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# A specific semi-markovian dynamic bayesian network estimating residual useful life

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**Abstract.** Degradation processes modelling is a key problem to perform any type of reliability study. Indeed, the quality of the computed reliability indicators and prognosis estimations directly depends on this modelling. Mathematical models commonly used in reliability (Markov chains, Gamma processes...) are based on some assumptions that can lead to a loss of information on the degradation dynamic. In many studies, Dynamic Bayesian Networks (DBN) have been proved relevant to represent multicomponent complex systems and to perform reliability studies. In a previous paper, we introduced a degradation model based on DBN named graphical duration model (GDM) in order to represent a wide range of duration models. This paper will introduce a new degradation model based on GDM integrating the concept of conditional sojourn time distributions in order to improve the degradation modelling. It integrates the possibility to take into account several degradation modes together and to adapt the degradation modelling in respect of some new available observations of either the current operation state or the estimated degradation level, to take into account an eventual dynamic change. A comparative study on simulated data between the presented model and the GDM will be performed to show the interest of this new approach.

**Keywords:** Dynamic Bayesian Networks, Graphical Duration Models, semi-markovian degradation process modelling, Reliability analysis, Residual Useful Life estimation.

## 1 Introduction

For the last fifty years, the complexity of most of industrial systems has constantly increased. If at best, their failure can lead to a temporary poorer performance of the system, a complete shutdown can also occur, inducing some potentially strong security risks. If the system fails, some components can have to be replaced, making the system unavailable for quite a long time which can be very costly. For these reasons, the research of decision support tools for the reliability analysis has become a key issue.

Many studies already dealt with this topic. Two approaches seems to be mainly used for degradation process modelling: the set of analytic degradation models



derived from the mechanic of the system, Lemaitre and Demorat [1], sometimes quite difficult to validate, and the use of stochastic tools, Aven and Jensen [2]. In this second approach, some models are directly based on probability distributions such as Bertholon model, Bertholon *et al.* [3], Weibull Freitas *et al.* [4] or exponential distributions... These modelling generally aim to focus on the failure time of the system. If one needs to evaluate the temporal behavior of a set of random variables, “dynamic” approaches will be preferred such as stochastic processes (Gamma process, Van Noortwijk [5], Poisson process, Hossain and Dahiya [6]...) or Probabilistic Graphical Models (PGM) such as Neuronal networks, Rajpal *et al.* [7], Petri nets, Volovoi [8], Dynamic Bayesian Networks, Weber and Jouffe [9]... In this paper, this last formalism was considered. Indeed, since some years, it has been proved as relevant to perform reliability studies since a degradation modelling based on discrete and finite states space is acceptable.

The simplest model of degradation process of a system using the DBN formalism is based on its ability to model a simple Markov chain. Then, this approach implies the strong assumption of geometrically distributed sojourn times in each state. To overcome this limitation, a specific DBN named Graphical duration model (GDM) was proposed, Donat *et al.* [10], to model the degradation of a discrete states system using any kind of discrete sojourn time distribution.

If this modelling provided some interesting results such as Bouillaut *et al.* [11], it assumes that sojourn times elapsed in each state are independent. The existence of several dynamics in the degradation process cannot therefore be identified neither taken into account. In this paper, an extension of the standard semi-markovian GDM modelling is proposed, managing the dependence between the sojourn times through the concept of conditional sojourn time distributions (CSTD). The aim is to be able to build a model that can describe a system whose dynamic is a mixture of some degradations modes and that can be adapted to observe changes in modes.

In the next section, the formalism of DBN and MGD will be briefly introduced and compared. Then, the proposed GDM with conditional sojourn time distributions will be detailed. Finally, a comparative study of the standard markovian approach with DBN and semi-markovian modelling (with GDM and MGD with CSTD) will be proposed for reliability analysis before some conclusions and prospects.

## **2 Probabilistic Graphical Models, a frame for reliability analysis**

### *2.1 From Bayesian Networks to Dynamic Bayesian Networks*

Formally, a Bayesian Network (BN) denoted by  $\mathcal{M}$  is defined as a pair  $(\mathcal{G}, \{p_n\}_{1 \leq n \leq N})$  where:

-  $\mathcal{G}=(\mathbf{X}, \varepsilon)$  is a directed acyclic graph giving a qualitative description of the BN. The graph nodes and the associated random variables are both represented by  $\mathbf{X}=\{X_1, \dots, X_N\}$ , with values in  $\mathcal{X}=\mathcal{X}_1 \times \dots \times \mathcal{X}_N$ .  $\varepsilon$  is the set of edges encoding the conditional independence relationships among these variables.

