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Experimental validation of theoretical methods to estimate the energy radiated by elastic waves during an impact

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Abstract

Estimating the energy lost in elastic waves during an impact is an important problem in seismology and in industry. We propose three complementary methods to estimate the elastic energy radiated by bead impacts on thin plates and thick blocks from the generated vibration. The first two methods are based on the direct wave front and are shown to be equivalent. The third method makes use of the diffuse regime. These methods are tested for laboratory experiments of impacts and are shown to give the same results, with error bars from 40% to 300% for impacts on a smooth plate and on a rough block, respectively. We show that these methods are relevant to establish the energy budget of an impact. On plates of glass and PMMA, the radiated elastic energy increases from 2% to almost 100% of the total energy lost as the bead diameter approaches the plate thickness. The rest of the

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lost energy is dissipated by viscoelasticity. For beads larger than the plate thickness, plastic deformation occurs and reduces the amount of energy radiated in the form of elastic waves. On a concrete block, the energy dissipation during the impact is principally inelastic because only 0.2% to 2% of the energy lost by the bead is transported by elastic waves. The radiated elastic energy estimated with the presented methods is quantitatively validated by Hertz's model of elastic impact.

Keywords: elastic waves, acoustic generation, impact source, energy budget

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Table 1: Nomenclature

B	Bending stiffness (J)
c_P, c_S, c_R	Longitudinal, shear and Rayleigh wave speeds (m s^{-1})
e	Coefficient of restitution (-)
e_{tot}, e_c, e_p	Bulk densities of total, kinetic and potential energies (J m^{-3})
$\tilde{e}_{tot}, \tilde{e}_c, \tilde{e}_p$	Time Fourier transform of e_{tot}, e_c, e_p , respectively (J m^{-2})
E	Young's modulus (Pa)
$E_c, \Delta E_c$	Energy of the impact and energy lost during the impact (J)
$E_{tot}(t)$	Total elastic energy radiated within the structure at time t (J)
f	Frequency (s^{-1})
$\tilde{G}_{zz}^P, \tilde{G}_{zz}^S, \tilde{G}_{zz}^R$	Vertical Green's functions associated with compressional, shear and Rayleigh waves ($\text{kg}^{-1} \text{s}^2$)
h	Plate thickness (m)
k	Wave number (m^{-1})
L, S, V	Length (m), surface area (m^2) and volume (m^3)
m	Bead mass (kg)
r, θ, z	Coordinates in the cylindrical reference frame (m)
S_{ij}	Strain tensor (-)
t	Time (s)
T_{ij}	Stress tensor (Pa)
\mathbf{u}_i	Normalized vector of direction i
u_i, v_i, a_i	Surface displacement, speed and acceleration in the direction \vec{u}_i (m; m s^{-1} ; m s^{-2})

$\tilde{U}_i, \tilde{V}_i, \tilde{A}_i$	Time Fourier transform of u_i, v_i and a_i , respectively (m s; m; m s ⁻¹)
v_g, v_ϕ	Group and phase velocities (m s ⁻¹)
V_z	Speed of a bead before impact (m s ⁻¹)
W_{el}, W_{el}^{th}	Radiated energy and theoretical radiated energy (J)
x, y, z	Coordinates in the Cartesian reference frame (m)
β, ξ	Parameters involved in energy calculations
γ	Attenuation coefficient of energy with distance (m ⁻¹)
λ, μ	Lamé coefficients of compression and shear (Pa)
ν	Poisson's coefficient (-)
π_P, π_S, π_R	Energy partitions among P, SV and Rayleigh waves (-)
$\pi_P^{surf}, \pi_S^{surf}, \pi_R^{surf}$	Surface energy partitions among compressional, shear and Rayleigh waves (-)
$\tilde{\Pi}$	Energy density flux (J m ⁻¹ s)
ρ	Density (kg m ³)
τ	Characteristic time of energy attenuation (s)
χ, η	Viscoelastic coefficients of compression and shear (Pa s)
ω	Angular frequency (s ⁻¹)

1. Introduction

The quantification of the energy emitted by a source in the form of elastic waves is a common problem in various fields such as vibroacoustics or shielding. In seismology, the problem was confronted long ago [1] and many approaches have since been developed to estimate the energy of natural sources such as earthquakes [see 2, 3, 4, 5], tremors [6], landslides and rockfalls [e.g. 7, 8, 9, 10, 11]. In the literature, the power spectral density (PSD) of the emitted signal is often measured to quantify the relative energy of different acoustic sources located at the same distance from the sensor and to compare their frequency content. For example, the temporal evolution of the PSD can provide information on river discharge and on the grain size of the bed load [e.g. 12]. The PSD can also be used to characterize crack formation in brittle [13, 14] or granular materials [see 15, for review] and other crackling or crumpling processes [e.g. 16, 17]. Finally, acoustic measurements can be useful in industry for particle sizing in powder transport and in particle streams [e.g. 18, 19]. However, the PSD does not provide an absolute estimate of the elastic energy radiated by the source because it depends on the distance of measurement.

There are three main approaches to determine the absolute radiated elastic energy from acoustic emissions. The first method consists in computing the energy flux crossing a surface surrounding the source. The integration of the energy flux over this surface gives the radiated power. This technique is applied in seismology to estimate the energy radiated in elastic waves during earthquakes [e.g. 5, 20] and rockfalls [e.g. 8, 9, 10].

The second technique to deduce the radiated elastic energy is based on

26 the estimation of the time dependence of the source force. Miller and Pursey
27 [21] and Goyder and White [22] thus estimated the power radiated in an
28 elastic half-space and in an infinite plate, respectively, by a monochromatic
29 harmonic force. In most cases, the force profile is generally unknown but it
30 can be retrieved from the deconvolution of the displacement field with the
31 Green's function tensor [3].

32 These two first methods can however be performed only when the emitted
33 wave front is not mixed with its reflections off the boundaries of the elastic
34 solid. If multiple side reflections occur, the transported energy becomes ho-
35 mogeneously distributed within the elastic solid and decreases exponentially
36 with time due to viscoelastic dissipation. This situation is commonly referred
37 to as a diffuse field in the literature [see 23, 24, 25]. A third energy estimation
38 method, called the diffuse method hereafter, thus consists in extrapolating
39 the radiated energy at the instant of the source from the exponential decrease
40 of the signal coda [see e.g. 25, 26, and references therein].

41 The energy flux, deconvolution and diffuse field methods to estimate the
42 energy lost in elastic waves are used separately by different communities
43 and are based on different assumptions. The first two methods require a
44 sufficiently large elastic solid so that the direct wave front can be clearly
45 distinguished from its reflections off the lateral sides of the elastic solid. On
46 the contrary, with the diffuse method, the elastic solid must be small enough
47 so that multiple side reflections occur. To our knowledge, no study has ever
48 compared these three methods in cases where all three can be applied.

49 The complex seismic signals generated by rockfalls, bed load transport in
50 rivers and granular flows are partially composed of waves generated by the

51 collisions of individual impactors (gravels, boulders,...). Therefore, if we hope
52 to understand these signals, we must first understand the energy budget of
53 individual impacts. The energy that is not radiated in elastic waves during
54 an impact is lost by plastic deformation i.e., not reversible, of the impactor
55 or of the surface [27], by local viscoelastic dissipation around the contact [28]
56 and by conversion into other degrees of freedom of the impactor's motion,
57 such as rotation and other displacement modes. Because of the significant
58 differences between the conditions of each impact on the field, it is however
59 not clear how the energy budget of the impactor depends on its size and
60 speed.

61 In this paper, we propose to use the three methods introduced above
62 to estimate the elastic energy radiated during an individual impact. Steel
63 beads of various diameters are dropped from different heights on two glass
64 and PMMA plates and on a concrete block and the vibration emitted by the
65 impacts is measured with piezoelectric accelerometers. Our main objective
66 is to quantify (i) the differences between the energy estimates and (ii) the
67 errors made using each of the methods. Thin plates are often used in labo-
68 ratory experiments because they are easier to manipulate than thick blocks.
69 In contrast, the problem of waves generation in thick blocks is that encoun-
70 tered on the field. We will show that the methods to estimate the radiated
71 elastic energy in these two geometries are different because different waves
72 are generated. An advantage of the laboratory experiments is that the total
73 energy lost by a rebounding bead can be easily measured from the ratio of
74 the bead velocity after rebound over the approach velocity, i.e. the coefficient
75 of restitution e [e.g. 28]. Therefore, we can establish the energy budget of

76 the impacts and observe how the percentages of energy radiated in elastic
77 waves and dissipated by inelastic processes vary for bead impacts of different
78 diameters and impact speeds on the thin plates and thick block investigated.

79 Section 2 of the paper presents the three methods to derive the energy
80 lost in elastic waves during an impact on thin plates and thick blocks from
81 the normal surface vibration. In section 3, the three methods are compared
82 for laboratory experiments of beads impacts. We also quantify the propor-
83 tion of the total energy radiated in elastic waves and dissipated in inelastic
84 processes. In section 4, we discuss the conditions of applicability of the pre-
85 sented methods. Finally, we evaluate the ability of the analytical model of
86 elastic impact of Hertz [29] [see 30] to predict the radiated elastic energy and
87 the ratio of this energy over the initial energy of the impactor when inelastic
88 dissipation occurs.

89 **2. Estimation of the radiated elastic energy**

90 *2.1. Thin plates*

91 A force $\mathbf{F}(t) = -F_z(t)\mathbf{u}_z$ is applied normally at a given position $(x, y, 0)$
92 on the surface ($z = 0$) of a homogeneous and isotropic thin plate (Figure 1).
93 The expression “thin plate” means that the impact duration is longer than
94 the two-way travel time of the compressional wave in the plate thickness. The
95 emitted elastic waves propagate radially from the impact location (direction
96 \mathbf{u}_r , Figure 1). We consider that the principal mode excited in plates is
97 the fundamental mode A_0 of Lamb, for which the direction of vibration is
98 mainly normal to the plate surface (i.e. direction \mathbf{u}_z , Figure 1) [e.g. 35]. This
99 assumption is verified experimentally in Appendix A. For all the methods

100 tested below, it is therefore assumed that the vibration is only along direction
 101 \mathbf{u}_z (Figure 1).

102 The mode A_0 of Lamb is highly dispersive at low frequencies, when the
 103 wavelength is much greater than the plate thickness h , i.e. within the limit
 104 $kh \ll 1$ where k is the wave number. Indeed, in this regime the mode
 105 A_0 behaves as a flexural wave for which the relation between the angular
 106 frequency ω and the wave number k , i.e. the dispersion relation is [35]:

$$107 \quad \omega = k^2 \sqrt{\frac{B}{\rho h}}, \quad (1)$$

108 where ρ is the plate density. The bending stiffness B is defined by $B =$
 109 $h^3 E / (12(1 - \nu^2))$, where E and ν are the Young's modulus and Poisson's ratio
 110 of the plate material, respectively. The propagation speed of the energy, i.e.
 111 the group velocity $v_g = \partial\omega / \partial k$, therefore also depends on the wave number
 112 k (i.e. on the angular frequency ω):

$$113 \quad v_g(\omega) = 2k \sqrt{\frac{B}{\rho h}}. \quad (2)$$

114 2.1.1. Energy flux method

115 The first method to estimate the radiated elastic energy is based on en-
 116 ergy flux conservation on the first wave arrival. The energy density flux
 117 $\tilde{\Pi}(\omega)$ at frequency ω is by definition the bulk density of the total energy
 118 $\tilde{e}_{tot}(\omega) = \tilde{e}_c(\omega) + \tilde{e}_p(\omega)$, integrated over plate thickness h , multiplied by the
 119 energy speed. But for elastic waves propagating in a homogeneous guide (for
 120 example a plate) such as the A_0 mode, the energy speed is equal to the group
 121 velocity $v_g(\omega)$ [35], so that:

$$122 \quad \tilde{\Pi}(\omega) \hat{=} v_g(\omega) \int_{-h/2}^{h/2} \tilde{e}_{tot}(\omega) dz. \quad (3)$$

123 Moreover, for guided waves the bulk densities of kinetic and potential energies
 124 $\tilde{e}_c(\omega)$ and $\tilde{e}_p(\omega)$ are equal [e.g. 35]:

$$125 \quad \tilde{e}_c(\omega) = \tilde{e}_p(\omega) = \frac{1}{2}\rho|\tilde{V}_z(r, \omega)|^2, \quad (4)$$

126 where $\tilde{V}_z(r, \omega)$ is the time Fourier transform of the surface vibration speed
 127 $v_z(r, t)$.

128 By definition, the elastic energy W_{el} radiated within the plate is given by
 129 [e.g. 35]:

$$130 \quad W_{el} \hat{=} \int_{-\infty}^{+\infty} F_z(\mathbf{r}_0, t)v_z(\mathbf{r}_0, t)dt, \quad (5)$$

131 where \mathbf{r}_0 is the position of force application. According to Parseval's theorem,
 132 this expression is equivalent to the integral over the frequencies ω of the
 133 radiated power, which is the flux $\tilde{\Pi}(\omega)$ integrated over a line surrounding the
 134 impact:

$$135 \quad W_{el} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left[\oint \tilde{\Pi}(\omega) r d\theta \right] d\omega \quad (6)$$

$$136 \quad W_{el} = \frac{1}{\pi} \int_0^{+\infty} \left[v_g(\omega) \iint_S \rho |\tilde{V}_z(r, \omega)|^2 r d\theta dz \right] d\omega. \quad (7)$$

137 As waves propagate radially from the source, one can integrate the surface
 138 element $r d\theta dz$ over a cylinder of height equal to the plate thickness h and
 139 of radius equal to the distance r between the impact and the position of
 140 measurement (Figure 1). In equation (7), the distance r compensates the
 141 geometrical attenuation in $1/r^{1/2}$ of the vibration amplitude $\tilde{V}_z(r, \omega)$ of the
 142 guided wave. In addition, other dissipation is due to the intrinsic viscosity of
 143 the plate. This dissipation can be modeled by $\exp(-\gamma(\omega)r)$, where γ is the
 144 coefficient representing the frequency-dependent attenuation of energy with

145 distance r in the plate (see Appendix B):

$$146 \quad W_{el} = \int_0^{+\infty} 2rh\rho v_g(\omega) |\tilde{V}_z(r, \omega)|^2 \exp(\gamma(\omega)r) d\omega. \quad (8)$$

147 Note that if we consider a constant group velocity v_g , we obtain an expression
 148 for W_{el} similar to that used by Hibert et al. [10] to estimate the energy of
 149 surface waves generated by rockfalls in a homogeneous surface layer of depth
 150 h in Dolomieu crater, Réunion Island.

151 2.1.2. Deconvolution method

152 As opposed to the energy flux method, here we compute the radiated elas-
 153 tic energy W_{el} using equation (5) from the estimation of the time dependence
 154 of the force of impact. Indeed, the energy W_{el} transferred into the plate at
 155 the point of application of a normal force $F_z(\mathbf{r}_0, t)$ is the time integral of the
 156 radiated power, which is given by Goyder and White [22]:

$$157 \quad \mathbf{F}(\mathbf{r}_0, t) \cdot \mathbf{v}(\mathbf{r}_0, t) = \frac{F_z(\mathbf{r}_0, t)^2}{8\sqrt{B\rho h}}. \quad (9)$$

158 Then, according to Parseval's theorem,

$$159 \quad W_{el} = \frac{1}{\pi} \int_0^{+\infty} \frac{|\tilde{F}_z(\omega)|^2}{8\sqrt{B\rho h}} d\omega. \quad (10)$$

160 We can deduce the normal force $\tilde{F}_z(\omega)$ in time Fourier domain from the
 161 expression of the first arrival of the vertical vibration speed $\tilde{V}_z(r, \omega)$ as a
 162 function of the plate Green's function $\tilde{G}_{zz}(r, \omega)$ [3]:

$$163 \quad \tilde{V}_z(r, \omega) = i\omega \tilde{G}_{zz}(r, \omega) \tilde{F}_z(\omega), \quad (11)$$

164 where the modulus of the plate Green's function can be approximated by,
 165 for $kr \gg 1$ [e.g. 36]:

$$166 \quad |\tilde{G}_{zz}(r, \omega)| = \frac{1}{8Bk^2} \sqrt{\frac{2}{\pi kr}} \quad (12)$$

167 Finally, the radiated elastic energy W_{el} is given by:

$$168 \quad W_{el} = \frac{1}{8\pi\sqrt{B\rho h}} \int_0^{+\infty} \omega^{-2} \frac{|\tilde{V}_z(r, \omega)|^2}{|\tilde{G}_{zz}(r, \omega)|^2} \exp(\gamma(\omega)r) d\omega. \quad (13)$$

169 where $\exp(-\gamma(\omega)r)$ models the viscoelastic dissipation.

