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Computation of the Mutual Inductance between Air-Cored Coils of Wireless Power Transformer

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Abstract. Wireless power transfer system is a modern technology which allows the transfer of electric power between the air-cored coils of its transformer via high frequency magnetic fields. However, due to its coil separation distance and misalignment, maximum power transfer is not guaranteed. Based on a more efficient and general model available in the literature, rederived mathematical models for evaluating the mutual inductance between circular coils with and without lateral and angular misalignment are presented. Rather than presenting results numerically, the computed results are graphically implemented using MATLAB codes. The results are compared with the published ones and clarification regarding the errors made are presented. In conclusion, this study shows that power transfer efficiency of the system can be improved if a higher frequency alternating current is supplied to the primary coil, the reactive parts of the coils are compensated with capacitors and ferrite cores are added to the coils.

1. Introduction

Wireless power transfer (WPT) systems allow the transfer of electric power between the air-cored coils of its transformer via high frequency magnetic fields to a consuming device (e.g. electric vehicle) [1–3]. However, due to its coil separation distance and misalignment, maximum power transfer is not guaranteed.

The computation of the mutual inductance between circular coils with and without misalignment is of fundamental practical interests to electrical engineers and physicists [1–6]. Several mutual inductance models based on the application of Maxwell's formula, Neumann's formula, Biot-Savart law and magnetic vector potential approach have been presented in [3–7]. These models are developed based on analytical or semi-analytical forms expressed over elliptical integrals of the first, second and third kinds, Heuman's Lambda function, Bessel functions and Legendre functions [6–10]. Nonetheless, a more advanced and relevant model for evaluating the mutual inductance between circular coils with misalignment have been formulated in [6, 7].

The aim of this paper is to rederive the mathematical models for evaluating the mutual inductance between circular coils with and without lateral and angular misalignment based on the more efficient and general model given in [6, 7]. Also, rather than presenting the computed results numerically as given in [7, 10], the results in this paper are graphically implemented using MATLAB codes. Lastly, the computed results are compared with the results of [7, 10] and clarification regarding the errors made are presented.



2. Advanced and Relevant Model for Mutual Inductance Computation

In [6], Grover formulated a more efficient and general model for computing the mutual inductance between two filamentary circular coils with misalignment (see Eq. (1) and Fig. 1). The model given in Eq. (1) was achieved based on the application of Neumann integral approach. In order to confirm the validity of the model, Kim used the magnetic vector potential approach to obtain another mutual inductance model (see Eq. (20) of [10]). However, using the same approach employed by Kim, Babic retrieved Grover's formula through well detailed derivations [7].

$$M = \frac{2\mu_0}{\pi} \sqrt{R_P R_S} \int_0^\pi \frac{[\cos\theta - \frac{d}{R_S} \cos\phi] \Psi(k)}{k\sqrt{V^3}} d\phi \quad (1)$$

where

$$\alpha = \frac{R_S}{R_P}, \quad \beta = \frac{c}{R_P}, \quad \xi = \beta - \alpha \cos\phi \sin\theta, \quad k^2 = \frac{4\alpha V}{(1+\alpha V)^2 + \xi^2}, \quad \Psi(k) = (1 - \frac{k^2}{2})K(k) - E(k)$$

$$V = \sqrt{1 - \cos^2\phi \sin^2\theta - 2\frac{d}{R_S} \cos\phi \cos\theta + \frac{d^2}{R_S^2}}$$

$$K(k) = \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1-k^2 \sin^2\theta}} d\theta \quad \text{and} \quad E(k) = \int_0^{\frac{\pi}{2}} \sqrt{1-k^2 \sin^2\theta} d\theta.$$

where μ_0 is the magnetic permeability of free space, R_P and R_S are the radii of the primary and secondary coils respectively, d is the lateral misalignment, α is the shape factor of the coils' physical geometry, c is the separation distance between the coils' centres, θ is the angular misalignment, k is a variable, parameter $\Psi(k)$ is function of k , ϕ is the angle of integration at any point of the secondary coil, β , V and ξ are dimensionless parameters, $K(k)$ and $E(k)$ are the complete elliptic integral of the first and second kinds respectively .

