



HAL
open science

Coupling between homogenization techniques and brittle mechanics for modelling the behaviour of micro-cracked refractory linings

Amna Rekik, Thi Thu Nga Nguyen, Alain Gasser

► **To cite this version:**

Amna Rekik, Thi Thu Nga Nguyen, Alain Gasser. Coupling between homogenization techniques and brittle mechanics for modelling the behaviour of micro-cracked refractory linings. 14th Biennial Worldwide Congress UNITECR 2015, Sep 2015, Vienne, Austria. hal-01213806

HAL Id: hal-01213806

<https://hal.science/hal-01213806v1>

Submitted on 31 Jan 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Coupling between homogenization techniques and brittle mechanics for modelling the behaviour of micro-cracked refractory linings

Amna Rekik^{1,*}, Thi Thu Nga Nguyen¹, Alain Gasser¹

¹ Univ. Orléans, INSA-CVL, PRISME, EA4229, 45072 Orléans, France
amna.rekik@univ-orleans.fr, alain.gasser@univ-orleans.fr

Abstract

During service, refractory linings are often subjected to severe thermo-mechanical loadings. Accordingly, these masonries may behave linearly or nonlinearly in a viscoelastic and/or viscoplastic manner. It is then interesting to investigate the creep behaviour and cracking of refractory linings. In the last decades, multi-level techniques have demonstrated their efficiency for predicting the global and local behaviour of masonry structures with low numerical cost. In this context and as a first step, we propose in this work to model masonry with safe bricks and a mortar following the Burgers or modified Maxwell rheological models at its safe and microcracked states. The proposed model is based on the coupling between linear homogenization technique and the Griffith theory. This allows the determination of the effective creep function of the microcracked mortar. The time-dependent macroscopic behaviour of the masonry is determined thanks to analytical periodic homogenization technique. The relevance of the proposed model is evaluated by reference to a numerical solution computed by finite elements method based on an incremental scheme. A similar methodology can be then adapted to viscoplastic masonries at safe and micro-cracked states using one of the available linearization schemes.

1. Introduction

In the iron and steel industry, the refractory lining of furnaces is often made of masonry in which joints between bricks may be either mortared or dry (i.e. without mortar [1]). In most cases, refractory brick linings are installed with mortar because its use to bond the brickwork provides more resistance to thermal shock and a cushion at the brick joint [7]. The temperature inside these structures can reach 1650 degrees which induces nonlinear mechanical behaviour for the masonry and even leads to the initiation and propagation of cracks in the joints. Note that a thick mortar tends to decrease the stiffness of structure and increases the likelihood of the possible penetration of process materials into the joints, resulting in the deterioration of the lining. So, the use of thin mortar joint is appropriate and necessary in designing the refractory brick lining system. Concerning the creep behaviour of the mortar which can be induced by severe service conditions, various rheological models namely the Maxwell, Kelvin-Voigt, Ross, Burgers and Modified-Maxwell models may be investigated [5]. A number of rheological models are examined to assess their ability to predict the creep of masonry. It was proved that the Modified Maxwell model is the most accurate. According to this result, the Burgers model, namely a Maxwell system connected in series with a Kelvin-Voigt one, and the Modified Maxwell scheme (a parallel combination of the Maxwell model and a spring) models are adopted in this paper to describe the mortar joint's creep. Moreover, in the literature, there are few works investigating the global and local behaviour of viscoelastic masonry. For instance, Cecchi and Tralli [3] used the asymptotic homogenization technique to deduce the behaviour of a safe (without cracks) viscoelastic periodic masonry cell. The effective behaviour of a masonry prism is provided due to periodic homogenization. In the present study, the coupling between the Griffith's theory and the dilute scheme [4] will be applied to provide the effective behaviour of a micro-cracked mortar [6]. In a second step, the expressions proposed in [3] are extended to determine the effective behaviour of a periodic microcracked viscoelastic masonry cell. As a first application, the case of a compressed wall was treated in order to assess the

relevance of the proposed model. Basic steps followed by the proposed model - an extension of the Cecchi & Tralli model [3] to microcracked viscoelastic masonries - are summarized in Figure (1).