-  $\{p_n\}_{1 \leq n \leq N}$  a set of Conditional Probability Distributions (CPD) associated with the random variables. These distributions aim to quantify the local stochastic behavior of each variable.

Besides, both the qualitative (i.e.  $\mathcal{G}$ ) and quantitative (i.e.  $\{p_n\}$ ) parts of  $\mathcal{M}$  can be automatically learnt, if some complete or incomplete data or experts opinions are available, Jensen [12]. Using BN is also particularly interesting because of the easiness for knowledge propagation through the network. Indeed, various inference algorithms allow computing the marginal distribution of any sub-set of variables.

In a dynamic behavior modeling point of view, the time extension of BN provide a convenient formalism to represent discrete sequential systems. Indeed, DBNs are dedicated to model data which are sequentially generated by some complex mechanisms (time-series data, bio-sequences, number of mechanical solicitations before failure...). It is therefore frequently used to model Markov chains. Formally, a DBN is defined by a pair of BN  $(\mathcal{M}_{ini}, \mathcal{M}^{\rightarrow})$  where:

-  $\mathcal{M}_{ini} = (\mathcal{G}^{ini}, \{p_n^{ini}\}_{1 \leq n \leq N})$  is a BN modeling the initial distribution of  $\mathbf{X}$ , denoted  $p^{ini}$ .

-  $\mathcal{M}^{\rightarrow} = (\mathcal{G}^{\rightarrow}, \{p_{t,n}^{\rightarrow}\}_{2 \leq t \leq T; 1 \leq n \leq N})$  defines the transition model of the considered process, i.e. the distribution of  $X_t$  knowing  $X_{t-1}$ , denoted  $p^{\rightarrow}$ .

Figure 1 introduces a DBN modeling the Markov Chain of the sequence  $\mathbf{X}=(X_1, \dots, X_N)$  taking its values in the set  $\mathcal{X}$ . This DBN is described by the pair:

$(\mathcal{M}_{ini}, \mathcal{M}^{\rightarrow}) = ((X_1, p_1), (\mathcal{G}^{\rightarrow}, \mathcal{Q}^{sys}))$  where  $\mathcal{Q}^{sys}$  denotes the transition matrix of a Markov Chain, quantifying the probability of  $X_t|X_{t-1}$ .

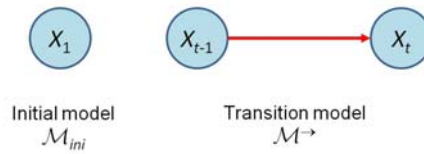


Fig. 1. DBN modelling a Markov Chain

If this approach is perfectly adapted to model the dynamic of systems, it induces a strong assumption on sojourn time distribution in each state of the system. Indeed, as all Markovian approaches, transition rates are assumed constant and, therefore, sojourn times are necessarily geometrically distributed. In many

industrial applications, such an assumption can introduce strong bias in the degradation modeling that cannot be foreseen in a context of reliability based maintenance optimization. To overcome this drawback, a specific DBN, named Graphical Duration model, was proposed and will be briefly introduced in the next paragraphs.

## 2.2 Graphical Duration Models

The Graphical Duration Model is a specific DBN, using a semi-Markov approach. The main idea is to deal with the couple  $(X_t, S_t)$  rather than the single variable  $\{X_t\}$  where  $S_t$  denotes the remaining time in the current state of  $X_t$ . Figure 2 introduces the structure of a DMN modeling a GDM. The solid lines define the basic structure of the GDM; dashed lines indicate optional items and red bold edges characterize dependencies between time slices.

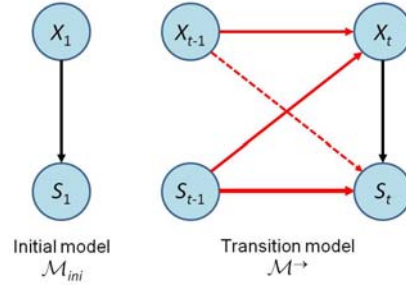


Fig. 2. Specific structure of a DBN modelling a Graphical Duration Model.