170 Interestingly, if we replace the Green's function $|\tilde{G}_{zz}(r, \omega)|$ by its expres-
 171 sion [equation (12)], we retrieve the same expression of W_{el} as for the energy
 172 flux method under the condition that $\omega = k^2\sqrt{B/\rho h}$, which is valid for
 173 $kh \ll 1$. Therefore, the two methods are equivalent at low frequencies
 174 $\omega \ll \sqrt{B/\rho h}/h^2$.

175 Note that the operation of dividing the amplitude of the vibration $|\tilde{V}_z(r, \omega)|$
 176 by the Green's function $|\tilde{G}_{zz}(r, \omega)|$ is not trivial because the inverse Green's
 177 function diverges when k (or ω) tends towards 0 [see e.g. 31, 37]. Therefore,
 178 we cannot deconvolve the signal and estimate the energy W_{el} below a cutoff
 179 frequency. In practice, we cut all frequencies below 3 kHz in the amplitude
 180 spectrum $|\tilde{V}_z(r, \omega)|$ before dividing it by the Green's function. Using a syn-
 181 thetic signal obtained by the convolution of the Hertz force for the elastic
 182 impact of bead diameters smaller than 20 mm with the Green's function in
 183 equation (12), we estimate that the energy W_{el} of the signal after the cut-
 184 off at 3 kHz is less than 5% smaller than the exact radiated elastic energy
 185 (Figure 2).

186 2.1.3. Diffuse method

187 This technique is derived from classical methods used in room acoustics
 188 [see e.g. 25, and references therein]. When the emitted wave is reflected off
 189 the boundaries many times, the elastic field becomes diffuse, i.e. homoge-
 190 neously distributed over the plate and equipartitioned. When the field is

191 equipartitioned, the potential and kinetic energy are equal. At a given time
 192 t , the average over several periods (noted \bar{x}) of the total energy $E_{tot}(t)$ within
 193 the plate therefore satisfies:

$$194 \quad \overline{E_{tot}(t)} \approx \rho h S \overline{v_z(t)^2}. \quad (14)$$

195 where ρ , h and S are respectively the plate density, thickness and surface
 196 and $\overline{v_z(t)^2}$ is the average of the normal squared vibration speed $v_z(r, t)^2$ over
 197 several periods. When the field is diffuse, energy losses due to viscoelastic
 198 dissipation are proportional to the total energy within the structure:

$$199 \quad \frac{d\overline{E_{tot}(t)}}{dt} \approx -\frac{\overline{E_{tot}(t)}}{\tau}, \quad (15)$$

200 with τ , the characteristic time of energy dissipation. In a narrow frequency
 201 range centered on ω_0 , this time equals $(\gamma(\omega_0)v_g(\omega_0))^{-1}$ (see Appendix B).
 202 As a consequence, the energy decreases exponentially with time:

$$203 \quad \overline{E_{tot}(t)} \approx \overline{E_{tot}(t_0)} \exp\left(-\frac{t-t_0}{\tau}\right), \quad (16)$$

204 where t_0 is the instant of the impact. The elastic energy radiated in the plate
 205 at the instant t_0 is therefore:

$$206 \quad W_{el} = \overline{E_{tot}(t_0)} \approx \rho h S \overline{v_z(t_0)^2}. \quad (17)$$

207 Knowing the instant of impact t_0 and the characteristic time τ is sufficient to
 208 determine the radiated elastic energy W_{el} . Note that $\overline{v_z(t_0)^2}$ may fluctuate
 209 with the position of vibration measurement depending on how the assembly
 210 of proper modes of the plate are excited. Equation (17) requires that only one
 211 mode is excited within the plate because the characteristic time τ of energy

212 attenuation depends on the mode. Therefore, we assume that no mode con-
 213 version occurs off the plate boundaries between the normally vibrating mode
 214 A_0 and transversal horizontal (TH) or longitudinal (S_0) modes. This hy-
 215 pothesis is valid provided that the plate boundaries are straight and smooth
 216 [e.g. 35].

217 2.2. Thick blocks

218 A force $\mathbf{F}(t) = -F_z \mathbf{u}_z$ is applied normally at a given position $(x, y, 0)$ over
 219 the surface ($z = 0$) of a homogeneous and isotropic thick block (Figure 3).
 220 The expression “thick block” means that the duration of impact is shorter
 221 than the two-way travel time of the compressional wave from the closest side
 222 of the block.

223 The problem of wave generation in a semi-infinite solid is commonly re-
 224 ferred as Lamb’s problem [1]. It has been treated many times for various
 225 sources below the surface [e.g. 1, 3, 38] and at the surface [e.g. 1, 21, 38, 39].
 226 The elastic energy W_{el} initially input by a normal surface force within blocks
 227 is distributed among three different modes: compressional wave P , shear ver-
 228 tical wave SV and surface Rayleigh waves. Sánchez-Sesma et al. [40] give the
 229 partitions π_P , π_S and π_R of energy radiated in P , SV and Rayleigh waves
 230 respectively, as a function of the Poisson ratio ν . For a concrete block with
 231 $\nu = 0.4$, the energy partition is $\pi_R \approx 61\%$ in Rayleigh waves, $\pi_S \approx 35\%$ in
 232 SV waves and only $\pi_P \approx 4\%$ in P waves.

233 The vibration propagating at the surface of the block contains Rayleigh
 234 waves but also compressional and shear waves as shown by the expression of
 235 the Green’s function \tilde{G}_{zz} owing to a normal surface force (Appendix C):

$$236 \quad \tilde{G}_{zz} = \tilde{G}_{zz}^P + \tilde{G}_{zz}^S + \tilde{G}_{zz}^R \quad (18)$$

237 where \tilde{G}_{zz}^P , \tilde{G}_{zz}^S and \tilde{G}_{zz}^R are the contributions of each mode:

$$238 \quad \tilde{G}_{zz}^P(r, \omega) \approx -\frac{i}{\mu} A_P \frac{k_1}{(k_1 r)^2} \exp(-i\omega r/c_P), \quad (19)$$

$$239 \quad \tilde{G}_{zz}^S(r, \omega) \approx -\frac{i}{\mu} A_S \frac{k_1}{(k_1 r)^2} \exp(-i\omega r/c_S), \quad (20)$$

$$240 \quad \tilde{G}_{zz}^R(r, \omega) \approx -\frac{i}{\mu} A_R k_1 \sqrt{\frac{2}{\pi k_1 r}} \exp\left(-i\left(\omega r/c_R - \frac{\pi}{4}\right)\right). \quad (21)$$

241 In these equations, A_P , A_S and A_R are functions of Poisson's ratio ν (Appendix C),
 242 c_P , c_S and c_R are the compressional, shear and Rayleigh wave speeds, respec-
 243 tively, μ is the Lamé shear modulus and $k_1 = \omega/c_P$ is the wave number. The
 244 expressions of these Green's functions show that the energy of compressional
 245 and shear waves at the surface decreases with frequency f and distance r
 246 as $(fr)^{-4}$ while the energy of Rayleigh waves varies as f/r because they are
 247 guided at the surface. Therefore, the Rayleigh waves dominate the signal at
 248 high frequencies and far from the impact [1, 21].

249 In the following, we apply the energy flux and deconvolution methods on
 250 the Rayleigh waves to deduce the absolute radiated elastic energy W_{el} . Con-
 251 sequently, we need to determine the percentage $\pi_R^{\text{surf}}(r)$ of Rayleigh waves in
 252 the energy at the position r from the impact. To that end, we compute the
 253 impact force from Hertz's elastic model [e.g. 30] (Figure 4a) and convolve it
 254 with the Green's functions \tilde{G}_{zz}^P , \tilde{G}_{zz}^S and \tilde{G}_{zz}^R and the total Green's function
 255 at $r = 20$ cm on concrete (Figure 4b) to obtain the synthetic vibration ac-
 256 celeration $a_z(r, t)$ associated with each mode (Figure 4c). The compressional
 257 wave arrives clearly before the other modes. However, shear and Rayleigh
 258 waves arrive roughly at the same time and are mixed together. The total
 259 vibration acceleration $a_z(r, t)$ is very similar to that of the Rayleigh waves
 260 with the exception of the small wavelet corresponding to the compressional

261 wave. Because shear and Rayleigh waves are out of phase, the maximum
 262 amplitude of the total vibration acceleration is 12% lower than that of the
 263 Rayleigh waves only and its squared integral is 18% lower.

264 The contribution of each mode n to the signal energy as a function of the
 265 frequency f is therefore simply $|\tilde{A}_z^n(r, f)|^2 / \sum_i |\tilde{A}_z^i(r, f)|^2$, where $|\tilde{A}_z^n(r, f)|$ is
 266 the amplitude spectrum of the signal $a_z^n(r, t)$ associated with the n^{th} mode
 267 (Figure 4d). Shear waves dominate the signal at low frequencies up to about
 268 $f = 7000$ Hz, where Rayleigh waves become overriding. The percentage of
 269 compressional waves is much smaller ($<10\%$) and decreases with frequency.
 270 For frequencies greater than 30 kHz, the surface vibration contains only
 271 Rayleigh waves. The integration of these energy partitions over the fre-
 272 quencies f gives the percentages of Rayleigh, compressional and shear waves
 273 at the surface (Figures 4e and 4f). For example, the percentages for a 5 mm
 274 diameter steel bead dropped from a height of 10 cm at $r = 20$ cm on a
 275 concrete block ($\nu = 0.4$) are respectively $\pi_R^{\text{surf}} = 98.5\%$, $\pi_P^{\text{surf}} = 0.1\%$ and
 276 $\pi_S^{\text{surf}} = 1.4\%$. Note that, at a given distance from the impact, the percentage
 277 π_R^{surf} of Rayleigh waves decreases as the bead diameter d increases (Figure
 278 4e) and the height of fall H decreases (Figure 4f). For example, at $r = 20$
 279 cm, Rayleigh waves represent 99.9% of the signal for $d = 1$ mm while only
 280 about 71% for $d = 20$ mm (Figure 4e). In other words, if we assume that the
 281 signal contains only Rayleigh waves at $r = 20$ cm from the impact, the error
 282 introduced in the energy W_{el} is negligible for a bead of diameter $d = 1$ mm
 283 but is about 30% for $d = 20$ mm. On the other hand, the influence of the
 284 height of fall H on this percentage is negligible over the range of heights
 285 investigated here (5 cm to 50 cm, Figure 4f).

286 For the last method, based on the diffuse field approximation, the parti-
 287 tions π_R and $\pi_R^{\text{surf}}(r)$ indicated above are no longer valid because the energy
 288 is distributed over the three directions of space x , y and z . In this case, we
 289 use the horizontal to vertical amplitude ratio

$$290 \quad \left(\frac{\mathcal{H}}{\mathcal{V}}\right)_{\text{diffuse}} = \frac{|\tilde{V}_x(r, \omega)| + |\tilde{V}_y(r, \omega)|}{|\tilde{V}_z(r, \omega)|}, \quad (22)$$

291 calculated by [40] for diffuse fields, to deduce the radiated elastic energy W_{el}
 292 from the normal surface vibration speed $\tilde{V}_z(r, \omega)$, using the same method as
 293 for plates (see section 2.1.3).

294 2.2.1. Energy flux method

295 We can estimate the absolute energy radiated in elastic waves W_{el} from
 296 the energy transported by Rayleigh waves. Because Rayleigh waves prop-
 297 agates radially from the impact location, their energy W_{el}^R is calculated
 298 similarly to the radiated elastic energy in plates [equation (7)]:

$$299 \quad W_{el}^R = \frac{1}{\pi} \int_0^{+\infty} \left[\rho v_g \iint_S |\tilde{V}^R(r, z, \omega)|^2 r d\theta dz \right] d\omega. \quad (23)$$

300 Rayleigh waves have an elliptical motion parallel to the direction of propaga-
 301 tion and normal to the surface, their vibration speed can therefore be written
 302 $\tilde{\mathbf{V}}^R = \tilde{V}_r^R \mathbf{u}_r + \tilde{V}_z^R \mathbf{u}_z$ (Figure 3) [e.g. 3]. The asymptotic amplitudes far from
 303 the source of the vibration speeds \tilde{V}_r^R and \tilde{V}_z^R are given as a function of
 304 depth z by Miller and Pursey [21]:

$$305 \quad |\tilde{V}_r^R(r, z, \omega)| \approx \omega \frac{\tilde{F}_z(\omega)}{\mu f_0'(x_0)} \sqrt{\frac{\pi k_1 x_0^3}{2r}} \left(2\sqrt{x_0^2 - 1} \sqrt{x_0^2 - \xi^2} e_\xi - (2x_0^2 - \xi^2) e_1 \right) \quad (24)$$

$$306 \quad |\tilde{V}_z^R(r, z, \omega)| \approx \omega \frac{\tilde{F}_z(\omega)}{\mu f_0'(x_0)} \sqrt{\frac{\pi k_1 x_0 (x_0^2 - 1)}{2r}} \left(2x_0^2 e_\xi - (2x_0^2 - \xi^2) e_1 \right), \quad (25)$$

307 where μ is the Lamé shear modulus, $k_1 = \omega/c_P$, with angular frequency $\omega =$
308 $2\pi f$ and compressional wave speed c_P , $f_0(x) = (2x^2 - \xi^2)^2 - 4x^2\sqrt{(x^2 - 1)(x^2 - \xi^2)}$,
309 x_0 is the positive root of f_0 (Figure 5), $\xi = \sqrt{2(1 - \nu)/(1 - 2\nu)}$, ν is Pois-
310 son's ratio, $e_\xi = \exp(-k_1 z \sqrt{x_0^2 - \xi^2})$ and $e_1 = \exp(-k_1 z \sqrt{x_0^2 - 1})$. From
311 these equations, we deduce that the total vibration speed \tilde{V}^R is related to
312 its vertical component \tilde{V}_z^R by:

$$313 \quad |\tilde{V}^R(r, z, \omega)|^2 = |\tilde{V}_z^R(r, z, \omega)|^2 \left[1 + \left(\frac{\mathcal{H}}{\mathcal{V}} \right)_R^2 \right] \quad (26)$$

314 with

$$315 \quad \left(\frac{\mathcal{H}}{\mathcal{V}} \right)_R = \frac{|V_r^R(r, z, \omega)|}{|V_z^R(r, z, \omega)|} = \frac{x_0}{\sqrt{x_0^2 - 1}} \frac{2\sqrt{x_0^2 - 1}\sqrt{x_0^2 - \xi^2}e_\xi - (2x_0^2 - \xi^2)e_1}{2x_0^2e_\xi - (2x_0^2 - \xi^2)e_1}. \quad (27)$$

316 Equation (25) also shows that \tilde{V}_z^R decreases exponentially with depth z as

$$317 \quad \tilde{V}_z^R(r, z, \omega) = \tilde{V}_z^R(r, z = 0, \omega) \frac{2x_0^2e_\xi - (2x_0^2 - \xi^2)e_1}{\xi^2}. \quad (28)$$

318 The integral over the surface S surrounding the impact in equation (23)
319 then becomes:

$$320 \quad \iint_S |\tilde{V}^R(r, z, \omega)|^2 r d\theta dz = 2\pi r \frac{|\tilde{V}_z^R(r, z = 0, \omega)|^2}{k_1} A(\nu), \quad (29)$$

321 where $A(\nu) = \int_0^{+\infty} \left[1 + \left(\frac{\mathcal{H}}{\mathcal{V}} \right)_R^2 \right] (2x_0^2e_\xi - (2x_0^2 - \xi^2)e_1)^2 / \xi^4 d(k_1 z)$ is a function
322 of Poisson's ratio ν only and equal to 1.6 for our concrete block with $\nu = 0.4$.

323 Furthermore, as discussed earlier, the squared vibration speed of Rayleigh
324 waves $|\tilde{V}_z^R(r, z = 0, \omega)|^2$ represents a proportion $\pi R^{\text{surf}}(r)$ of the vertical squared
325 vibration speed $|\tilde{V}_z(r, z = 0, \omega)|^2$, that also includes the effects of compres-
326 sional and shear waves. Thus, using equations (23), (26) and (29), we express

327 the energy W_R of Rayleigh waves as a function of the sole vertical component
 328 of the vibration speed measured at the surface of the block:

$$329 \quad W_{el}^R = 2\rho r v_g c_p \pi_R^{\text{surf}}(r) A(\nu) \int_0^{+\infty} |\tilde{V}_z(r, z=0, \omega)|^2 \omega^{-1} \exp(\gamma(\omega)r) d\omega, \quad (30)$$

330 where $\exp(\gamma(\omega)r)$ counterbalances the viscoelastic dissipation of energy. In
 331 practice, we cut the frequencies below 3 kHz in the amplitude spectrum
 332 $|\tilde{V}_z(r, z=0, \omega)|$ to avoid the divergence of the term within the integral as ω
 333 tends towards 0 (see section 2.1.2).

