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► To cite this version:

Ronan Fablet, François Rousseau. Joint interpolation of multi-sensor sea surface geophysical fields using non-local and statistical priors. 2015. hal-01216762

HAL Id: hal-01216762

<https://hal.science/hal-01216762>

Preprint submitted on 16 Oct 2015

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Joint interpolation of multi-sensor sea surface geophysical fields using non-local and statistical priors

Ronan Fablet, *Senior Member, IEEE*, and François Rousseau

Abstract

This work addresses the joint analysis of multi-source and multi-resolution remote sensing data for the interpolation of high-resolution geophysical fields. As case-study application, we consider the interpolation of sea surface temperature fields. We propose a novel statistical model, which combines two key features: an exemplar-based prior and second-order statistical priors. The exemplar-based prior, referred to as a non-local prior, exploits similarities between local patches (small field regions) to interpolate missing data areas from previously observed exemplars. This non-local prior also sets an explicit conditioning between the multi-sensor data. Two complementary statistical priors, namely a prior on the spatial covariance and a prior on the marginal distribution of the high-resolution details, are considered as sea surface geophysical fields are expected to depict specific spectral and marginal features in relation to the underlying turbulent ocean dynamics. We report experiments on both synthetic data and real SST data. These experiments demonstrate the contributions of the proposed combination of non-local and statistical priors to interpolate visually-consistent and geophysically-sound SST fields from multi-source satellite data. We further discuss the key features and parameterizations of this model as well as its relevance with respect to classical interpolation techniques.

I. INTRODUCTION

The ocean surface is monitored with a dense network of satellites. Various satellites record images of multiple ocean parameters at different resolutions. As a peculiar example, sea surface temperature (SST), which is the temperature of the thin ocean's upper layer, is a peculiar example. Microwave (MW) radiometry provides low resolution observation (0.5° - 0.25° , 25/50km) while infrared (IR) sensor delivers high resolution SST measurements (up to 0.02° , 1km). SST monitoring is of high interest as SST observations are key observations for a wide range of studies, including weather forecasting, ocean circulation, global warming impacts.

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Beyond their intrinsic resolution, the different satellite sensors also differ in their sensitivity to the atmospheric conditions, and especially the cloud coverage. Whereas microwave radiometers involve very low missing data rates, IR sensor may result in high missing data rates (up to 90% over several consecutive days in some regions). Operational level-4 products provide interpolated fields, which typically rely on covariance-driven kriging-based approaches (e.g., [2], [28]). The joint exploitation of the multi-source multi-scale data remains however a challenge to produce geophysically consistent interpolated fields.

In this context, we address here the interpolation of missing data in high-resolution geophysical fields under the assumption that a simultaneous low-resolution observation is available. Such multi-scale interpolation is expected to depict geophysically consistent features, with a view to conforming to the underlying ocean turbulence characteristics, especially: i) consistent high-resolution textured patterns, ii) non-Gaussian marginals and iii) specific spectral signatures. We propose a novel model to address this multi-scale interpolation jointly accounting for the above-mentioned constraints. The key idea is to exploit a non-local patch-based framework [14], [31]. Such exemplar-based representation provides an implicit texture model, which directly exploits available observations to reconstruct consistent geophysical patterns.

II. PROBLEM STATEMENT AND RELATED WORKS

The state-of-the-art techniques for missing data interpolation in ocean remote sensing mainly involve two categories of approaches. The most popular techniques, by far, involve covariance-based geostatistical filtering and are exploited in most operational products [2], [28]. This category includes kriging methods as well as optimal interpolation. These approaches relies on the definition and/or estimation of the spatial covariance of the field to be interpolated. The interpolation is stated by as the computation for each grid point of the conditional mean of the field given the observed grid points and the considered covariance model. This geostatistical interpolation simultaneously performs the interpolation and some smoothing of the original data. By nature, through the parameterization of the covariance model, it involves some trade-off between the fidelity to the observed data and the reconstruction of a spatially-consistent field. Hence, interpolated fields typically involve a noticeable smoothing of the truly observed data. The same difficulty arises from the second category of approaches which involves matrix factorization [1], especially EOF-based (Empirical Orthogonal Functions, also referred to as PCA in the image analysis community) approaches, which relies on the projection onto a precomputed basis functions. By contrast, we develop here a different point of view. We rather aim at reconstructing or emulating a spatially- and geophysically-consistent field with a strong fidelity of the observed data. This is referred to as image inpainting in the image processing literature [10], [22]. This is also viewed as a mean to truly handle multi-source data acquired at different resolutions.

In the image processing field, Image inpainting and super-resolution have been particularly active research topics over the last decade [10], [14], [16], [30], [31]. A variety of models and algorithms have been introduced, including for instance variational, Bayesian, non-local setting. As geophysical fields may be viewed as textured images, we focus on texture-related models. Among others, we may distinguish two categories of approaches: exemplar-based or patch-based models and explicit statistical models. Exemplar-based schemes state the inpainting and super-resolution

issue as the sampling from a collection of local patches (also referred to as a patch dictionary). From the original work of Efros et al. [12], the formulation of these exemplar-based strategies as non-local models [5], [29], [31] has become the state-of-art for a wide range of inverse problems in image processing. Applications to remote sensing data have been investigated [11], [24]. By contrast, explicit statistical models rely on specific statistical features such as spatial covariances (or equivalently, spectral power densities) [15], [7], marginal distributions [18], scale-invariance [9].

Regarding ocean surface geophysical fields, such statistical features are particularly relevant as ocean turbulence is expected to depict specific spectral and non-Gaussian characteristics. Besides, for any given case study region, existing image time series provide the mean for building representative dictionaries of exemplars. Therefore, the combination of non-local models and statistical priors appears as a natural path to address sea surface field reconstruction from multi-source and multi-scale satellite data. In this respect, our main contributions are three-fold:

- the exploitation of a multi-scale decomposition of the geophysical field with a view to explicitly dealing with satellite data associated with different spatial resolution;
- the combination of non-local and statistical priors for the texture-based interpolation and blending of multi-source satellite observations;
- the introduction of a mathematically-sound formulation as a minimisation of projection constraints onto different subspaces.

We detail in the subsequent these different contributions.

III. PROPOSED APPROACH

Formally, the considered missing data interpolation is stated as an inverse problem, i.e. the reconstruction of unknown fields $X^{(1)}$ and $X^{(2)}$ from partial observations $Y^{(1)}$ and $Y^{(2)}$, where the subscripts refers to satellite-derived gridded fields from two different sensors. These sensors may differ both in the associated spatial resolution and in the missing data rate. We consider all fields are gridded and interpolated on some reference grid Ω , referred to hereafter as the high-resolution grid. We denote respectively by $\Omega^{(1)}$ and $\Omega^{(2)}$ the missing data region for each sensor, and, $r^{(1)}$ and $r^{(2)}$ the actual resolution associated with each sensor. Subscript (1) refers to the sensor with the lowest resolution. By convention, we assume below for the sake of simplicity that $r^{(1)} \geq 1$ and $r^{(2)} = 1$. This amounts to considering below that all scales are defined up to the actual resolution grid.

In this section, we describe the proposed framework as follows. We first introduce the considered multi-resolution representation. Second, we introduce non-local and patch-based prior. Third, the considered statistical priors are detailed. Fourth, we present the proposed interpolation model along with the associated numerical scheme.

A. Multi-resolution representation

With a view to handling information sources at different resolutions, we consider a wavelet decomposition of the analysed fields [26]. It comes to decompose field X over an orthonormal basis of unctons $\{\Psi_{s,i}\}$, which are

scaled and translated version of a mother wavelet $\Phi(\cdot)$:

$$\forall p \in \Omega, X(p) = \sum_{s,i} X_{s,i} \Psi_{s,i}(p) \quad (1)$$

where $\{X_{s,i}\}$ are the wavelet coefficients for field X . Indices s and i refer respectively to a scale index and to a spatial position at scale s . We consider here a dyadic decomposition, such that scale indices are of the form 2^j . For a given field, the computation of the associated wavelet coefficients are issued from the projections onto the basis functions, which are implemented as decimation and filtering steps iterated from the fine scale to the coarsest scale. It may be noticed that the wavelet decomposition guarantees the exact reconstruction of field X from its wavelet coefficients $\{X_{s,i}\}$. We let the reader refer to [26] for further details on the wavelet decomposition. We introduce the associated wavelet-based projection operator \mathcal{P}_W^{s*} for scale s^* , which refers to the reconstruction of field X from all scales greater than s^*

$$\forall p \in \Omega, \mathcal{P}_W^{s*}(X)(p) = \sum_{s>s^*,i} X_{s,i} \Psi_{s,i}(p) \quad (2)$$

It may be noted that, by construction, field $X - \mathcal{P}_W^{s*}(X)$ refers to a detail field, denoted by $\delta\mathcal{P}_W^{s*}(X)$ which only involves information at scales finer than s^* .

