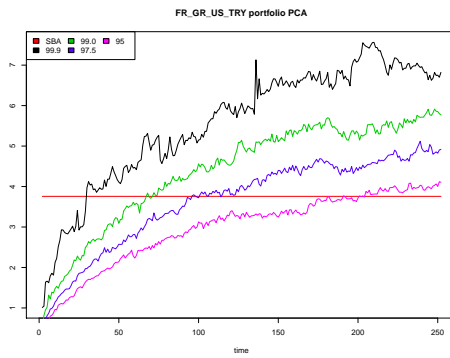


Figure 49: FR-GR-US-TRY PCA vs SBA

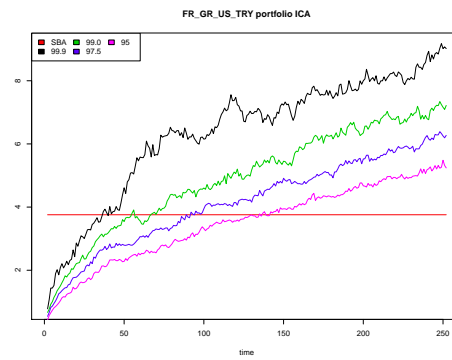


Figure 50: FR-GR-US-TRY ICA vs SBA

IV.3. Comparing results

In order to reduce the fluctuation, the results were interpolated to a logarithmic function resulting in the following:

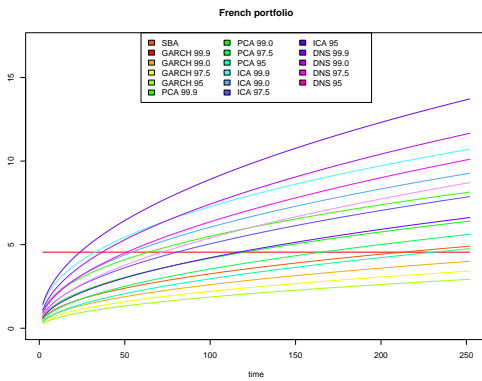


Figure 51: FR capital charge

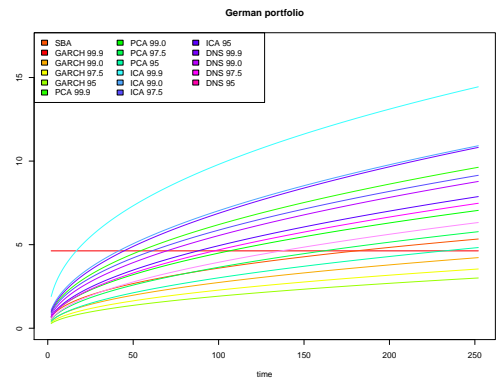


Figure 52: GR capital charge

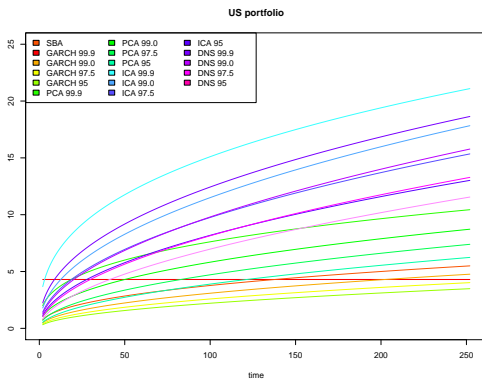


Figure 53: US capital charge

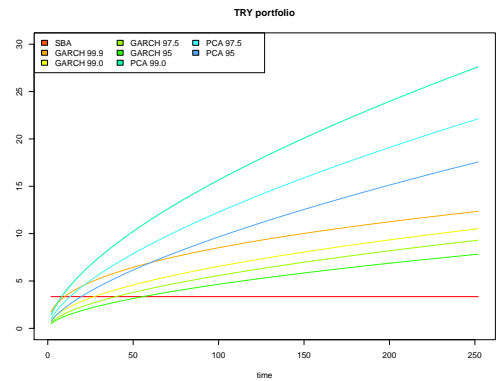


Figure 54: TRY capital charge

Based on the single currency portfolios, GARCH model shows a permanent 'low' level of capital requirement compared to Basel and the other methods. This could be explained by the correlations' exclusion between different tenors in this first method.