334 Finally, the energy W_{el}^R of Rayleigh waves represents only a percentage
 335 π_R of the total elastic energy W_{el} radiated within the block, thus:

$$336 \quad W_{el} = \frac{W_{el}^R}{\pi_R} = 2\rho r v_g c_p \frac{\pi_R^{\text{surf}}(r)}{\pi_R} A(\nu) \int_0^{+\infty} |\tilde{V}_z(r, z=0, \omega)|^2 \omega^{-1} \exp(\gamma(\omega)r) d\omega. \quad (31)$$

337 2.2.2. Deconvolution method

338 Miller and Pursey [21] deduced an analytical expression for the radiated
 339 elastic energy W_{el} from the surface deformation created by the action of a
 340 point force $\tilde{F}(\omega)$ (in the time Fourier domain) on the surface of a semi-infinite
 341 solid, using equation (5):

$$342 \quad W_{el} = \frac{\xi^4 \beta}{2\pi^2 \rho c_p^3} \int_0^{+\infty} \omega^2 |\tilde{F}(\omega)|^2 d\omega, \quad (32)$$

343 where β is the imaginary part of

$$344 \quad \int_0^X \frac{x \sqrt{x^2 - 1}}{f_0(x)} dx, \quad (33)$$

345 with $f_0(x) = (2x^2 - \xi^2)^2 - 4x^2 \sqrt{(x^2 - 1)(x^2 - \xi^2)}$, $\xi = \sqrt{2(1 - \nu)/(1 - 2\nu)}$
 346 and X a number greater than the real root x_0 of f_0 . The coefficient β depends

347 only on the Poisson ratio ν (see Appendix D for details on the calculation
348 of β).

349 In our case, the impact force $\tilde{F}(\omega)$ is vertical and can be obtained from the
350 normal surface vibration speed $\tilde{V}_z^R(r, z = 0, \omega)$ using equation (11) with the
351 Green's function of Rayleigh waves [equation (21)]. Therefore, the radiated
352 elastic energy W_{el} is given by:

$$353 \quad W_{el} = \frac{\xi^4 \beta \pi_R^{\text{surf}}(r)}{2\pi^2 \rho c_p^3} \int_0^{+\infty} \frac{|\tilde{V}_z^R(r, z = 0, \omega)|^2}{|\tilde{G}_{zz}^R(r, \omega)|^2} \exp(\gamma(\omega)r) d\omega \quad (34)$$

354 To compute the radiated elastic energy, we perform the same operation as
355 in section 2.1.2 because the inverse Green's function $1/\tilde{G}_{zz}$ also diverges as
356 ω tends toward 0.

357 If we replace $|\tilde{G}_{zz}^R(r, \omega)|$ by its expression in equation (21), we obtain:

$$358 \quad W_{el} = 2\rho r v_g c_p \pi_R^{\text{surf}}(r) \frac{\beta x_0}{8\pi A_R^2} \int_0^{+\infty} |\tilde{V}_z(r, z = 0, \omega)|^2 \omega^{-1} \exp(\gamma(\omega)r) d\omega \quad (35)$$

359 Note that the energy W_{el} calculated with the energy flux method [equation
360 (31)] and the energy calculated from the impact force [equation (35)] are pro-
361 portional to the same integral. The discrepancy between the energies com-
362 puted with the two methods can be estimated by the ratio of the coefficients
363 in front of the integral in equations (31) and (35), i.e. $\beta x_0 \pi_R / 8\pi A_R^2 A(\nu)$,
364 which equals 1 ± 10^{-4} regardless of Poisson's ratio ν . The two methods are
365 therefore equivalent.

366 2.2.3. Diffuse method

367 After many reflections of the wave front off the block boundaries, we
368 assume that the energy within the block is distributed along the three direc-
369 tions of space, i.e. that the field is diffuse [e.g. 25]. The ratio of horizontal

370 to vertical amplitude at the surface of a semi-infinite medium under a dif-
 371 fuse field approximation is given by Sánchez-Sesma et al. [40] for a normal
 372 loading force as a function of the Poisson ratio ν : $\left(\frac{\mathcal{H}}{\mathcal{V}}\right)_{\text{diffuse}} \approx 1.245 + 0.348\nu$.
 373 For our concrete block ($\nu = 0.4$), $\left(\frac{\mathcal{H}}{\mathcal{V}}\right)_{\text{diffuse}} \approx 1.38$. From the hypothesis of
 374 energy equipartition, we obtain an expression for the radiated elastic energy
 375 W_{el} that is similar to that previously demonstrated for plates [equation (17)]:

$$376 \quad W \approx \left(1 + \left(\frac{\mathcal{H}}{\mathcal{V}}\right)_{\text{diffuse}}^2\right) \rho V \overline{v_z(t_0)^2}, \quad (36)$$

377 where V is the block volume. In the case of blocks, the factor $1 + \left(\frac{\mathcal{H}}{\mathcal{V}}\right)_{\text{diffuse}}^2$
 378 compensates the energy distribution over the three directions of space.

379 **3. Experimental test**

380 *3.1. Setup*

381 We conduct impact experiments on two thin plates and a thick block to
 382 test the three methods presented in section 2. Piezoelectric charge shock
 383 accelerometers (type 8309, *Brüel & Kjaer*) record the normal acceleration
 384 generated by impacts at various positions. The surface vibration is digitalized
 385 with an acquisition rate of 0.3 MHz. The accelerometers have a rather flat
 386 response over a wide range of frequencies (1 Hz to 54 kHz). Note that only one
 387 accelerometer is necessary to measure the radiated elastic energy regardless
 388 of the method used because the radiated wave field is isotropic. Nevertheless,
 389 several sensors are placed at different distances from the impact to measure
 390 wave dispersion (Appendix A) and energy attenuation, i.e. the coefficients
 391 γ and τ (Appendix B and Table 2).

392 The impactors are spherical steel beads of density 7800 kg m^{-3} and di-
393 ameter ranging from 1 mm to 20 mm. The beads are dropped from various
394 heights from 2 cm to 25 cm, without initial velocity and rotation, on a cir-
395 cular glass plate with a radius of 40 cm and thickness of 1 cm, on a 1.2×1
396 m^2 PMMA plate with a thickness of 1 cm and on a $3 \times 1.5 \times 0.6 \text{ m}^3$ concrete
397 block. The properties of these structures are presented in Table 2.

398 *3.2. Description of the measured signals*

399 The two plates and the block were selected to check as comprehensively
400 as possible the assumptions made in the previous section to calculate the
401 radiated elastic energy. On the one hand, after each bead impact on the
402 glass plate and on the concrete block, the accelerometers record a long coda
403 owing to the multiple side reflections off the lateral sides of the structure
404 (Figure 6a and 7a). In these two structures, there are enough reflections for
405 a diffuse field to be set up and we can apply the diffuse method to estimate
406 the radiated elastic energy. However, it is not possible to use this method
407 on the PMMA plate because side reflections are too attenuated (Figure 8a).
408 After about 30 side reflections in the glass plate and 10 in the concrete block,
409 the averaged squared vibration amplitude $|\overline{a_z(r, t)}|^2$ decreases exponentially
410 with time, until it reaches the noise level (Figures 6b and 7b). We can thus
411 estimate the characteristic time τ of energy attenuation in these structures
412 (see Appendix B and Table 2).

413 On the other hand, the two plates and the block are sufficiently large to
414 record a majority of the first arrival of the emitted vibration before the return
415 of the first side reflection (Figures 6c, 7c and 8a). We can therefore apply
416 the methods based on the first arrival i.e., the energy flux and deconvolution

417 methods, to determine the elastic energy radiated by the impacts on each
418 investigated structure.

419 The time Fourier transform of the first arrival gives the amplitude spec-
420 trum $|\tilde{A}_z(r, f)|$ (Figures 6d, 7d and 8c). Impacts of beads excite a wide
421 frequency range up to about 80 kHz and are characterized by an energy peak
422 with a central frequency between 2 kHz and 40 kHz (Figure 9). The dura-
423 tion of impact increases with the bead diameter and consequently the peak
424 frequency of the generated vibration decreases. Interestingly, for impacts of
425 beads of diameter smaller than 5 mm on the glass plate, the peak frequency
426 is constant and equals 34 kHz. This is discussed in section 4.2.

427 3.3. Radiated elastic energy

428 For experiments of bead impacts on the glass and PMMA plates, the
429 energy flux and deconvolution methods give almost identical results (Figure
430 10a and 10c). The energy obtained with deconvolution is 2% greater than
431 that obtained with the energy flux method on the glass plate and 5% greater
432 on PMMA. On the glass plate, we also observe a fair agreement between the
433 energy estimated using the energy flux method and the diffuse method (Fig-
434 ure 10b). The lower signal to noise ratio for small beads (i.e. for $W_{el} < 10^{-7}$
435 J, Figure 10b) leads to an error of +20% on the radiated elastic energy W_{el}
436 with the diffuse method with respect to the energy flux method. However,
437 the discrepancy between the methods is lower than the uncertainties on the
438 energy W_{el} (± 1 standard deviation). The error is about $\pm 37\%$ with the en-
439 ergy flux method, $\pm 36\%$ with the deconvolution method and $\pm 53\%$ with the
440 diffuse method. The error is greater ($\pm 60\%$) for beads smaller than 2 mm
441 (i.e. for $W_{el} < 10^{-7}$ J, Figure 10) because of the lower signal to noise ratio.

442 For impacts on the concrete block, the radiated elastic energy W_{el} ob-
 443 tained with the deconvolution method is equal to that computed with the
 444 energy flux method, as discussed in section 2.2.2 (Figure 11a). The energy
 445 estimation error with these two methods is that of the integral $\int_0^{+\infty} |\tilde{V}_z(r, z =$
 446 $0, \omega)|^2 \omega^{-1} \exp(\gamma(\omega)r) d\omega$ in equations (31) and (35) and is about $\pm 75\%$. We
 447 cannot use the diffuse method for beads smaller than 2 mm in diameter
 448 because not enough side reflections can be recorded. For larger beads, the
 449 energy measured with the diffuse method is between 0.3 to 3 times that ob-
 450 tained with the other methods (Figure 11b). Error bars with the diffuse
 451 method are between $\pm 70\%$ and $\pm 300\%$ and are of the same order of magni-
 452 tude as the difference between the methods.

453 Let us discuss the possible source of errors in our experiments. For the
 454 energy flux and deconvolution methods, the error bars are greater on the
 455 block ($\approx 75\%$) than on the plates ($\approx 36\%$). This is probably because we
 456 can less clearly identify the first emitted wave train from the side reflections
 457 in the concrete block than in the plates (Figures 6c, 7c and 8b). Moreover,
 458 the rough surface of the concrete block is a likely cause for greater scattering
 459 of the results than on the smooth glass and PMMA plates, in particular for
 460 beads of diameter $d < 3$ mm for which the depth of penetration into the
 461 concrete is of the same order of magnitude as the surface roughness. The
 462 diffuse method is based on statistical assumptions that induce additional
 463 errors. First, the diffuse regime is reached after at least 30 side reflections
 464 in the glass plate and 10 in the concrete block. Consequently, if damping
 465 is important, as it is the case in concrete, the diffuse field is not completely
 466 set, the exponential decay of the energy is not clear and the characteristic

467 time τ of energy dissipation is not well estimated (Figure 7b). The error on
468 τ therefore leads to either overestimate or underestimate the radiated elastic
469 energy. Secondly, an exponential decay of the energy assumes that the energy
470 dissipation is frequency independent, which is not completely the case here
471 (Table 2).

472 3.4. Elastic transfer efficiency

473 We measure the total energy ΔE_c lost by the beads from their vertical
474 coefficient of restitution e [e.g. 28, 41]. The proportion of energy radiated in
475 elastic waves W_{el} with respect to the lost energy ΔE_c , i.e. the elastic transfer
476 efficiency, increases with bead diameter up to $d = 5$ mm and decreases for
477 $d \geq 10$ mm (Figure 12a). The ratio $W_{el}/\Delta E_c$ does not depend on the fall
478 height H for impacts on the PMMA plate and concrete block (Figure 12b).
479 On the glass plate, for bead diameters d between 2 mm and 5 mm and
480 fall heights $H > 5$ cm, the radiated elastic energy W_{el} is greater than the
481 lost energy ΔE_c , which is impossible. We will explain this discrepancy in the
482 discussion section. More energy is converted into elastic waves for impacts on
483 the glass plate and on the PMMA plate than on the concrete block. Indeed,
484 the ratio $W_{el}/\Delta E_c$ is never greater than 2% on the concrete block while on
485 the PMMA plate, almost all the lost energy is radiated elastically for bead
486 diameters $d \geq 5$ mm (Figure 12a), regardless of the fall height H (Figure
487 12b).

488 4. Discussion

489 4.1. Comparison between the different methods

490 It is valid to use the energy flux and deconvolution methods when the
491 first wave arrival can be discerned from side reflections or when the side re-
492 flections are very attenuated. The diffuse method is applicable provided that
493 enough side reflections occur to equipartition the energy. The diffuse method
494 therefore becomes very efficient in a small structure. Another advantage of
495 the diffuse method is that there is no assumption on the direction of the
496 impact force.

497 The three methods can be used with only one sensor to measure the
498 radiated elastic energy but the precision of the energy estimation can be
499 enhanced when several sensors are used. For the direct wave methods, the
500 use of several sensors can take into account an anisotropic emission. For the
501 diffuse method, it can compensate for a not completely equipartitioned field
502 because we estimate the averaged value of the energy over the surface of the
503 structure.

504 4.2. Comparison with Hertz's model of elastic impact

505 Impacts of spherical beads on a plane surface are often compared with
506 Hertz's [29] theory of elastic impact [e.g. 19, 30, 31, 28, 32, 33]. For example,
507 using equation (5) with an expression of the impact force $F_z(\mathbf{r}_0, t)$ based on
508 Hertz's theory, Hunter [32] and Reed [33] estimated the theoretical value W_{el}^{th}
509 of the elastic energy emitted by beads impacting thick elastic blocks. How-
510 ever, their approach has never been extended to the case of impacts on thin
511 plates. Moreover, if inelastic energy dissipation occurs during the impact,

512 the amplitude of the impact force is expected to decrease with respect to
513 the elastic case [30, 28, 34] and Hertz’s model may overestimate the radiated
514 elastic energy.

515 To interpret our results, we compare the measured signals and amplitude
516 spectra with synthetic signals obtained by convolution of the Green’s function
517 [equations (12) and (18)] with Hertz’s force of elastic impact (Figures 6c, 6d,
518 7c, 7d, 8b and 8c). Moreover, we also compare the measured radiated elastic
519 energy W_{el} with the energy W_{el}^{th} of the synthetic signal (Figure 13).

520 A good agreement with elastic theory is observed for the PMMA plate
521 in terms of amplitude and frequencies (Figures 8b and 8c). The measured
522 radiated energy W_{el} in PMMA is generally of the same order of magnitude
523 but smaller than the theoretical one W_{el}^{th} by up to a factor of 3 (Figures 13a
524 and 13b). We used a laser Doppler vibrometer to measure the exact vibra-
525 tion displacement of the glass plate surface after a bead impact (Figure 14).
526 This reveals that the system constituted by the accelerometer and the glass
527 plate shows a resonance frequency around 38 kHz. As a consequence, the
528 accelerometer records a greater amplitude than that of the generated vibra-
529 tion at frequencies close to 38 kHz (Figure 14). This is clearly visible both
530 on the temporal signal and amplitude spectrum when we compare them with
531 their synthetic counterparts (Figures 6c and 6d). Indeed, the measured sig-
532 nal lasts much longer than the synthetic signal (Figure 6c) and the measured
533 spectrum has a higher amplitude than the synthetic spectrum around the
534 resonance frequency (Figure 6d). Because of the resonance, the measured
535 radiated elastic energy W_{el} is up to 4 times greater than W_{el}^{th} for impacts of
536 beads of diameter $d < 10$ mm on the glass plate, regardless of the fall height

537 H (Figures 13a and 13b). More importantly, W_{el} is even greater than the
 538 lost energy ΔE_c (Figures 12a and 12b), which is impossible owing to energy
 539 conservation. This resonance seems excited by impacts of beads of diameter
 540 $d \leq 5$ mm because the peak frequency of the amplitude spectrum generated
 541 by the impacts of these beads is constant and equals 34 kHz (Figure 9),
 542 while it should increase for decreasing bead diameter d [31]. The origin of
 543 this resonance is still under study.

544 It is not clear whether the resonance is also observed for impacts on
 545 the concrete block because the synthetic signal is very different from the
 546 measured signal (Figures 7c and 7d). For example, we can discern the com-
 547 pressional wave and the Rayleigh wave in the synthetic signal but not in
 548 the measured signal (Figure 7c). That said, on concrete, the peak frequency
 549 of the amplitude spectrum decreases for increasing bead diameter d , which
 550 does not suggest resonance (Figure 9). The measured signal on concrete has
 551 smaller frequencies than the synthetic signal, probably because the duration
 552 of the impact of steel beads on this block is longer than that predicted by
 553 Hertz (Figures 7c and 7d). On the concrete block, the measured radiated
 554 energy W_{el} is smaller than the theoretical energy W_{el}^{th} by up to a factor of 7
 555 for bead diameters $d < 5$ mm and $d > 10$ mm (Figures 13a and 13b).