For the targeted issue, we denote by s_{LR} the reference low-resolution scale in the dyadic wavelet decomposition, given as the dyadic scale $\lceil \log_2 r^{(1)} \rceil$ the closest to the low resolution $r^{(1)}$ ². Given that the two sensors observe to the same geophysical condition, we assume that the two fields $X^{(1)}$ and $X^{(2)}$ share the same low-resolution projection at scale s^{LR} such that we impose

$$\mathcal{P}_W^{s_{LR}}(X^{(1)}) = \mathcal{P}_W^{s_{LR}}(X^{(2)}) \quad (3)$$

B. Multimodal patch-based interpolation and non-local projection

As stressed in the introduction, patch-based and non-local priors are particularly appealing to develop data-driven image processing techniques, by contrast to model-driven techniques [24], [11], [14]. This appears especially relevant when the definition of explicit analytical *a priori* models is a complex issue, as for sea surface turbulent dynamics whose characteristics may vary in space and time. We here focus on non-local regularization or projection [5], [29], [31]. Given a field X , it comes to compute a non-local mean $\mathcal{P}_{NLM}(\{\mathcal{A}_k\})(X)$, with respect to a reference set of patches based on patch similarities. The non-local mean for a patch around a pixel $p \in \Omega$ is given by

$$\mathcal{P}_{NLM}(\{\mathcal{A}_k\})(X)(p) = \sum_{k \in \mathcal{K}} w_k(X, \{\mathcal{A}_l\}, p) \mathcal{A}_k \quad (4)$$

where k is a patch index within the reference patch dictionary, \mathcal{A}_k the k^{th} $P \times P$ patch in this dictionary. Typical values for the patch width P are between 5 and 13. $w_k(I, \mathcal{A}_k, p)$ is a weighing factor which weighs the contribution

¹By convention, throughout the manuscript, notation \mathcal{P} refers to a projection operator in the image domain. Here, notation \mathcal{P}_W refers to a wavelet-based projection operator. In the subsequent, we introduce other projection operators corresponding to the non-local, covariance-based and margin-based priors.

² $\lceil \cdot \rceil$ is the ceiling function, such that $\lceil a \rceil$ is the integer part of number a plus one

of the k^{th} reference patch \mathcal{A}_k in the non-local mean at point p . It relies on the similarity between the $P \times P$ patch around pixel p in field X , denoted by $X(\mathcal{N}_p)$, and reference patch \mathcal{A}_k . Patch similarities are generally evaluated according to a classical Euclidean distance [5], [31], such weight $w_k(X, \{\mathcal{A}_l\}, p)$ resorts to a normalised weight as follows:

$$w_k(X, \{\mathcal{A}_l\}, p) = \frac{g(\|X(\mathcal{N}_p) - \mathcal{A}_k\|)}{\sum_l g(\|X(\mathcal{N}_p) - \mathcal{A}_l\|)} \quad (5)$$

where $g(\cdot)$ is a kernel function. Non-local regularisation [5], [29], [31] typically considers a Gaussian kernel $g(x) = \exp(-\gamma \cdot x^2)$ with γ a smoothing parameter to be defined. In texture synthesis, nearest-neighbor kernels may be preferred to avoid some over smoothing of textured details. Different strategies may also be considered for the combination of the non-local means computed for different image positions. Non-local denoising and segmentation [5], [31] typically involves the computation of the non-local mean for every pixel to derive the non-local image projection. By contrast, in texture synthesis and inpainting [10], [14], [22], one usually achieves the non-local projection for partially or non-overlapping patches with a predefined, optimized or randomized selection of the pixels at which the non-local means are actually evaluated (Eq.4). The later, which is the more generic, is selected in this work. We apply the non-local means to randomly selected positions and, for each selected position p , the update the non-local image projection with the $(P - B) \times (P - B)$ interior of the non-local mean $P \times P$ patch (4) with B the width at the boundary. It may noted that, in our implementation, we use for each non-local projection a different random sequence of patch positions with a view to ensuring shift invariance [26].

Here, we apply the non-local setting to the joint interpolation of the two fields $X^{(1)}$ and $X^{(2)}$ associated with two different sensors. Hence, the evaluation of patch similarities naturally relies on the information from each field and the reference patch dictionary $\{\mathcal{A}_k^{(1)}, \mathcal{A}_k^{(2)}\}$ is formed by concomitant patch exemplars from each sensor. Non-local projection (4) becomes a multi-modal non-local projection. For the sake of simplicity and readability, we drop the explicit reference to patch dictionary $\{\mathcal{A}_k^{(1)}, \mathcal{A}_k^{(2)}\}$ in our notations. For field $X^{(1)}$, we denote by $\mathcal{P}_{NLM, X^{(2)}}(X^{(1)})$ its non-local projection given reference patch dictionary $\{\mathcal{A}_l^{(1)}, \mathcal{A}_l^{(2)}\}$ and field $X^{(1)}$: $\forall p \in \Omega$

$$\mathcal{P}_{NLM, X^{(2)}}(X^{(1)})(\mathcal{N}_p) = \sum_{k \in \mathcal{K}} w_k(X^{(1)}, X^{(2)}, p) \mathcal{A}_k^{(1)} \quad (6)$$

where the normalized non-local weights involve joint similarities for the two fields

$$\begin{aligned} w_k(X^{(1)}, X^{(2)}, p) \propto & g\left(\alpha^{(1)} \left\|X^{(1)}(\mathcal{N}_p) - \mathcal{A}_k^{(1)}\right\| \right. \\ & \left. + \alpha^{(2)} \left\|X^{(2)}(\mathcal{N}_p) - \mathcal{A}_k^{(2)}\right\| \right) \end{aligned} \quad (7)$$

where $\alpha^{(1)}$ and $\alpha^{(2)}$ are scalar values which weigh the relative contribution of each sensor. Regarding the targeted issue, we explore different strategies for the multi-modal non-local projection:

- Projection $\mathcal{P}_{NLM, X^{(2)}}(X^{(1)})$: the non-local projection of field $X^{(1)}$ given field $X^{(2)}$;
- Projection $\mathcal{P}_{NLM, \|\nabla X^{(2)}\|}(\delta X^{(1)})$: the non-local projection of detail field $\delta X^{(1)}$ given field $\|\nabla X^{(2)}\|$. This strategy is motivated by recent observations that the variability of the high-resolution detail $\delta X^{(1)}$ is driven by the magnitude of the low-resolution gradient $\|\nabla X^{(2)}\|$ [13].

In the reported experiments, we compare these different strategies and evaluate their pros and cons.

C. Statistical constraints

Geophysical fields are expected to depict specific statistical features [33], [20], [4], [6], which may be used as additional priors to further constrain their reconstruction from partial and noisy observations. In this work, we focus on two categories of statistical features: covariance (or equivalently spectral) features and non-Gaussianity.

Sea surface geophysical processes, especially for the mesoscale range (*i.e.*, for horizontal scales from tens of kilometres to hundreds of kilometres), involve turbulent processes characterised by specific spectral signatures [20], [25]. Their radial power spectral density generally involves power laws [33], [20]. A typical example is the homogeneous and isotropic turbulence for incompressible flows [19] which resorts to Kolmogorov's 5/3 power law for the power spectral density of the energy of the velocity field. We then set a power law prior on the radial spectral density of geophysical fields $X^{(1)}$ and $X^{(2)}$ for the fine-scale range (*i.e.*, scales lower than $r^{(1)}$). Let us denote by R^* the reference radial power spectral density and $S_{X,X}$ the power spectral density of field X . The radial power spectral density of field X , denoted by R_X is a radial integration of power spectral density $S_{X,X}$:

$$R_X(r) = \int_{\|\omega\|=r} S_{X,X}(w) dw \quad (8)$$

It may be noted that power spectral density $S_{X,X}$ is the Fourier transform of the spatial covariance of field X . Reference density R^* is set as power law:

$$R^*(r) = \gamma \cdot r^\alpha \quad (9)$$

where γ and α are the parameters of this spectral prior. In the reported experiments, we calibrate these parameters from the spatial covariance of field $X^{(1)}$ for scales greater than $r^{(1)}$ (typically, greater than 100km for the considered experiments on SST data). Given this prior, the fine-scale spectral constraint for field $X^{(2)}$ amounts to:

$$R_{X^{(2)}}(r) = R^*(r), \forall r \in [r^{(2)}, r^{(1)}] \quad (10)$$

This spectral constraint can be solved for in the Fourier domain through the computation of a transformed field $\mathcal{T}_{R^*,r^{(1)}}(X)$ such that its Fourier transform $\mathcal{T}_{R^*,r^{(1)}}(X)$ is given by:

$$\widehat{\mathcal{T}_{R^*,r^{(1)}}(X)} = \sqrt{\frac{R^*}{R_X}} \hat{X} \quad (11)$$

Numerically, the Fourier transform will be computed here using a windowed FFT. It might be pointed out that the Fourier transform amounts to a linear filtering and a projection onto the manifold of the two-dimensional fields accounting for spectral constraint (10).

