Probabilistic simulation for the certification of railway vehicles

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Abstract

The present dynamic certification process, built thanks to experts' experience is essentially based on experiments. The introduction of the simulation in this process would be of great interest. However an accurate simulation of complex, non-linear systems is complicated, in particular when rare events (unstable behaviour for example) are considered. After having analysed the system and the richness of the present procedure, this paper proposes a method to achieve, in some particular cases, a numerical certification. It focuses on the need for precise and representative excitations (running conditions) and on their variable nature. A probabilistic approach is therefore proposed and illustrated by an example.

First the paper presents a short description of the vehicle / track system and of the experimental procedure. The proposed numerical process is then described. The necessity of analysing a set of running conditions at least as large as the one tested experimentally is moreover explained. In the third section a sensitivity analysis of the system is reported, to determine the most influential parameters. Finally the proposed method is summarized and an application is given.

Keywords

Certification, railway dynamic simulation, complex, non-linear, variable system, uncertainty propagation.

1 INTRODUCTION

Thanks to the improvement of models and computing power, simulations are increasingly used in many industrial fields. They are indeed more and more representative of the observed physical behaviour and can thus replace or complete the experiments analysing a wider range of running conditions, especially around critical situations that cannot be tested experimentally.

Simulations have, for example, been used for a long time in certification processes of offshore fields and nuclear plants. Usually experiments are achieved on subsystems and simulation is used on the whole system to estimate the behaviour of the structure under nominal loading as well as under extreme loadings. For nuclear plants a probabilistic approach is required in order to represent the uncertainty and variability of different input parameters (earthquake loading for example) but also to prove that the probability of encountering nuclear meltdown is under a given threshold.

In railways, the certification is essentially based on experiments, however the expected benefits of the numerical methods are multiple. The aim of this article is thus to propose adapted methods and processes for a computer-aided certification. It will reduce the number of physical tests and the influence of uncontrolled conditions.

To be relevant, the virtual certification process has to allow a representation of the dynamic response of the system at least as precise as the one given by the measurement. One of the main difficulties is thus to build a representative set of excitation to achieve the simulations. Indeed, as the system is non-linear, a poor representation of the inputs can lead to important errors on the output. These probabilistic considerations are taken into account in the experimental certification process defined in the EN14363: the certification criteria are computed on several portions of track of different track designs and with different track qualities. The measurements are then statistically processed to estimate a maximum as explained section 2.2.

The paper first explains the main characteristics of the vehicle / track system and briefly describes the experimental procedure. The proposed numerical process is then presented. In the third section a sensitivity analysis of the system is reported. This highlights the modelling parameters that play an important role on the certification criteria. Finally the proposed method is summarized and an application is given.

2 DESCRIPTION OF THE RAILWAY / VEHICLE SYSTEM AND OF THE CURRENT CERTIFICATION PROCESS

2.1 Mechanical characteristics of the system: non-linear system containing variability sources

The vehicle / track system is a highly coupled system. Furthermore, frequency analysis of the dynamic response of vehicles shows that the modal contents depend on the velocity and on the amplitude of the track irregularities [1,12]. This dependency underlines the non-linearity of the wheel/rail contact, of the behaviour of some suspensions and also of the sub-structure.

In addition, during the running of a train, some excitations are variable by nature. This is easily demonstrated when comparing the response of a train running twice, the same day, on the same track. One can mention:

- wind gusts or passing trains that can significantly modify the behaviour of the train,
- track irregularities and stiffness,
- rail profile and the friction coefficient which can be very different from one place to another, and can evolve, during the day or during longer periods,
- the velocity of the train that is never exactly constant, and that causes longitudinal excitation of the train.

The mechanical characteristics can also be different among a fleet of vehicles of the same type, because of the number of passengers or of the mass of the goods, because of process uncertainties (especially for elastomers) and also because the components are stemming from different suppliers and are built with different processes. Damage and wear moreover add variability during the life-cycle of the vehicle so that the scatter of behaviour of nominally identical components can vary significantly.

Some of these sources of variability can be characterized by measurement (track irregularities for example), but it is more difficult to have access to others (friction coefficient for example).

2.2 Description of the present experimental certification procedure

In order to guarantee safety and comfort, and to avoid infrastructure damage, new rolling stock, is certified for a specified network through on-track tests. The measured vehicle reactions (accelerations of different bodies and wheel / rail forces) are thus representative of the behaviour of the new train on the considered network since the running conditions are those the train will face. These on-track tests are prescribed by the European leaflet EN14363 [2].

According to this procedure, the train has to run at different speeds on several types of sections having different track radii, cant deficiency, track geometry quality in terms of alignment and longitudinal levels, rail profiles. These portions are then sorted in four zones (depending on the magnitude of the track radius) each one containing at least 25 sections.

In each section, the 99.85 percentile of the filtered vehicle reactions (Y/Q ratio of the lateral and vertical loads in the contact, car-body lateral and vertical acceleration, sum of Y on a wheelset) is then estimated assuming a normal distribution. From these percentiles, each corresponding to a different section, is then derived an "estimated maximum" in each zone, computing on these values the 99.85 percentile. This estimated maximum is finally compared to a limit value prescribed by the EN. As an example, Figure 1 shows the Y/Q ratio on 5 sections and the associated percentiles. Figure 2 shows a bar plot of 99.85 percentile on 47 sections. The horizontal blue line is the percentile of the 47 values, and the red line, the limit value prescribed by EN14363 (2005). In this case, the "estimated maximum" 0.79 is very near the limit of 0.8, as the vehicle is a freight wagon in an empty configuration, which is a configuration prone to derailment.

This procedure naturally takes into account the variability of the system described in section 2.1 and achieves a probabilistic post-treatment.