556 For impacts on the thin plates, the variation of the energy ratio W_{el}/E_c
 557 with diameter d is well reproduced by Hertz's theory up to $d = 10$ mm,
 558 but the agreement is not quantitatively good on the glass plate, probably
 559 due to the resonance (Figures 13c and 13d). For larger beads, however,
 560 Hertz's theory leads to values of the radiated elastic energy W_{el}^{th} greater than
 561 the impact energy E_c , which is impossible (Figure 13c). On the concrete

562 block, Hertz's model fails to reproduce the variation of the ratio W_{el}/E_c
 563 with bead diameter d (Figure 13c). Indeed, for an elastic impact, the ratio
 564 W_{el}^{th}/E_c is independent of the bead diameter d while the measured ratio
 565 W_{el}/E_c first increases, reaches a maximum for $d = 5$ mm and then decreases
 566 (Figure 13c). Similarly, the measured ratio W_{el}/E_c is roughly independent
 567 of the fall height H while theory predicts it should increase (Figure 13d).
 568 The average measured ratio W_{el}/E_c on the block is between 0.1% and 2%,
 569 which is in agreement with previous bead-drop experiments on thick blocks
 570 [32, 33, 34]. This is however several orders of magnitude higher than the
 571 ratios $W_{el}/E_c = 10^{-5}$ to 10^{-3} measured for rockfalls in the field, for which
 572 plastic deformation is much more important [9, 10].

573 To sum up, it is valid to use Hertz's force of elastic impact to qualitatively
 574 predict the variation of the radiated elastic energy W_{el} with bead diameter
 575 d and fall height H on a smooth plate when the bead diameter d is smaller
 576 than the plate thickness h . However, the small ratio of W_{el} to the lost energy
 577 ΔE_c for beads of diameter $d < 3$ mm and $d > 10$ mm suggests that our
 578 experiments involve a range of bead diameters and impact speeds in which
 579 viscoelastic and plastic dissipation may occur (Figure 12a). Hertz's model
 580 does not take into account inelastic dissipation during impact, which can
 581 reduce the amplitude of the impact force and thereby decrease the amount
 582 of energy radiated by elastic waves [see 30]. The difference observed between
 583 the measured radiated elastic energy W_{el} and that predicted by Hertz's model
 584 W_{el}^{th} can therefore be partly explained by the presence of inelastic dissipation.
 585 A more complex model is therefore needed to account for these energy losses,
 586 as discussed in the next paragraph.

587 *4.3. Inelastic energy dissipation*

588 For a viscoelastic impact, Ramírez et al. [42] showed that the coefficient
589 of restitution e decreases with the impact speed V_z as $1 - cV_z^{1/5}$ where c is
590 a constant depending on bead diameter. This scaling law agrees well with
591 our experimental results on the glass and PMMA plates but not with those
592 on the concrete block (Figure 15). Some energy may therefore be dissipated
593 viscoelastically on plates. Although not explicitly indicated by the authors,
594 the model of Ramírez et al. [42] shows that the energy lost by viscoelastic
595 dissipation is greater for small beads. This is in agreement with our data
596 because the discrepancy between the measured and the theoretical energy
597 is larger as the bead diameter d decreases (Figure 12a). Additional energy
598 losses may also occur for the smallest beads investigated ($d < 3$ mm) due
599 to surface imperfections and adhesion [31]. These effects are even greater on
600 the concrete block with its surface roughness of ≈ 0.5 – 1 mm. Therefore, the
601 energy that is not radiated in elastic waves for beads of diameter $d < 5$ mm
602 is likely dissipated in viscoelasticity as well as in adhesion and rotational
603 and translational modes. On the PMMA plate, this inelastic dissipation
604 represents from 99% to 10% of the lost energy with increasing diameter d
605 from 1 mm to 4 mm (Figure 12a). On the concrete block, this represents
606 almost all the lost energy because the percentage of lost energy radiated in
607 elastic waves is very small (0.1%-2%) (Figure 12a).

608 The minimum impact speed necessary to deform a structure plastically
609 is very low ($\simeq 0.1$ m s⁻¹ for steel impacting steel [30]) and this velocity
610 is clearly exceeded in all our experiments. However, the minimum impact
611 speed to cause fully plastic deformation is much higher and such impacts

612 are characterized by a coefficient of restitution e that decreases with impact
 613 speed as $V_z^{-1/4}$ [30]. Our data do not fit this scaling law, even for the largest
 614 beads investigated (Figure 15). The impacts in our experiments are therefore
 615 elastic-plastic but not fully plastic. Plastic deformation is more likely to
 616 occur for the largest beads because higher stresses are developed during the
 617 impact. As a matter of fact, plastic deformation is evidenced on glass and
 618 concrete, but not on PMMA, by the presence of small indentations on the
 619 surface after impacts of beads larger than 10 mm. As a consequence, the
 620 elastic transfer efficiency decreases for beads of diameter $d > 5$ mm (Figure
 621 12a). For a given bead diameter $d > 10$ mm, the impact seems more elastic
 622 on PMMA than on glass or on concrete because the ratio $W_{el}/\Delta E_c$ decreases
 623 less on PMMA than on the other structures (Figure 12a). As suggested by
 624 McLaskey and Glaser [31], PMMA is a more compliant material than glass
 625 and concrete and thereby the impacts lasts longer and over a larger area
 626 of contact, reducing the maximum stresses applied on the surface. On the
 627 plates, we estimate that the plastic deformation represents up to 20% of the
 628 lost energy for $d = 20$ mm (Figure 12a). This is however not quantifiable on
 629 the concrete block because the surface roughness may contribute to a high
 630 proportion of the energy losses.

631 Finally, note that even when inelastic dissipation occurs, the three meth-
 632 ods of energy calculation compared in this paper give very similar results
 633 (Figures 10 and 11). However, plastic deformation (or surface roughness)
 634 may generate an impact force with a greater tangential component, as sug-
 635 gested by Buttle and Scruby [18]. This can therefore affect our estimation of
 636 the radiated elastic energy because we make the assumption that the impact

637 force is normal to the surface. For example, Sánchez-Sesma et al. [40] showed
638 that the stronger the tangential force is on the surface of a semi-infinite block,
639 the smaller the generated vertical displacement is with respect to the radial
640 displacement.

641 5. Conclusions

642 We presented and validated experimentally three methods to estimate
643 the elastic energy radiated by an impact on a thin plate and a thick block
644 from the measurement of the surface normal vibration at a single location.
645 The energy flux method and deconvolution methods are based on the direct
646 wave between the impact and are shown to give the same results for both
647 plates and blocks. The diffuse method makes use of the diffuse coda during
648 which multiple reflections occur off the structure's borders. This last method
649 slightly overestimates the radiated elastic energy with respect to the other
650 methods on plates (+5–20%), but gives results of the same order of magni-
651 tude (i.e. within a factor of 3) as the other methods when applied to blocks.
652 The differences between the estimates are however less than the uncertainty
653 of each method, with standard deviations between 40% and 70% for the en-
654 ergy flux and deconvolution methods and between 50% and 300% for the
655 diffuse method.

656 The presented methods have the major advantage of estimating the radi-
657 ated elastic energy independently with respect to the other energy dissipation
658 processes, without knowledge of the impact force. This allowed us to estab-
659 lish an energy budget for the impacts:

- 660 • On thin plates, the percentage of energy lost in elastic waves increases
661 with the bead diameter. This percentage is less than 2% of the total
662 energy lost when the bead diameter is smaller than 10% of the plate
663 thickness. The rest of the energy lost by the bead is likely dissipated by
664 viscoelasticity. On the other hand, almost all the lost energy is radiated
665 in elastic waves for bead diameters greater than the plate thickness and

666 the rest is lost in plastic deformation (up to 20% in our experiments).

- 667 • On rough thick blocks, the radiated elastic energy represents only be-
668 tween 0.2% to 2% of the lost energy, regardless of the bead diameter and
669 fall height. Inelastic dissipation (i.e. viscoelastic, plastic, rotational,...)
670 is therefore the major energy consumption process.

671 The elastic impact model of Hertz well reproduces the measured radi-
672 ated elastic energy on thin plates for bead diameters smaller than the plate
673 thickness, but overestimates the energy for larger beads. On thick blocks,
674 the model gives quantitatively good results but overestimates the radiated
675 elastic energy by a factor of 2 to 10 when inelastic dissipation occurs.

676 Further work is required to investigate how surface roughness affects the
677 amount of energy radiated in elastic waves and dissipated by inelastic pro-
678 cesses during an impact. For example, it would be interesting to establish
679 the energy budget of beads impacts on thick blocks with a surface as smooth
680 as that of the thin plates.

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691 **Appendix A. Experimental determination of the relations of dis-**
692 **persion**

693 In this section, we detail how to determine the relations of dispersion
694 of the structures used for impacts experiments. In order to observe wave
695 dispersion, we measure the emitted wave front at several distances r from
696 a given bead impact (e.g. for PMMA, Figure 16a). The double Fourier
697 transform in time and space of the vibration acceleration $a_z(r, t)$ allows to
698 deduce the relation between the angular frequency ω and the wave number
699 k , i.e. the dispersion relation (Figures 16b and 16c).

700 As expected, for the plates of PMMA and glass the dispersion relation
701 corresponds exactly to that of the fundamental mode A_0 of Lamb (Figures
702 16c and 17a). At low frequencies, i.e. for $kh < 1$, the dispersion relation
703 can be approximated by $\omega \approx 5.5k^2$ in PMMA and $\omega \approx 13.8k^2$ in glass, thus
704 satisfying equation (1) with a bending stiffness $B = 357$ J and $B = 4760$
705 J, respectively. On the other hand, the mode A_0 is not dispersive at higher
706 frequencies, for $kh > 1$. Indeed, the relation between the frequency and the
707 wave number becomes roughly linear and the group velocity $v_g = \partial\omega/\partial k$
708 tends towards the Rayleigh wave velocity that is ≈ 1400 m s⁻¹ for PMMA
709 and ≈ 3100 m s⁻¹ for glass [35].

710 For the glass and PMMA plates, we estimate the energy associated with
711 the longitudinal S_0 mode with an accelerometer on the plate border. In
712 both plates, the energy of this mode is about 0.2% of that of the vertical
713 A_0 mode and is consequently negligible. The plates vibration is therefore
714 mostly normal to the surface. The lowest secondary mode in plates is the
715 mode A_1 that has a cutoff frequency equal to $c_S/4h \approx 82$ kHz in glass and

716 22 kHz in PMMA, where c_S is the shear wave speed. The accelerometers
717 record frequencies up to 80 kHz, therefore we do not measure modes higher
718 than the A_0 mode in glass. In PMMA, however, the mode A_1 may be present
719 but its amplitude is too low to be detected in the dispersion curve $\omega = f(k)$
720 (Figure 16c).

721 For the concrete block, the relation between the angular frequency ω
722 and the wave number k is roughly linear with a slope of 1530 m s^{-1} that
723 corresponds to both the phase v_ϕ and group v_g velocities (Figure 17b).

724 **Appendix B. Energy dissipation model in a viscoelastic solid**

725 In this Appendix, we show that the viscous dissipation of energy with
 726 distance r in a Kelvin-Voigt viscoelastic solid can be modeled by a factor
 727 $\exp(-\gamma r)$ where $1/\gamma$ is a characteristic length of energy dissipation that de-
 728 pends on frequency. To that end, we have to demonstrate the equation of
 729 energy conservation in such a solid. We start from the equation of momentum
 730 conservation in the solid, stating that:

$$731 \quad \rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial T_{ij}}{\partial x_j}, \quad (\text{B.1})$$

732 where u_i is the wave displacement and T_{ij} is the stress tensor. The summation
 733 on repeated indices is implicit. In a homogeneous and isotropic viscoelastic
 734 solid modeled by Kelvin-Voigt model, Hooke's law is [35]:

$$735 \quad T_{ij} = T_{ij}^{el} + T_{ij}^{inel}, \quad (\text{B.2})$$

736 with

$$737 \quad T_{ij}^{el} = \lambda \delta_{ij} S + 2\mu S_{ij}, \quad (\text{B.3})$$

738 and

$$739 \quad T_{ij}^{inel} = \chi \delta_{ij} \frac{\partial S}{\partial t} + 2\eta \frac{\partial S_{ij}}{\partial t}, \quad (\text{B.4})$$

740 where $S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$ is the strain tensor and $S = \partial u_j / \partial x_j$. The con-
 741 stants λ , μ and χ , η are the elastic and viscous coefficients associated to
 742 compression and shear, respectively. Note that these coefficients generally
 743 depend on frequency f .

744 Multiplying equation (B.1) by $\frac{\partial u_i}{\partial t}$, we obtain:

$$745 \quad \frac{\partial e_c}{\partial t} = \frac{\partial T_{ij}^{el}}{\partial x_j} \frac{\partial u_i}{\partial t} + \frac{\partial T_{ij}^{inel}}{\partial x_j} \frac{\partial u_i}{\partial t}, \quad (\text{B.5})$$

746 where e_c is the bulk density of kinetic energy.

747 We can develop the second term of equation (B.5) noting that:

$$748 \quad \frac{\partial T_{ij}^{el}}{\partial x_j} \frac{\partial u_i}{\partial t} = \frac{\partial}{\partial x_j} \left(T_{ij}^{el} \frac{\partial u_i}{\partial t} \right) - T_{ij}^{el} \frac{\partial^2 u_i}{\partial t \partial x_j}, \quad (\text{B.6})$$

749 According to Royer and Dieulesaint [35], the Poynting vector is defined
750 by:

$$751 \quad P_j = -T_{ij}^{el} \frac{\partial u_i}{\partial t}. \quad (\text{B.7})$$

752 and verifies, for guided waves:

$$753 \quad \frac{\partial P_j}{\partial x_j} = c_j \frac{\partial e_{tot}}{\partial x_j}, \quad (\text{B.8})$$

754 where c_j is the energy speed, i.e. the group velocity, in the direction \mathbf{x}_j
755 and $e_{tot} = e_c + e_p$ is the bulk density of total energy within the structure.
756 Moreover, because of the symmetry $S_{ij} = S_{ji}$ of the strain tensor, we can
757 show that:

$$758 \quad T_{ij}^{el} \frac{\partial^2 u_i}{\partial t \partial x_j} = \frac{1}{2} (\lambda \delta_{ij} + 2\mu) \frac{\partial}{\partial t} \left(\frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \right), \quad (\text{B.9})$$

759 which is the derivative of the bulk density of potential energy e_p .

760 Injecting equations (B.6), (B.8) and (B.9) in equation (B.5), we obtain:

$$761 \quad \frac{\partial e_{tot}}{\partial t} + c_j \frac{\partial e_{tot}}{\partial x_j} = \frac{\partial T_{ij}^{inel}}{\partial x_j} \frac{\partial u_i}{\partial t}, \quad (\text{B.10})$$

762 where the last term can be developed using equation (B.4).

763 If we assume that the wave is longitudinal and propagates in direction \mathbf{x}_1
764 ($u_2 = 0$), the wave displacement is:

$$765 \quad u_1 = A_1 \sin(\omega(t - x_1/c_P)), \quad (\text{B.11})$$

766 where A_1 is the amplitude, ω is the angular frequency and c_P is the com-
 767 pressional wave speed. Thus we get:

$$768 \quad \frac{\partial T_{ij}^{inel}}{\partial x_j} \frac{\partial u_i}{\partial t} = -(\chi + 2\eta) A_1^2 \frac{\omega^4}{c_P^2} \cos^2(\omega(t - x_1/c_P)). \quad (\text{B.12})$$

769 If we remark that the bulk density of energy e_{tot} is equal to

$$770 \quad \rho \left(\frac{\partial u_1}{\partial t} \right)^2 = \rho \omega^2 A_1^2 \cos^2(\omega(t - x_1/c_P)), \quad (\text{B.13})$$

771 we obtain:

$$772 \quad \frac{\partial T_{ij}^{inel}}{\partial x_j} \frac{\partial u_i}{\partial t} = -(\chi + 2\eta) \frac{\omega^2}{\rho c_P^2} e_{tot}. \quad (\text{B.14})$$

773 Using equations (B.10) and (B.14), we have finally demonstrated that
 774 the equation of energy conservation of a longitudinal wave propagating in a
 775 viscoelastic solid is:

$$776 \quad \frac{\partial e_{tot}}{\partial t} + \mathbf{v}_g \cdot \nabla e_{tot} = -\frac{e_{tot}}{\tau}, \quad (\text{B.15})$$

777 with $v_g = c_P$, the group speed and τ , the characteristic time of energy
 778 dissipation [see e.g. 35, 43]:

$$779 \quad \tau = \frac{\rho c_P^2}{(\chi + 2\eta)\omega^2} = \frac{1}{\gamma v_g}. \quad (\text{B.16})$$

780 In equation (B.15), the term $-e_{tot}/\tau$ represents energy dissipation with
 781 time when the source force is not acting on the structure any more [e.g. 36].
 782 Multiplying this equation by $\exp(t/\tau)$ gives:

$$783 \quad \left(\frac{\partial e_{tot}}{\partial t} + \frac{e_{tot}}{\tau} \right) \exp\left(\frac{t}{\tau}\right) + \mathbf{v}_g \cdot \nabla e_{tot} \exp\left(\frac{t}{\tau}\right) = 0. \quad (\text{B.17})$$

784 Writing $e'_{tot} = e_{tot} \exp(t/\tau)$ leads to:

$$785 \quad \frac{\partial e'_{tot}}{\partial t} + \mathbf{v}_g \cdot \nabla e'_{tot} = 0 \quad (\text{B.18})$$

786 Thus energy $e'_{tot} = e_{tot} \exp(t/\tau) = e_{tot} \exp(\gamma r)$ is conserved. Therefore, mul-
 787 tiplying the energy by the factor $\exp(\gamma r)$ compensates the viscoelastic dissi-
 788 pation of energy with distance.