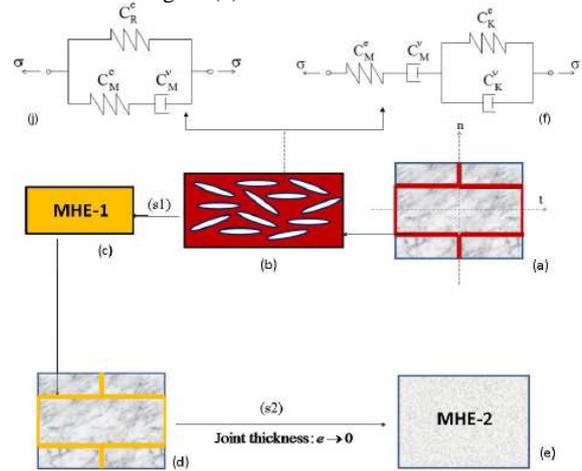


Fig. 1: Main steps of the proposed model

2. Main hypothesis

Microcracked mortar's effective behaviour. Let us firstly recall that the effective stiffness of an elastic porous medium with a homogeneous solid phase tensor C^s is:

$$\tilde{C} = C^s : (I - \phi A^p) \quad (1)$$

where A^p is the average of the strain localisation tensor $A(z)$ over the pore space p and ϕ is the porosity volume fraction. For a spheroid - usual 3D crack model - of aspect ratio $w \ll 1$, the Eshelby tensor S^E is a function of w . Accordingly the components A_{ikl}^p and A_{ikik}^p of the dilute localization tensor $A_{DL}^p = (I - S^E)^{-1}$ in the pore are of the order of $1/w$ and therefore is the ratio of the normal strain ε_{nn} to the macroscopic strain E . Possible none negligible variations of $1/w$ is in contradiction with the assumption of linearity of the localization relationship. To overcome this difficulty, it was proposed [4] to consider the rate-type formulation of the problem

i.e. the strain localization tensor should be replaced by a strain rate localization as in the following $\dot{\epsilon}^p = A(z) : \dot{E}$. Similarly, the rate-type formulation of the Eshelby problem for a spheroidal cavity reads:

$$\dot{\epsilon}^p = (I - S^E(w))^{-1} : \dot{E} \quad (2)$$

Such hypothesis implies that the use of A^p in the homogenized stiffness (1) leads to an estimate of the tangent effective stiffness. Moreover, since the crack porosity ϕ is proportional to w [4], the tangent effective stiffness is mathematically independent of w . This renders the effective behaviour linear elastic. Note that the rate-type hypothesis is indispensable [4] to also avoid troubles related to possible large strain in the direction normal to the crack.

On the other hand, the extension of the linear homogenization schemes to non-aging viscoelasticity is based on the Laplace-Carson (LC) transform [8]. The effective stiffness $\tilde{C} = \langle C : A \rangle$ becomes

$$\tilde{C}^* = \langle C^* : A \rangle \quad (3)$$

in the LC space where C_* is the apparent elastic stiffness. The presence of microcracks implies the existence of nonlinearity at the local scale in the relationship between the crack strain and the macroscopic strain. Accordingly the homogenization of a viscoelastic cracked medium is not as straightforward as (3). The basic idea consists in anticipating that both the microscopic strain field and the displacement discontinuity vectors $[u]_i$ linearly depend on the macroscopic stress. Such hypothesis, confirmed by [6], justifies the use of the LC transform which can be applied to the macroscopic strain related to the microscopic strain and the displacement discontinuity vectors $[u]_i$ between two lips of crack. In the framework of the stress-based dilute scheme [4], the displacement jump $[u]_i$ is linearly related to Σ . Indeed, the normal displacement jump (mode I) at the lips of a crack in an infinite matrix submitted to an isotropic asymptotic macroscopic stress $\Sigma^* = \Sigma^* i$ can be written:

$$[u_n]^* = \frac{4(1-\nu_s^*)}{\pi} \frac{\Sigma^*}{\mu_s^*} \sqrt{l^2 - \rho^2} \quad (4)$$

where l is the crack's radius and ρ is the position of a point M pertaining to the crack's plane. The tangential displacement jump (mode II) under an asymptotic shear stress $\Sigma^* = \Sigma^* n \otimes t$ where t is parallel to the crack's plane reads:

$$[u_t]^* = \frac{4(1-\nu_s^*)}{\pi(2-\nu_s^*)} \frac{\Sigma^*}{\mu_s^*} \sqrt{l^2 - \rho^2} \quad (5)$$