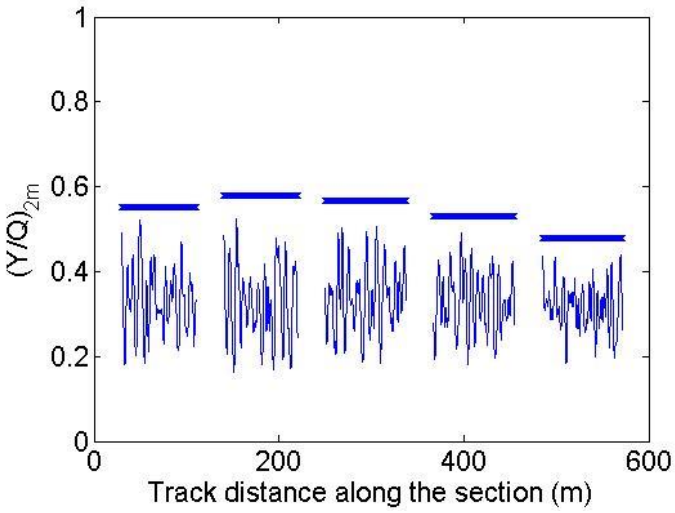

Fig. 1: Histogram of 2m-filtered Y/Q ratio on 6 sections and 99.85 % percentile

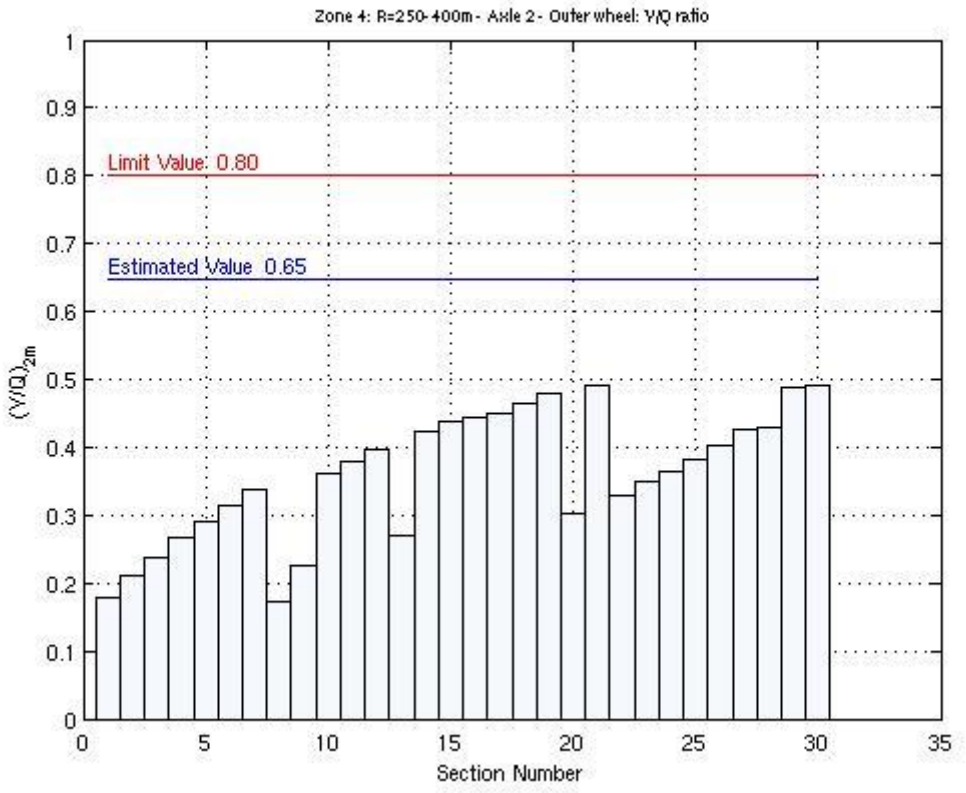

Fig. 2: Percentiles on each section of Y/Q ratio; estimated value based on 30 sections and limit value

3 DESCRIPTION OF THE PROPOSED NUMERICAL STRATEGY

3.1 Proposed simulation strategy: validation of the robustness of the modelling and probabilistic simulation

The non-linearity of the system and the presence of variability sources make both the validation of the model and the representation of the physical behaviour of the system with simulation difficult [13]. Indeed it is necessary to evaluate the modelling on representative running conditions, that is to say to compare simulations to on-track measurements [9], [19]. However, as explained in section 2.1, the running conditions are not exactly known because of different variability sources, making the deterministic comparison difficult. Indeed, the unknown parameters have to be chosen to achieve the simulations, these may be different from the on-line parameters. The simulations results will thus be different from the measurements even if the model is precise. [22] proposes alternative methods.

The validated model of the system can then be changed to evaluate the behaviour of the studied train when running in “other conditions” (other network) or to evaluate the behaviour of a “slightly modified vehicle”. **The new**

system can however **not be too different** from the original one, otherwise the validity of the model cannot be ensured anymore. To verify that the systems are not too different, we propose to introduce the following robustness criterion: if the mean or extreme behaviour of the two systems, usually considered for the certification, are too different, then further experimental test to check the modelling should be achieved. Based on a few studied cases, limit values for these differences on mean and extreme values have been proposed in the project. Further studies should however been achieved to specify more relevant limits.

The representation of the dynamic behaviour of the system moreover requires a study of the system when it is submitted to a **large set of representative (on-track) running conditions as** it is done in the certification procedure. This is the case even if we are only interested in the mean behaviour of the train: this one is indeed not obtained simply by an estimate at the nominal or mean input parameters.

In order to have a good description of the running dynamics of the system thanks to simulation, it seems important to introduce variability in the modelling. In fact moving from an experimental test to a deterministic simulation would indeed lead to an impoverished knowledge of the vehicle behaviour: variability naturally introduced during tests would not be considered. We therefore propose to introduce **probabilistic simulations** to better reproduce the experimental behaviour of the system.

Probabilistic simulations have been used for some decades in different industries, especially in nuclear field. The classical method used is described in Figure 3, and has been applied in the presented study. The mechanical model chosen for the simulations is the classical multi-body model with analytical evaluation of the non-linear contact forces. Different software have been used to perform simulations: Vampire®, Voco and Simpack®.

