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# A study of Permanent magnet couplings with Progressive magnetization using analytical exact formulation

J.F. Charpentier and G. Lemarquand

*Abstract*— In this paper, original structures of permanent magnet cylindrical air gap couplings with progressive magnetization are studied. In these couplings, parallelepipedal magnets with non classical magnetization direction are used and stuck in ironless cores. The magnetization direction of each magnet is chosen to provide an optimal repartition of the induction in the air gap. This progressive magnetization allows to obtain very high values of pull out torque. These types of structures with ironless cores and very high pull-out torque seem very advantageous solutions to minimize the inertia and maximize the pull-out torque of a magnetic coupling. A very efficient method to calculate the torque of such a coupling is presented. This method is based on the use of analytical forces formulas between magnets. This way of calculation allows to evaluate exactly the performance of the studied couplings when the mains dimensions of the coupling are varying with small calculation time. In this paper the influence of the number of pole pairs, the influence of the number of magnets by pole, the influence of the magnets thickness, the influence of the air gap radius and the influence of the length of the structure are studied and discussed.

## I. INTRODUCTION

PERMANENT Magnet magnetic couplings are used in many industrial applications. They allow to transmit torque from a rotating part to another rotating part without any mechanical contact. They are used in sealed equipment to transmit a movement through a separation wall. They are also very useful in high security applications to avoid failures due to torque overload.

The more useful characteristics for a Permanent magnet coupling are the pull-out torque, the inertia, the overall dimensions and the cost of the device. So to design efficiently a Permanent Magnet coupling, we want to maximize the pull-out torque, minimize the inertia and reduce the cost for a given set of overall dimensions.

This paper concerns the study of Permanent Magnet couplings with progressive magnetisation. In these structures Permanent Magnet with non classical magnetisation direction are used and stuck in ironless cores [1]. This type of devices allows to obtain very high value for the pull-out torque with a small magnet volume. The use of ironless cores allows to have very low inertia for the device and is

not a disadvantage in the magnetic point of view. This solution seems a very interesting way to maximize the pull-out torque, minimize the inertia and reduce the cost of the coupling.

A classical tool to study P.M. couplings is F.E. analysis [2], [3] which allows to calculate the pull-out torque of the device for a given geometry of the structure, but this method is very time consuming. This calculation time is an important limitation for a systematic study of the device.

Some analytical methods exist to calculate the torque of P.M. couplings They are based on 2D or 3D hypothesis.

- Furlani have developed 2D and 3D model to compute the torque of radial couplings[4], [5], [6]. Nevertheless this formulation can be only used with cylindrical tye magnets and use numerical integration in the 3D case. So this formulation can not be use in the case of progressively magnetized structures.
- Some authors have presented formulations to compute forces between cuboidal and parallelepipedic magnets [7], [8]. Recently a formulation has been proposed by Lemarquand [?]. This Formulation allows the calculation of the torque of couplings where classically magnetized magnets are used with ironless materials [9], [?].

In this paper we show that the formulation proposed by Lemarquand allows to evaluate easily and very quickly the pull-out torque of a P.M. progressive magnetization coupling. Then the influence of the the variation of each characteristic dimension of the coupling on the performance of the device have been studied. This systematic study allow us to determine some general rules which allows to design efficiently a coupling with progressive magnetisation.

## II. STRUCTURES PRESENTATION

In this paper some structures of permanent magnets coupling with progressive magnetization are studied. In these structures parallelepipedic rare earth Permanent Magnet with a non classical magnetization direction and 1 Tesla remanence are stuck in ironless cores. This type of magnet presents a magnetization direction which is not parallel to one of the edge of the magnet as shown in Fig. 1 The magnetization direction of each magnet is chosen to be progressively oriented from one pole to one other. This disposition of magnets magnetizations allows to create an optimal induction in the air-gap of the coupling [1], [10]. As an example of magnetization direction repartition, the angle  $\theta_i$ , between the magnetization direction and the tangential

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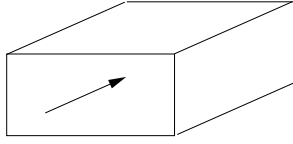


Fig. 1. Magnet with non-classical magnetization

axe, for the  $i^{th}$  magnet of a south pole of the inner rotor of a coupling with same size magnets, as shown in Fig. ??, is chosen at the following value :

$$\theta_i = \frac{\Pi}{2N} + (i - 1) \frac{\Pi}{N} \quad (1)$$

where  $N$  is the number of magnets by pole of the coupling.