Under respectively an isotropic and deviatoric loading and assuming that all cracks have the same radius l , an integration over all orientations on the unit sphere yields the total crack contribution and allows the determination of the apparent effective bulk's and shear moduli as follows

$$\left\{ \begin{array}{l} \frac{1}{\tilde{k}_{DL}^*} = \frac{1+d_c Q^*}{k_s^*} \quad \text{where} \quad Q^* = \frac{16(1-\nu_s^{*2})}{9(1-2\nu_s^*)} \\ \frac{1}{\tilde{\mu}_{DL}^*} = \frac{1+d_c M^*}{2\mu_s^*} \quad \text{where} \quad M^* = \frac{32(1-\nu_s^*)(5-\nu_s^*)}{45(2-\nu_s^*)} \end{array} \right. \quad (6)$$

in which the crack density parameter and the symbolic

Poisson's ratio of the safe matrix reads respectively:

$$d_c = Nl^3 \quad \text{and} \quad \nu_s^* = \frac{3k_s^* - 2\mu_s^*}{6k_s^* + 2\mu_s^*}. \quad \text{Since the expressions}$$

(6) can not be satisfied rigorously, it was proposed in [6] to identify the best approximation of the effective behaviour in the class of Burgers' (Modified Maxwell's) model if the mortar in its safe state follows the Burgers' (Modified Maxwell's) model. The idea is to satisfy the series expansion of the dilute estimates of the bulk's (6)-a and shear (6)-b moduli to the first order at $p = 0$ and $p \rightarrow \infty$ such that

$$\lim_{t \rightarrow 0} f(t) = \lim_{p \rightarrow \infty} f^*(p), \quad \lim_{t \rightarrow \infty} f(t) = \lim_{p \rightarrow 0} f^*(p) \quad (7)$$

The dilute symbolic moduli of a non-aging linear viscoelastic (n.a.l.v.) microcracked mortar following the MM's model can be approached by expressions similar respectively to those available for a safe mortar following the MM's rheological model

$$\begin{aligned} k_{MM}^* &= k_R(d_c) + \frac{pk_M(d_c)\eta_M^s(d_c)/3}{k_M(d_c) + p\eta_M^s(d_c)/3} \\ \mu_{MM}^* &= \mu_R(d_c) + \frac{p\mu_M(d_c)\eta_M^d(d_c)/2}{\mu_M(d_c) + p\eta_M^d(d_c)/2} \end{aligned} \quad (8)$$

Following idea (7), it is possible to identify the MM's six parameters [9] and accordingly to determine an approximate creep function of a microcracked mortar which matrix follows the MM's model reads

$$\begin{aligned} J_{MM}^{app}(t, d_c) &= \frac{1}{9k_R(d_c)} \left(1 - \frac{k_M(d_c)}{k_R(d_c) + k_M(d_c)} e^{-t/\tau_{MM}^s(d_c)} \right) \\ &+ \frac{1}{3\mu_R(d_c)} \left(1 - \frac{\mu_M(d_c)}{\mu_R(d_c) + \mu_M(d_c)} e^{-t/\tau_{MM}^d(d_c)} \right) \end{aligned} \quad (9)$$

in which the characteristic times of the spherical and deviatoric parts are respectively

$$\begin{aligned} \tau_{MM}^s(d_c) &= \frac{\eta_M^s(d_c)(k_R(d_c) + k_M(d_c))}{3k_M(d_c)k_M(d_c)} \\ \tau_{MM}^d(d_c) &= \frac{\eta_M^d(d_c)(\mu_R(d_c) + \mu_M(d_c))}{2\mu_M(d_c)\mu_M(d_c)} \end{aligned} \quad (10)$$