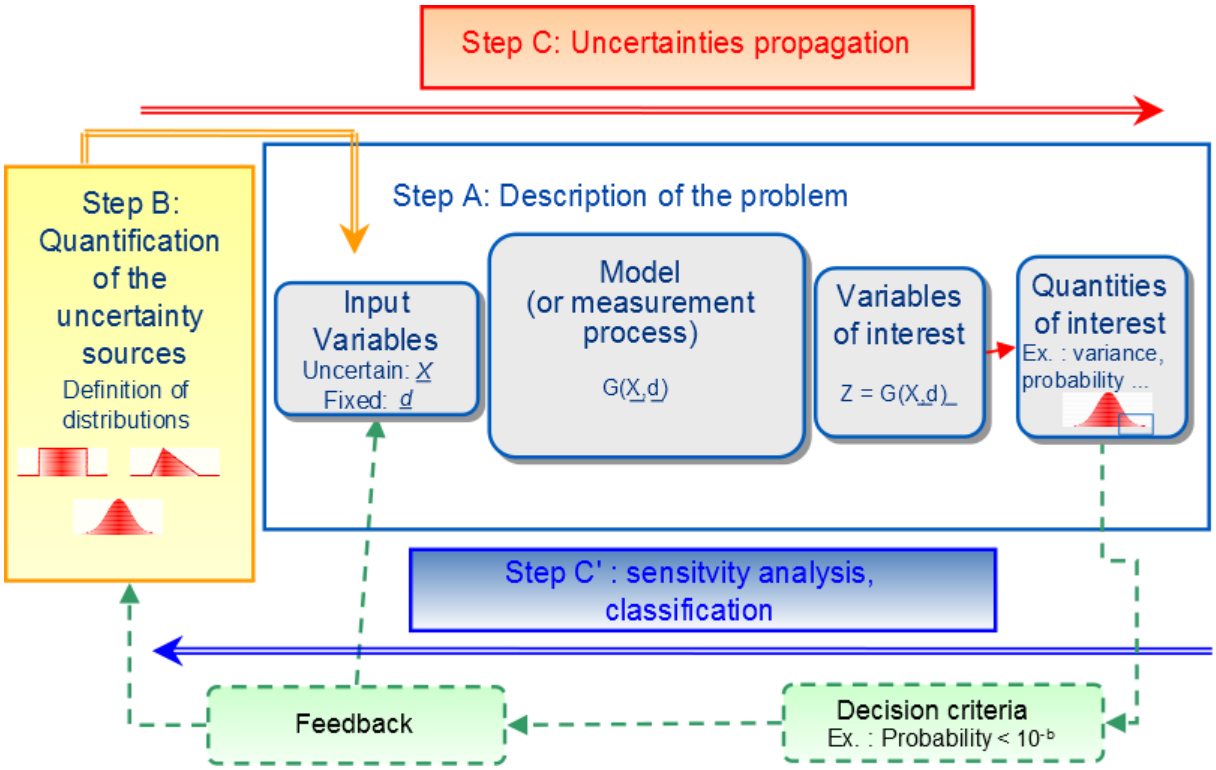

Fig. 3: Diagram of the method to introduce uncertainty in simulation [3]

3.2 Description of the uncertainty sources

The first step of the probabilistic simulation is the description and the quantification of the variability sources. Some of them can be measured: it is the case for example of the rail profile and stiffness and damping of components, other are more difficult to characterize (track stiffness or wind gusts for example). Some of the parameters are moreover scalar (masses), others are vector valued (the track stiffness varies along the track).

For the scalar parameters affected by uncertainty, direct methods (the parameters of a chosen distribution are identified [4]) or indirect methods (the distribution is computed by a transformation of a chosen distribution [5]) can be chosen to define the statistical distribution describing the quantity affected by uncertainty. When only the mean value and the standard deviation are known, it is possible to demonstrate, thanks to the Maximum Entropy Principle [14,15], that the most adapted distribution is a Gaussian. When only bounds are known, the best distribution is a uniform distribution. For the vector valued parameters affected by uncertainty, random fields [16, 17] have to be identified.

Additionally an extremely important issue is the statistical dependencies between the input variables (i.e inertia and vehicle mass are extremely dependent); to guarantee a realistic evaluation of the vehicle behaviour it is necessary to properly assess these relationships.

In this study we have considered variability of the vehicles' mechanical parameters (i.e masses, stiffness and damping of components etc.), the friction coefficient and the rail profile. The vehicle parameters have been modelled with uniform distributions (bounds given by the manufacturers). The friction coefficient and the rail profile have been considered as constant in each studied track portion, but different from one section to another. Indeed, the available information is not sufficient to identify a random field for the friction coefficient, and the interpolation between rail profiles often causes problem during the simulation. The friction coefficient distribution is given Figure 4 and details can be found in section 4.2.2. Measured rail profiles have been randomly picked in the WP1 database paying attention to the zone (radius and cant deficiency) as well as to the low and high rail.

The tracks have been built according to the EN14363 requirements thanks to the Virtual Test Track environment enhanced during the project and described in details in the WP6 joint paper of the same authors.

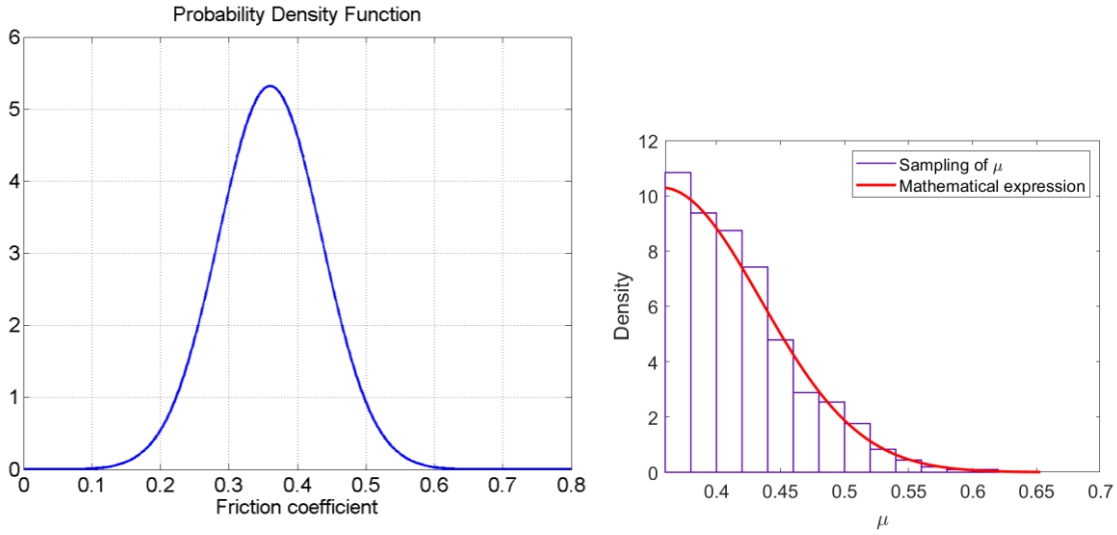

Fig. 4: Statistical distribution of friction coefficient between wheel and rail (right figure added in 2024: should have been used instead of left figure!)

3.3 Numerical post-treatment of the simulations and computation of the quantities of interest

The simulation results are then processed automatically in the same way as it is prescribed by the standard: after a verification of the track characteristics, the simulation outputs are filtered and statistically processed first in each section and then in each zone. Since the number of sections in each zone prescribed by the leaflet is relatively small compared to the expected percentile, it is not easy to identify the distribution of the percentiles stemming from each section. However, since high percentiles are considered, they should follow an extreme value distribution [18]. The estimated maxima are thus computed assuming both a Gaussian and an extreme value distribution in order to account for a more realistic distribution of data. As shown in figure 5 the obtained quantity

of interest can be quite different and the Gaussian distribution can happen to be less conservative than the extreme one. This underlines how important is the knowledge of the process under assessment.