PCA and ICA methods show a close trend however, the ICA approach is more conservative and demands higher requirements. In all portfolios, DNS converges very rapidly, equaling the SBA on a short time horizon. The results differ as follows for multi-currencies portfolios:

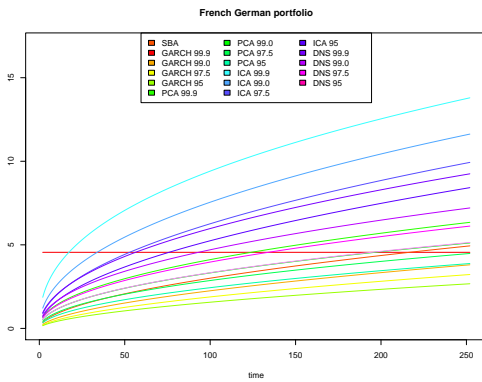


Figure 55: FR-GR capital charge

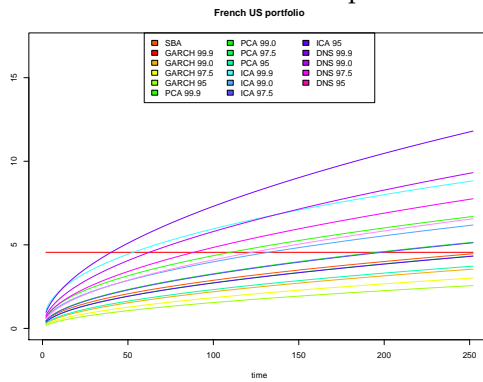


Figure 56: FR-US capital charge

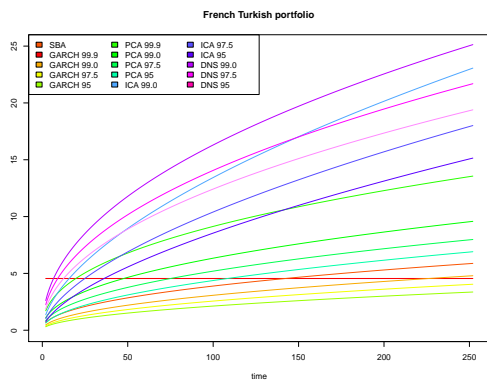


Figure 57: FR-TRY capital charge

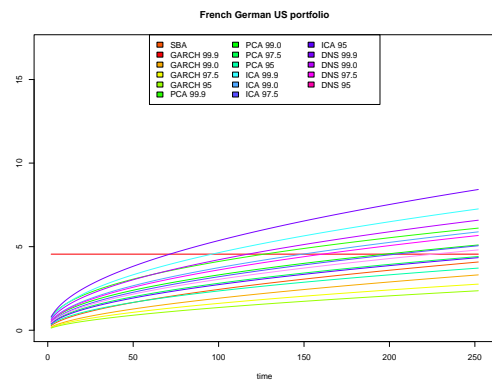


Figure 58: FR-GR-US capital charge

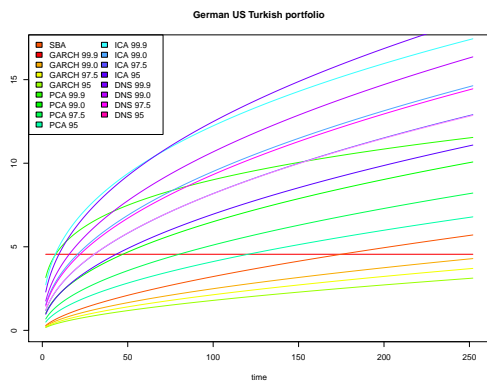


Figure 59: GR-US-TRY capital charge

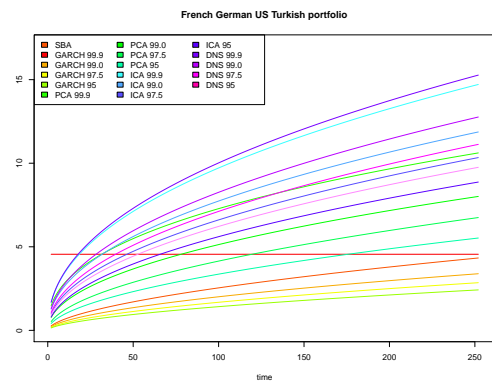


Figure 60: FR-GR-US-TRY CC

IV.4. Back-testing

In order to decide on the best approach for the capital requirement computation, a backtest analysis is a must. We used the typical 'number of hits' approach (number of violations) based on a scrolling window of 252 dates projecting them 100 notches (daily or weekly) and comparing the results with the actually observed rates. Our findings are summarized in the following table.

Results reject the GARCH model being too unrealistic and raises question marks regarding the DNS model for the daily projection.

		expected	daily	weekly
ARCH	FR	5.00%	7.00%	8.00%
	GR	5.00%	0.00%	5.00%
	US	5.00%	20.00%	11.00%
	TRY	5.00%	22.00%	14.50%
PCA	FR	5.00%	2.00%	4.00%
	GR	5.00%	3.00%	4.00%
	US	5.00%	4.00%	3.50%
ICA	FR	5.00%	0.00%	0.50%
	GR	5.00%	0.00%	2.50%
	US	5.00%	0.00%	0.00%
DNS	FR	5.00%	1.00%	0.00%
	GR	5.00%	9.00%	1.00%
	US	5.00%	9.00%	0.50%