Masonry's periodic cell global behaviour. For the viscoelastic periodic problem, the auxiliary problem reads

$$\left\{ \begin{array}{l} \dot{E} + \varepsilon(\dot{u}^{per}) = \varepsilon(\dot{u}) \\ \text{div}(\sigma(t)) = 0 \\ \sigma(t) \cdot n \\ u^{per} \\ \langle \sigma(t) \rangle = \Sigma(t) \end{array} \right. \quad \begin{array}{l} \text{anti-periodic on } \partial Y \\ \\ \\ \text{periodic on } \partial Y \end{array} \quad (11)$$

where $\sigma(t)$ is the microscopic stress tensor state, $\varepsilon(\dot{u}(y))$ is the microscopic strain tensor state, u^{per} is the periodic displacement field and \dot{E} is the macroscopic in-plane strain tensor rate. Following the procedure of Cecchi and Tralli [3] accounting for the joint thickness, the expression of the homogenized tensor \tilde{C}^R of the periodic cell is provided in the same quoted reference. If

only the mortar is assumed to be viscoelastic such that its stiffness tensor reads $C^m(t) = C^m(1 + \varphi_m(t))$, the main idea consists to substitute the Young's modulus E^m by $J_m^{-1}(t) = E^m(1 + \varphi_m(t))$.

3. Results

The proposed model is applied to the case of a masonry with hybrid mortar's joints which material's properties are given in tab-1. The properties of the safe bricks are the following: $\nu^m = \nu^b$ and $E^b = 100E^m$ with dimensions $55 \times 250 \text{ mm}^2$. Mortar's joints thicknesses are $e_h = e_v = 10 \text{ mm}$. Note that the characteristic times

$E^M \text{ (MPa)}$	$\tau_M \text{ (s)}$	$E^R \text{ (MPa)}$	$\tau_R \text{ (s)}$	ν^R
4038	46490	2112	90866	0.22

Tab. 1: Elastic and viscous moduli of hybrid mortar [5]

for the spherical and deviatoric hybrid mortar's behaviour are assumed to be equal $\tau^s = \tau^d = \tau$ for the MM's and Burgers' models. The Young's modulus E^R and Poisson's ratio ν^R coincide respectively with E^K and ν^K , properties of the Kelvin-Voigt's spring.

Mortar's effective creep function. Figure (2) and (3) illustrate the evolutions of normalized inverse dilute creep functions of respectively the MM's and Burgers' models with respect to the damage parameter dc and the time t .

For a given crack density parameter $dc = 0; 0.1$ or 0.2 , Figure (2) demonstrates that the MM's model yields to a constant function $J_m^{-1}(t, d_c) / J_m^{-1}(0, 0)$ with variation of the time beyond $t = 10^6 \text{ (s)}$ (i.e. almost 11 days).

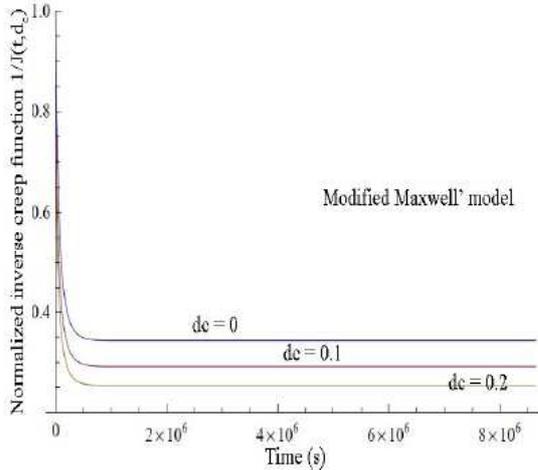


Fig. 2 : Mortar following the MM's model: Variation of $J_{MM}^{-1}(t, d_c) / J_{MM}^{-1}(0, 0)$ versus time for safe mortar ($dc = 0$) and microcracked mortars ($dc = 0.1, dc = 0.2$).