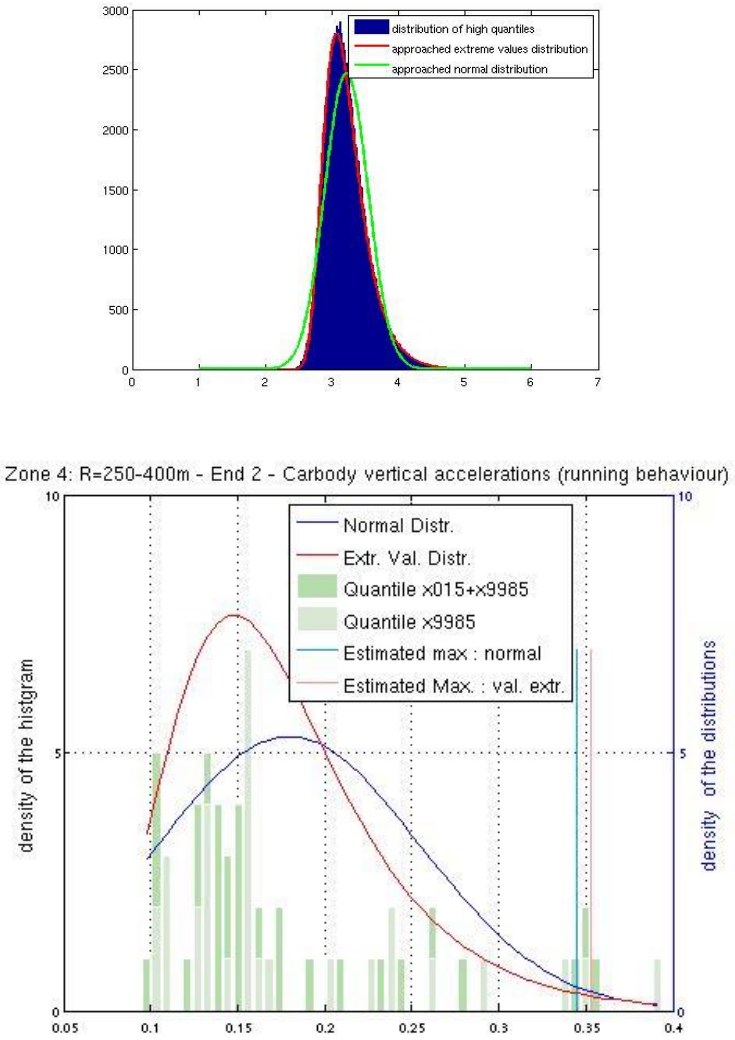

Fig. 5: Comparison of the estimated maximum computed from a Gaussian and from an extreme value distribution

4 ANALYSIS OF THE SENSITIVITY OF THE CERTIFICATION CRITERIA TO THE VEHICLE PARAMETERS AND TO THE RUNNING CONDITIONS

The track / vehicle system is complex. In order to choose the mechanical parameters that have to be carefully modelled to obtain a good precision on the certification criteria (maximal behaviour) sensitivity analysis has been carried out.

4.1 Description of different methods to analyse the sensitivity of a complex system

The vehicle / track system is non-linear, very sensitive to the input data and the response surfaces of the different certification criteria are rough. The use of methods linearizing the surface around a functioning point (FORM/SORM for example) is not adapted to propagate the variability through the modelling. The classical Monte-Carlo method is used which presents the advantage of giving access to a confidence level, even if it requires a large number of simulations. However during the project it has been demonstrated that, in some specific cases, coupling the Monte-Carlo approach together with a design of experiments (DOE) approach it is possible to obtain still reliable results decreasing considerably the computation effort [10].

To compute the sensitivity of the certification criteria to the input data two methods have been used. The first one is the Morris method [6] which is adapted to systems containing a very large number of parameters. This one-at-a time method (see scheme Figure 6) is used to determine the parameters that are the most influential. Several sets of input parameters (black points) are raised at random, each of the parameters is then varied one-at-a time of an amplitude Δp (green, red and blue points) and the simulations are raised for all these sets of parameters. The method proposes then two estimates: the first one represents the overall effect of the parameter, the other estimates the higher order effects.

The right hand side of Figure 6 presents, for example, the result of the analysis for the derailment criterion Y/Q. In this example all the vehicle parameters were considered as variable, as well as the friction coefficient and the rail profile. It appears that the most influential parameters are the rail profile, the friction coefficient and the mechanical properties of the lateral bumpstop.

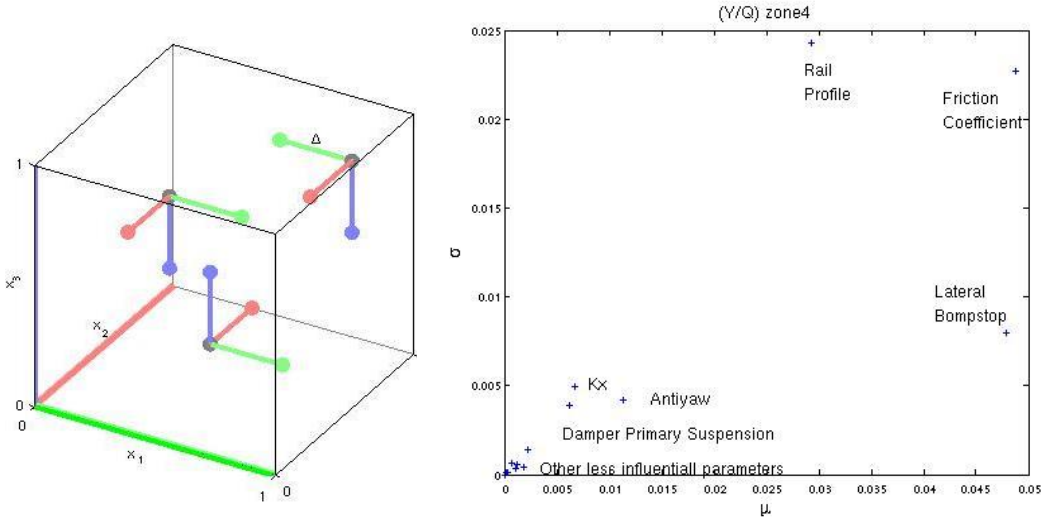

Fig. 6: Sketch of the Morris method – Example of result: Morris method for the Y/Q criterion in zone 4

The Sobol indices [7] are then computed to give access to the relative importance of the chosen parameters on the certification criteria. The results for the derailment criterion are given Figure 7, quantifying the results of the Morris method.