A GDM is therefore defined by the pair  $(\mathcal{M}_{ini}, \mathcal{M}^{\rightarrow})$  with:

-  $\mathcal{M}_{ini} = (\mathcal{G}^{ini}, (\alpha_1, F_1))$  where  $\alpha_1$  and  $F_1$  denote respectively the initial distribution of  $X_1$  and  $S_1$ .

-  $\mathcal{M}^{\rightarrow} = (\mathcal{G}^{\rightarrow}, (Q^{\rightarrow}, F^{\rightarrow}))$  characterized by two transition distributions:  $Q^{\rightarrow}$  is the natural states changes distribution and  $F^{\rightarrow}$  is the sojourn time distribution, both described by the following equations (1) and (2).

$F^{\rightarrow}$  is the distribution of the remaining sojourn times in the current state  $X_t$ . If a natural transition occurs at  $t$  (i.e.  $S_{t-1}=1$ ), this distribution is defined by a given conditional probability distribution  $F^{sys}$ . On the other hand,  $F^{\rightarrow}=C$  where this matrix simply decrement the sojourn time of 1 unit at each iteration, until a natural transition occurs.

$$\begin{aligned}
 F^{\rightarrow} &= p(S_t = s | X_t = x, S_{t-1} = s') \\
 &= \begin{cases} F^{sys}(x, s) = p(S_t = s | X_t = x, S_{t-1} = 1) \\ C(s', s) = p(S_t = s | S_{t-1} = s') \\ \quad = \begin{cases} 1 & \text{if } s = s' - 1 \\ 0 & \text{otherwise} \end{cases} \end{cases} \quad (1)
 \end{aligned}$$

$Q^{\rightarrow}$  is the distribution of state transitions. If a natural transition occurs at  $t$ , this distribution is defined by the previously introduced transition matrix  $Q^{\text{sys}}$ . On the other hand,  $Q^{\rightarrow}=I$  enforcing the variable  $X$  in the current state until a natural transition occurs.

$$\begin{aligned}
Q^{\rightarrow} &= p(X_t = x | X_{t-1} = x', S_{t-1} = s') \\
&= \begin{cases} Q^{\text{sys}}(x', x) = p(X_t = x | X_{t-1} = x', S_{t-1} = 1) \\ I(x', x) = p(X_t = x | X_{t-1} = x', S_{t-1} > 1) \\ = \begin{cases} 1 & \text{if } x' = x \\ 0 & \text{otherwise} \end{cases} \end{cases} \quad (2)
\end{aligned}$$

Besides, the structure of a GDM introduced in figure 2 shows the process  $(X_t)$  (respectively  $(S_t)$ ) is not Markovian since  $X_{t-1} \not\perp\!\!\!\perp X_{t+1} | X_t$  (respectively  $S_{t-1} \not\perp\!\!\!\perp S_{t+1} | S_t$ ); where  $A \not\perp\!\!\!\perp B$  denotes that variables  $A$  and  $B$  are not statistically independent. On the other hand, the GDM structure leads to

$$(X_{t-1}, S_{t-1}) \perp\!\!\!\perp (X_{t+1}, S_{t+1}) | (X_t, S_t) \quad (3)$$

So, the set  $(X_t, S_t)$  engendered by a GDM is Markovian, despite  $(X_t)$  is not. On the practical point of view, this approach allows specifying arbitrary state sojourn time distributions by contrast with a classic Markovian framework in which all durations have to be exponentially distributed. This modeling is therefore particularly interesting as soon as the question is to capture the behavior of a given system subjected to a particular context and a complex degradation distribution. More details on this GDM (quantitative description, optional context description ...) can be found in Donat *et al.* [10].

As an illustration of the contribution of GDM for reliability analysis, the simple and “standard” academic 3 serial-parallel components system will be considered.  $C_2$  and  $C_3$  are parallel components, taking their values in  $\{ok, failed\}$  whereas the serial component  $C_1$  takes its values in  $\{ok, small\ defect, failed\}$ . The system fails when  $C_2$  and  $C_3$  are simultaneously failed or when  $C_1$  fails.

Figure 3 introduces the sojourn time distributions, considered in this example, for all “not failed” states. Parameters of a DBN modeling a Markov Chain and of GDM are learnt for the three components, using a database with 1000 sojourn times in each state. This learning phase provides transition rates for the standard DBN modeling and discretized sojourn time distributions for the GDM modeling, conditioned by the parameter  $T_{max}$ , defined as the higher bound of sojourn times. The settlement of this parameter is a fundamental point in the GDM approach. Indeed, if it is underestimated, the learning of sojourn time distributions can be strongly biased. On the other hand, the complexity of the considered bayesian network can induce algorithmic problems. In this paper,  $T_{max}=200$ .

