

## **Assigned to creativity: didactical contract negotiation and technology**

Valérie Émin, Nataly Essonnier, Pedro Lealdino Filho [1], Christian Mercat, and Jana Trgalová

S2HEP EA4148, Université de Lyon, Université Claude Bernard Lyon 1, École Normale Supérieure de Lyon, France, [pedrolealdino@gmail.com](mailto:pedrolealdino@gmail.com)

*The mcSquared European project aims at studying Social Creativity among pedagogical resources designers and Creative Mathematical Thinking in their users, through technology, namely a creative e-book software infrastructure and resources called “c-book units”. This article focuses on a study carried out in the framework of this project in France experimenting such a c-book unit and highlights a particular obstacle created by a didactical contract effect related to creativity in the French mathematics classroom: high achieving students perform well on content related to an official assignment but have difficulties engaging in unusual creative problems. The article concludes on possible ways to circumvent it in order to foster the unleashing of mathematical creativity in all students.*

*Keywords: c-book technology, creative mathematical thinking, didactical contract,*

### **INTRODUCTION**

Mathematicians profess that performing mathematics is a creative activity (Hadamard, 1954). While “capital C” creativity is clearly of the essence, can “small c” creativity (Csikszentmihalyi, 1996) be implemented in the classroom in a way to transpose professional activity as a learning tool? Technology supported inquiry based learning is a possible way to put students in situations where their creativity is needed and can be expressed (Blumenfeld et al., 1991). Yet, many obstacles pave its way (Edelson et al., 1999). Despite these obstacles, inquiry based learning is rather put in practice and somehow familiar to students in sciences ; but it might not be so in mathematics. This article focuses on didactical contract effects that may deter its adoption, in a manner similar to (Brandl, 2011) in the realm of giftedness.

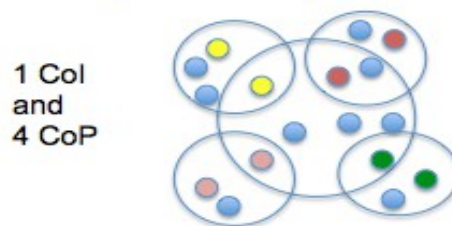
In this article, we first introduce the mcSquared European project [2] and its structure in Communities of Interest (CoI) (Fischer, 2001) aiming at designing electronic books for teaching mathematics enhancing Creative Mathematical Thinking (CMT). We then present the educational resource under consideration in the reported experiment, the “Velocity” c-book, designed by the French CoI. Then we describe and analyze the experimentation carried out in two Grade 9 classrooms, where 14-15 years old students were invited to work on the Velocity c-book math activities. This lead to a didactical contract clash that we describe, followed by an outline of possible remediation. We then conclude on possible ways to improve acceptability and devolution of activities aiming at promoting creativity in mathematics classroom.

## THE MC SQUARED EUROPEAN PROJECT

The mcSquared project aims at designing and developing an intelligent computational environment, a new genre of authorable e-book, which we call 'the c-book' (c for creative), extending e-book technologies to include diverse dynamic widgets, an authorable data analytics engine and a tool supporting asynchronous collaborative design of pedagogical resources, which we call 'c-book units'. The c-book environment aims at stimulating and enhancing creative designs for fostering mathematical creativity in mathematics classes.

Creativity is studied in two complementary ways: Creative Mathematical Thinking (CMT) in students using technology and Social Creativity (SC) in the design of c-book units intended to enhance CMT in the users. The c-book units are produced by four different Communities of Interest (CoI), organised by consortium partners' countries (France, Greece, Spain and UK), bringing together stakeholders from different professional domains, such as publishers, game developers, math education researchers, school educators... The French CoI is composed of representatives of several Communities of Practice (CoP) (Wenger, 1998), mainly gathered around the IREM [3] in Lyon and in Grenoble, and a few individuals (Fig. 1).

Following one CoI fed by several CoP



**Figure 1: Several CoPs around a CoI**

In this paper we only consider the creative mathematical thinking part of the project.