In addition to this spectral constraint, ocean turbulence is widely acknowledged to depict non-Gaussian marginals in multiscale decomposition [6], [13], [27] as considered here. Turbulent flows tend to advect tracers to form fronts or high-gradient structures at different scales. This in turn result in non-Gaussian heavy-tailed marginals of detail fields of multi-scale representation. Generalized Gaussian distributions $\mathcal{GG}_{\sigma,\alpha}$ appear as relevant, yet simple, parametric

model to account for the observed non-Gaussian signatures:

$$\mathcal{GG}_{\sigma,\alpha}(x) \propto \exp\left(-\left(\frac{x}{2\sigma}\right)^\nu\right) \quad (12)$$

where σ is a scaling parameter and ν a power exponent. For $\alpha = 2$, it resorts to a standard zero-mean Gaussian distribution. For a given field $X^{(2)}$, we fit a generalized Gaussian distribution to the detail field $\delta X^{(2)} - \mathcal{P}_W^{sLR}(X^{(2)})$ according to a maximum likelihood (ML) criterion. Let us denote by $\mathcal{GG}_{\sigma^*,\alpha^*}$ the reference distribution. The marginal constraint then amounts to:

$$X^{(2)} - \mathcal{P}_W^{sLR}(X^{(2)}) \propto \mathcal{GG}_{\sigma^*,\alpha^*} \quad (13)$$

To solve for this constraint for field $X^{(2)}$, we proceed as follows. Given current fields $X^{(1)}$ and $X^{(2)}$, we fit a generalized Gaussian distribution $\mathcal{GG}_{\tilde{\sigma},\tilde{\alpha}}$ to the current detail field $X^{(2)} - X^{(1)}$ according to a ML criterion. The transformed field $\mathcal{T}_{\mathcal{GG}^*}(X^{(2)})$ resorts to:

$$\begin{aligned} \mathcal{T}_{\mathcal{GG}^*}(X) &= \mathcal{P}_W^{sLR}(X^{(2)}) \\ &+ \Psi_{\sigma^*,\alpha^*}\left(\Psi_{\tilde{\sigma},\tilde{\alpha}}^{-1}(X^{(2)}(p) - \mathcal{P}_W^{sLR}(X^{(2)})(p))\right) \end{aligned} \quad (14)$$

where $\Psi_{\sigma,\alpha}$ is the cumulative distribution function of generalized Gaussian distribution $\mathcal{GG}_{\tilde{\sigma},\tilde{\alpha}}$, $\Psi_{\tilde{\sigma},\tilde{\alpha}}^{-1}$ the associated inverse cumulated distribution function.

D. Joint interpolation and blending framework

The joint interpolation and blending of two fields $X^{(1)}$ and $X^{(2)}$ from partial observations $Y^{(1)}$ and $Y^{(2)}$ is stated as the following joint non-local projection:

$$\begin{cases} \widehat{X^{(1)}} = \arg \min_{X^{(1)}} \left\| X^{(1)} - \mathcal{P}_{NLM,X^{(2)}}(X^{(1)}) \right\|_{\Omega^{(1)}}^2 \\ \widehat{X^{(2)}} = \arg \min_{X^{(2)}} \left\| X^{(2)} - \mathcal{P}_{NLM,X^{(1)}}(X^{(2)}) \right\|_{\Omega^{(2)}}^2 \end{cases} \quad (15)$$

where $\|\cdot\|_{\Omega}^2$ stands for the quadratic norm restricted to spatial domain Ω . The above minimization is to be solved for under the four following constraints:

$$\left\{ \begin{array}{l} \text{Interpolation constraints:} \\ X^{(1)}(\Omega \setminus \Omega^{(1)}) = Y^{(1)}(\Omega \setminus \Omega^{(1)}) \\ X^{(2)}(\Omega \setminus \Omega^{(2)}) = Y^{(2)}(\Omega \setminus \Omega^{(2)}) \\ \\ \text{Low-resolution constraint:} \\ \mathcal{P}_W^{sLR}(X^{(1)}) = \mathcal{P}_W^{sLR}(X^{(2)}) \\ \\ \text{Spectral constraints:} \\ R_{X^{(1)}} = R^* \\ \\ \text{Marginal constraint:} \\ X^{(2)} - \mathcal{P}_W^{sLR}(X^{(2)}) \propto \mathcal{GG}_{\sigma^*, \alpha^*} \end{array} \right. \quad (16)$$

Problem formulation (15) refers to the application of the non-local projections to field $X^{(1)}$ given field $X^{(2)}$. As described in Section III-B, for field $X^{(2)}$, we consider a different strategy with non-local projection $\mathcal{P}_{NLM, \|\nabla X^{(2)}\|}(\delta X^{(1)})$ applied to the detail field.

To solve for this constrained minimization, we consider an iterative procedure. From given initialization $X^{(1),0}$ and $X^{(2),0}$, it comes to iterate the non-local projections of the current solutions $X^{(1),n}$ and $X^{(2),n}$ and the projection of the resulting fields $\mathcal{P}_{NLM, X^{(2)}}(X^{(1),k})$ and $\mathcal{P}_{NLM, X^{(1)}}(X^{(2),k})$ onto the four additional constraints (16). Numerically, this approach involves three key components:

- The projection onto interpolation and low-resolution constraints (first-two constraints in (16)): we solve for this projection using a gradient-based descent to retrieve the field accounting for the interpolation constraint while minimizing the quadratic norm to the expected low-resolution field (see Algorithm (1) in Annex for details);
- The projection onto the constraints on the spectral and marginal constraints. Based on transforms (14) and (11), we use a gradient-based descent to jointly solve for these two constraints. Each iteration of this gradient-based descent involves a projection onto interpolation and low-resolution constraints (see Algorithm (2) in Annex for details);
- The overall algorithm iterates three main steps: i) non-local projections of fields $X^{(1)}$ and $X^{(2)}$ followed by a projection onto interpolation and low-resolution constraints, ii) projections of resulting field updates onto the marginal and spectral constraints, iii) the update of the reference low-resolution field as the mean of the low-resolution projection of the updated fields $X^{(1)}$ and $X^{(2)}$ as well as the update of reference generalised Gaussian distribution $\mathcal{GG}_{\sigma^*, \alpha^*}$ for the high-resolution details of field $X^{(1)}$ (cf. (13)). The procedure is described in the annex by Algorithm (1).