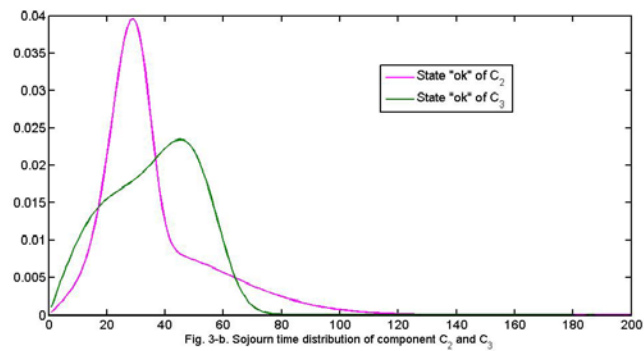
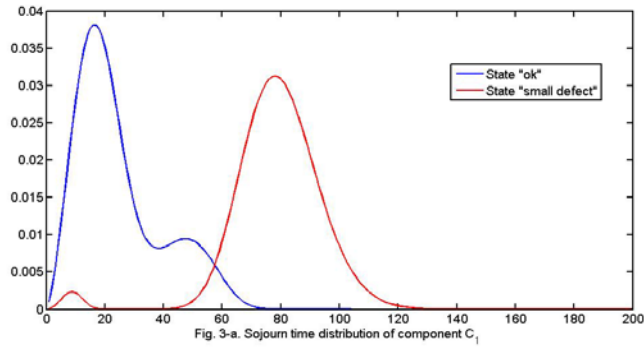


Fig. 3. Sojourn time distributions in non failure states for components  $C_1$ ,  $C_2$  and  $C_3$ .

When all parameters are learnt, all kind of reliability indicators can be easily estimated, such as instantaneous availability, reliability, cumulative distribution function... The following figure introduces the estimation of this last indicator by both modeling “standard” DBN and GDM for the 3 components system.

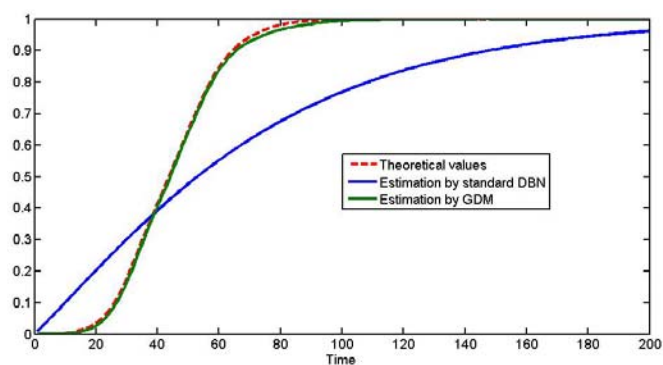


Fig. 4. Sojourn time distributions in non-failure states for components  $C_1$ ,  $C_2$  and  $C_3$ .

The red line introduces the theoretical behavior of the considered reliability indicator. One can note that, due to the stochastic properties of the sojourn time distributions characterizing our system, a Markovian approach cannot fit the exact behavior of the system when the GDM provides a good formalism. This first result focuses on the impact of the assumptions made during the degradation modeling on the accuracy of our reliability analysis.

In the next section, an extension of GDM is introduced, allowing taking into account several dynamics in the degradation process modeling. Indeed, as we can note in figure 3, the sojourn times distributions seems to consist in the merging of different dynamics. This is particularly identifiable on figure3-a where two behaviors can be observed. With the standard GDM, this information cannot be taken into account...

### 3 Introduction of conditional sojourn time distributions in GDM

#### 3.1 Structure and main properties of GDM-CSTD

Figure 5 shows the graphical structure of the DBN modeling a GDM with conditional sojourn time distributions. A variable encoding the current degradation mode, denoted  $M_t$ , is added to the couple  $(X_t, S_t)$  used by the standard MGD. This change induces updating the transition distributions introduced in (1) and (2).