### **Creative mathematical thinking**

Based on the existing literature review on creativity (Guilford, 1950; Kaufman & Sternberg, 2010), mathematical creativity (Sriraman, 2004, 2005; Leikin & Lev, 2007) and mathematical thinking (Tall, 2002; Blinder, 2013), we understand CMT as the combination of divergent and convergent thinking in **mathematics**.

Divergent thinking in mathematics is characterized by:

- fluency: number of solutions;
- flexibility: number of categories (representations and settings) of solutions;
- originality: statistical frequency of solutions;
- elaboration: depth and detail of solutions.

Convergent thinking is characterized by :

- mathematical correctness or conventional answers,

- use of cognitive processes to produce one or very few possible solutions.

Besides, CMT can be fostered by adequate feedback regarding its different dimensions: fluency, flexibility, originality/novelty, appropriateness and usefulness, provided by the c-book unit, teachers, fellow students... CMT is associated with an individual and relative to a given community, a given context, in which the process is envisioned to be used.

We can also integrate the social aspect of creativity in CMT dealing with motivation of participants, the issue of informal norms that promote cooperation and assistance, the social recognition of one's work value. The present article treats specifically this point and some obstacles that hinder creativity, namely didactical contract effects.

### **THE VELOCITY C-BOOK UNIT**

First, a c-book unit is a digital pedagogical resource developed in a specific environment of the project, the c-book technology, viewable and editable in any modern Internet browser, organised in a set of pages which bundle together texts and communicating widgets from different origins enabling a great variety of affordances. They can be movie or sound players, 2D and 3D object viewers, but also constructionist bricks of software such as dynamic geometry software (Geogebra, Cinderella...), dynamic algebra software (epsilonwriter), programming environments (eSlate Logo TurtleWorlds, javaScript, cindyScript, GeogebraScript...), specialized visualization constructs ("widget factories", spreadsheets, graph of a function, algebraic expression editor, calculator...). These widgets can be saved and shared in a particular state, ranging from an empty canvas to a finished full fledged "press and play" interactive resource, as well as half-baked micro-worlds (Kynigos, 2007) to be appropriated and worked upon by the students.

The idea of the Velocity c-book unit stems from the Community of Practice called TraAM (Mutualised Academic Works [4]) group in IREM Lyon, mainly composed of secondary mathematics teachers, focusing on the design of open-ended problems and problem-solving with technology. Their aim is to develop a shared repertoire of resources based on interesting use of ICT in the tackling of interdisciplinary open-ended problems in everyday life situations. They collaboratively design resources that they cross-experiment in their own classes. This CoP production takes place into a national framework coordinated by the ministry of education.

After a presentation and a quick *a priori* analysis of the c-book unit, we present the results of the experimentation carried out in two classes, with different average achievement levels in mathematics.

### **Learning goal**

The Velocity c-book unit, aimed for Grade 9 students, has as the main learning goal the notion of speed as distance divided by time leading to the notion of derivation as a faraway objective.

The main targeted competency is modelling real world situations, but a series of sub-competences useful in this context and promoting creativity are also at stake: Understand a problem, engage in a research, show initiatives, be original; suggest answers, propose hypotheses or conjectures, formulate questions; prove that something is true or that it is false; communicate, orally or in a written form.

The idea behind this c-book unit is to let students gather information from the real world in order to analyse what they see with a “scientific eye”, including the need to “be true to the data”, reflecting the fact that real life does not provide you with polished data that makes sense at once: real data is full of glitches and does not follow exactly the model that you want to force on it. Specifically for this c-book unit, speed is the topical notion at work. Therefore an important goal is to make the students realize that position and time can be defined exactly only in theory but nevertheless, that a crude approximation is enough to be able to make science and take decisions based on it. We know from the start that this is a real change in didactical contract (Brousseau 1988) for most of the students, used to be fed artificial exercises with a unique well defined answer to a question they know they can answer with readily available tools. This experiment focuses on the study of the devolution phase.