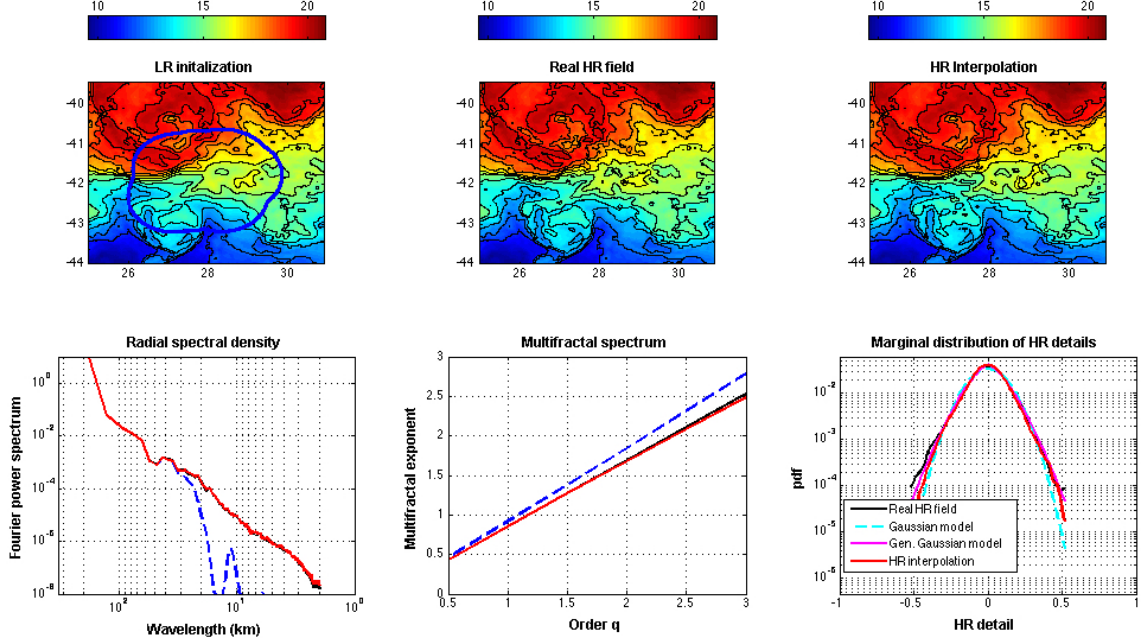


Fig. 1. Interpolation results for a large simulated missing data area: first row, low-resolution SST field (subsampling factor of 16 from a 8-Daubechies wavelet of the ODYSSEA SST field) with superimposed the contour of the missing data area in blue (left), high-resolution ODYSSEA SST field (resolution grid of 2km) (center), interpolated field using the proposed model (right); second row, radial spectral densities of the low-resolution (blue, -), real high-resolution (black,-) and interpolated high-resolution (red, -) fields (left), multifractal spectrum of the low-resolution (blue, -), real (black,-) and interpolated high-resolution fields (center), distribution of high-resolution detail $\delta\mathcal{P}_W^{sLR}(X^{(2)})$ for the real real (black,-) and interpolated (red, -) fields along with the target generalized Gaussian distribution (magenta, -) and a Gaussian distribution with the same variance (cyan,-). We superimpose image level-lines to illustrate the underlying spatial geometry.

The initialization of fields $X^{(1)}$ and $X^{(2)}$ from partial observations $Y^{(1)}$ and $Y^{(2)}$ may exploit classical interpolation techniques. In the reported experiments, we consider a initial variational interpolation using the AMLE (Absolutely Minimizing Lipschitz Extension) [8]. An additional random component is added in missing data area $\Omega^{(2)}$ for high-resolution field $X^{(2)}$. We use a white Gaussian noise, such that initial field $X^{(2)}$ involves the expected variance for high-resolution detail $X^{(2)} - \mathcal{P}_W^{sLR}(X^{(2)})$.

IV. RESULTS

We report numerical experiments to evaluate the proposed framework two three types of data: i) high-resolution SST data from ODYSSEA analysis [2] for which we simulate missing data and a low-resolution condition, ii) real AMSR-E and SEVIRI daily SST data.

A. ODYSSEA SST data

ODYSSEA SST fields are gridded and interpolated composite fields produced by Ifremer/CERSAT³[2] onto a grid with a 0.02° spatial resolution (~ 2 km). ODYSSEA analysis combines both microwave sensor data (including TMI

³www.ifremer.fr/cersat

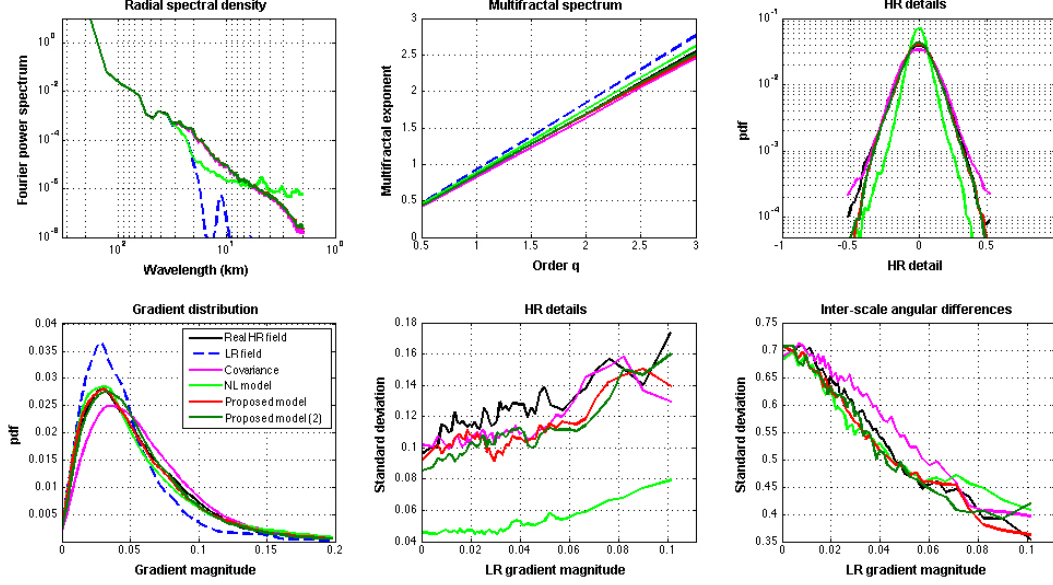


Fig. 2. Statistical analysis of interpolation results for the synthetic case considered in Fig.1: first row, radial spectral density (left), multifractal spectrum (center), distribution of high-resolution detail $\delta\mathcal{P}_W^{sLR}(X^{(2)})$ (right); second row, distribution of the norm of the gradient $\|\nabla X\|$ (left), standard deviation of high-resolution detail $\delta\mathcal{P}_W^{sLR}(X^{(2)})$ as a function of the low-resolution gradient $\|\nabla X^{(1)}\|$ (center), standard deviation of the angular difference between gradient fields $\nabla X^{(1)}$ and $\nabla X^{(2)}$ as a function of the low-resolution gradient $\|\nabla X^{(1)}\|$ (right). For each subplot, we report the features associated with the real high-resolution field (black,-), the low-resolution field (blue,-), and four different interpolation models, namely using only the covariance and marginal constraints (magenta,-), using only the non-local projection (light green,-), using the proposed model (red,-), using the proposed model with the non-local projection applied to detail field $\delta\mathcal{P}_W^{sLR}(X^{(2)})$ rather than field $X^{(2)}$ (dark green,-).

and AMSR-E data) and infrared sensor data (including AVHRR, VISSR and SEVIRI data). It involves different processing steps, especially sensor-specific bias correction, sensor inter-calibration and optimal interpolation of missing data areas. The selected field (January, 8th 2012) and case study area, off South Africa, are chosen for the quality of the composite field and the richness of the observed SST structures (e.g., filaments, fronts).

From the considered ODYSSEA SST field, we first evaluate the relevance of the proposed model and algorithm for the interpolation of a high-resolution field given a known low-resolution condition, *i.e.* observation $Y^{(1)}$ does not involve missing data and $X^{(1)} = Y^{(1)}$. We proceed as follows. For a given dyadic downsampling factor κ , typically 16 (*i.e.*, $r^{(1)} = 2km$ and $r^{(1)} = 32km$ and $s^{LR} = 4$), we derive the low-resolution condition $X^{(1)}$ as $\mathcal{P}_W^{sLR}(X^{(2)})$ the low-resolution projection of the considered ODYSSEA field with a Daubechies-8 wavelet transform at scale $s_{LR} = \log_2(\kappa)$. Regarding high-resolution observation $Y^{(2)}$, we generate randomly missing data. With a view to evaluating the performance of the model and the relative importance of the non-local and statistical priors, we consider two types of random missing data patterns: a large missing data area and small but numerous missing data areas.