789 Note that if the wave is transversal and polarized along direction \mathbf{x}_2 and
 790 propagates along direction \mathbf{x}_1 , we have:

$$791 \quad \frac{\partial T_{ij}^{inel}}{\partial x_j} \frac{\partial u_i}{\partial t} = -\eta A_2^2 \frac{\omega^4}{c_S^2} \cos^2(\omega(t - x_2/c_S)), \quad (\text{B.19})$$

792 and we retrieve the conservation equation (B.15) with a different coefficient
 793 $\tau = \rho c_S^2 / \eta \omega^2$, with c_S , the shear wave speed. Practically, the waves propagat-
 794 ing in thin plates and thicks blocks are a complex combination of longitudi-
 795 nal and transversal waves. If we consider only one of these modes, either the
 796 mode A_0 of Lamb or the Rayleigh waves, the equation (B.15) of energy con-
 797 servation is still verified provided that we integrate it over the depth [35] but
 798 the expression of the characteristic coefficient τ is much more complicated.

799 Here, we validate experimentally the model of energy attenuation in
 800 $\exp(-t/\tau)$ or in $\exp(-\gamma r)$ in the thin plates and the thick block investi-
 801 gated. To do so, we estimate the coefficient γ by measuring the first arrival
 802 of the emitted vibration at different distances r from an impact (Figure 18a)
 803 and filtering this vibration in different frequency ranges. For example in the
 804 PMMA plate, the squared amplitude of the A_0 mode decreases with distance
 805 r as $\frac{1}{r} \exp(-\gamma r)$ (Figures 18b to 18d). We deduce the value of γ as a function
 806 of frequency f (Figure 18e).

807 When the first arrival can not be separated from the side reflections or
 808 when numerous side reflections occur in the structure after an impact, we
 809 can determine energy attenuation with an other method. For example on
 810 the glass plate, after an impact the envelope of the squared signal averaged

811 over several periods decreases exponentially with time as:

$$812 \quad \overline{A(t)^2} = \overline{A(t=0)^2} \exp\left(-\frac{t}{\tau}\right), \quad (\text{B.20})$$

813 where $t = 0$ is the impact time (Figure 19a). The characteristic time τ at
814 frequency f is simply the inverse of the slope of $\overline{A(t)^2}$ in semi-log scale, filtered
815 in a frequency range centered on f (Figures 19b to 19d). We thus show how
816 the characteristic time τ decreases as the frequency f increases (Figure 19e).
817 Note that for a diffuse field, the inverse of τ is given by the average of the
818 inverse of the characteristic times τ of each modes of propagation weighed
819 by their percentage of partition.

820 **Appendix C. Green's functions owing to a vertical load at the**
821 **surface of an elastic half-space**

822 Here we recall the expression of the time Fourier's transform of the
823 Green's function $\tilde{G}_{zz}(r, \omega)$ at the surface of a half-space owing to a verti-
824 cal load on the surface.

825 Miller and Pursey [38] determined the exact expression of the surface
826 vertical displacements $\tilde{U}_z(r, z, \omega)$ generated at a distance r by a normal force
827 $\mathbf{F} = \tilde{F}_z(\omega)\mathbf{u}_z$ on the surface of an elastic half-space [equation (72) of their
828 paper with $z = 0$]:

$$829 \quad \tilde{U}_z(r, \omega) = \frac{\tilde{F}_z(\omega)\xi^2}{\pi a\mu} \int_0^{+\infty} \frac{\sqrt{x^2 - 1}}{f_0(x)} J_1(k_1 ax) J_0(k_1 rx) dx, \quad (\text{C.1})$$

830 where a is the radius of the loading area, μ the Lamé shear modulus, $k_1 =$
831 ω/c_P , with the angular frequency $\omega = 2\pi f$ and the compressional wave speed
832 c_P , $f_0(x) = (2x^2 - \xi^2)^2 - 4x^2\sqrt{(x^2 - 1)(x^2 - \xi^2)}$, $\xi = \sqrt{2(1 - \nu)/(1 - 2\nu)}$ and
833 ν is Poisson's ratio. J_0 and J_1 are the Bessel's functions of the first kind.

834 For very small values of the radius of contact a , $J_1(k_1 ax)$ can be approx-
835 imated at a first order by $k_1 ax/2 + O(a^2)$ so that

$$836 \quad \tilde{U}_z(r, \omega) \approx \frac{\tilde{F}_z(\omega)\xi^2}{2\pi\mu} k_1 \int_0^{+\infty} \frac{x\sqrt{x^2 - 1}}{f_0(x)} J_0(k_1 rx) dx. \quad (\text{C.2})$$

837 A first order approximation of the integral in equation (C.2) was calcu-
838 lated by Miller and Pursey [21] for large values of $k_1 r = 2\pi fr/c_P$. From
839 a practical viewpoint, this approximation is valid for impact problems be-
840 cause the impact generates high frequencies $1 \text{ kHz} < f < 80 \text{ kHz}$ (Figures
841 6d, 8c and 7d) and $k_1 r \gg 1$ even for small distances r from the impact
842 location. Using this computation, we can show that the vertical Green's

843 function $\tilde{G}_{zz}(r, \omega) = \tilde{U}_z(r, \omega)/\tilde{F}_z(\omega)$ is the sum of contributions of compres-
 844 sional, shear and Rayleigh waves, respectively, \tilde{G}_{zz}^P , \tilde{G}_{zz}^S and \tilde{G}_{zz}^R :

$$845 \quad \tilde{G}_{zz} = \tilde{G}_{zz}^P + \tilde{G}_{zz}^S + \tilde{G}_{zz}^R \quad (\text{C.3})$$

846 with

$$847 \quad \tilde{G}_{zz}^P(r, \omega) \approx -\frac{i}{\mu} A_P \frac{k_1}{(k_1 r)^2} \exp(-i\omega r/c_P), \quad (\text{C.4})$$

$$848 \quad \tilde{G}_{zz}^S(r, \omega) \approx -\frac{i}{\mu} A_S \frac{k_1}{(k_1 r)^2} \exp(-i\omega r/c_S), \quad (\text{C.5})$$

$$849 \quad \tilde{G}_{zz}^R(r, \omega) \approx -\frac{i}{\mu} A_R k_1 \sqrt{\frac{2}{\pi k_1 r}} \exp\left(-i(\omega r/c_R - \frac{\pi}{4})\right), \quad (\text{C.6})$$

850 where c_P , c_S and c_R are the compressional, shear and Rayleigh waves speeds,
 851 respectively, and where A_P , A_S and A_R are only functions of Poisson's ratio
 852 ν (Figure 20):

$$853 \quad A_P(\nu) = \frac{\xi^2}{2\pi(2 - \xi^2)^2}, \quad (\text{C.7})$$

$$854 \quad A_S(\nu) = \frac{2(\xi^2 - 1)}{\pi\xi^3}, \quad (\text{C.8})$$

$$855 \quad A_R(\nu) = \frac{\xi^2 \sqrt{x_0(x_0^2 - 1)}}{2 f_0'(x_0)}, \quad (\text{C.9})$$

856 with x_0 , the real positive root of $f_0(x)$.

857 **Appendix D. Detailed calculation of coefficient β**

858 We detail here the calculation of the coefficient β that appears in the
 859 expression of the elastic energy W_{el} radiated in a block [equation (35)]. β is
 860 defined as the imaginary part of

$$861 \int_0^X \frac{x\sqrt{x^2-1}}{f_0(x)} dx, \quad (D.1)$$

862 where $f_0(x) = (2x^2 - \xi^2)^2 - 4x^2\sqrt{(x^2-1)(x^2-\xi^2)}$, $\xi = \sqrt{2(1-\nu)/(1-2\nu)}$
 863 and ν the Poisson ratio of the block. X is a number greater than the real
 864 root x_0 of f_0 , which is represented in Figure 5.

865 Let the function f be:

$$866 f : x \longrightarrow \frac{x\sqrt{x^2-1}}{(2x^2 - \xi^2)^2 - 4x^2\sqrt{(x^2-1)(x^2-\xi^2)}}. \quad (D.2)$$

867 For most materials, the Poisson ratio ν is between 0 and 0.5, correspond-
 868 ing to values of ξ from 1.4 to 10. To calculate β we have to look at the
 869 definition of f over the intervals $[0, 1[$, $[1, \xi[$ and $x \geq \xi$:

- 870 • For $x \in [0, 1[$, $x^2 - 1 < 0$ and $x^2 - \xi^2 < 0$, then we can then write
 871 $\sqrt{x^2-1} = i\sqrt{1-x^2}$ and $\sqrt{x^2-\xi^2} = i\sqrt{\xi^2-x^2}$ where i is the complex
 872 number $\sqrt{-1}$. Over this interval, $f(x)$ is a pure imaginary number:

$$873 f(x) = \frac{ix\sqrt{1-x^2}}{(2x^2 - \xi^2)^2 + 4x^2\sqrt{(1-x^2)(\xi^2-x^2)}} \quad (D.3)$$

874 and

$$875 \text{Im}(f(x)) = f_1(x) = \frac{x\sqrt{1-x^2}}{(2x^2 - \xi^2)^2 + 4x^2\sqrt{(1-x^2)(\xi^2-x^2)}}. \quad (D.4)$$

876 Regardless of the value of ξ , f_1 is continuous over $[0, 1]$ with $f_1(0) =$
 877 $f_1(1) = 0$ and f_1 is C^∞ over $[0, 1[$.

- 878 • For $x \in [1, \xi[$, $x^2 - 1 > 0$ and $x^2 - \xi^2 < 0$, therefore $\sqrt{x^2 - \xi^2} =$
 879 $i\sqrt{\xi^2 - x^2}$. Over this interval:

$$880 \quad f(x) = \frac{x\sqrt{x^2 - 1}}{(2x^2 - \xi^2)^2 - 4ix^2\sqrt{(x^2 - 1)(\xi^2 - x^2)}}. \quad (\text{D.5})$$

881 Multiplying the numerator and the denominator by the complex con-
 882 jugate of the denominator leads to:

$$883 \quad f(x) = \frac{x\sqrt{x^2 - 1} \left[(2x^2 - \xi^2)^2 + 4ix^2\sqrt{(x^2 - 1)(\xi^2 - x^2)} \right]}{(2x^2 - \xi^2)^4 + 16x^4(x^2 - 1)(\xi^2 - x^2)} \quad (\text{D.6})$$

884 and

$$885 \quad \text{Im}(f(x)) = f_2(x) = \frac{4x^3(x^2 - 1)\sqrt{\xi^2 - x^2}}{(2x^2 - \xi^2)^4 + 16x^4(x^2 - 1)(\xi^2 - x^2)}. \quad (\text{D.7})$$

886 Regardless of the value of ξ , f_2 is continuous over $[1, \xi]$ with $f_2(1) =$
 887 $f_2(\xi) = 0$ and f_2 is C^∞ function over $[1, \xi[$.

- 888 • For $x \geq \xi$, $x^2 - 1 > 0$ and $x^2 - \xi^2 > 0$, therefore f is a real function over
 889 this interval and its imaginary part is null, except for the contribution
 890 of the pole x_0 of f_0 , which is always greater than ξ (Figure 5). The
 891 integral of f over this interval is due to half of its residue in x_0 :

$$892 \quad \int_{\xi}^X f(x)dx = -i\pi \frac{x_0\sqrt{x_0^2 - 1}}{f_0'(x_0)}. \quad (\text{D.8})$$

893 Finally, $\beta = \int_0^1 f_1(x)dx + \int_1^{\xi} f_2(x)dx - \pi \frac{x_0\sqrt{x_0^2 - 1}}{f_0'(x_0)}$. β is represented as a
 894 function of the Poisson ratio ν in Figure 21.

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Table 2: Physical values used for calculation of the radiated elastic energy in the glass plate and the concrete block: density ρ , Young's modulus E , Poisson ratio ν , compressional and shear wave speeds c_P and c_S , bending stiffness B , characteristic distance $1/\gamma$ and time τ of energy attenuation, group velocity v_g (that depends on the frequency f (in Hz)), phase velocity v_ϕ and coefficient β . Glass parameters are from Fuegel [44] and PMMA parameters from the MIT material properties database [45]. Elastic parameters E and ν of concrete are estimated from the compressional and shear wave velocities measured through the block and the density ρ of concrete is from Elert [46].

material		ρ (kg m ⁻³)	E (GPa)	ν -	c_P (m s ⁻¹)	c_S (m s ⁻¹)	B (J)	γ (1/m)	τ (s)	v_g (m s ⁻¹)	v_ϕ (m s ⁻¹)	β -
glass	$kh < 1$	2500	74	0.2	5730	3500	4760	$0.014f^{1/6}$	$3.8f^{-2/3}$	$18.6f^{1/2}$	$9.3f^{1/2}$	-
	$kh > 1$							$8.5 \times 10^{-5}f^{2/3}$		3100	3100	
PMMA	$kh < 1$	1180	4.4	0.37	1920	860	357	1	$0.09f^{-1/2}$	$11.7f^{1/2}$	$5.8f^{1/2}$	-
	$kh > 1$							$4.8 \times 10^{-3}f^{2/3}$		$0.15f^{-2/3}$	1400	
concrete	-	2200	16.3	0.4	4030	1620	-	$2.3 \times 10^{-5}f$	$28f^{-1}$	1530	1530	0.3

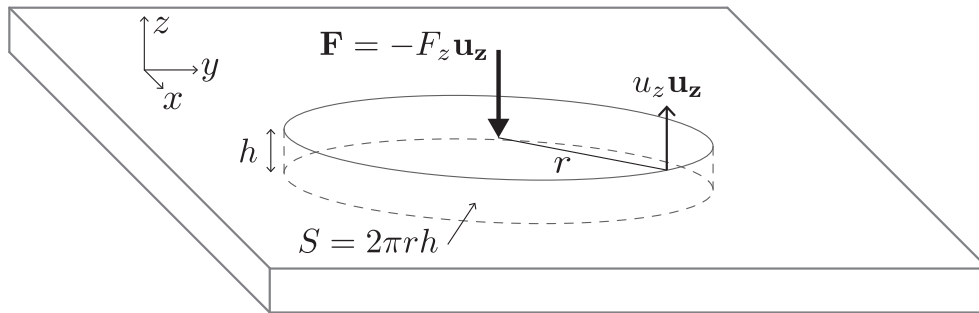


Figure 1: Sketch of the thin plate of thickness h , characterized by Cartesian coordinates x, y, z . $z = 0$ corresponds to the plate free surface. When a normal impact force $-F_z \mathbf{u}_z$ excites the plate at the origin $(0, 0, 0)$, Lamb waves are emitted radially and generate a displacement field $\mathbf{u} \approx u_z(r, t) \mathbf{u}_z$. S is a closed section of the plate, surrounding the impact position and corresponds here to a cylinder of radius r and height equal to the plate thickness h .