### Description of the learning situation

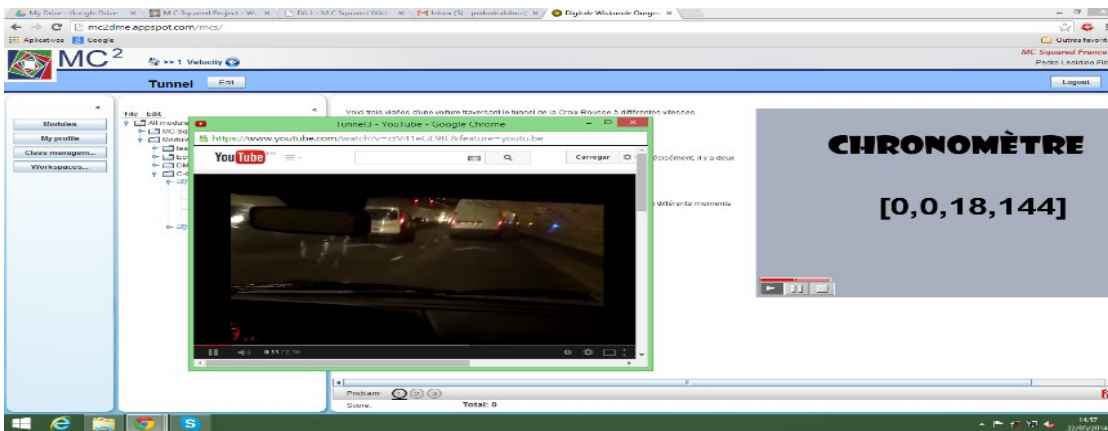
The c-book unit comprises a series of four activities organized in 11 c-book pages:

Activities	“widgets” used	Questions asked to students
Tunnel (3 pages)	Three videos taken from a car driving through a tunnel, Cinderella chronometer, GeoGebra	According to the video, will the car driver receive a fine for exceeding the speed limit in the tunnel?
Particles (4 pages)	Simulation of a particle in Cinderella, Graph2, GeoGebra, Microsoft Kinect	Dance your way as a function: graph the movement of a particle, and move to replicate a given graph.
Control (3 pages)	Picture of a paper disk used in trucks to monitor their speed along the day, Geogebra	According to the tachograph, what is the total distance driven by the truck driver that day?
Average speed (1 p)	GeoGebra	Give the average speed of a car in a given condition.

**Table 1: Velocity c-book unit activities.**

Each activity presents raw data of some sort and a very simple question that should engage the students in making sense of the data in order to answer these questions (see Table 1). Initiatives, which are manifold, have to be taken in order to overcome the limitations inherent to real world phenomena and reach definitive conclusions despite uncertainty. Being able to validate hypotheses such as “the car was driving too fast”, or “the truck drove for more than 500 km”, without knowing everything precisely is a goal that is attainable but requires mathematical creativity from the

students. In what follows, we present in more details the Tunnel activity and its brief *a priori* analysis.



**Figure 2: The tunnel activity**

In this activity, students have to analyse videos taken from within a car while driving through a tunnel and try to figure out whether the speed limit is reached or not. They have to look for clues, such as the trails of lights or chevrons in order to estimate the car speed, by tabulating the timing and positions of these events, computing the average speed, dealing with imprecision and confidence intervals. Three different videos are provided (Fig. 2), with computer adjusted speed profiles falling in three different scenarios showing two types of speed camera: a fixed speed camera and an average speed measuring camera, requiring the notions of instantaneous and average speed. In the first scenario, the car is not caught by any of the two speed cameras, in the second one it is caught only by the fixed speed camera, and in the third scenario it is caught only by the average speed measuring camera.

The total length of the tunnel is 1757 m. The students can identify chevrons and lights that are evenly spaced, with a space prescribed in some official documents that can be found on the internet, or estimated with the number of such events and the total length. The position of the fixed speed camera should be found as well. Then a chronometer available on the c-book page can be used to record some events like the moment a chevron or a light disappear from the screen.

The expected students' behaviour is to search for the total length of the tunnel, to view the video and to measure with the embedded chronometer the total time from the beginning to the end and finally to infer the average speed for the first question.