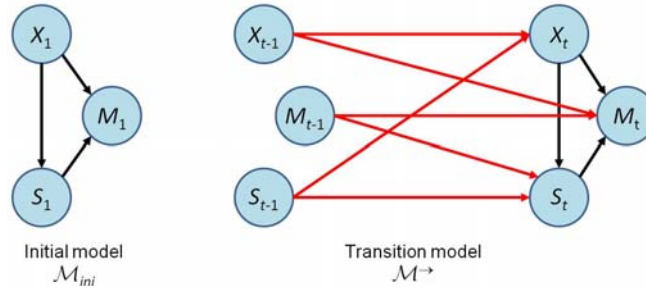


Fig. 5. Specific structure of a DBN modelling a GDM integrating conditional sojourn time distributions.

Since  $M_{t-1}$  does not influence  $X_t$ , the state transition distribution introduced by (2) will not be modified. The impact of the modes variable on the sojourn time variable  $S_t$  leads to the following adaptation of (1).

$$\begin{aligned}
 F^{\rightarrow} &= p(S_t = s \mid X_t = x, S_{t-1} = s', M_{t-1} = m') \\
 &= \begin{cases} F^{sys}(x, s, m') & \text{if } s' = 1 \\ C(s', s) & \text{if } s' > 1 \end{cases} \quad (4)
 \end{aligned}$$

If no natural transition occurs between  $t-1$  and  $t$ , the matrix  $C$  decrements the remaining sojourn time by 1 unit whereas if  $s'=1$ , the state of the system changes between  $t-1$  and  $t$ . Then, the sojourn time in the new current state is



chosen in respect  $F^{sys}$ , the probability distribution of the sojourn time for each state, in each mode.

Finally, (5) quantifies the transition of modes. If the state of the system does not change between  $t-1$  and  $t$ , the considered mode at  $t-1$  is conserved using the matrix  $I$ . If a state transition occurs, the distribution  $W^{sys}$  defines, through the elapsed time in the previous state, the most probable mode for the current state.

$$\begin{aligned} W^{-} &= p(M_t = m \mid X_{t-1} = x', X_t = x, S_t = s, M_{t-1} = m') \\ &= \begin{cases} W^{sys}(x', x, s, m', m) & \text{if } x' \neq x \\ I(x', x, m') & \text{if } x' = x \end{cases} \end{aligned} \quad (5)$$

The learning of CSTD consists in three phases. First, the number of modes, denoted  $n_m$ , has to be determined. This can be done either by expert's advice or using a criterion such as BIC - *Bayesian Information Criterion*, Schwarzze [13], that will determine, through a return of experience (REX) database  $D_X$ , the optimal number of mixtures. Then, through  $D_X$ , the EM algorithm, Dempster *et al.* [14], is used to estimate the sojourn time distributions in the initial state for each of the  $n_m$  modes. This learning also provides a segmentation of  $D_X$  in  $n_m$  sub-bases dedicated to each mode, denoted  $D_X^m$  with  $m \in [1..n_m]$ . Finally, the sojourn time distributions for each mode in all other states are learnt using the right  $D_X^m$ .

To illustrate this modeling, a four states system is considered, taking its values in  $\mathcal{X} = \{ok, state2, state3, failed\}$ , with two degradation modes, periodically observed ( $T_{obs}$ ). To set a 1000 trajectories learning database, each sojourn time distribution follows a Weibull distribution that parameters are under mentioned:

- *ok*: mode 1  $\sim \mathcal{W}(2,15)$  and mode 2  $\sim \mathcal{W}(6,33)$
- *state2*: mode 1  $\sim \mathcal{W}(6,10)$  and mode 2  $\sim \mathcal{W}(9,25)$
- *state3*: mode 1  $\sim \mathcal{W}(6,5)$  and mode 2  $\sim \mathcal{W}(15,15)$

Then, considering the previously introduced learning procedure, conditional probability distributions are learnt from the sampled database for both MGD and MGD-CSTD approaches. This learning phase underlined the bias that might be introduced in the estimation of the failure time of a periodically observed system when the coexistence of several dynamics in the degradation process is not taken into account. Indeed, if these sojourn time distributions are used to estimate the remaining useful life of our system, in the standard MGD approach, the observation of a short sojourn time in the first state does not impact the estimation of the sojourn time in the next states. The following subsection will introduce some illustrative results on this point.

### 3.2 Estimation of the remaining useful life of periodically observed systems

In this last illustration, the 4 states system introduced in section 3.1 will be considered with a  $T_{obs}$  periodic monitoring, providing the current state of the system each 5 time step.