We report in Fig.1 the interpolation results for a subsampling factor of 16 between the low-resolution and high-resolution fields and a large missing area corresponding to the center of the image and accounting for about

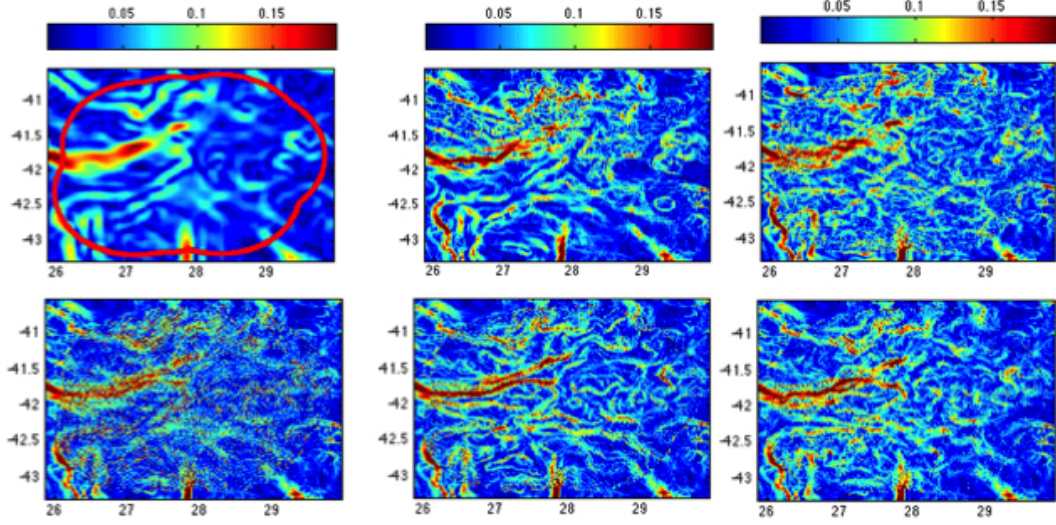


Fig. 3. Gradient fields of the interpolation compared in Fig.2: from left to right and top to bottom, low-resolution field with superimposed the contour of the missing data area, reference high-resolution field and four different interpolation models, namely using only the covariance and marginal constraints, using only the non-local projection, using the proposed model, using the proposed model with the non-local projection applied to detail field $\delta\mathcal{P}_W^{sLR}(X^{(2)})$ rather than field $X^{(2)}$.

20% of the overall grid. These interpolation results are issued from the non-local projection of field $X^{(2)}$ given low-resolution field $X^{(1)}$. As reference power spectral density, we consider the radial spectral density of the high-resolution field. We select 5000 11×11 patches off the missing data area. The non-local means is computed for the first five neighbors. The scale parameter of the Gaussian kernel is set to 1.

As illustrated, we emulate a high-resolution field which is visually consistent in terms of perceived high-resolution details and geometrical patterns. To quantitatively analyze the relevance of the proposed interpolation, we evaluate six different features of the interpolated field with respect to the reference field as illustrated in Fig.2: the radial spectral density, the multifractal spectrum, the marginal distribution of high-resolution details $\mathcal{P}_W^{sLR}(X^{(2)})$, the distribution of the magnitude of gradient $\nabla X^{(2)}$, the standard deviation of high-resolution detail $\mathcal{P}_W^{sLR}(X^{(2)})$ as a function of the norm of low-resolution gradient $\nabla X^{(1)}$ and the standard deviation of the angular differences between low-resolution and high-resolution gradients as a function of the norm of low-resolution gradient $\nabla X^{(1)}$. These different features are selected to analyse the extent to which we recover the inter-scale characteristics of the reference field. These results clearly stress the relevance of the combination of a non-local prior with covariance and marginal priors. The sole use of a non-local prior leads to high-resolution field which depicts too much fine-scale energy. The combination with the covariance-based prior allows us to further regularize the non-local projections and to match the expected spectral properties. It also leads to a better fit in terms of gradient statistics and inter-scale angular statistics, as the non-local prior exploits an actual conditioning of the high-resolution detail from the low-resolution field. This is further illustrated by the visual inspection of the gradient field of the different interpolation results (Fig.3). Little difference is observed between the proposed model whether we apply the non-local projection

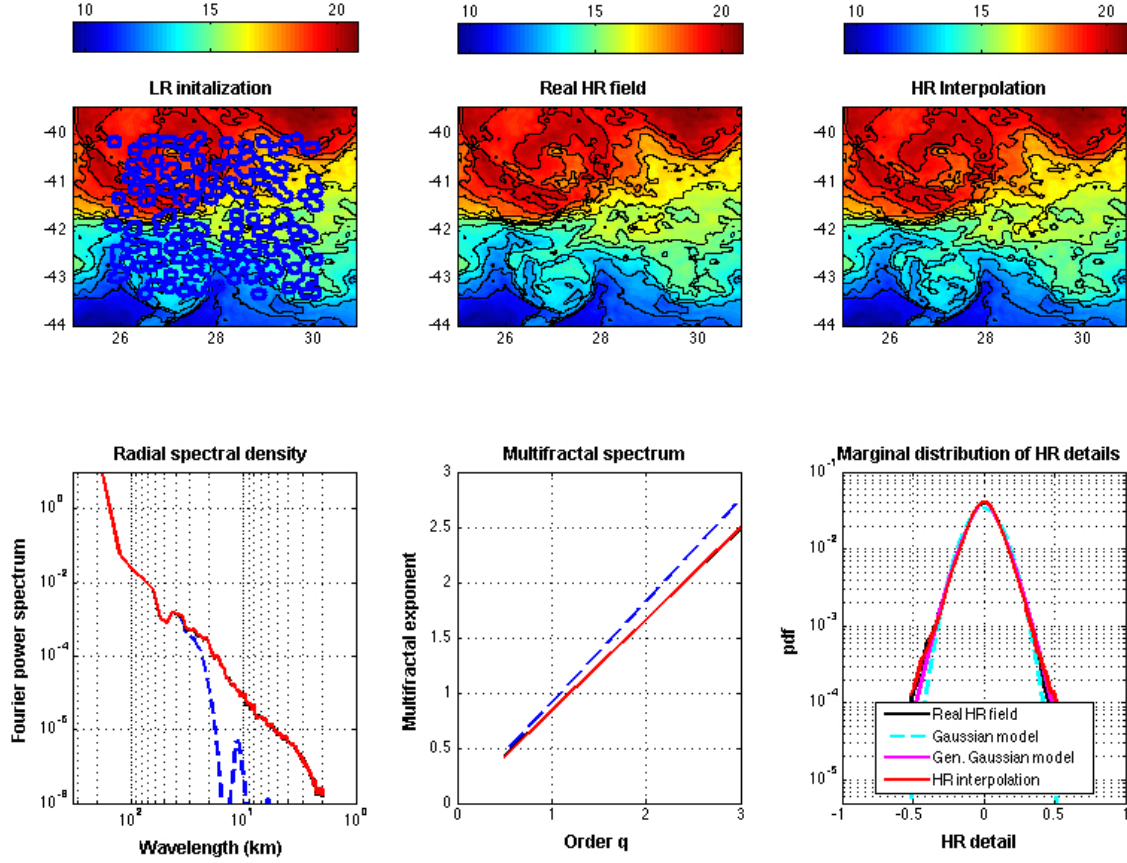


Fig. 4. Interpolation results for small randomly sampled missing areas: first row, low-resolution SST field (subsampling factor of 16 from a 8-Daubechies wavelet of the ODYSSEA SST field) with superimposed the contour of the missing data area (left), high-resolution ODYSSEA SST field (center), interpolated field using the proposed model (right); second row, radial spectral densities of the low-resolution (blue, -), real (black,-) and interpolated (red, -) high-resolution field (left), multifractal spectrum of the low-resolution (blue, -), real (black,-) and interpolated high-resolution fields (center), distribution of the high-resolution details $\delta \mathcal{P}_W^{sLR}(X^{(2)})$ for the real real (black,-) and interpolated (red, -) fields along with the target generalized Gaussian distribution (magenta, -) and a Gaussian distribution with the same variance (cyan,-). We superimpose image level-lines to illustrate the underlying spatial geometry.

to high-resolution field $X^{(2)}$ or to high-resolution detail field $\mathcal{P}_W^{sLR}(X^{(2)})$.

We perform a second synthetic experiments for the same high-resolution field but a different missing data pattern (Fig.IV-B). For a similar missing data rate as in Fig.1, we simulate many small missing data area. The interpolation for the proposed model is reported in Fig.IV-B. Overall, the same conclusions hold for this experiment with better performance of the combination of the non-local and statistical priors. It might be noted that in that case the sole use of the non-local prior can actually retrieve a good match with the expected marginal distribution of the high-resolution details, but still involve too much fine-scale energy compared to the expected spectral pattern.