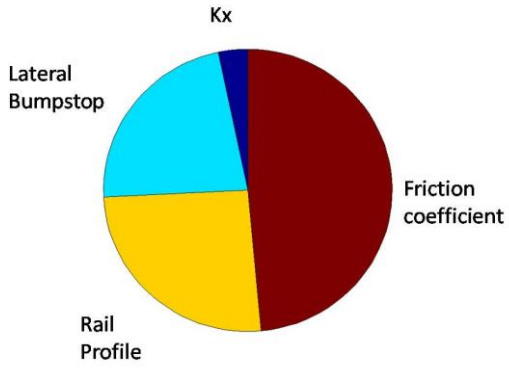

Fig. 7: Sobol indices for the Y/Q criterion

4.2 Analysis of the sensitivity of contact parameters on the certification criteria

In this section, the influence of wheel/rail contact is investigated. The equivalent conicity is often not sufficient to characterize the vehicles' dynamic response. This can easily be shown analysing the dynamic response of a vehicle / track system equipped with measured wheel and rail profile pairs leading to the same equivalent conicity. Figure 9 presents for example, the maximum of the non-dimensional quantity $\frac{(\Sigma Y)}{(\Sigma Y)_{\text{limit}}}$ on different tracks and for two different contacts having the same conicity (see Figure 10) which are very different. We will therefore directly consider measured profiles.

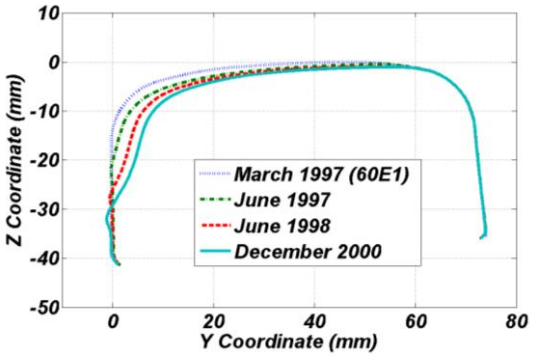

Fig. 8: Measured rail profile taken from the high rail of a 985 m curve.

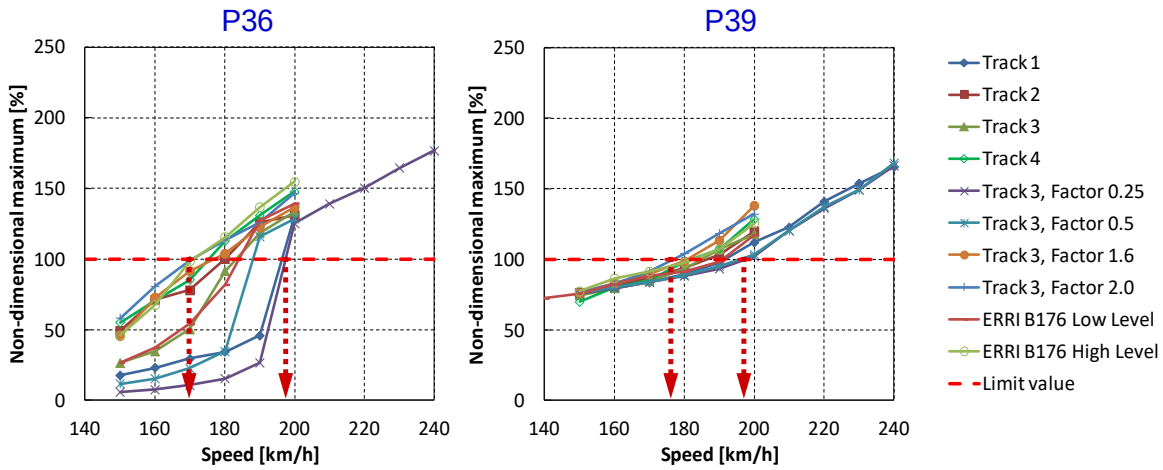

Fig. 9: Maximum of the non-dimensional quantity $(\Sigma Y)/(\Sigma)_{limit}$ on different tracks. Contact P36 on, the left, P39 on the right.

Two variables are studied simultaneously: the rail profile and the friction coefficient between wheel and rail. Both data are usually not measured during the track geometry evaluation, and even if they are identified, they may vary with time. The wheel profiles considered in this study are the ones measured on the studied vehicle which allows the comparison with measured reactions. The wheel profiles however also plays a very important role (see [12]).

4.2.1 Description of the rail profile variability

Curved track rail profiles can differ significantly, depending on the age of the rail, on the service of the line and on the grinding operations. Figure 10 shows an example of the modification of the geometry of the rail profile due to wear. Even if the curvature is moderate (985 m), the rail profile significantly changed over the years. Additionally, worn wheel profiles exhibit thinner flange, wear tends to increase the play between wheel and rail. Another effect of wear is the change of effective conicity. In sharp curves, lowering the conicity will decrease the steering ability of the vehicle: as a result the stability of the vehicle will be altered.

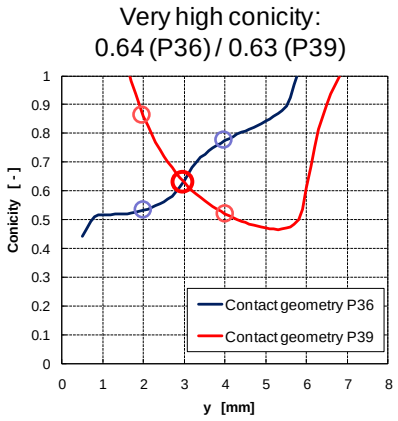

Fig. 10: Conicity of contacts P36 et P39.

4.2.2 Description of the friction coefficient variability

The European leaflet EN14363 (2005) prescribes a derailment limit value of 0.8 on the Y/Q ratio. This value corresponds to an adhesion of 0.6 in Nadal's formula [8]. The standard value of the friction coefficient is 0.36. In the simulation, it is a common practice to choose this constant value. Choosing a constant value of 0.6 would certainly lead to unrealistic results. It is proposed here to use a statistical distribution of the friction coefficient as suggested by the draft norm prEN14363:2012 thanks to measurements achieved in Great Britain [5]. The friction coefficient follows a one sided normal distribution representative of measured dry conditions, with mean value 0.36 and standard deviation 0.075. In the same manner as with rail profiles, a different friction coefficient, constant in each section, has been introduced in the simulation. The coefficients are raised at random in the normal distribution. The friction coefficients are different on the two rails.