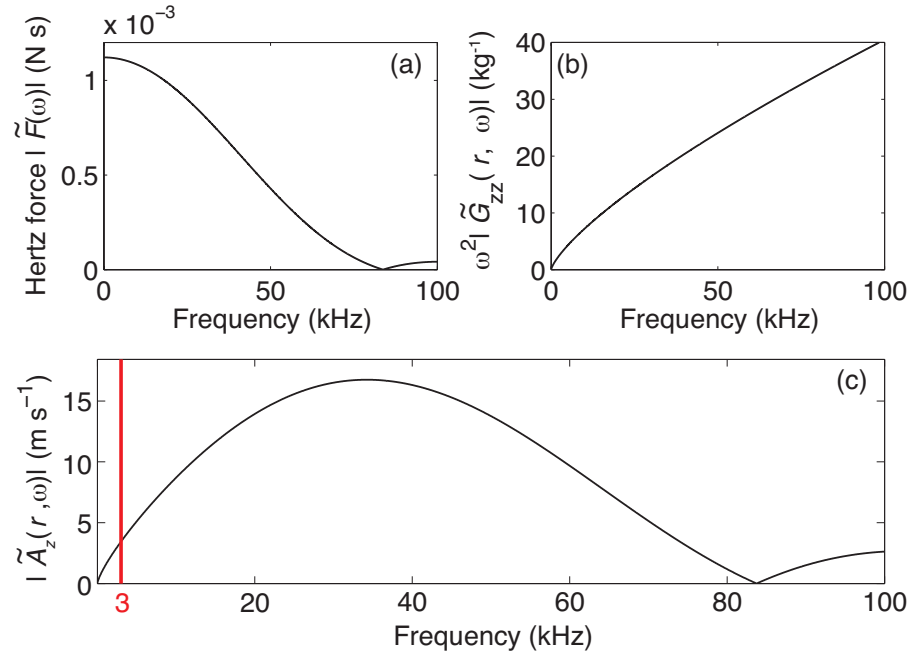


Figure 2: (a) Fourier transform $|\tilde{F}(\omega)|$ of the ideal Hertz's force of elastic impact of a 4-mm diameter steel sphere on PMMA. (b) Green's function $|\tilde{G}_{zz}(r, \omega)|$ [equation (12)] at $r = 10$ cm multiplied by ω^2 (c) Synthetic amplitude spectrum $|\tilde{A}_z(r, \omega)|$ obtained by the product of the force in (a) and the function in (b).

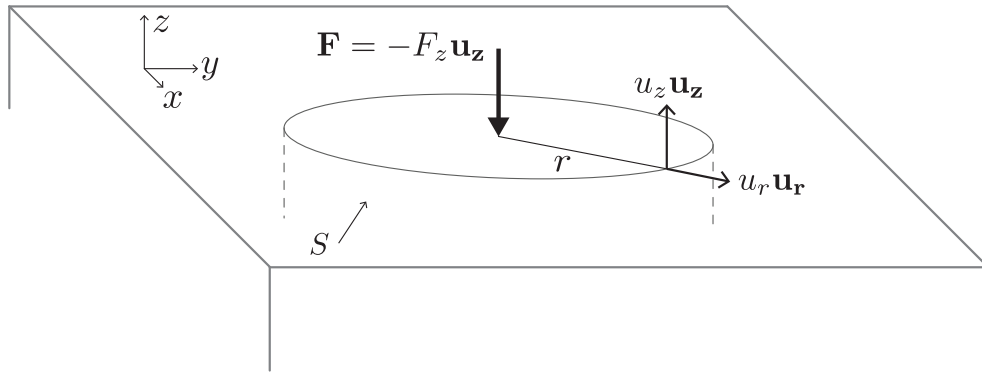


Figure 3: Sketch of the thick block configuration, characterized by Cartesian coordinates x, y, z . $z = 0$ corresponds to the block free surface. When a normal impact force $-F_z \mathbf{u}_z$ excites the block normally at the origin $(0, 0, 0)$, Rayleigh waves are emitted radially at the surface and generate a displacement field $\mathbf{u} = u_r(r, z, t)\mathbf{u}_r + u_z(r, z, t)\mathbf{u}_z$ with an amplitude that decreases exponentially with depth z (see text). S is a closed section of the block, surrounding the impact position, and corresponds here to a cylinder of radius r and infinite height.

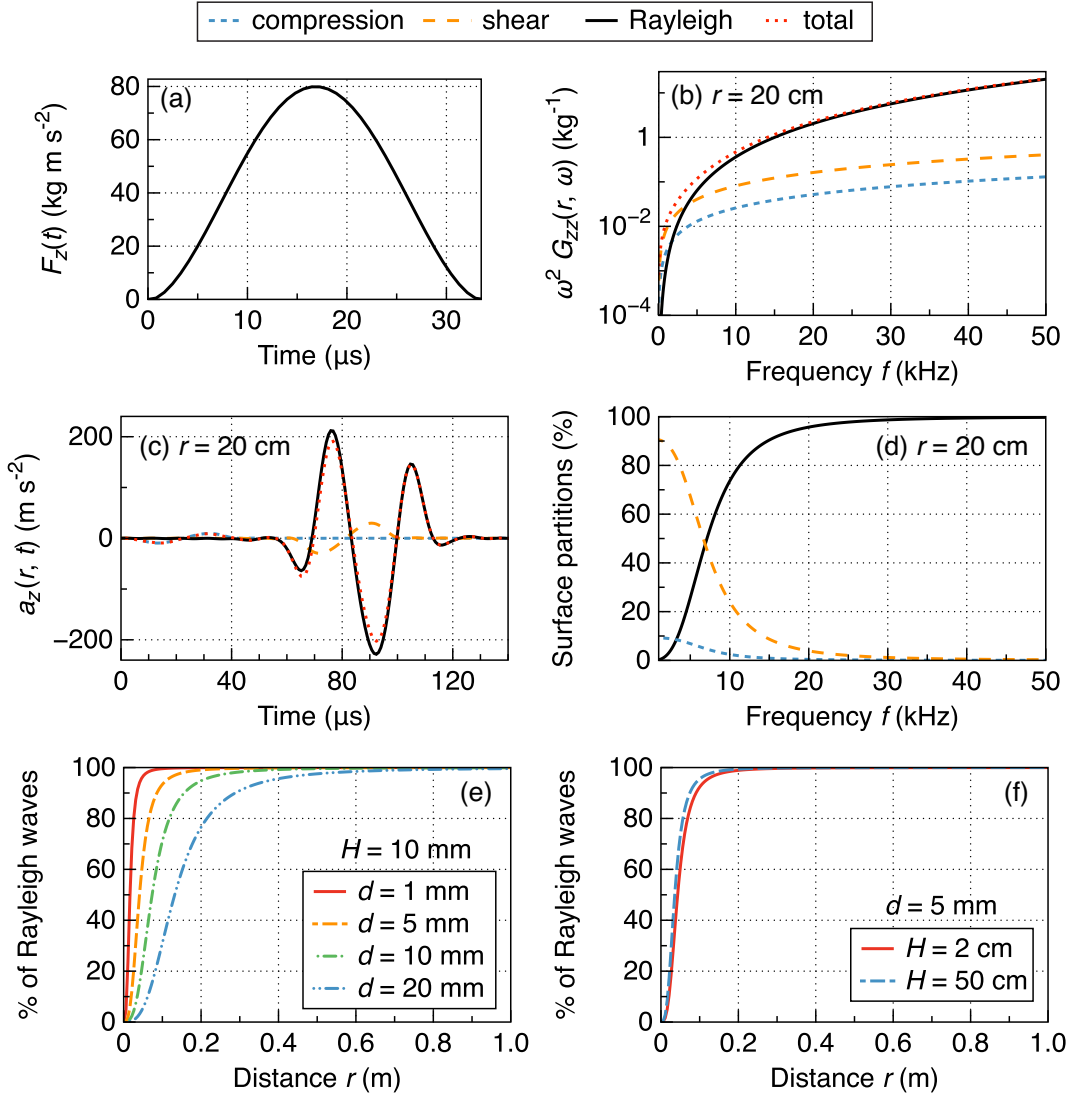


Figure 4: (a) Hertz's force of elastic impact of a steel bead of diameter $d = 5$ mm dropped from height $H = 10$ cm on a concrete block is convolved with (b) the Green's functions \tilde{G}_{zz}^P , \tilde{G}_{zz}^S and \tilde{G}_{zz}^R [equations (19), (20) and (21), respectively], multiplied by ω^2 , at $r = 20$ cm from the impact to obtain (c) the synthetic vertical vibration acceleration $a_z(r, t)$ of each mode at the surface. (d) Percentage of the energy transported by compressional, shear and Rayleigh waves at $r = 20$ cm from the impact as a function of frequency f . (e) Percentage $\pi_{\text{surf}}^R(r)$ of Rayleigh waves in the surface vibration as a function of the distance r from the impact for (e) a fall height $H = 10$ cm and different bead diameters d and (f) for a bead diameter of $d = 5$ mm and fall heights $H = 5$ cm and $H = 50$ cm.

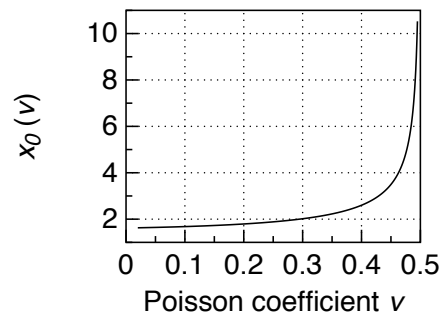


Figure 5: Value of the real solution x_0 of $f_0(x) = 0$ as a function of Poisson's ratio ν .

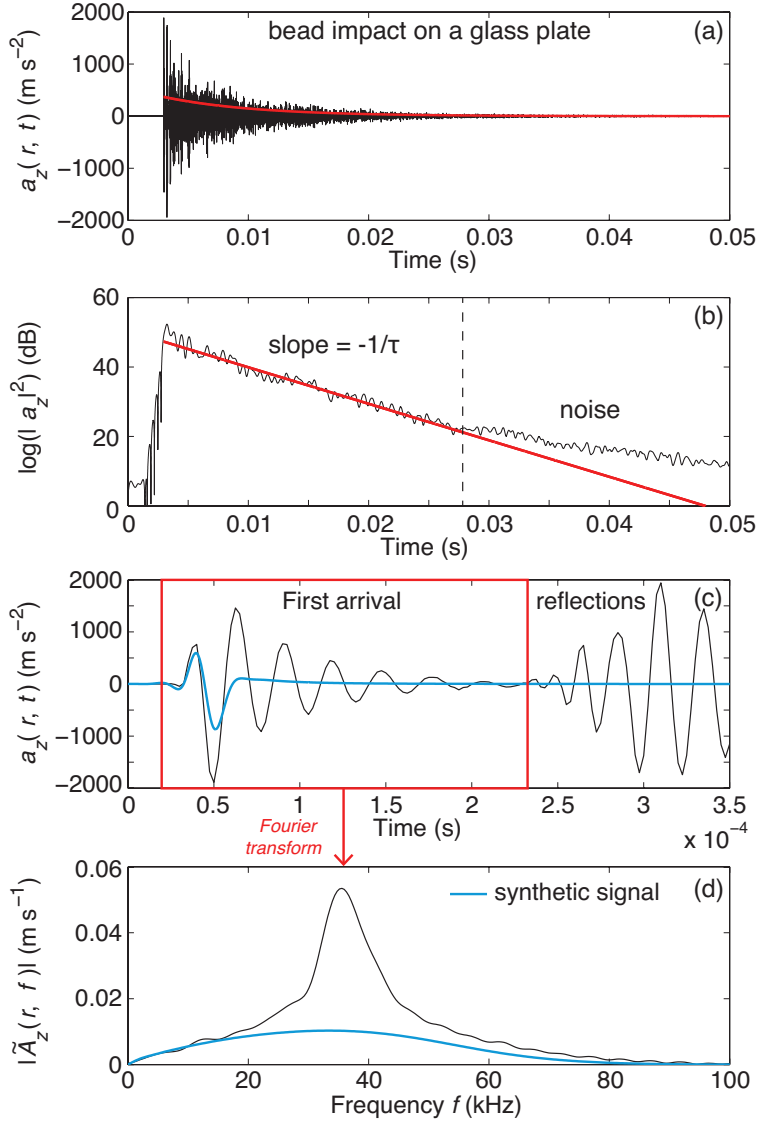


Figure 6: (a)-(c) Normal surface acceleration $a_z(r, t)$, filtered below 100 kHz, recorded at $r = 6$ cm from the source after the impact of a steel bead of diameter 4 mm on the glass plate. (a) and (b) The wave reflects many times off the plate lateral sides and the energy decreases exponentially with time due to viscoelastic dissipation (red line). In (b), $a_z(r, t)$ is squared, filtered below 2000 Hz and plotted in semi-log form. (c) The plate is sufficiently large to record the first wave arrival entirely (red frame) before the return of the first side reflections. (d) Time Fourier transform $|\tilde{A}_z(r, f)|$ of the first wave arrival as a function of frequency f . The thick blue line in (c) and (d) is a synthetic signal obtained with the convolution of the Green's function in equation (12) with the force of Hertz. The discrepancy of the measured signal with theory is discussed in section 4.

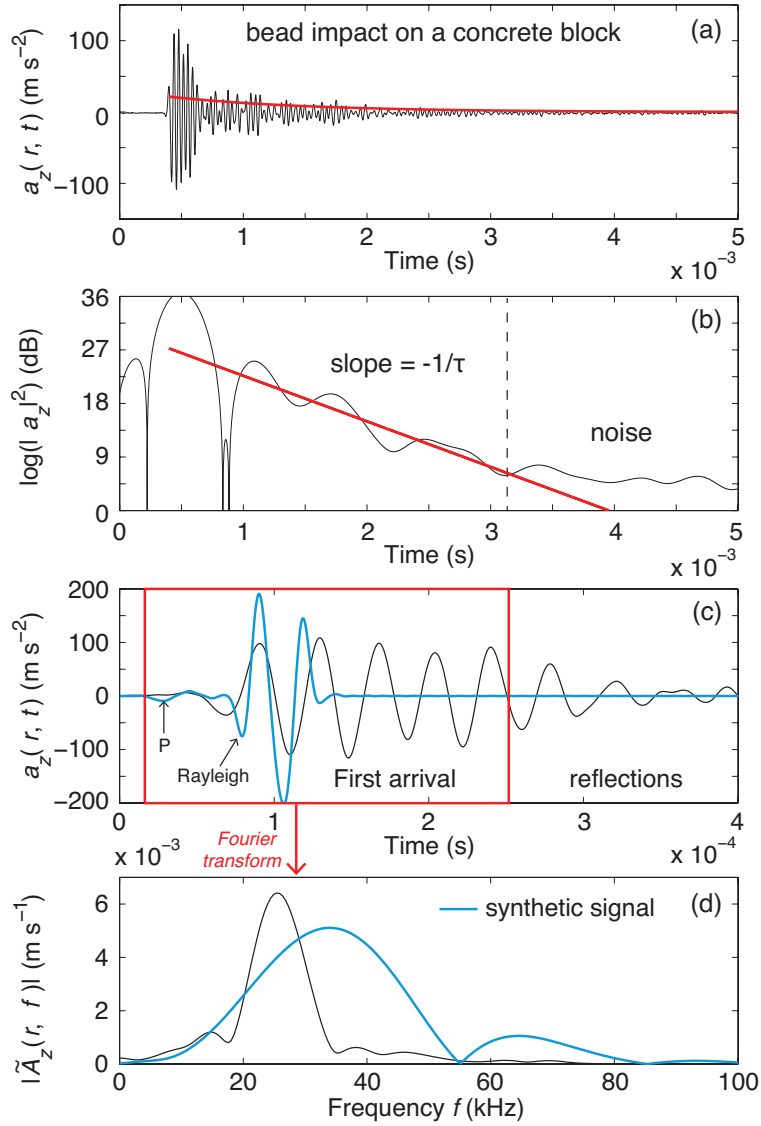


Figure 7: (a)-(c) Normal surface acceleration $a_z(r, t)$ recorded at $r = 20$ cm from the source after the impact of a steel bead of diameter 5 mm on the concrete block. (a) and (b) The wave reflects several times off the block boundaries and the energy decreases exponentially with time due to viscoelastic dissipation (red line). In (b), $a_z(r, t)$ is squared, filtered below 2000 Hz and plotted in semi-log form. (c) The block is sufficiently large to record most of the first wave arrival (red frame) before the return of the first side reflection that should arrive on the right side of the red frame. (d) Time Fourier transform $|\tilde{A}_z(r, f)|$ of the first wave arrival as a function of frequency f . The thick blue line in (c) and (d) is a synthetic signal obtained with the convolution of the Green's function in equation (18) with the force of Hertz. In the temporal synthetic signal in (c), we can discern the compressional wave (noted P) and the Rayleigh waves. The discrepancy of the measured signal with theory is discussed in section 4.