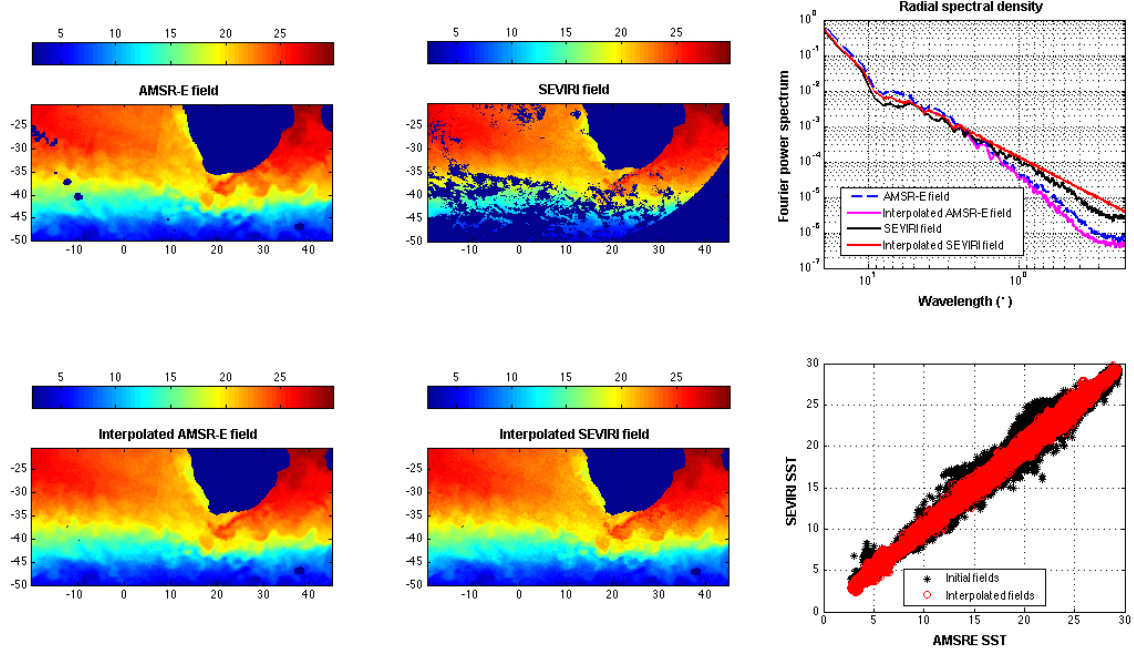


Fig. 5. Reconstruction of a daily SST field from a joint analysis of SEVIRI and AMSRE data: first row, AMSR-E field with few missing data (left), SEVIRI field (center), radial power spectral density of the different fields (right); second row, interpolated AMSR-E field (left), interpolated SEVIRI field (center), scatterplot of AMSR-E and SEVIRI data for the original (black, *) and interpolated (red, o) fields (right).

B. Joint analysis of AMSR-E and SEVIRI SST data

We consider an application of the proposed framework to the joint analysis of AMSR-E and SEVIRI SST fields. AMSR-E SST field involves a typical resolution grid of 0.2° , whereas we consider daily composite SEVIRI field with a 0.1° grid resolution. From Fig.5, one can notice that their radial power spectral density differ from about 1° , with a significantly lower spectral energy of the AMSR-E field. This difference relates to the interpolation and filtering applied to AMSR-E fields. The situation reported in Fig.5 depicts a low missing data rate for the AMSR-E field, but a much higher missing data rate, especially off South Africa for the SEVIRI field. Besides, due to the associated geostationary orbit, no SEVIRI data is available, whatever the atmospheric conditions, for the extreme south-east area of the case-study region. In a similar way, contrary to SEVIRI data, AMSR-E data do not provide any information close to the sea shore. Regarding the parameterization of the proposed model, we proceed as follows. For the reference radial power spectral density, we consider a power law fitted for scales $[0.8^\circ, 3^\circ]$ and extrapolated to the finer scales. Here, in Algorithm (V), the spectral constraint is applied globally and not only in the missing data area with a view to filtering noisy patterns observed in the daily SEVIRI composite. As patch dataset, we randomly sample joint patches in the observed AMSR-E and SEVIRI fields for the considered data and the two preceding days. Other parameters are set similarly to the experiments on ODYSSEA SST data (Fig.).

We report in Fig.5 the interpolation results issued from the proposed model with a non-local projection applied to detail field $\delta\mathcal{P}_W^{sLR}(X^{(2)})$. Though the AMSR-E field involve few missing data, we benefit from the greater

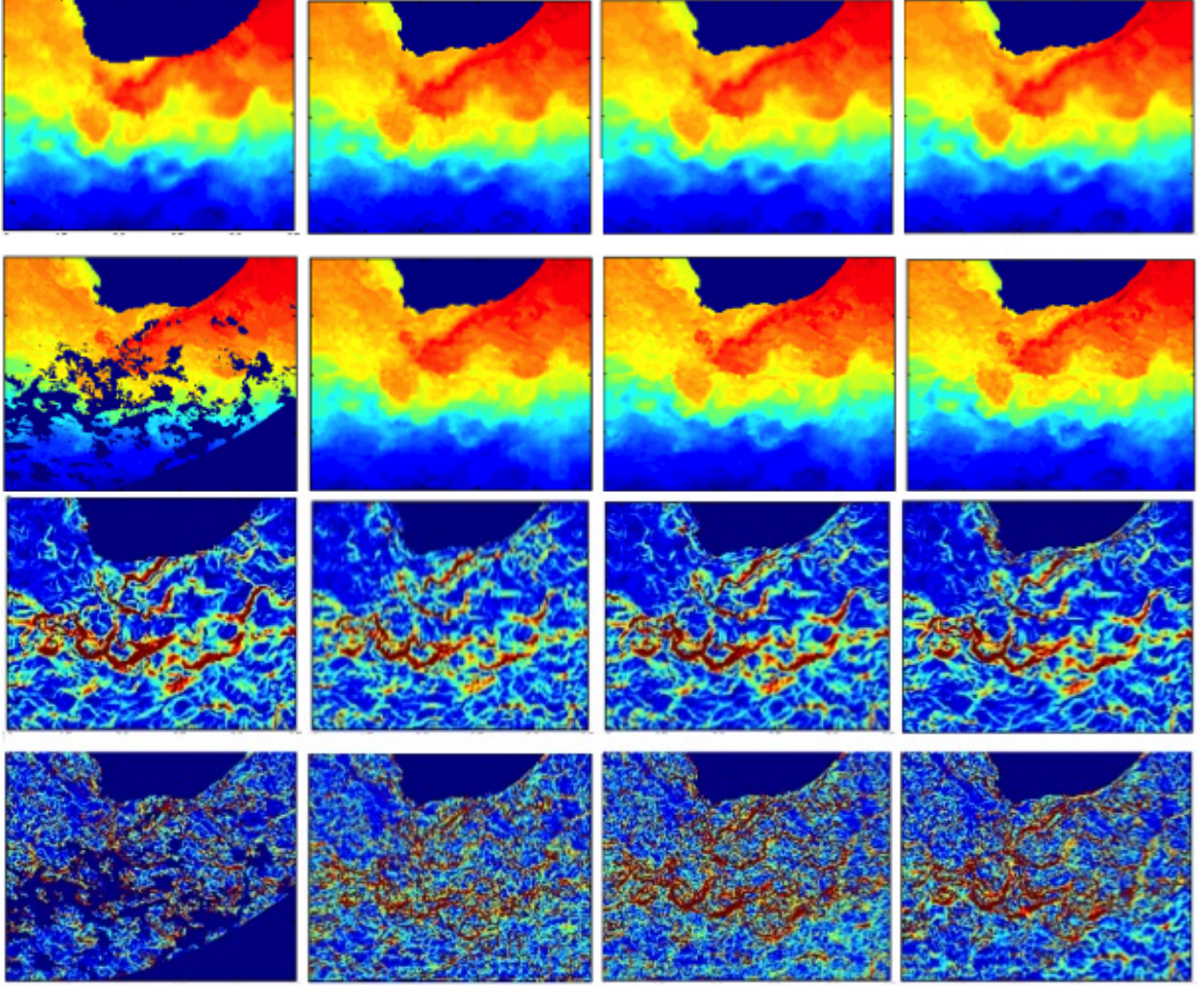


Fig. 6. Zoom of joint AMSR-E/SEVIRI interpolation results for the situation considered in Fig.5: first row, observed and interpolated AMSR-E fields; second row, observed and interpolated SEVIRI fields; third row, gradient of the AMSR-E fields, fourth row, gradient of the SEVIRI fields. For each row, we depict the observed data, the interpolation results using the proposed model with a non-local projection applied to field $X^{(2)}$, the interpolation results using the proposed model with a non-local projection applied to detail field $\delta \mathcal{P}_W^{sLR}(X^{(2)})$, the interpolation results using only the statistical priors.