4.2.3 Description of the sensitivity analysis strategy (see Figure.11) and results

The virtual track considered has been built with the Virtual Test Track [4] from measurements achieved in the project. The track is compliant with the EN14363 requirements for the considered freight wagon at the studied speed. The friction coefficient and rail profile are raised at random for each section. Finally the simulations are raised for several sets (10) of contact conditions and the different certification criteria.

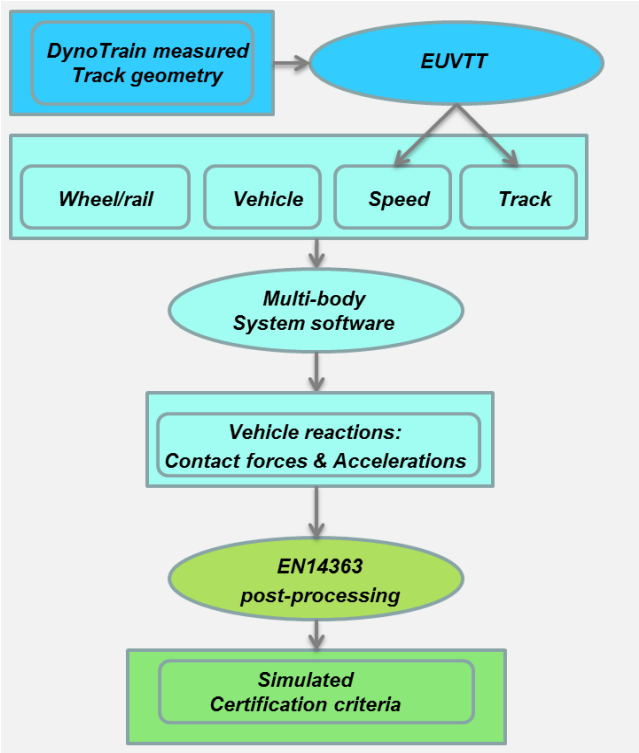

Fig. 11: Certification of rolling stocks: simulation road.

In curves, the track lateral forces can vary from 30% between worn geometry and new ones. It is therefore essential to consider both representative wheel and rail profiles for virtual certification.

This variability of friction coefficient changes the estimated maximum of the lateral loads up to 20%. One can moreover notice that the higher derailment criteria are obtained for low friction coefficient on the outer rail and high coefficient on the inner rail (see Figure. 12).

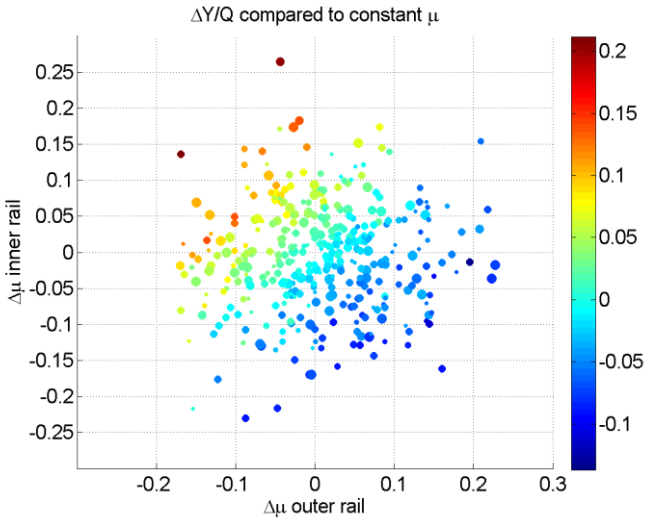

Fig. 12: Scatter plot of variation of Y/Q as a function of friction values on outer and inner rails

The same type of analysis has been achieved with other parameters. Some results are given for the track irregularities Figure 13.

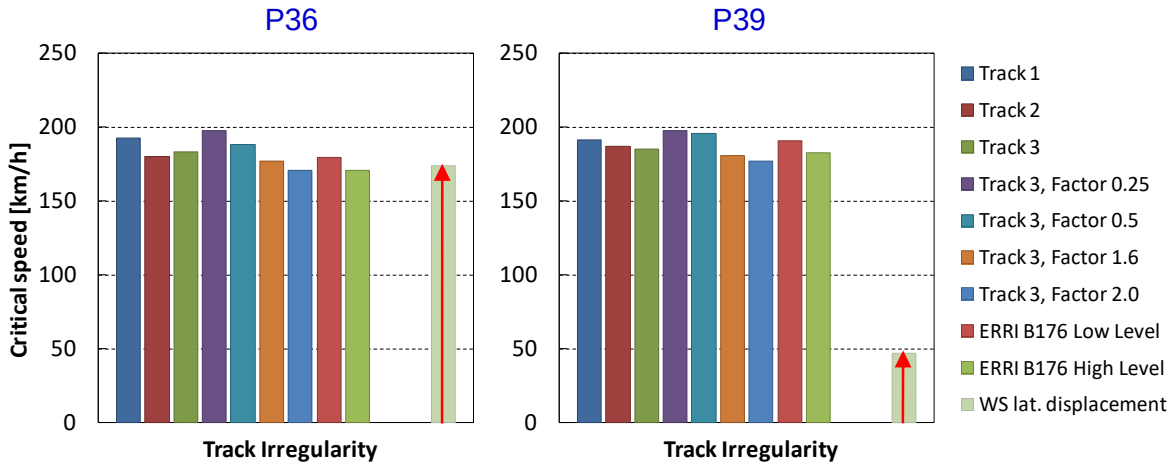

Fig. 13: Effect of the track irregularities on the critical speed

It has also been shown in this study that a higher number of track sections would add consistency to the studied certification quantities. Figure 14 shows an example of convergence analysis. The ordinate gives the mean increase in percentage of the estimated maxima compared to the limit values.