Several issues are expected to be encountered. What about the format Hours : Minutes : Seconds . Milliseconds, how to work with this data in order to make some math? The students are supposed to paste the minutes and seconds in two columns and elaborate a formula that gives the total amount in seconds. Another issue is the accuracy of the data. When two students record the same events, the data can vary. How can they account for it? Shall they take the mean of the different values between students? Before that they have to agree on a starting time and a translation in time



might be in order. Once this is done, they can begin to do some calculations, but they were already involved in mathematics: modelling a phenomenon, transforming it to observations which can be quantified, is part of mathematics. A more difficult problem is the comparison of data recording different events: especially the ones that record timings of chevrons and timings of lights: it is only after the plotting of both position graphs overtime that the comparison can be performed, because the discrete set of data for both events do not mingle that easily on the tables of numerical values. Only the graphical plot of position with respect to time (and not the rank of item!), which are both discrete sequences of the same integer rank, can show similarities. This is a difficult issue because plotting position against the item rank (the line number in the spreadsheet) is the obvious way to plot a sequence. And the item ranks of the chevrons and lights are related to the associated positions through a linear transformation (the respective distances between two occurrences). This requires a deep understanding of the notion of function of one variable: the position is a function of time but given as values in a sequence with given ranks and time itself is a function of the same rank. Therefore we can infer values of the position as a function of time and forget about the rank as an intermediate variable, artefact of our modelling.

Starting with the raw data, we produced different elaborate constructions and each page in the c-book unit unravels some possible new features, including widgets of many different kinds. We count on the orchestration of a teacher, knowing a good portion of what is feasible given the available technology and examples of implementations that are proposed in the c-book, to help the students leapfrog from one instrumented situation to another. But it is our hope that some students will eventually go beyond the proposed implementations, or in totally new unexpected directions to answer the first question.

### **THE EXPERIMENTATION**

Although our objective was to design learning situations in which the use of the c-book unit could be autonomous by groups of students, the pilot study reported in this section is regulated by a teacher, in order not to “spoil the fun” and yet see progress. Moreover, mastering all the aspects of the powerful tools afforded by the c-book technology is not obvious. For the time being the transition from one page to a next one is not automatic and should be controlled by a teacher orchestrating the activity. The pedagogical context in which we conducted the experimentation was provided by a teacher with her students. Two Grade 9 classes (secondary 3<sup>rd</sup> in French school system, called therefore 3C and 3D in what follows) with 14-15 years old students participated in the study, working in groups of 3-4, equipped with computers and a beamer that could project the work of a given group. The two classes are very different, the first one is composed of students which perform well in mathematics

and the second one of students that have difficulties coping with the core of the curriculum and who circumvent frontal confrontation with mathematics.

We report only on the “Tunnel” activity. Investigation on the average speed, different speed controls and some research on the tunnel was given as homework before the classroom session during which the students worked together on the activity.

### **3D class - lower achieving students**

The students were relating the scenario to their own experience and discussed about the issue, it really meant something for them. After a few viewings of the video, they understood how to compute the average speed by measuring the time and the distance, having learnt how to do it themselves in similar situations. They addressed, discussed and solved the issues, difficult for them, of converting the ratio of a distance by a time from m/s to km/h, using proportionality and dealing with decimal representation of time instead of the usual hexadecimal HHMMSS representation. They showed progress in the direction of defining a protocol suitable to estimate the instantaneous speed (lights, marks on the road) but nobody came to a definite answer, only the average speed was computed and related to their own estimates based on their experience. The session was nevertheless felt as successful by all parties.

### **3C class - higher achieving students**

When comparing the assigned videos between groups, the class, in a very homogeneous fashion, realized that it was the same one, tuned to fit special purposes. They answered at once without doing computations, sorting correctly the three videos into three different scenarios regarding the fine. They gave the answer they thought the teacher was expecting, minimizing their effort and failing to engage into the activity. When pressed, they correctly explained the measurements and computations that had to be done, both for the average and instantaneous speeds but nobody actually did them. What refrained them from doing anything is the obvious fact that only crude estimates were possible and that the answer had to be somehow unique, giving that an approximation would have been wrong and was not in order. The issue of giving error margins was debated. Incited once again to make measurements, they were forced to reluctantly take decisions, measure events, and give answers. They had to admit that, whereas their actual numbers were indeed different, the final conclusions were the same for each group on a given video. The expressed feeling was that of an abuse of power and a lousy work ethic on the part of the teacher who went beyond her right in asking such questions, that this was not mathematics and that nothing of that sort was ever asked at the junior high school final national exam “brevet des collèges”. This students’ behaviour can be interpreted as a reaction to the didactical contract break-up (Brousseau, 1988).