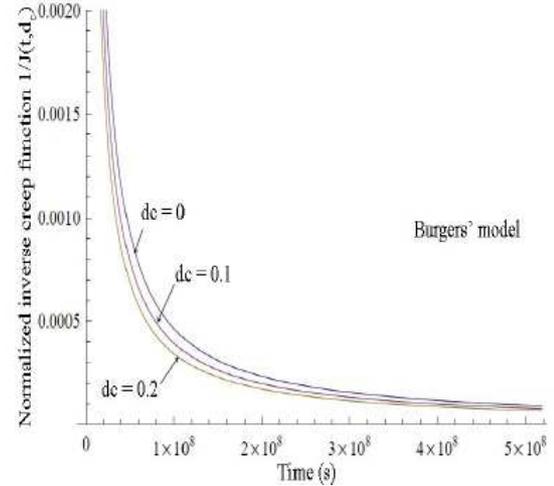


Fig. 2 : Mortar following the Burgers model: Variation of $J_{Bu}^{-1}(t, d_c) / J_{Bu}^{-1}(0, 0)$ versus time for safe mortar ($dc = 0$) and microcracked mortars ($dc = 0.1, dc = 0.2$).

As expected, the increase of dc decreases the level of the asymptotic limits reached by this function. The difference between the MM's curves for different dc is not negligible (around 15%) unlike that observed for the Burgers curves which are very close especially at short and long terms. Figure (3) shows again that the Burgers model leads to vanishing inverse creep functions for $t \geq 3 \times 10^8 \text{ (s)}$ i.e. 1157 days for every crack density value dc .

Case of a compressed masonry panel. We study the case of a masonry panel of dimensions $L = 1560 \text{ mm}$ (length) and $H = 1040 \text{ mm}$ (height) studied in [3] subjected to boundary conditions BC1 with three distributed loads at the top and two lateral edges (see Figure (4)).

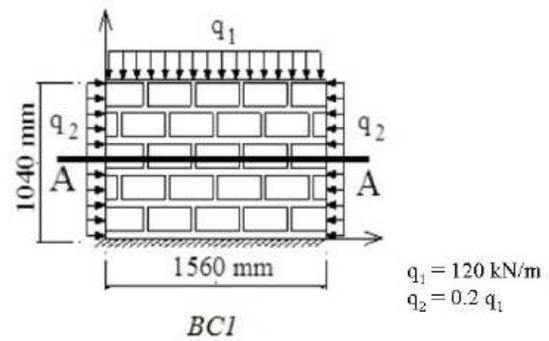


Figure 4: Boundary conditions BC1 applied to the masonry panel [3]

Bricks are assumed to be elastic or rigid. The mortar inside the joints is assumed to be microcracked with a matrix which obeys to linear viscoelastic behaviour following the Burgers or Modified Maxwell's model. As the arrangement of the bricks is regular, the effective behaviour of the panel is assumed to be well estimated by that of a periodic cell (see Figure 1-a). The panel can then be modelled as a homogeneous material which properties coincide with those of the equivalent material MHE-2.

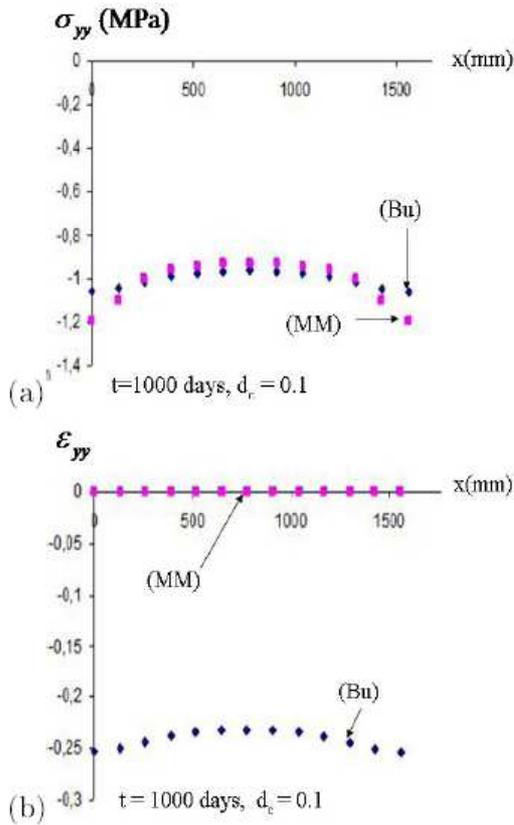


Fig. 5: BC1 boundary conditions: Burgers and MM's models predictions of the stresses σ_{yy} (a) and strain ϵ_{yy} (b) evolutions versus the panel's axis x for: $d_c = 0.1$ at $t = 1000$ days.