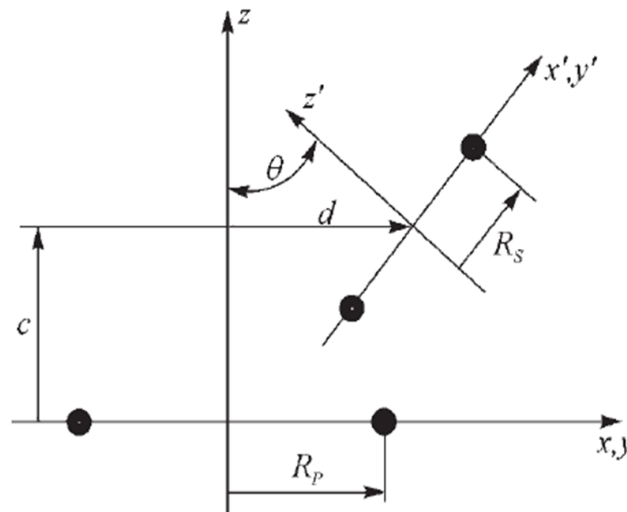


Figure 1. Filamentary circular coils with angular and lateral misalignment [6].

3. Rederived Mathematical Models for Mutual Inductance Computation

It is important to note that in Eq. (1), the number of turns (i.e., N_P and N_S) for the primary and secondary coils are not expressed. Therefore, taking into account N_P and N_S based on the example considered in [7], the rederived models are presented in the following subsections (see Figs. 2-4):

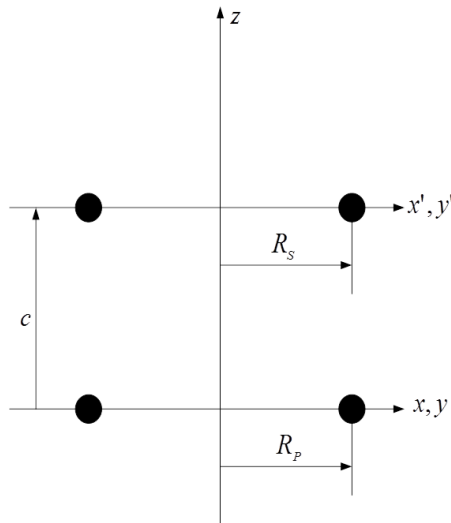


Figure 2. Case 1: without misalignment

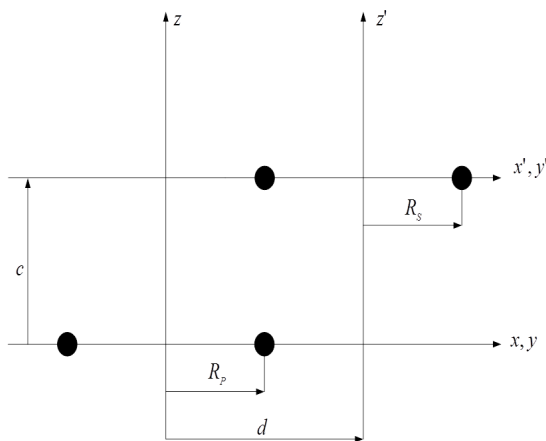


Figure 3. Case 2: with lateral misalignment.

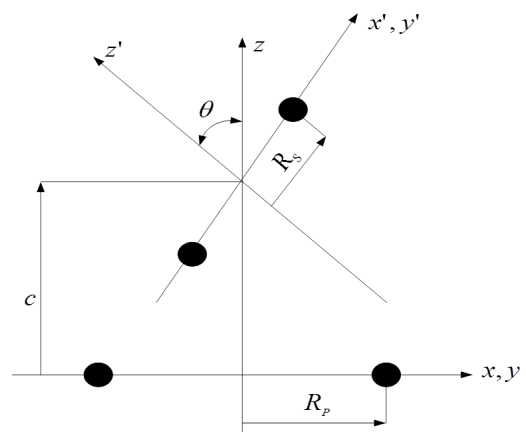


Figure 4. Case 3: with angular misalignment.