Aiming to estimate the remaining useful life (RUL) of the system and to update it when a new observation is available, the structure of the MGD-CSTD introduced in figure 5 was adapted, adding two variables  $D_t$  and  $\delta_t$  that represent respectively the diagnosis of  $X_t$  by the monitoring device, taking its values in  $\mathcal{X} \cup \{\emptyset\}$  and an activation variable controlling if the diagnosis is active or not.

In the considered example,  $\delta_t$  is  $T_{obs}$  periodically activated. In the other cases, the monitoring device is not active and  $D_t$  returns no information on  $X_t$  through the state  $\emptyset$ . Figure 6 introduces the structure of this new DBN.

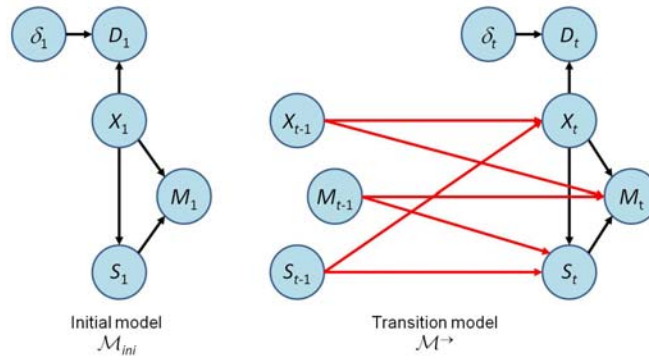


Fig. 6. Structure of the DBN used to estimate the RUL of a periodically observed system with a GDM-CSTD approach.

During the initialization phase, a sojourn time in state  $ok$  is obtained with  $F_1$ , determining the most probable current mode  $m_1$ . Then, knowing  $m_1$ , sojourn time distributions (or conditional sojourn time distribution in the MGD-CSTD approach) in states  $state2$  and  $state3$  (contained in  $F^{sys}$ ) provide the initial estimation of the RUL.

When a new diagnosis is available, if a natural transition is observed, in the MGD-CSTD approach, the current mode is eventually corrected in respect of the most probable mode knowing the elapsed time in the previous state. Then, with the same reasoning that in the initialization phase, the RUL is updated.

Before trying to integrate this RUL estimation algorithm in a wider specific DBN, the proposed methodology has to be briefly evaluated. This was the aim of the results introduced in this end of paper.

Figure 7 introduces two illustrative results, underlying the behavior of MGD and MGD-CSTD approaches for the RUL estimation of the periodically observed 4 states system.

The first one focuses on the interest of using the conditional sojourn time distributions concept. Indeed, one can observe that the information contained in the degradation mode (approach MGD-CSTD) allows using a more accurate sojourn time range to estimate the RUL. This is the reason why the estimation is closer to the real value of the remaining useful life that when the standard MGD is considered.

In the second draw, another interest of MGD-CSTD is underlined. Indeed, in this run, the MGD based estimation looks better in the first observations since it is closer to the real RUL. The explanation of this unusual situation is that, during the initial phase, the mode  $m_1$  was chosen when the right one was  $m_2$ . For this reason, the wrong part of the sojourn times was used in the MGD-CSTD approach, explaining the less precise estimations. We can note that at  $t=35$ , the situation changes. Between 30 and 35, the state turns from *ok* to *state2*. Then, knowing the elapsed time in *ok*, the algorithm is able to correct its initial mistake by correcting the most probable mode. Then, the MGD-CSTD uses the right part of the sojourn time range while the MGD approach still work with the complete sojourn time domain.

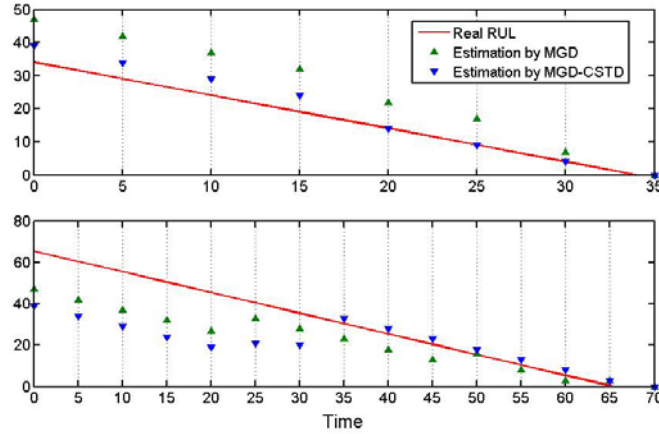


Fig. 7. Examples of RUL estimations using GDM and GDM-CSTD approaches.