resolution of the SEVIRI field close to the shore to interpolate the AMSR-E field in these areas. This is particularly noticeable off South Africa for the Agulhas current, characterized by warm waters going southward along the eastern coast of South Africa (see also, the first row of Fig.6). It may also be outlined that the constraint of a common low-resolution component between the interpolated AMSR-E and SEVIRI fields result in more coherent scatterplots of the low- vs high-resolution data (Fig.5, bottom right). From the visual inspection of the zoom onto an area off South Africa, involving an important missing data rate for the SEVIRI field (Fig.6), the gradient fields clearly stress the greater relevance of the combination of non-local and statistical priors in the south-east of the considered area. Besides, it may be noted that the application of the non-local projection to field $X^{(2)}$ produces

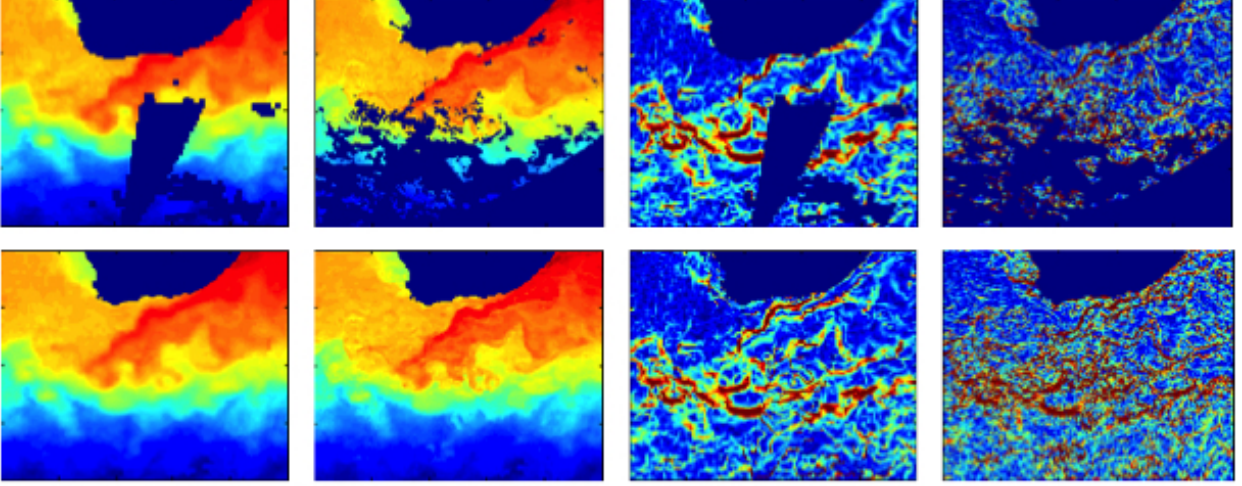


Fig. 7. Joint AMSR-E/SEVIRI interpolation results for a second situation involving large missing data areas for both sensors: first row, observed fields, second row, interpolated fields. For each row, we depict from left to right, the AMSR-E field, the SEVIRI field, the gradient of the AMSR-E field and the gradient of the SEVIRI field. We report a zoom on the same area as in Fig.6 off South Africa, which involves the largest missing data rate. The reported interpolation results are issued from the proposed model with a non-local projection applied to detail field $\delta\mathcal{P}_W^{sLR}(X^{(2)})$.

unexpected tiling effects. This is due to a very low number of cold patches in the reference dataset. By contrast, applying the non-local projection to field $\delta\mathcal{P}_W^{sLR}(X^{(2)})$ leads to much more satisfactory interpolation results, as we are provided with more representative patch datasets for the detail field and a conditioning by the low-resolution gradient. The gradient fields also clearly stress the difference between the information carried out by AMSR-E and SEVIRI fields in terms of presence and location of frontal structures, with a much coarser resolution of the AMSR-E fields.

We report a second example of joint AMSR-E/SEVIRI interpolations for a date at which AMSR-E field also involves a large missing data area off South Africa (Fig.7). Following the conclusions drawn from the reported experiments, we only depict the zoom on the region off South Africa and the interpolation results for the proposed model with a non-local projection applied to detail field $\delta\mathcal{P}_W^{sLR}(X^{(2)})$. This example shows that we can interpolate visually-consistent fields both at AMSR-E and SEVIRI resolutions and jointly benefit from the two types of data to jointly improve the interpolation of the low-resolution and high-resolution fields.

V. DISCUSSION

In this work, we have presented a novel model for the joint interpolation and blending of multi-source and multi-resolution remote sensing images. As case-study, we have focused on sea surface geophysical fields, and more particularly sea surface temperature. The key contributions are two-fold: the combination of a non-local exemplar-based approach and of statistical priors. The later is particularly motivated by the fact that sea surface fields, as most geophysical fields, are expected to depict specific fine-scale patterns associated with turbulent dynamics. To

our knowledge, this is the first work which addresses such combination between patch-based models and statistical priors for multi-resolution interpolation issues.

Reported results for both synthetic and real data demonstrate the relevance of this model compared to the sole use of non-local or statistical priors. Regarding the later, the prior on the spatial covariance of the high-resolution field is of greater importance than the prior on the marginal of the high-resolution detail. We have also evaluated that the non-local projection applied to the detail field, rather than to the high-resolution field, offers a greater robustness with respect to possibly weakly represented high-resolution patches (in our case, patches corresponding to cold waters). The representativeness of the reference patch dataset is obviously a key issue for the effectiveness of the non-local prior. In this respect, we considered reference dataset with 5000 patches. Larger datasets may be considered at the expense of a greater computational cost. One may also explore spatialized version of the patch dataset, where patch similarities account not only for visual similarities but also for space and/or time similarities. Regarding the application to ocean remote sensing data, the classical tools mainly rely on covariance-based interpolations. By nature, such interpolations involve some smoothing of the observed data with respect to the considered covariance structure. This covariance structure models some fine-scale noise to be filtered out. The resulting interpolated fields resort to some trade-off between the fidelity to the observed data and the interpolation of the missing data areas. By contrast, we adopt here a different strategy. We aimed at keeping all the relevant observations, while filling the missing data areas with visually-consistent and geophysically-sound field patterns. Future work will further explore and evaluate this intrinsic difference with the classical geostatistical tools. In particular, we may consider interpolation ensembles from multiple random initializations. This would provide a mean to evaluate a mean field with remnants (deterministic) structures and some associated variabilities.

From a methodological point of view, our future work will also explore several extensions. Regarding the spectral prior, we only consider here radial (isotropic) covariance structure. The extension to parametric and non-parametric two-dimensional covariance structure could be explored. The same holds for other parametric models of the marginal distribution of the high-resolution detail. The extension to spatio-temporal fields will also be of key interest, including exemplar-based data assimilation issues [32]. Such extension might require manifold learning and kernel learning strategies to define patch similarity measure adapted to the considered space-time processes [34].

Regarding the application to ocean remote sensing data, the extension of the proposed model to the joint interpolation of three or more data sources should be investigated. For instance, for SST data, operational ODYSSEA product typically aims at merging the daily data issued from at least 6 different sensors [2]. Such developments should involve particular methodological issues and could benefit from a classical coarse-to-fine implementation strategy to decompose this multi-sensor/multi-scale fusion as an iteration of two-scale fusions. Beyond the joint processing of multi-sensor data for same geophysical fields (here, SST), one could also explore the relevance of the proposed model for the joint analysis of different geophysical fields for which a mutual conditioning might be expected, but difficult to explicit model. The joint interpolation of SST and ocean colour data may provide a typical example, where could expect to benefit from multi-sensor SST data to constrain the interpolation of ocean colour data, which often involve very large missing data rate in temperate areas.

ACKNOWLEDGEMENTS

This work was supported by ANR (Agence Nationale de la Recherche, grant ANR-13-MONU-0014). We are grateful to E. Autret, B. Chapron, J.F. Piolle from Ifremer/CERSAT for providing the SST data.