3.1. Case 1: without Misalignment

Figure 2 shows the case without misalignment (i.e., $\theta = 0$ and $d = 0$). Thus, the mutual inductance model between air-cored filamentary circular coils without misalignment is given by

$$M = \frac{2N_P N_S \mu_0}{\pi} \sqrt{R_P R_S} \int_0^\pi \frac{\Psi(k)}{k} d\phi \quad (2)$$

where

$$\alpha = \frac{R_S}{R_P}, \quad \beta = \frac{c}{R_P}, \quad \xi = \beta, \quad k^2 = \frac{4\alpha}{(1+\alpha)^2 + \xi^2}, \quad \Psi(k) = \left(1 - \frac{k^2}{2}\right)K(k) - E(k), \quad K(k) = \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1-k^2 \sin^2 \theta}} d\theta \quad \text{and} \quad E(k) = \int_0^{\frac{\pi}{2}} \sqrt{1-k^2 \sin^2 \theta} d\theta.$$

3.2. Case 2: with Lateral Misalignment only

Figure 3 shows the case with lateral misalignment only (i.e., $\theta = 0$). Thus, the mutual inductance model between air-cored filamentary circular coils with only lateral misalignment is given by

$$M = \frac{2N_P N_S \mu_0}{\pi} \sqrt{R_P R_S} \int_0^\pi \frac{[1 - \frac{d}{R_S} \cos\phi] \Psi(k)}{k \sqrt{V^3}} d\phi \quad (3)$$

where

$$\alpha = \frac{R_S}{R_P}, \quad \beta = \frac{c}{R_P}, \quad \xi = \beta, \quad k^2 = \frac{4\alpha V}{(1+\alpha V)^2 + \xi^2}, \quad \Psi(k) = (1 - \frac{k^2}{2})K(k) - E(k)$$

$$K(k) = \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1-k^2 \sin^2\theta}} d\theta, \quad E(k) = \int_0^{\frac{\pi}{2}} \sqrt{1-k^2 \sin^2\theta} d\theta \quad \text{and} \quad V = \sqrt{1 - 2\frac{d}{R_S} \cos\phi + \frac{d^2}{R_S^2}}$$

3.3. Case 3: with Angular Misalignment only

Figure 4 shows the case with angular misalignment only (i.e., $d = 0$). Thus, the mutual inductance model between air-cored filamentary circular coils with only angular misalignment is given by

$$M = \frac{2N_P N_S \mu_0}{\pi} \sqrt{R_P R_S} \cos\theta \int_0^\pi \frac{\Psi(k)}{k \sqrt{V^3}} d\phi \quad (4)$$

where

$$\alpha = \frac{R_S}{R_P}, \quad \beta = \frac{c}{R_P}, \quad \xi = \beta - \alpha \cos\phi \sin\theta, \quad k^2 = \frac{4\alpha V}{(1+\alpha V)^2 + \xi^2}, \quad \Psi(k) = (1 - \frac{k^2}{2})K(k) - E(k)$$

$$V = \sqrt{1 - \cos^2\phi \sin^2\theta}, \quad K(k) = \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1-k^2 \sin^2\theta}} d\theta \quad \text{and} \quad E(k) = \int_0^{\frac{\pi}{2}} \sqrt{1-k^2 \sin^2\theta} d\theta$$

3.4. Case 4: with Lateral and Angular Misalignment

Figure 1 shows the case with both misalignment. Thus, taking into account N_P and N_S , the mutual inductance model between air-cored filamentary circular coils with lateral and angular misalignment is given by

$$M = \frac{2N_P N_S \mu_0}{\pi} \sqrt{R_P R_S} \int_0^\pi \frac{[\cos\theta - \frac{d}{R_S} \cos\phi] \Psi(k)}{k \sqrt{V^3}} d\phi \quad (5)$$

4. Presentation and Discussion of Results

The results obtained for the computation of the mutual inductance between air-cored circular coils with and without lateral, d and angular, θ misalignment are shown in Figs. 5-9. MATLAB software is used to compute the rederived models given in (2), (3), (4) and (5). In addition, the results are presented graphically (see Figs. 5-9) and numerically tabulated (see Table 1).

The result shown in Fig. 5 is compared with the numerical results obtained in [7, 10] (see Table 1). This is so because the example studied in [7, 10] considered the case with only lateral misalignment. Although different coil separation distances (i.e., $c = 0, 0.05 \text{ m}$ and 0.1 m) are studied in Fig. 5, the authors of [7] and [10] focused only on $c = 0$.