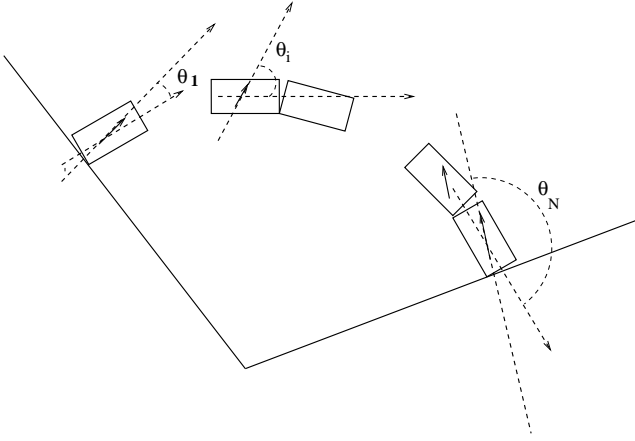


Fig. 2. angles of magnetization in an inner rotor south pole

In all this study all the magnet of a rotor have the same dimensions and the number of magnets by pole can vary.

In Fig. 3 and Fig. 4 two examples of 4 poles coupling with respectively 4 and 5 magnets by pole are presented.

The main parameters of such a coupling are :

- The number of pole pairs :  $n_p$
- The number of magnets by pole :  $N$
- The Length of the structure in the third dimension :  $L$
- The mean Radius of the air gap :  $R_a$
- The mechanical air gap :  $a$ . This air gap is the minimal distance between the inner and the outer rotor.
- The magnet thickness of the inner and outer rotor :  $Th_i$  and  $Th_o$

In Fig. 5 A pole of a 5 magnets by pole coupling is represented and the main dimensions of the coupling are shown. To simplify the study, in all the considered cases the inner magnets and the outer magnets have the same thickness ( $Th_i = Th_o$ ). This choice allows to avoid the demagnetization problems.

With such a progressive magnetization the magnetic flux is mainly contained in the magnet area and the presence of

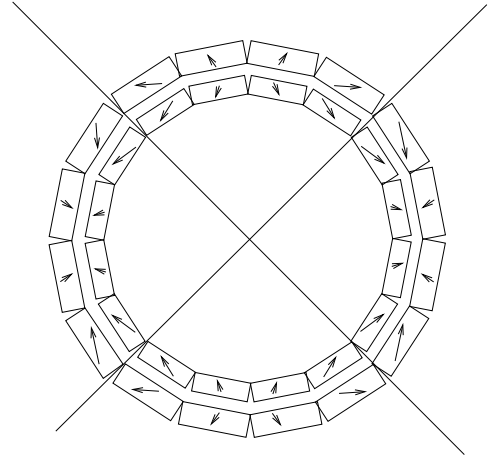


Fig. 3. 4 poles coupling with 4 magnets by pole

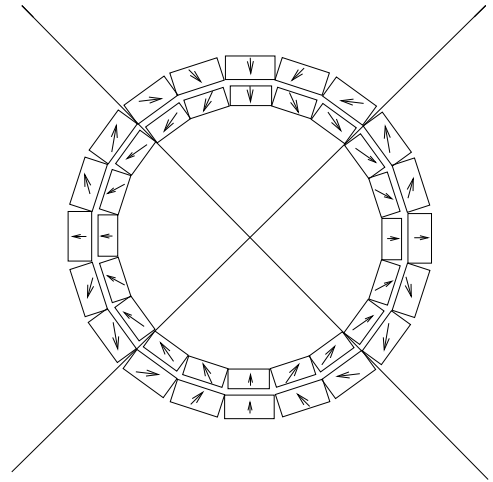


Fig. 4. 4 poles coupling with 5 magnets by pole

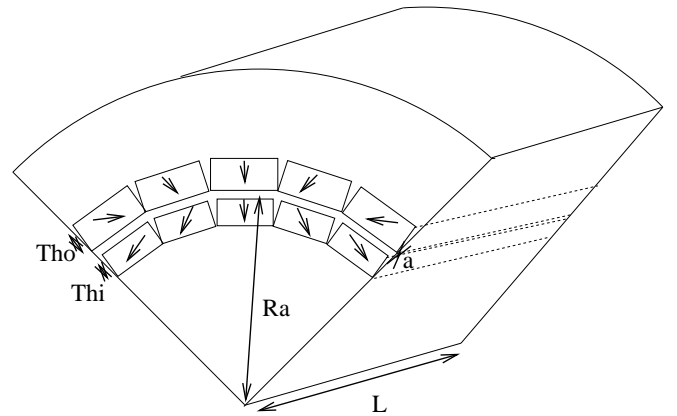


Fig. 5. Main dimensions of a coupling

an iron core does not increase very much the flux created by each rotor in the air gap [10]. That means that the use

of ironless cores is not a disadvantage in the magnetic point of view. In other hand ironless cores allow to have a very low inertia for the inner and the outer rotor [?]. Therefore, in all the studied devices the magnets are stuck in ironless cores. This solution allows computation of the torque by the way of force analytical formulas proposed by Lemarquand. Indeed this formulas are only available when there is no soft ferromagnetic material.