	TRY	5.00%	0.00%	0.00%

V. CONCLUSION

We have compared in this paper different methods for computing the capital charge of a commercial bank based on a Eurobonds portfolio example and we have explored the performance in an out-of-sample forecasting based on the number of violations.

These approaches might be used as internal models compared to Basel's SBA in order to define a chosen time horizon and confidence level vis a vis the 'standard method'.

The following table summarizes the encountering points (in days) between the SBA and the different other methodologies studied for the ten portfolios.

	GARCH			PCA			ICA			DNS		
	95.0%	97.5%	99.0%	95.0%	97.5%	99.0%	95.0%	97.5%	99.0%	95.0%	97.5%	99.0%
FR	252	252	252	231	165	120	117	81	54	72	51	37
GR	252	252	252	232	162	106	88	61	42	137	98	70
US	252	252	202	121	83	50	28	19	13	41	30	20
TRY	56	40	27	20	13	8						
FR_GR	252	252	252	252	252	196	202	179	76	194	134	93
FR_US	252	252	252	252	252	196	252	199	132	123	88	63
FR_TRY	252	252	226	108	75	48	36	25	16			
FR_GR_US	252	252	252	252	252	198	252	204	149	226	161	117
GR_US_TRY	252	252	252	120	80	47	43	30	21			
FR_GR_US_TRY	252	252	252	176	120	78	66	46	32			

Figure 61: Encounter days with the SBA capital requirement

Trying to make sense out of these data we conclude the following:

	GARCH	PCA	ICA	DNS
	95.0%			
FR	12.00	11.00	5.57	3.43
GR	12.00	11.05	4.19	6.52
FR_GR	12.00	12.00	5.62	9.24
US	12.00	5.76	1.33	1.95
FR_US	12.00	12.00	12.00	5.86
FR_GR_US	12.00	12.00	12.00	10.76
FR_GR_US_TRY	12.00	8.38	3.14	
GR_US_TRY	12.00	5.71	2.05	
FR_TRY	12.00	5.14	1.71	
TRY	2.67	0.95		