The numerical results obtained in [7] showed that from $d = 0$ to 15.5 mm (i.e., for the cases where the secondary coil is located inside the primary coil), the value of the mutual inductance M increases from 0.152875989 mH to 0.181647101 mH . This outcome is in agreement with the results obtained in this paper (see Fig. 5 and Table 1). However, there are discrepancies compared to the results obtained in [10] (see Table 1). Furthermore, from $d = 69.5 \text{ mm}$ to 1000 mm (i.e., the region where the secondary coil is located outside the primary coil), the results of [7] showed that negative values are obtained for M and for larger d (i.e., from $d = 200 \text{ mm}$ to

1000 mm), its value approached zero. This outcome is in agreement with the results obtained in this paper (see Fig. 5 and Table 1). This behaviour corresponds with the theory, as the flux lines linked by the secondary coil change their orientation outside the primary coil [6–8]. However, in [10], M slowly approached zero when the smaller coil is outside the bigger coil and never reached negative values, which is not correct according to the theory. Thus, confirm with the authors of [7] that Eq. (20) of [10] is erroneous.

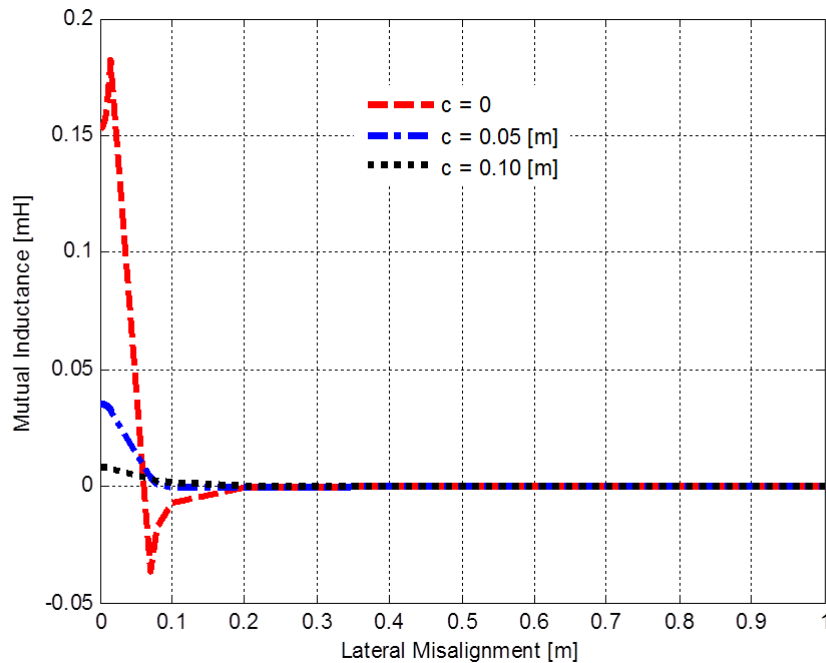


Figure 5. Case 2: with lateral misalignment

Table 1. Example studied for $R_P = 42.5 \text{ mm}$, $R_S = 20.0 \text{ mm}$, $N_P = 150$, $N_S = 50$

$d \text{ (mm)}$	$c \text{ (mm)}$	$M \text{ (} 10^{-4} H \text{) [7]}$	$M \text{ (mH): This paper}$	$M \text{ (} 10^{-4} H \text{) [10], Eq. (20)}$	Discrepancy [%], Eq. (3) vs. Eq. (20)
0.0	0	1.52875989	0.152875989	1.52875989	0.00
1.0	0	1.52967999	0.152967999	1.53063181	0.062
2.5	0	1.53453878	0.153453878	1.54049796	0.388
5.0	0	1.55229043	0.155229043	1.57627671	1.545
7.0	0	1.57597265	0.157597265	1.62337093	3.001
10.0	0	1.63029226	0.163029226	1.72877701	6.041
15.0	0	1.79299153	0.179299153	2.02559287	12.973
15.5	0	1.81647101	0.181647101	2.06644759	13.762
69.5	0	-0.36397730	-0.036397730	1.57888396	—
80.0	0	-0.17767980	-0.017767980	1.05074054	—
100.0	0	-0.07231342	-0.007231342	0.60979570	—
200.0	0	-0.00718415	-0.000718415	0.13810829	—
500.0	0	-0.00043535	-0.000043535	0.02158082	—
1000.0	0	-0.00005401	-0.000005401	0.00537760	—

This study shows that as the coil separation distance and misalignment increase, the mutual inductance between the coils decreases (see Figs. 5-9). Thus, an increased value of mutual inductance can be achieved when a higher frequency AC current is supplied to the primary coil, the reactive parts of the coils are compensated with capacitors and ferrite cores are incorporated around the coils.