### **Remediation**

A possible way out of this didactical contract clash was found by making explicit a list of competencies taken from the national curriculum. This list of competencies, one of the results of the EvaCoDICE project [5], is now introducing the c-book, in order to be self-evaluated throughout the activity. For example, students have to choose, for the competency labelled “Understand the problem, do some research, take initiatives, be original” an item in the following list:

- “I don't understand what we are looking for, I can not begin”,
- “I understand what we are looking for but I don't know how to begin, I don't have any good idea”,
- “I understand what we are looking for, I am trying but I make mistakes in my research, I have some ideas”,
- “I understand what we are looking for, I make some experiments, I have ideas”.

Showing the students a table with the list of competencies reassured them in the fact that, whereas these are indeed never assessed at a national exam, they are nevertheless officially expected from them. But on the other hand, this remediation leads to a reinforcement of the didactical contract that only what is explicitly and officially required is to be used in the classroom. In a system where national assessments explicit the expectations for all partners of the educational system (students, teachers, parents...), and which sticks to technical and standardized tasks, promoting CMT is a real challenge when the achievement measured by these assessments does not correlate easily with CMT.

## CONCLUSION

Fostering Creative Mathematical Thinking in the classroom needs a tailored “ecology” (Barquero, Serrano, & Serrano, 2010), a trained teacher, a rich *milieu* and a special didactical contract (Chevallard, 2012; Wozniak, 2012) to be negotiated: *it's alright to think! Make some guess! Explore!* Such competencies are seldom valued in the curriculum.

In the agenda for progress in math education, Schoenfeld (2011) states that “*assessments that are consistent with [mathematically rich content and sense making activities]*” is one of the conditions to achieve the goal of a “meaningful engagement with powerful mathematics for all children”. This condition might be the most taxing in the didactical contract effect that was observed here: high achieving students tend to minimize their effort and see no direct interest in engaging into what they see as exceeding their job as a student. Teacher training has as well to address the evaluation of competencies.

It is all the more true with technology enhanced learning especially in unsupervised situations: in order to earn student's interest, we might have to put our activities in the cyber-space perspective in which they live and which is so engaging for them. Adding social-networking and timing might turn a dull set of marks and assessments into a friendly competition; the 21<sup>st</sup> century didactic engineer should identify,



alongside the didactical variables, the playful appealing ones, which can turn a mathematical task into a game where devolution of the task means not trying to please the teacher but to have fun, where fellow students cooperate online, turn upside down their assignments and boast their achievements on social networks. Recent works (Pelay, 2011) and political stands (Vallaud-Belkacem, 2014) tend to point in the right direction!

## NOTES

1. Supported as well by the CAPES - Proc . N° 0791/14-8, Ministry of Education of Brazil.
2. MC SQUARED European project ICT STREP 100712. <http://mc2-project.eu>. The research leading to these results has received funding from the European Union Seventh Framework Programme (FP7/2007-2013) under grant agreement n° 610467 - project “M C Squared”. This publication reflects only the author’s views and Union is not liable for any use that may be made of the information contained therein.
3. IREM: Institute for Research in Math Education, a network of 28 such institutes in France is devoted to studying math education and math teachers training. <http://www.univ-irem.fr>
4. TraAM (Travaux Académiques Mutualisés) are the IREM groups under regional educational authorities that develop and share resources aiming at supporting the use of technology in classrooms.
5. EvaCoDICE project (Évaluation par compétences dans les démarches d’investigation au collège et à l’école) <http://ife.ens-lyon.fr/lea/le-reseau/les-differents-lea/evacodice>

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