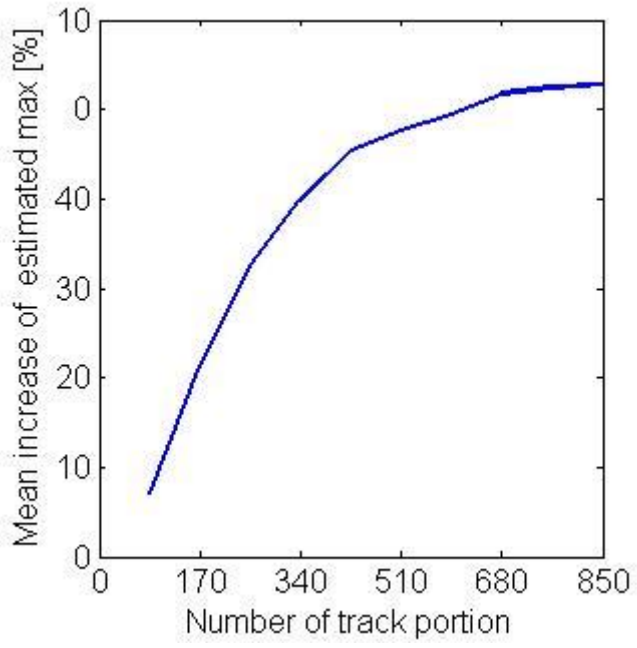

Fig. 14: Convergence analysis on the number of track portions

4.3 Global sensitivity analysis of the certification criteria to the different input parameters

A global analysis of the sensitivity of the certification criteria to the different input parameters have moreover been achieved, with the Morris method and the Sobol indices.

Two vehicles were considered: a wagon and a locomotive. All the input parameters were considered jointly: vehicle parameters (i.e masses, centre of gravity, suspension parameters) as well as contact parameters (i.e friction coefficients, rail profiles etc.). The input variability was modelled with uniform laws for the vehicle parameters, the friction coefficient by a normal law, as described in section 3.2.

The results show that the importance of parameters on the assessment quantity depends on the studied criterion, the value of interest (acceleration or load) and the quantity of interest (high or medium quantile). See for example figures 6 and 15. Indeed the parameters acting during exceptional events are different from the ones acting during normal service scenario.

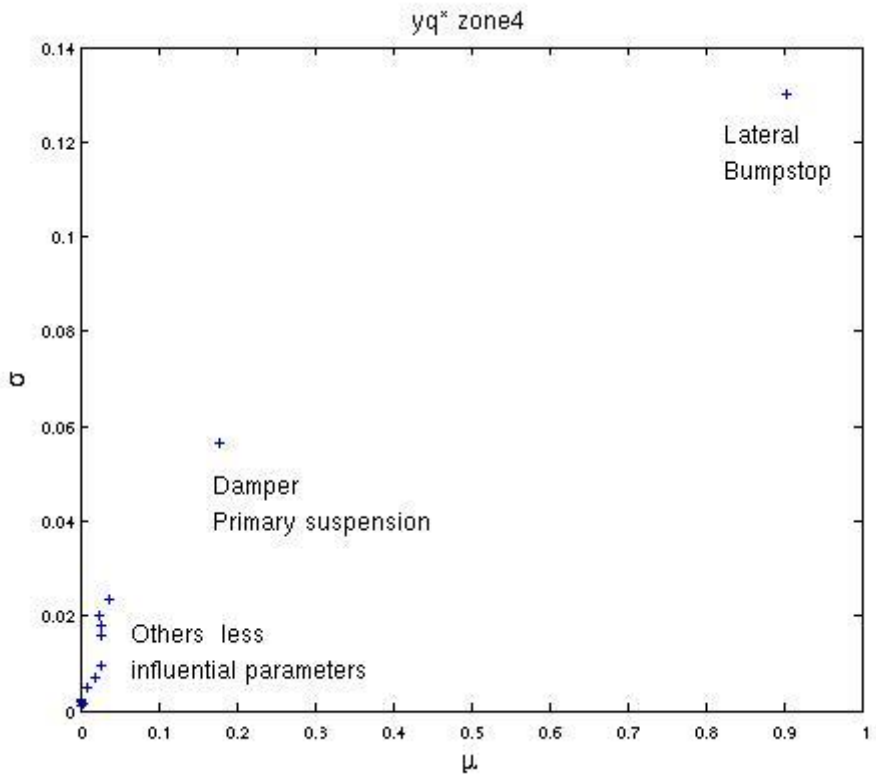

Fig. 15: Results for the lateral carbody acceleration in zone 4

As an example, for the lateral loads, the parameters that have the most significant influence are the rail profile and the fiction coefficient as well as some suspension elements. The lateral bumpstops moreover play an important role on the lateral acceleration even if they almost never act during classical rides.

5 SUMMARY OF THE PROPOSED NUMERICAL CERTIFICATION PROCEDURE AND APPLICATION TO EXAMPLES

5.1 Conditions in which the virtual certification is possible

The vehicle / track system being very complex, the use of simulation is only possible when the modelling can be extensively validated against on-track dynamic reactions both in the time domain and in the frequency domain. It mainly concerns three cases:

- A train that had already been experimentally certified and has been "slightly modified" outside the ranges allowing dispensation given in EN14363. A large set of measurements are available to validate the modelling and the modifications only lead to small behaviour changes as defined in 3.1. If the suspension has been changed, static tests of the new vehicle are required.

- All the track requirements (except the high cant deficiencies) were not met during the testing. The first tests are used to validate the model and the simulation can be used to complete the certification. In order to treat this,[20] proposes another method fully based on measurements.

- The train has been certified for a network, and has to be certified for other running conditions (other network, or other type of tracks). The behaviour of the train on the new running condition is not too different from the behaviour observed during certification (see 3.1 for details).

In these three cases, the behaviour of the simulated system has to be "similar" to the behaviour of the tested system. This similarity can be assumed if the train structure has not changed (no change of the type of suspension elements for example) and if the responses of the two systems are similar. So, to exploit the advantages of the simulations, and contrary to the requirements given in the lambda table for the simplified tests [2], we propose to give requirements on the trains' reactions rather than on the train or track characteristics.

The flowchart of the numerical certification process described here is given in Figure 16.