To be complete, we have to point on some possible situations that will have to be investigated in further works. Figure 8 introduces what can happen when the sojourn time in a given state is located exactly in the intersection range of two modes. In that example, the mode was correctly initialized in  $m_1$ . Then the first estimations with MGD-CSTD are very satisfactory. But, when the transition to state *state2* is detected at  $t=30$ , the probability of such a sojourn time in state *ok* for mode  $m_1$  is so weak that an inappropriate mode change is adopted. Then, during the two next observations, the RUL estimation by MGD-CSTD is really poor. Fortunately, in that case, the transition *state2-state3* allows correcting the mode and therefore improving the last RUL estimation.

To give a better idea of the global behavior of the proposed approach, figure 9 introduces the RUL estimation error in respect of time for the trajectories of the considered database obtained with the MGD-CSTD approach, respectively for the 500 trajectories in mode 2 and 93.2% of the 500 trajectories in mode 1. The considered time indices in these drawings correspond to the number of observation before the system fails.

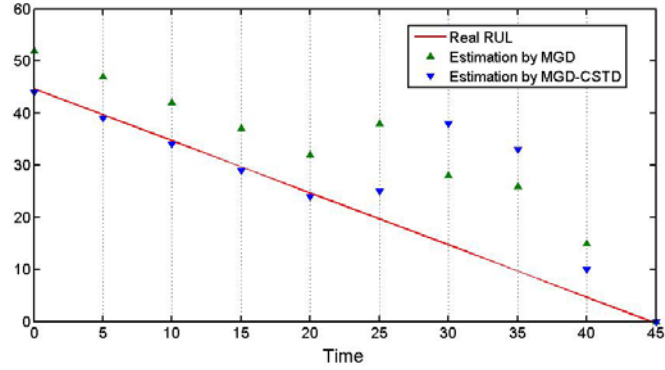


Fig. 8. Examples of RUL estimations using GDM and GDM-CSTD approaches - Unfavourable case.

One can note that, for mode 2 trajectories, the initial estimation of the RUL is quite poor. This is due to the fact that, in the current version of the algorithm, the variable  $M_t$  is systematically initialized in mode 1. Then, the first estimations are based on the wrong sojourn time distributions, inducing strong errors. In most of cases, when the first natural state transition is observed, the most probable degradation mode is uploaded and the RUL estimation is improved.

We also can note that the confidence in the estimation increases when the failure time comes and the RUL estimation becomes quite informative. Only very few trajectories are long enough to have more than 14 observations, this is the reason why there are a weak number of points for temporal indices upper than 15.

For mode 1 trajectories, i.e. faster degradation scenarios, we can observe the same global behavior of the RUL estimation algorithm. Nevertheless, this case also illustrates the drawback of the proposed approach, introduced through figure 8 that might be improved in further works. Globally, 34 of the 500 trajectories in mode 1 have at least one sojourn time in the intersection range of two modes, inducing a wrong estimation of the most probable degradation mode, such a way that the adaptation ability of the algorithm, underlined in figure 8, cannot process in these cases and the RUL estimation is absolutely unusable.

Nevertheless, even if the proposed algorithm shows some interesting adaptation abilities when mode estimation errors occur, this final example points out some improvements that will have to be investigated in further works, to make more robust the proposed approach.

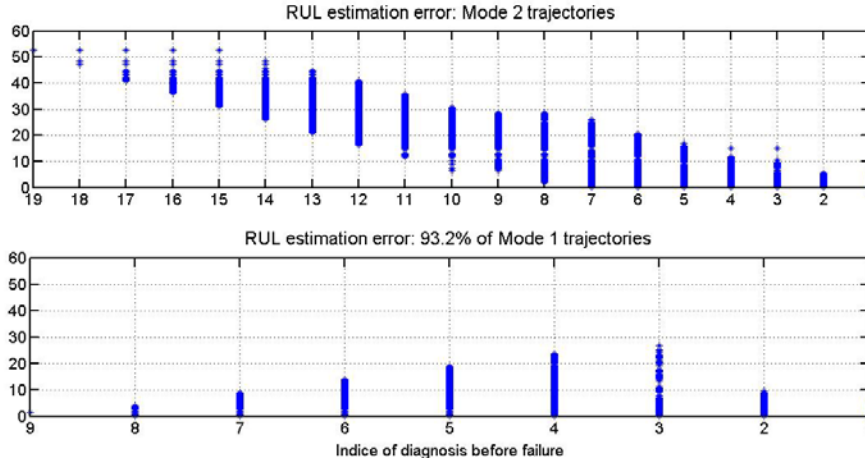


Fig. 9. Global behaviour of the RUL estimation algorithm. Estimation error for each degradation mode in respect of time