Under boundary conditions BC1, the stress (strain) fields σ_{yy} and σ_{xy} (ϵ_{yy} and ϵ_{xy}) are concentrated at both left and right superior corners of the compressed wall. The magnitude of the stress field σ_{yy} is similar for both the MM's and Burgers' models. However, the Burgers estimates of the stress fields σ_{xx} and σ_{xy} are superior to those predicted by the MM's model. These trends are confirmed by the evolutions of the stress components with respect to the axis x as shown on Figures (5). Note that unlike the stress (strain) fields σ_{yy} and σ_{xx} (ϵ_{yy} and ϵ_{xx}) which are symmetric by reference to the axis of symmetry of the panel ($x = L/2$), the shear stress σ_{xy} (shear strain ϵ_{xy}) is anti-symmetric. For the MM's model, it is observed that the strain fields increase slightly with time and damage. Moreover, this model predicts small strains unlike the Burgers model. This rheological model predicts for the masonry large strains similarly to the case of polymer materials. This quantitative aspect for the Burgers predictions is available at $t \geq 25$ days in a safe or damaged state. This result violates the hypothesis of small strains adopted in this study. These results motivate to avoid the use of a mortar following the Burgers model in masonries since such a model leads to vanishing masonry's stiffness's and large strains increasing thus the risk of failure.

4. Conclusions and perspectives

In this paper an extension of the Cecchi & Tralli's model [3] was proposed for microcracked non-aging viscoelastic masonry. This model allows the prediction and localization of mostly stressed and deformed areas in a loaded masonry wall or structure. At long term and for every damage level ($d_c \in [0; 2]$), this study demonstrates that the Burgers rheological model leads to vanishing effective properties by opposition to the Modified Maxwell. Moreover, it has shown that the Burgers model seems to violate the hypothesis of small strains unlike the MM's model. At last, this study should be improved by accounting for possible creep of bricks which often occurs under high temperatures. Other creep models for the mortar joints can be tested. More complex loadings and/or a masonry structure (a refractory lining for e.g.) could also be simulated using the proposed model. These proposed perspectives will be investigated in future works.

References

- [1] Andreev, K., Sinnema, S., Rekik, A., Allaoui S., Blond E., Gasser A., Compressive behaviour of dry joints in refractory ceramic masonry. *Construction and Building Materials* (2012) 34, 402-408.
- [2] Huy Duong, B., *Mécanique de la rupture fragile*. Edition Masson, (1978).
- [3] Cecchi, A., Tralli, A., A homogenized viscoelastic model for masonry structures. *International Journal of Solids and Structures* (2012) 49, 1485-1496.
- [4] Dormieux L., Kondo D., Stress-based estimates and bounds of effective elastic properties: the case of cracked media with unilateral effects. *Computational Materials Science* (2009) 46 (1), 173-179.
- [5] Ignoul, S., Schueremans, L., Tack, J., Swinnen, L., Feytons, S., Binda, L., Gemert, D. V., Balen, K. V., Creep behaviour of masonry structures - failure prediction based on a rheological model and laboratory tests. *Structural Analysis of Historical Constructions* (2007), Conference, New Delhi.
- [6] Nguyen, S. T., Propagation de fissures et endommagement par microfissures dans un milieu visco-élastique linéaire non vieillissant. PhD, University Paris-Est, France (2010).
- [7] Schacht, Ch., *Refractory linings: thermo-mechanical design and applications*. New York: Marcel Dekker Inc. (1995).
- [8] Schapery, R. A., Approximate methods of transform inversion for viscoelastic stress analysis. *Proc. U.S. Nat. Congr. Appl. Mech. ASME 4th. Vol. 2.* (1962), 1075-1085.
- [9] Nguyen T. T. N., Approche multi-échelles pour des maçonneries visco-élastiques. PhD University of Orleans, France (2015).
- [10] Nguyen T. T. N., Rekik A., Gasser A., A multi-level approach for micro-cracked masonry. 11th World Congress on Computational Mechanics (WCCM XI), Barcelona, July (2014).