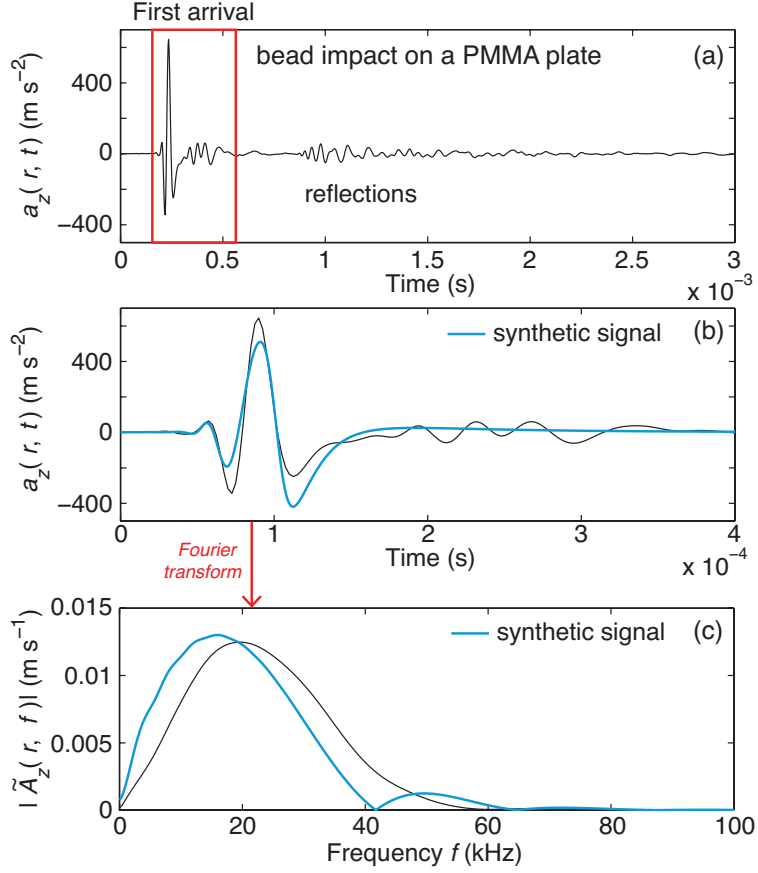


Figure 8: (a) and (b) Normal surface acceleration $a_z(r, t)$, filtered below 100 kHz, recorded at $r = 10$ cm from the source after the impact of a steel bead of diameter 3 mm on the PMMA plate. (a) The direct wave front (red frame) is clearly separated from its reflections off the plate lateral sides. (b) Zoom on the first wave arrival. (c) Time Fourier transform $|\tilde{A}_z(r, f)|$ of the first wave arrival as a function of the frequency f . The thick blue line in (b) and (c) is a synthetic signal obtained with the convolution of the Green's function in equation (12) with the force of Hertz. The discrepancy of the measured signal with theory is discussed in section 4.

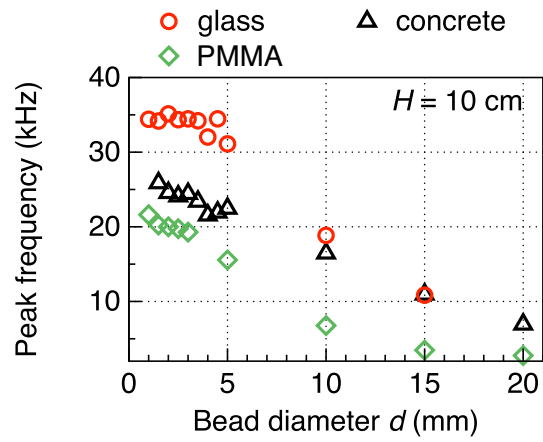


Figure 9: Frequency of the maximum of the amplitude spectrum $|\tilde{A}_z(r, f)|$, or peak frequency, for impacts of steel beads of different bead diameters d on the glass plate, PMMA plate and concrete block. The peak frequency is independent of the fall height in the range investigated.

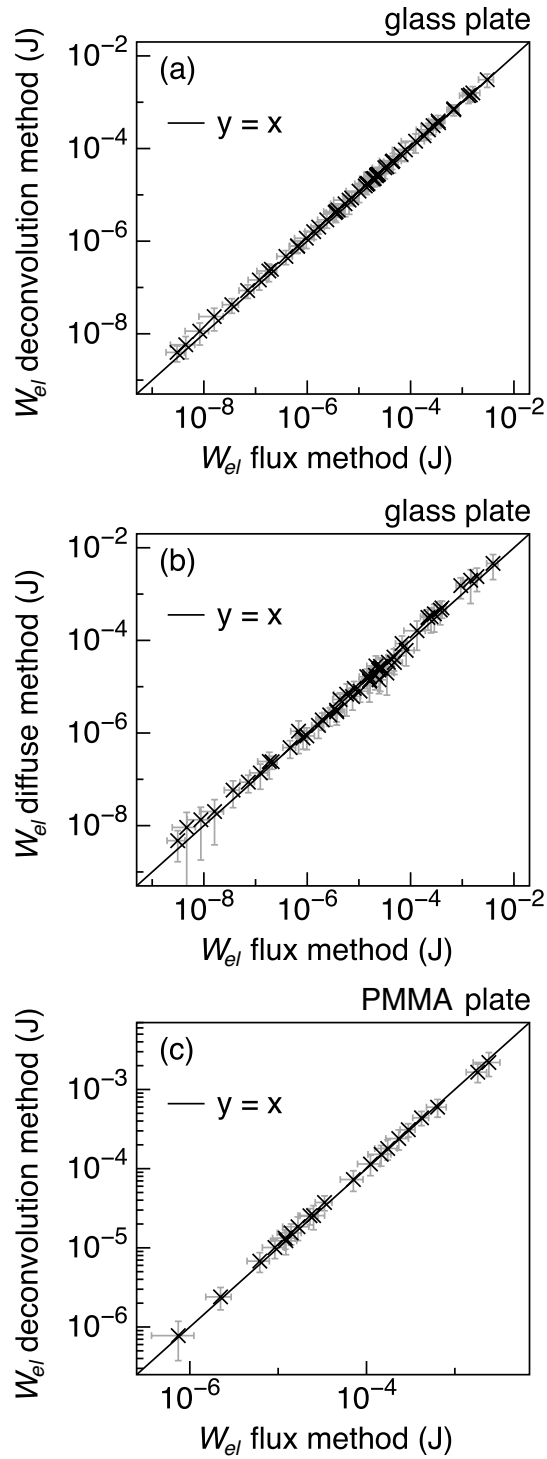


Figure 10: Comparison of the radiated elastic energy W_{el} calculated using the three methods [equations (8), (13) and (17)] for impacts of steel beads of various diameters from 1 mm to 20 mm dropped from various heights from 2 cm to 25 cm on (a) and (b) the glass plate and (c) the PMMA plate. Error bars (± 1 standard deviation) are estimated from reproducibility tests conducted on a series of 12 identical experiments.

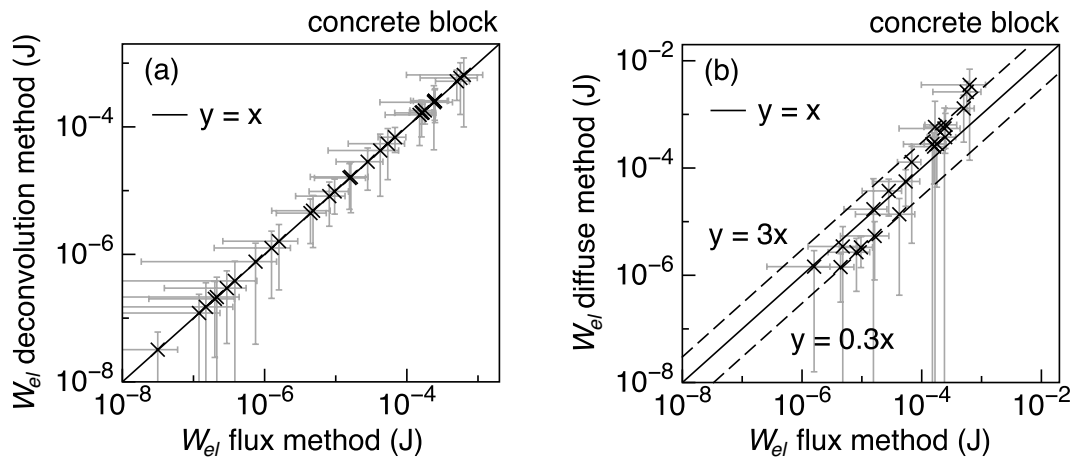


Figure 11: Comparison of the radiated elastic energy W_{el} calculated using the three methods [equations (31), (35) and (36)] for impacts of steel beads of various diameters from 2 mm to 20 mm dropped from various heights from 5 cm to 43 cm on the concrete block. Error bars (± 1 standard deviation) are estimated from reproducibility tests conducted on a series of 12 identical experiments.

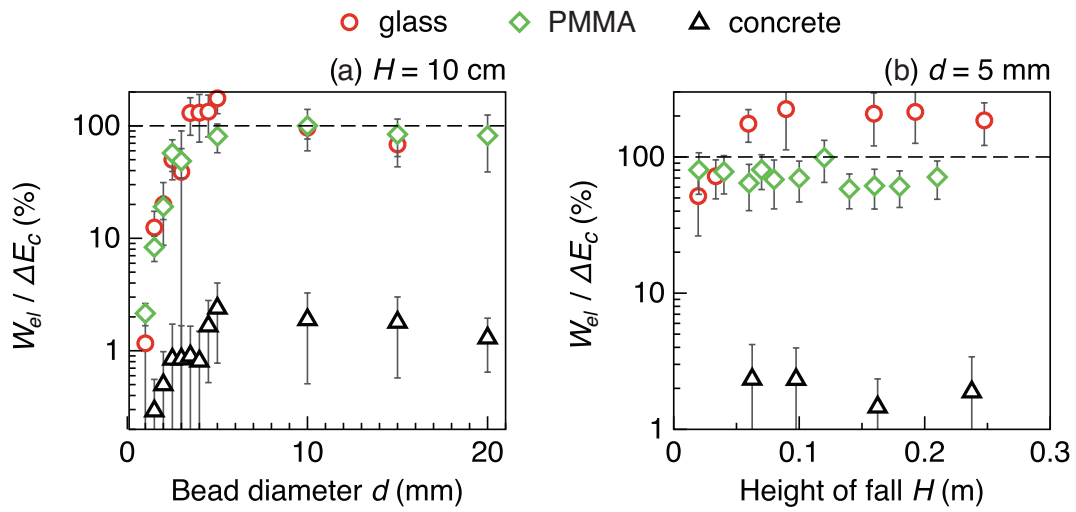


Figure 12: Ratio of the radiated elastic energy W_{el} to the energy lost during the impact ΔE_c , as a function of (a) the bead diameter d for drops tests from height $H = 10$ cm and (b) the fall height H for a bead diameter $d = 5$ mm, on the glass plate, PMMA plate and concrete block. Error bars (± 1 standard deviation) are estimated from reproducibility tests conducted on a series of 12 identical experiments.

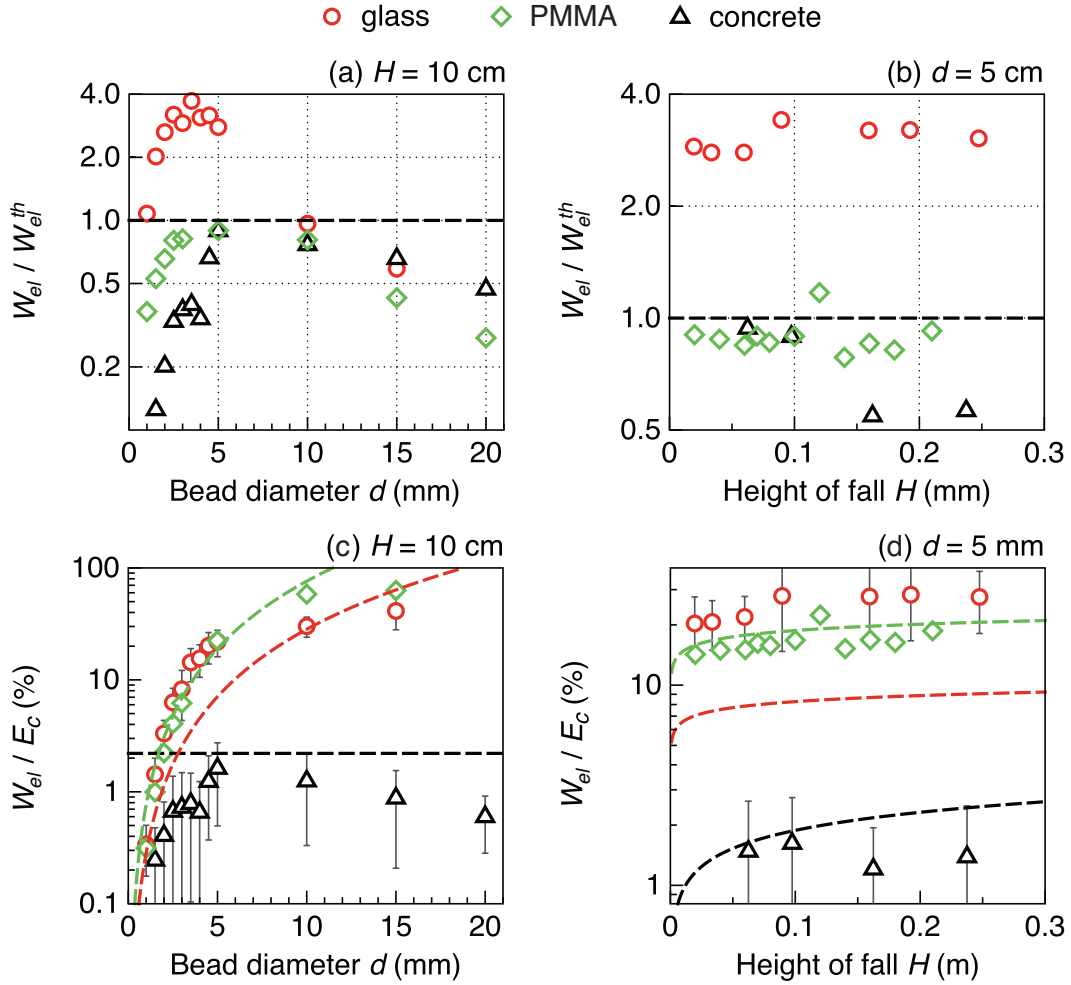


Figure 13: Ratio of the radiated elastic energy W_{el} measured with the energy flux method (a)-(b) to the theoretical radiated energy W_{el}^{th} and (c)-(d) to the energy of the impact $E_c = \frac{1}{2}mV_z^2$, with m , the bead mass and V_z , the impact speed for impacts of steel beads of (a)-(c) different diameters d for a fall height $H = 10$ cm and (b)-(d) different fall heights H for a diameter $d = 5$ cm, on the glass plate, PMMA plate and concrete block. In Figures (c) and (d), the dashed lines represent the ratio of the theoretical radiated elastic energy W_{el}^{th} to E_c .

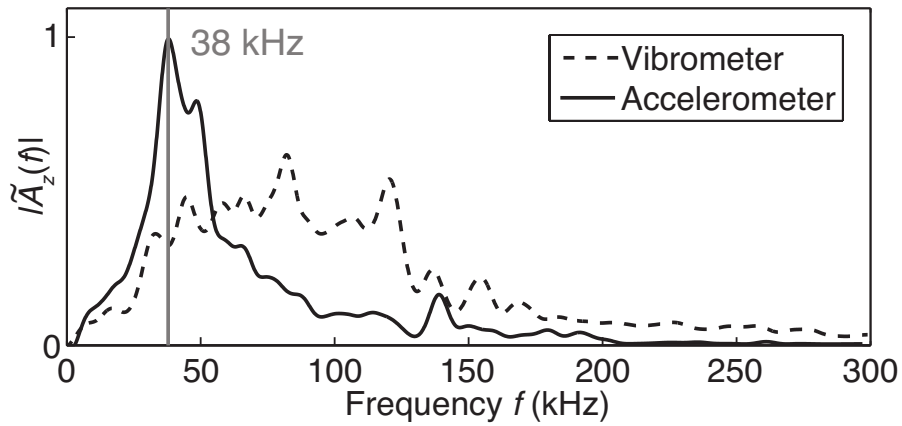


Figure 14: Normalized vibration acceleration $|\tilde{A}_z(f)|$ generated by the impact of a 3 mm steel bead on the surface of the glass plate. The vibration is recorded by a laser Doppler vibrometer (dashed line) and by the accelerometer used in this study (full line). The system constituted by the glass plate and the accelerometer shows a resonance around 38 kHz. Note that the accelerometer is not very sensitive to the frequencies higher than 100 kHz. However, most of the impacts investigated here generate signals with frequencies lower than 100 kHz. Practically, the laser Doppler vibrometer has a much lower signal to noise ratio than the accelerometer and therefore was not used in this study.