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ANNEX

We provide in this annex the detailed iterative algorithm used to solve for constrained minimization (15)). More precisely, we describe in the subsequent:

- Algorithm (1): the algorithm of function *projection_Interp_LR* which solves for the projection onto the interpolation and low-resolution constraints (first two constraints in (16));
- Algorithm (2): the algorithm of function *projection_PSD_PDF* used to solve for the projection onto the spectral and marginal constraints (last two constraints in (16));
- Algorithm (3): the general iterative algorithm of the proposed approach for the joint interpolation and blending of two multi-sensor fields

Algorithm 1 Description of the function which solves for the projection of field X onto interpolation constraint $X(\Omega) = Y(\Omega)$ and low-resolution constraint $\mathcal{P}_W^s(X) = X_{LR}$ (the first two constraints in (16)). It uses as input parameter high-resolution field X , low-resolution field X_{LR} , missing data region Ω^* , scale s , coefficient λ and precision ϵ .

function PROJECTION_INTERP_LR($X, X_{LR}, \Omega, s, \lambda, \epsilon$)

$diff \leftarrow \epsilon + 1$

$X^0 \leftarrow X$

$n \leftarrow 0$

while $diff > \epsilon$ **do**

$n \leftarrow n + 1$

$dX \leftarrow \mathcal{P}_W^s(X^{n-1}) - \mathcal{P}_W^s(X_{LR})$

$X^{n+1} \leftarrow X^n$

$X^{n+1}(\Omega^*) \leftarrow X^n(\Omega^*) - \lambda \cdot dX(\Omega^*)$

$diff \leftarrow \|X^{(n+1)} - X^{(n)}\|_{\Omega^*}^2$

end while

return X^{n+1}

end function

Algorithm 2 Description of the function which solves for the projection of field $X^{(1)}$ and $X^{(2)}$ onto marginal and spectral constraints (the last two constraints in in (16). It uses as inputs fields $X^{(1)}$ and $X^{(2)}$, missing data regions $\Omega^{(1)}$ and $\Omega^{(2)}$, marginal distribution $\mathcal{GG}_{\sigma^*, \alpha^*}$, radial power spectral density R^* , scale s , coefficients λ_I , λ_M and λ_S and precision ϵ .

```

function PROJECTION_PDF_PSD( $X^{(1)}, X^{(2)}, \Omega^{(1)}, \Omega^{(2)}, \mathcal{GG}_{\sigma^*, \alpha^*}, R^*, s, \lambda_M, \lambda_S, \epsilon$ )
     $diff \leftarrow \epsilon + 1$ 
     $X^{(1),0} \leftarrow X^{(1)}$ 
     $X^{(2),0} \leftarrow X^{(2)}$ 
     $X^{LR} \leftarrow \mathcal{P}_W^s(X^{(2)})$  ▷ Common low-resolution component at scale  $s$ , cf. (3)
     $n \leftarrow 0$ 
    while  $diff > \epsilon$  do
         $n \leftarrow n + 1$ 
         $X^{(1),n+1} \leftarrow X^{(1),n}$ 
         $X^{(2),n+1} \leftarrow X^{(2),n}$ 

        %% Marginal constraint (13)
         $dX \leftarrow X^{(2),n} - X^{LR}$ 
         $dX_{New} \leftarrow \mathcal{T}_{\mathcal{GG}^*}(dX)$  ▷ Transform associated with marginal constraint (13), cf. (14)
         $X^{(2),n+1}(\Omega^{(2)}) \leftarrow X^{(2),n+1}(\Omega^{(2)}) - \lambda_M \cdot (dX(\Omega^{(1)}) - dX_{New}(\Omega^{(2)}))$  ▷ Gradient-based update of  $X^{(2)}$ 
         $X^{(2),n+1} \leftarrow \text{PROJECTION\_INTERP\_LR}(X^{(2),n+1}, X^{LR}, \Omega^{(2)}, s, \lambda_I, \epsilon)$ 

        %% Spectral constraint (13)
         $X^{(2),n+1}(\Omega^{(2)}) \leftarrow X^{(2),n+1}(\Omega^{(2)}) - \lambda_S \cdot (X^{(2),n+1}(\Omega^{(2)}) - \mathcal{T}_{R^*}(X^{(2),n+1})(\Omega^{(2)}))$  ▷ Gradient-based update of  $X^{(2)}$ 

        %% Projection onto interpolation and low-resolution constraints (first two constraints in (16))
         $X^{(1),n+1} \leftarrow \text{PROJECTION\_INTERP\_LR}(X^{(1),n+1}, X^{LR}, \Omega^{(1)}, s, \lambda_I, \epsilon)$ 
         $X^{(2),n+1} \leftarrow \text{PROJECTION\_INTERP\_LR}(X^{(2),n+1}, X^{LR}, \Omega^{(2)}, s, \lambda_I, \epsilon)$ 
         $diff \leftarrow \|X^{(1),n} - X^{(1),n+1}\|_{\Omega^{(1)}}^2 + \|X^{(2),n} - X^{(2),n+1}\|_{\Omega^{(2)}}^2$ 
    end while
    return  $X^{(1),n+1}$  and  $X^{(2),n+1}$ 
end function

```

Algorithm 3 Description of the algorithm for the proposed joint interpolation and blending of two fields $X^{(1)}$ and $X^{(2)}$ from constrained minimization (15). It uses as inputs fields initial fields $X^{(1),0}$ and $X^{(2),0}$, missing data regions $\Omega^{(1)}$ and $\Omega^{(2)}$, marginal distribution $\mathcal{GG}_{\sigma^*, \alpha^*}$, radial power spectral density R^* , scale s , coefficients λ_I , λ_M and λ_S , precision ϵ and iteration number N .

```

procedure JOINT_INTERPOLATION( $X^{(1)}, X^{(2)}, \Omega^{(1)}, \Omega^{(2)}, \mathcal{GG}_{\sigma^*, \alpha^*}, R^*, s, \lambda_I, \lambda_M, \lambda_S, \epsilon, N$ )
  for  $n = 0$  to  $N - 1$  do
    %% Non-local projection for  $X^{(1)}$  followed by the projection onto interpolation and low-resolution constraints, cf.(16)
     $X^{(1), n+1} \leftarrow \mathcal{P}_{NLM, X^{(2)}, n}(X^{(1), n})$  ▷ Non-local projection for field  $X^{(1)}$ , cf. (6)
     $X^{(1), n+1} \leftarrow \text{PROJECTION\_INTERP\_LR}(X^{(1), n+1}, X^{LR}, \Omega^{(1)}, s, \lambda_I, \epsilon)$ 

    %% Non-local projection for  $X^{(2)}$  followed by the projection onto interpolation and low-resolution constraints, cf.(16)
     $X^{(2), n+1} \leftarrow \mathcal{P}_{NLM, X^{(1)}}(X^{(2), n})$  ▷ Non-local projection for field  $X^{(2)}$ , cf. (6)
     $X^{(2), n+1} \leftarrow \text{PROJECTION\_INTERP\_LR}(X^{(2), n+1}, X^{LR}, \Omega^{(2)}, s, \lambda_I, \epsilon)$ 

    %% Projection onto marginal and spectral constraints, cf. (16)
     $X^{(2), n+1}, X^{(2), n+1} \leftarrow \text{PROJECTION\_PDF\_PSD}(X^{(1)}, X^{(2)}, \Omega^{(1)}, \Omega^{(2)}, \mathcal{GG}_{\sigma^*, \alpha^*}, R^*, s, \lambda_I, \lambda_M, \lambda_S, \epsilon)$ 

    %% Update of the common low-resolution field and resulting high-resolution fields
     $X^{(1,2), n} \leftarrow \mathcal{P}_W^{sLR}((X^{(1), n+1} + X^{(2), n+1})/2)$  ▷ Mean low-resolution field
     $X^{(1), n+1} \leftarrow X^{(1), n+1} - \mathcal{P}_W^{sLR}(X^{(1), n+1}) + X^{(1,2), n}$  ▷ Update of field  $X^{(1)}$  given the new low-resolution field
     $X^{(2), n+1} \leftarrow X^{(2), n+1} - \mathcal{P}_W^{sLR}(X^{(2), n+1}) + X^{(1,2), n}$  ▷ Update of field  $X^{(2)}$  given the new low-resolution field
     $\mathcal{GG}_{\sigma^*, \alpha^*} \leftarrow \text{ML estimate of the generalized Gaussian distribution of } Y^{(1)} - X^{(1,2), n+1}(\Omega^{(1)})$ 
  end for
end procedure

```