	GARCH	PCA	ICA	DNS
	97.5%			
FR	12.00	7.86	3.86	2.43
GR	12.00	7.71	2.90	4.67
FR_GR	12.00	12.00	8.52	6.38
US	12.00	3.95	0.90	1.43
FR_US	12.00	12.00	9.48	4.19
FR_GR_US	12.00	12.00	9.71	7.67
FR_GR_US_TRY	12.00	5.71	2.19	
GR_US_TRY	12.00	3.81	1.43	
FR_TRY	12.00	3.57	1.19	
TRY	1.90	0.62		

	GARCH	PCA	ICA	DNS
	99.0%			
FR	12.00	5.71	2.57	1.76
GR	12.00	5.05	2.00	3.33
FR_GR	12.00	9.33	3.62	4.43
US	9.62	2.38	0.62	0.95
FR_US	12.00	9.33	6.29	3.00
FR_GR_US	12.00	9.43	7.10	5.57
FR_GR_US_TRY	12.00	3.71	1.52	
GR_US_TRY	12.00	2.24	1.00	
FR_TRY	10.76	2.29	0.76	
TRY	1.29	0.38		

Figure 62: Color coded encounter dates as months

Approaching Basel's method, the results show:

- Except for the Turkish market, GARCH method computes a similar capital requirement as the SBA for a minimum of one year time horizon.
- Comparing PCA and ICA we can conclude that the ICA is more restrictive for single currency denoted and mixed portfolios.
- PCA gives an eight months, 97.5% adequacy parallel to the four months given by the ICA for FR and GR portfolios. We add a PCA requirement of 4 months horizon for the US portfolio facing a one month limit given by the ICA.
- In the mixed portfolios PCA sets (at 97.5%) a limit of one year except when the Turkish lira is involved, ICA shows a lower encounter point of 9 months.
- The DNS on the 97.5% gives an adequate capital charge between two and four months.
- In the mixed portfolios DNS method on the 97.5% requires a time horizon between four and seven months.

Based on the previous, our recommendations are:

- > For the Eurozone:
 - 1 year GARCH capital charge would be equivalent to the SBA for a level of 97.5 %.
 - GARCH does not account for inter-maturities correlations therefore an ICA or PCA approach would be more rational:
 - * 8 months PCA 97.5 % on a country level and 1 year when combined.
 - * 3 months ICA 97.5 % on a country level and 8 months when combined.
 - DNS would inquire an average of 3 months for each country and 6 months for a multi-European portfolio.
- > For the US:
 - GARCH imposes a 1 month horizon for 97.5% confidence level.
 - 97.5% PCA for less than 4 months capital charge would do it and a 1 month ICA.
 - An average of 2 months DNS results in a close capital charge as the SBA's requirement.
- > For the Turkish market:
 - TRY is too volatile to be adequately represented by ICA or DNS models: it can be used for very short term : one or two months PCA (97.5%).
- > When combining US and Euro markets:
 - GARCH results could remain applicable.
 - PCA method time horizons' increases significantly to an average of one year.
 - ICA method time horizons' increases to 9 months.
 - DNS approaches a limit between 5 and 7 months.
- > When combining the Turkish lira with any of the US dollar or Euro portfolio: PCA approaches an average of 4 months and ICA an average of 2 months.

The goal of this paper was to provide banks with a tool that explains 'mathematically' Basel's SBA approach in order to fix the time horizon and confidence level of their capital requirement in the trading portfolio. In addition, these models could provide an internal approach with customized coefficients and parameters.

In June 2015, a new consultative document was issued by the BCBS on the 'Interest rate risk in the banking book, presenting new approaches to handle this book's capital charge computation and suggests dividing this amount between the first and second pillar. Incorporating the banking book in the first pillar is a new approach, because that segment was reserved for the trading book. Doing so, a similar methodology to the SBA would be inquired for the banking book. Our next step is to try and construct an internal model that mimics the proposed approach to compute the local parameters for the capital charge computation and interest rate shock scenarios applications.

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