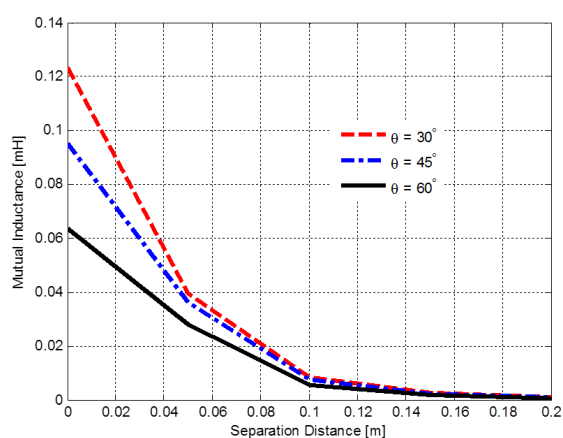


Figure 6. Case 3: with angular misalignment.

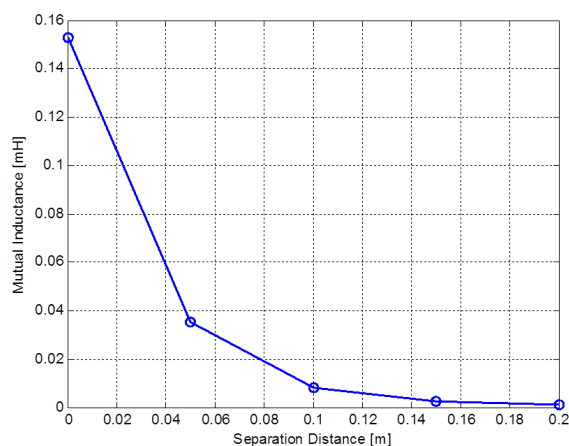


Figure 7. Case 1: without misalignment.

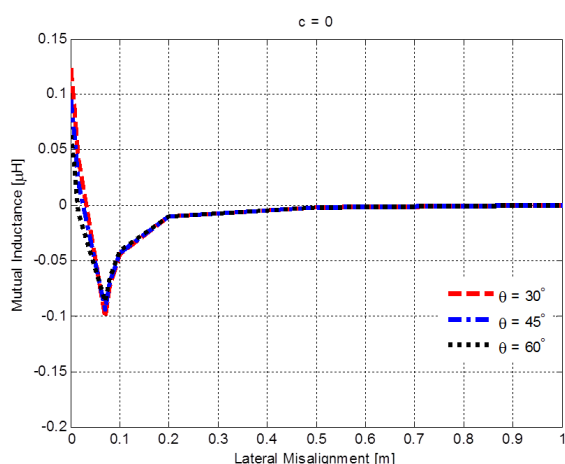


Figure 8. Case 4: with misalignments.

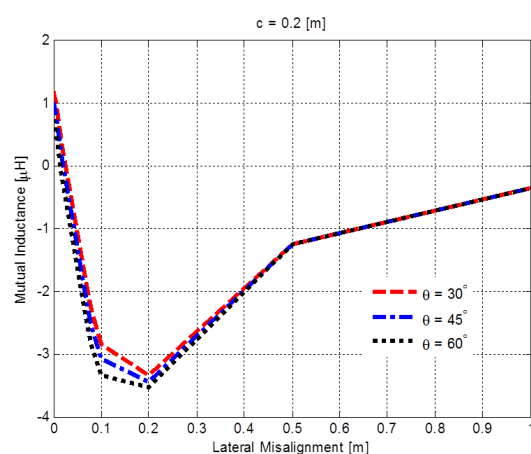


Figure 9. Case 4: with misalignments.

5. Conclusion

In this paper, rederived models for the mutual inductance between circular coils with and without misalignment are expressed and computed using MATLAB codes. The results obtained are compared with the published ones and clarification regarding the errors made are presented.

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