## Conclusions

In this paper, the formalism of Probabilistic Graphical Models was investigated to determine degradation process modeling for systems with discrete and finite states space. If the “classic” Markovian approach consisting in the modeling of a Markov Chain by a Dynamic Bayesian Network induces necessarily the assumption of sojourn times in each state exponentially distributed, a semi-Markovian approach was proposed using a specific DBN structure, named Graphical Duration Model, that allows considering all kind of sojourn time distribution without any assumption on the stochastic properties of the degradation process.

In this approach, sojourn times in each state are supposed independent. In some applications, especially when the degradation process is the merging of several dynamics, such an assumption can induce strong bias in the estimation of reliability indicators. To illustrate this problem, an example was introduced dealing with the estimation of the remaining useful life of a periodically observed system.

Then, an extension of the standard GDM approach was proposed, integrating the notion of conditional sojourn time distributions by a new random variable managing the most appropriate degradation mode, knowing the elapsed time in the previous state. Then, both MGD and MGD-CSTD were used and compared for the estimation of the remaining useful life of a multi-states system. If the MGD-CSTD approach offers a very interesting global behavior, its main drawback lies in the existence of intersection ranges for several modes, inducing some potential wrong degradation mode estimations and, in the worst cases, an inability of the algorithm to readjust the RUL estimation before the failure time.

Some improvements of the proposed algorithm are currently in progress on this key point.

## Acknowledgment

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## References

1. J. Lemaitre, and R. Demorat, Engineering Damage mechanics, Berlin Springer, 2005
2. T. Aven, J. Jensen, Stochastic models in reliability. Stochastic modelling and applied probability, 1999.
3. H. Bertholon, N. Bousquet, and G. Celeux, An alternative competing risk model to the Weibull distribution for modelling aging in lifetime data analysis. Lifetime Data Analysis, 12(4), pp. 481-504, 2006.
4. M. Freitas, M. de Toledo, E. Colosimo, and M. Pires, Using degradation data to assess reliability: a case study on train wheel degradation. Quality and Reliability Engineering International, 25(5), pp. 607-629, 2009.
5. J. Van Noortwijk, A survey of the application of gamma processes in maintenance. Reliability Engineering & System Safety, 94(1), pp. 2-21, 2009.
6. S. Hossain and R. Dahiya, Estimating the parameters of a non-homogeneous Poisson process model for software reliability. IEEE Transactions on Reliability, 42(4), pp. 604-612, 1993.
7. P.S. Rajpal, K.S. Shishodia and G.S. Sekhon, An artificial neural network for modeling reliability, availability and maintainability of a repairable system, Reliability Engineering and System Safety, 91(7), pp. 809-819, 2006.
8. V. Volovoi, Modeling of System Reliability Using Petri Nets with Aging Tokens, Reliability Engineering and System Safety, 84, pp. 149-161, 2004.
9. P. Weber and L. Jouffe, Reliability modelling with dynamic bayesian networks. In proceedings of the 5th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes, 2003.
10. R. Donat, P. Leray, L. Bouillaut, and P. Aknin, A dynamic Bayesian network to represent discrete duration models, Neurocomputing, 73(46), pp. 570-577, 2010.
11. L. Bouillaut, O. François, and S. Dubois, A Bayesian network to evaluate underground rails maintenance strategies in an automation context. Proceedings of the Institution of Mechanical Engineers, Part O, Journal of Risk and Reliability, 227(4), pp.411-424, 2013.
12. F.V. Jensen, An introduction to Bayesian networks. UCL Press, 1996.
13. G. Schwarz, Estimating the dimension of a model, Annals of Statistics, 6(2), pp. 461-464, 1978.
14. A. Dempster, N. Laird and D. Rubin, Maximum likelihood from incomplete data via the EM algorithm. Journal of the Royal Statistical Society, B(39), pp. 1-38, 1977.