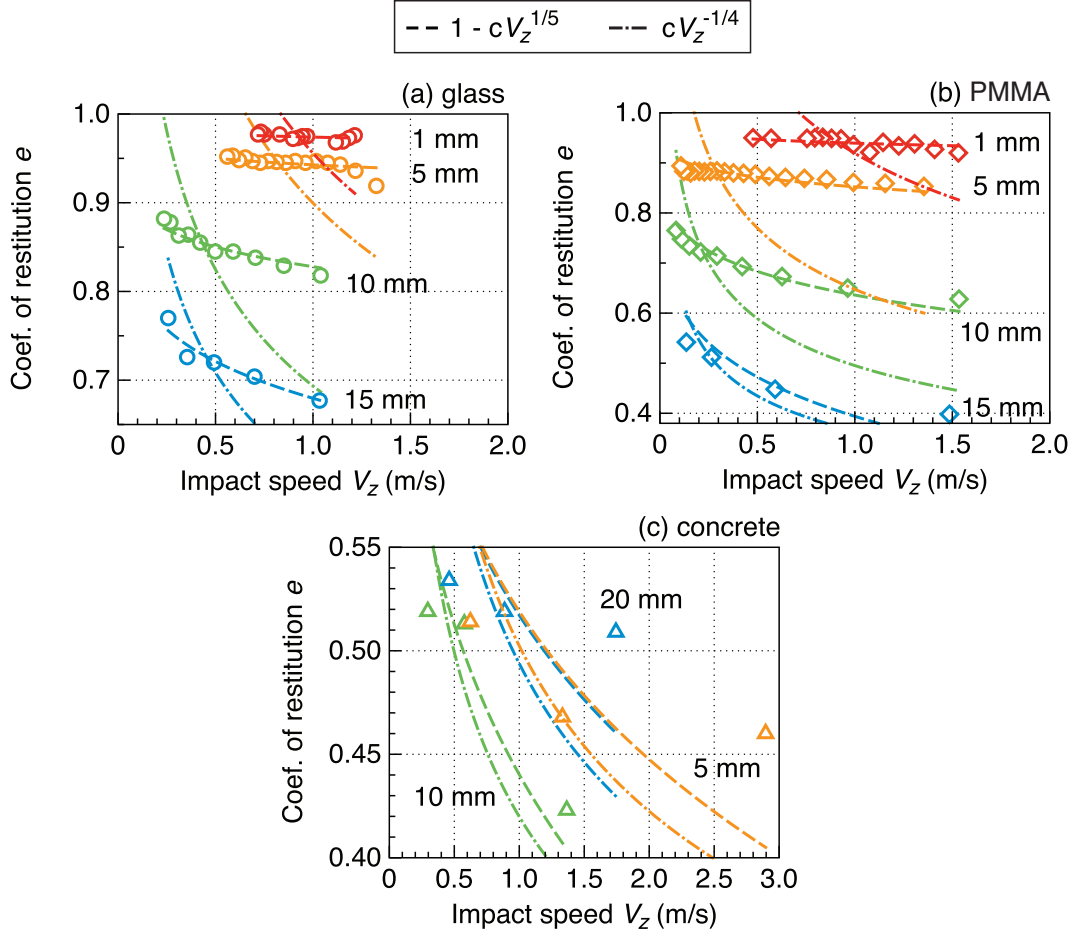


Figure 15: Coefficient of restitution e as a function of the impact speed V_z for different bead diameters d (different colors) on the (a) glass plate, (b) PMMA plate and (c) concrete block. The dashed and dash-dotted lines represent the fitting of the experimental data with the scaling laws $e = 1 - cV_z^{1/5}$ and $e = cV_z^{-1/4}$, respectively, where c is a constant that depends on the bead diameter.

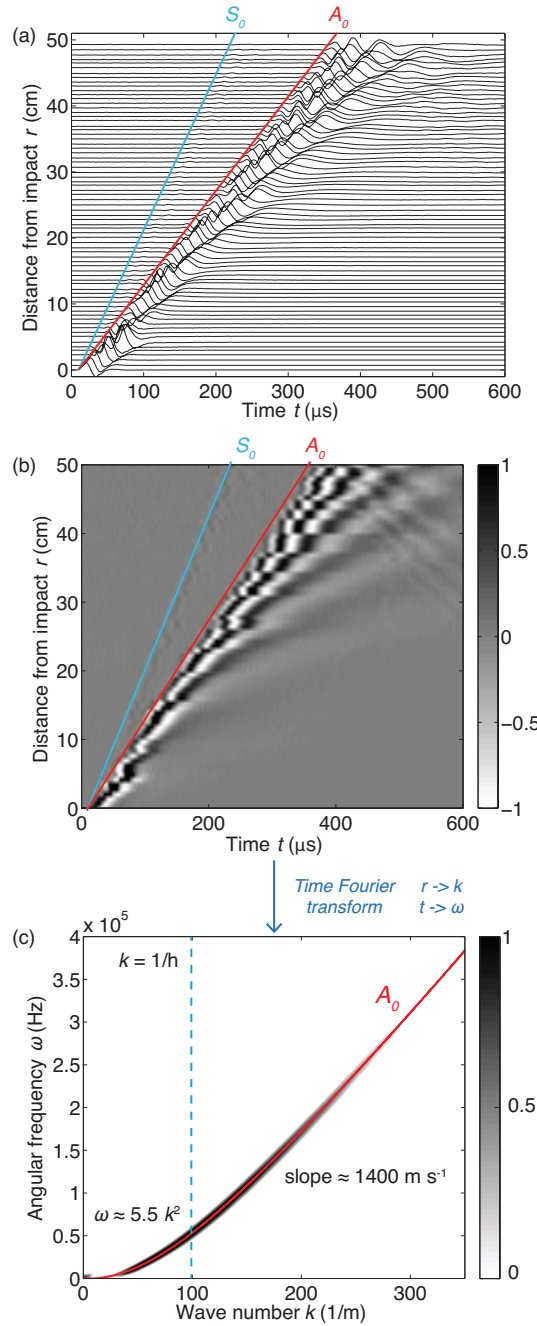


Figure 16: (a) Vibration acceleration $a_z(r, t)$ recorded at different distances r from the impact of a 2-mm steel bead on the PMMA plate. The amplitude of each signal is normalized by its maximum value. The red and blue lines indicate the arrival of the Lamb modes A_0 and S_0 , respectively. (b) Matrix representation of the signals of (a). (c) Relation between the angular frequency ω and the wave number k (i.e. dispersion relation), obtained by time and space Fourier transforms of the matrix in (b). Light and dark shading represent respectively low and high power spectral energy (normalized). Red line: theoretical dispersion relation for the fundamental mode of Lamb A_0 in a PMMA plate of thickness $h = 1$ cm and elastic parameters reported in Table 2.

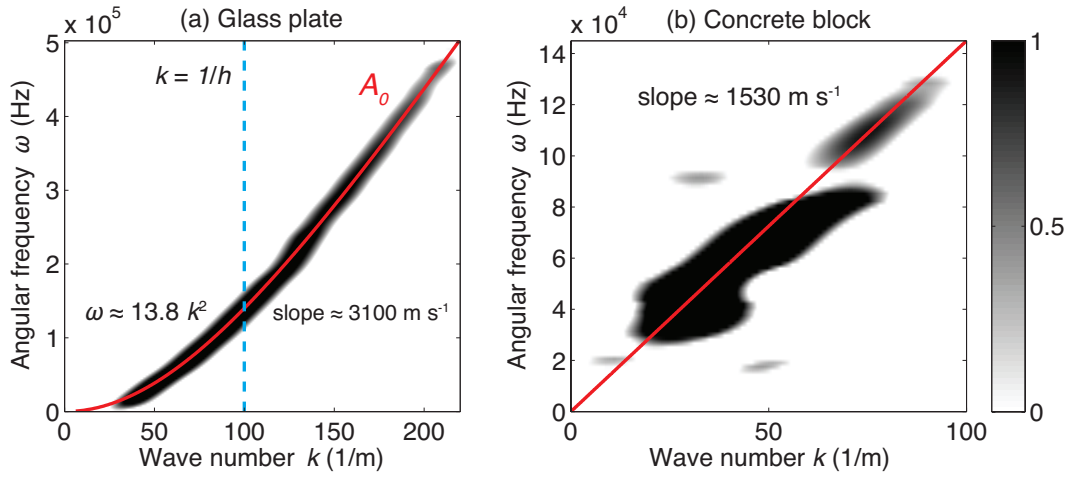


Figure 17: Relation between the angular frequency ω and the wave number k (i.e. dispersion relation) for the direct wave front in (a) the glass plate and (b) the concrete block. Light and dark shading represent respectively low and high power spectral energy (normalized). Red line: (a) theoretical dispersion relation for the fundamental mode of Lamb A_0 in a glass plate of thickness $h = 1 \text{ cm}$ and elastic parameters reported in Table 2; (b) Linear fit of the data. In the concrete block, the group velocity $v_g = \partial\omega/\partial k$ equals the phase velocity $v_\phi = \omega/k$ and is about 1530 m s^{-1} .

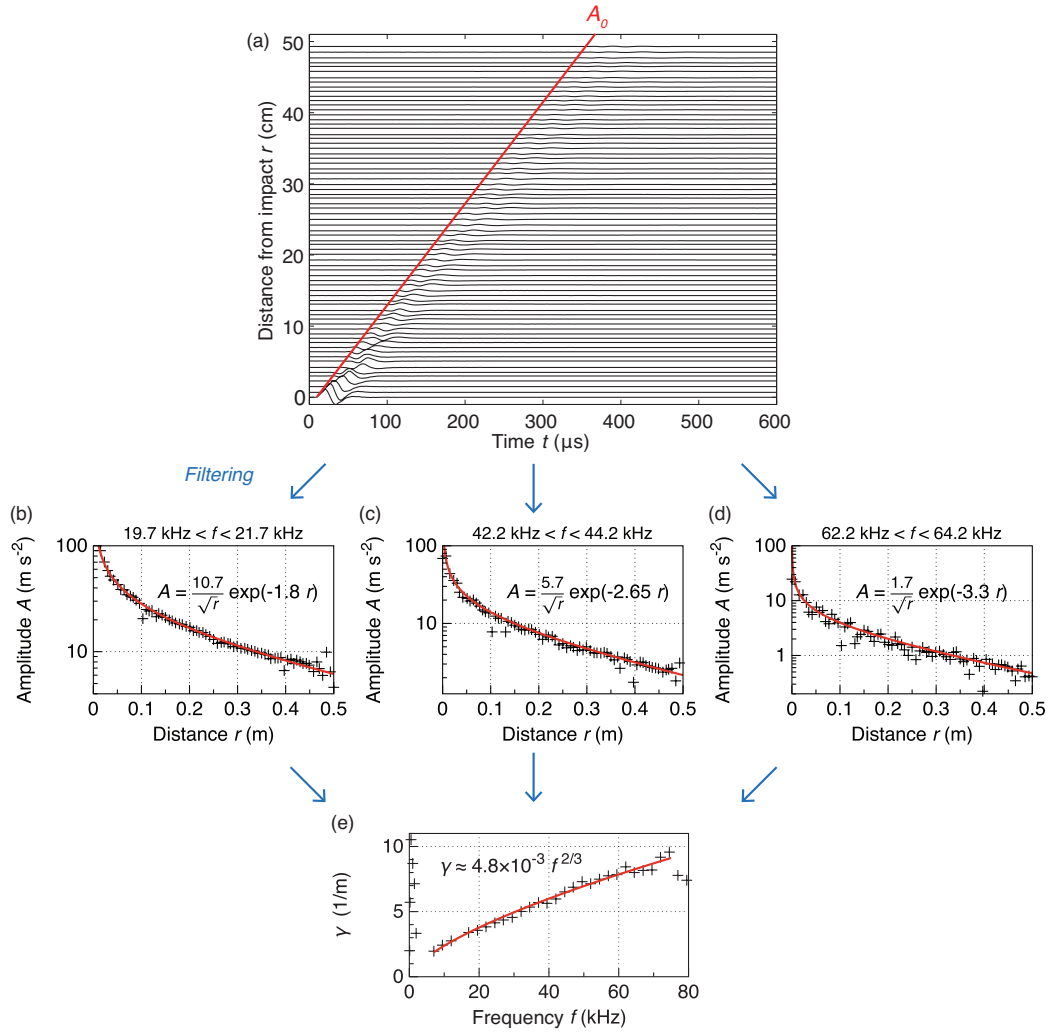


Figure 18: (a) Vibration acceleration $a_z(r, t)$ recorded at different distances r from the impact of a 2-mm steel bead on the PMMA plate. (b), (c) and (d) The signals are filtered in different frequency ranges and their maximum amplitude is represented as a function of the distance r . (e) Attenuation coefficient γ in the PMMA plate as a function of frequency f .

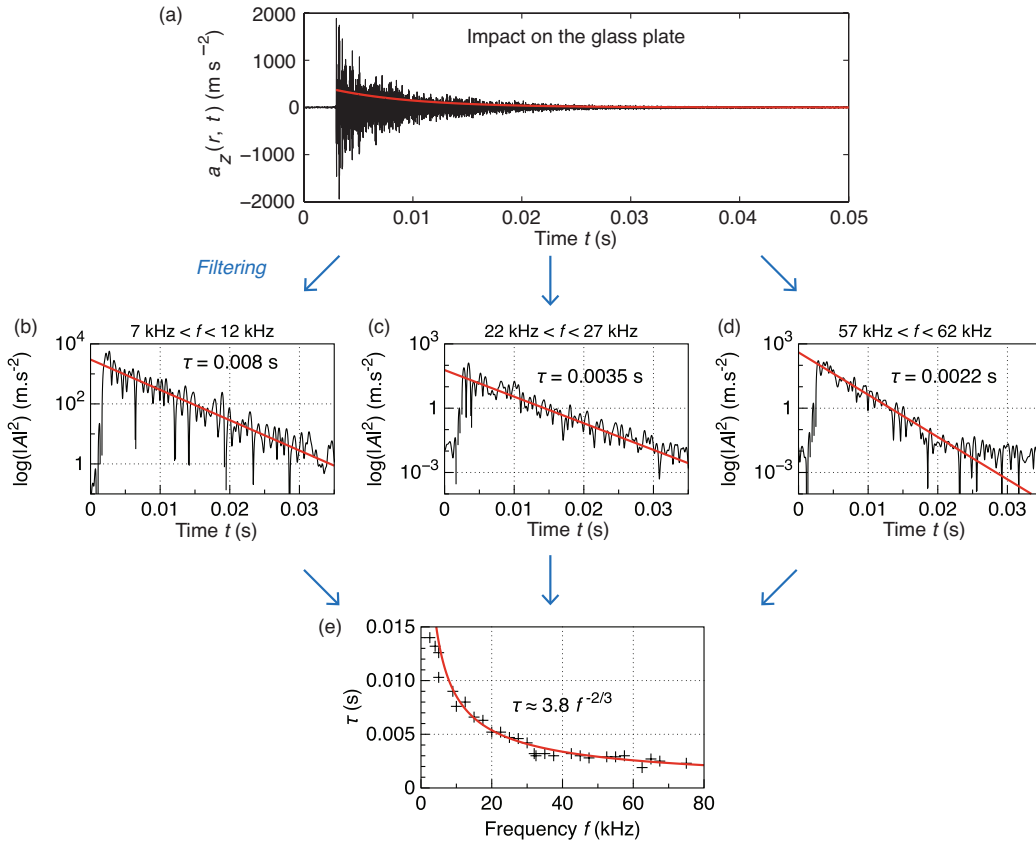


Figure 19: (a) Vibration acceleration $a_z(r, t)$ generated by the impact of a 4-mm steel bead on the glass plate. (b), (c) and (d) The vibration in (a) is filtered in different frequency ranges. The envelope of the squared vibration averaged over several periods decreases exponentially with time and the inverse of the slope in semi-logarithmic scale (red line) is the characteristic time τ . (e) τ as a function of frequency f in the glass plate.

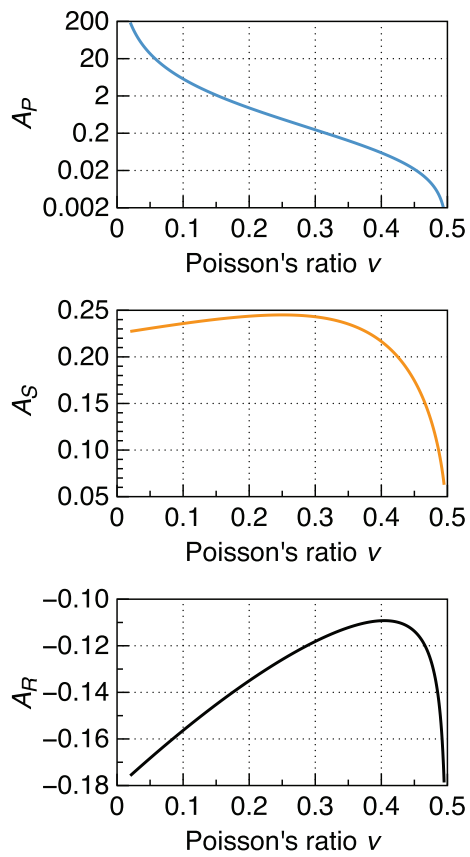


Figure 20: Values of the coefficient A_P , A_S and A_R as a function of Poisson's ratio ν .

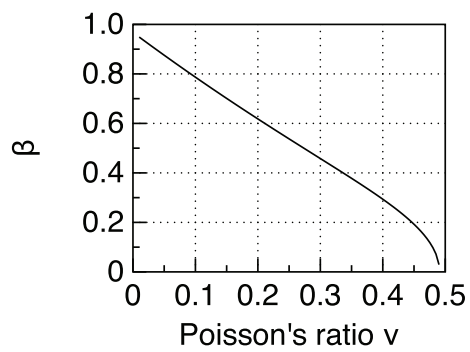


Figure 21: Coefficient β defined by equation (33) as a function of the Poisson ratio ν .