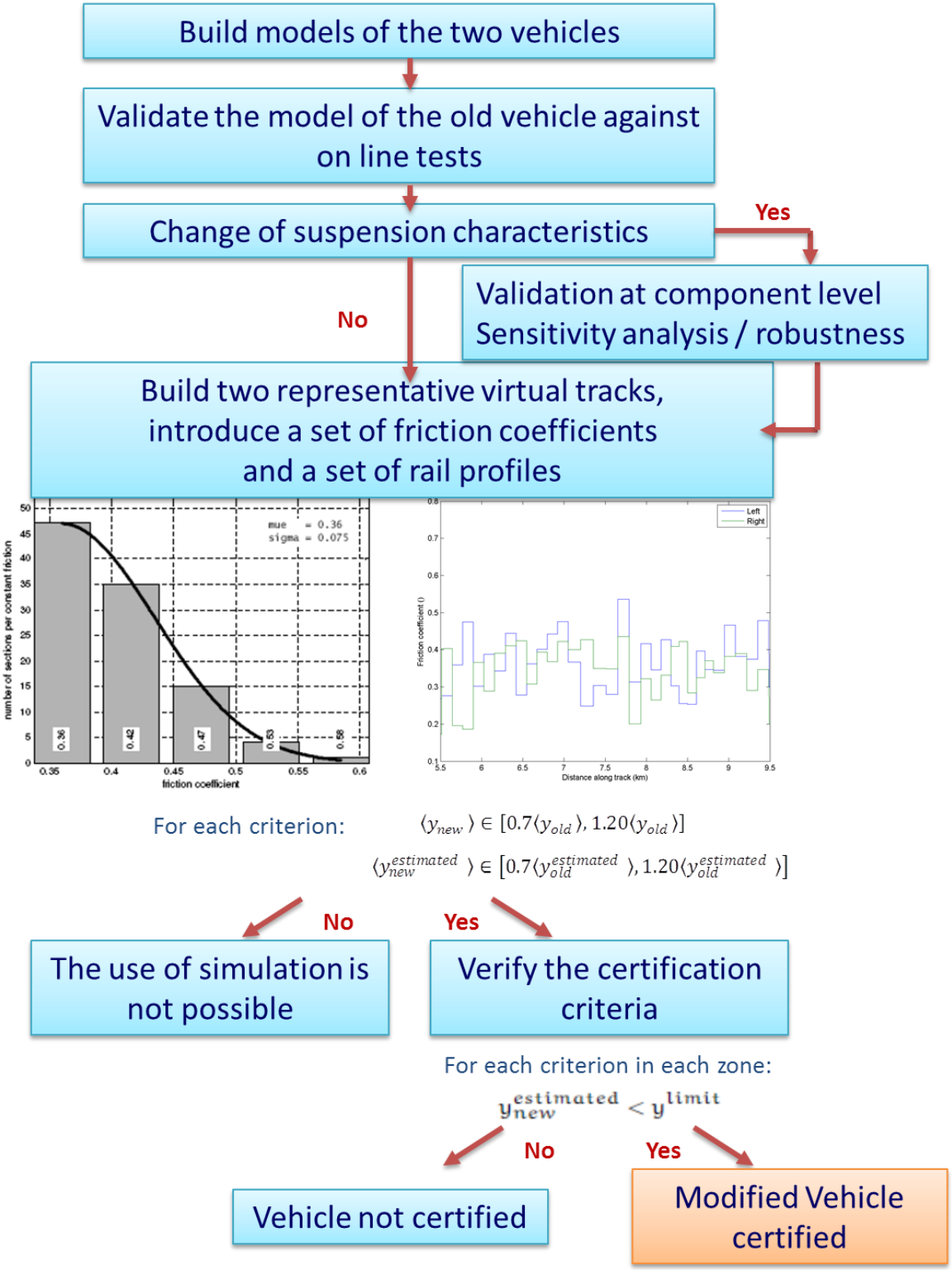

Fig. 16: Certification of Rolling Stock: Proposed System Sequence

5.2 Application of the procedure to examples

The full procedure has been applied to different application example. First the experimental procedure has been compared for two freight vehicles (Sgns691 and Laas wagon [21]) tested during the project. For the Sgns691, the average relative difference between the certification quantities obtained from measurement and simulation is about

20% and is quite different from one criterion to another (it is between 0 and 60 %). However, for the Y/Q criterion, a difference of 30% was observed for the experimental when comparing two different runs on the same track, and 20% for the lateral accelerations. These vehicles are moreover very non-linear and difficult to model.

The procedure has then been applied to a slightly modified: a first vehicle was built to run in one country and was certified experimentally. The train was then slightly modified to run in another country. The modifications of the vehicle fulfilled the proposed requirements and the dynamic simulated behaviour of the new vehicle appeared to be better than the original one. In this case, and if the modelling was validated, the prescribed method would have accepted the numerical certification.

6 CONCLUSIONS

The vehicle/track is a **complex non-linear** system containing several **variability sources**: the track geometry, its quality and its stiffness, the contact conditions (friction coefficient, rail profile) and the mechanical properties of the vehicle (mass and inertia, suspension characteristics). To characterise this non-linear system, several measurements have to be achieved in different locations and have to be post-treated using a **statistical** processing.

In the nuclear field and in the off-shore energy fields, complex systems are already certified using **experiments on sub-systems** and **simulation on the whole system**. When there are high safety concerns the system is often re-certified regularly during its life cycle. In the energy field, the state moreover asks the operators to certify that the probability of incident is lower than a given threshold. A full probabilistic approach is then required.

Some of these methods have been applied to the virtual certification of different track/vehicle systems. These are always based on a three step approach: the description of the mechanical problem, the **identification of the uncertainties** and the **propagation** through the modelling. In this context, it has been shown that the track geometry, the stiffness and the contact conditions play a key role on the estimated maxima studied during the certification. In order to accurately take into account these effects, a method to generate representative virtual tracks has been proposed. These tracks are built from the **concatenation of measured track sections**, according to the standard requirements. The variability of the rail profiles and of the friction coefficient has also to be introduced (one constant friction coefficient and one constant rail profile raised at random for each concatenated track section). We moreover showed that a **larger number of track sections** would enhance the precision of the estimated values.

Taking into account the variability of vehicle parameters would also be interesting. For practical purposes, the proposed procedure is only based on the assumption that a normalized and validated model for the train is available, for which mechanical properties have been accurately identified from experimental data. Nevertheless, in order to verify the robustness of the vehicle model and also to be sure that the parameters important for the certification criteria are well modelled, we insist on the importance of achieving a **sensitivity analysis before the whole virtual certification procedure**.

The sensitivity analyses conducted have pointed out that **the wear and the temporal modifications of the mechanical characteristics** of the train can strongly modify its dynamic response during its life cycle. Taking advantage of the numerical possibilities, it would be interesting to investigate the response of new trains but also to predict the behaviour of this train over its whole life. However in this case the limit values should be changed since they include a margin to allow for change in behaviour over the normal maintenance cycle.

In order to validate the proposed numerical procedure, the complete virtual certification of two trains tested in the project has been achieved in this work. The certification results computed from the simulated results have then been compared to the results obtained from measurements. Even if the mean values are quite well reproduced, differences have been noticed when analysing the extreme percentiles (2%-60%). When validating the model for certification purposes, special attention therefore has to be paid not only on the mean response of the sub-systems of the train but also to the rare events.

To add consistency to this work, the full proposed procedure has also been applied to a case that would have met the required conditions of the virtual certification.

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