### III. TORQUE CALCULATION METHOD

The performance of the studied couplings can be evaluated by the computation of the pull-out torque of each coupling. This calculation is based on analytical formulas developed by Lemarquand which allows to calculate the components of the forces exerted on a parallelepipedic magnet by another parallelepipedic magnet [?].

#### A. Force between two magnets with a non classical magnetization

This Formulation of forces allows to compute exactly the forces in two direction ( y and z ) between two magnets when the magnetization direction is parallel to the z-edge of the magnet and when one edge of the first magnet is parallel to one edge of the second magnet. The analytical expressions of this forces is given in appendix. The forces are given as a function of the position of the corner  $O'$  of the second magnet in the first magnet local coordinate system (Oxyz), of the dimensions of the two magnets and of the angle  $\Theta$  between the Oxy and Ox'y' planes. This configuration is illustrated by Fig. ???. In the studied case the

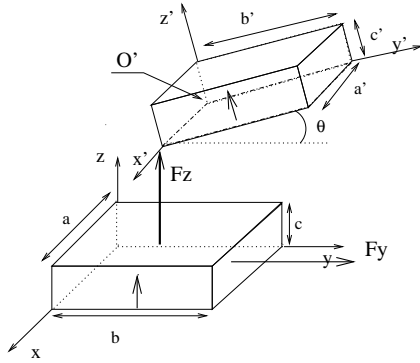


Fig. 6. y and z forces between two magnets

magnetization directions of the magnets are not parallel to the z-axis in the local coordinates of the magnets. However the magnetization of one of the studied magnet can be decomposed in two directions along the y and z edge of the magnet. That means that a magnet  $M$  with a non classical magnetization direction, can be considered as the superposition of two magnets of same size  $M_z$  and  $M_y$  magnetized with the magnetization  $\vec{J}_z$  and  $\vec{J}_y$  where  $\vec{J}_z$  is collinear to the z-axis and  $\vec{J}_y$  to the y-axis in the local coordinates of the

magnet as shown at Fig. 7. Then the magnetization  $\vec{J}$  of the magnet  $M$  is equal to :

$$\vec{J} = \vec{J}_z + \vec{J}_y \quad (2)$$

This assumption is true because of the linearity of the problem.

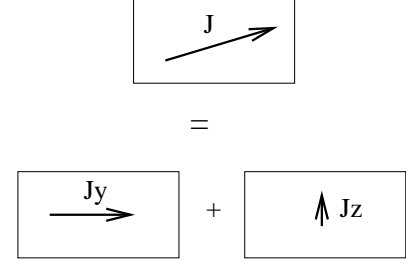


Fig. 7. Superposition of two classically magnetized magnets

Then the force exerted on the magnet  $M$  by another magnet  $M'$  can be expressed as the sum of the forces exerted on the magnet  $M_z$  and  $M_y$  by the magnets  $M'_z$  and  $M'_y$ . Where  $M_z$  and  $M_y$ ,  $M'_z$  and  $M'_y$  are the equivalent magnets resulting of the decomposition of the magnet  $M$  and  $M'$  respectively. If the magnet  $M$  is one magnet of the inner rotor and the magnet  $M'$  is a magnet of the outer rotor, then the tangential component of the force exerted by the magnet  $M'$  on the magnet  $M$  can be easily calculated. This component is the only one which contributes to the torque and can be calculated as the sum of the z-component of the forces exerted on the magnet  $M_y$  by the magnets  $M'_y$  and  $M'_z$  and the y-component of the forces exerted on the magnet  $M_z$  by the magnets  $M'_y$  and  $M'_z$  as shown in the figure 8.

Then the expression of the tangential component of the force exerted by an outer rotor magnet in an inner rotor magnet can be easily established using the analytical formulation of forces. This expression can be expressed as a function of the main dimensions, the position and the magnetization direction of the two magnets.

#### B. Torque Calculation

Then the total torque of a first, second or third type coupling can be calculated as the product of the sum of the tangential forces exerted in each magnet of the inner rotor by all the magnets of the outer rotor, with the average radius of the magnets of the inner rotor.

$$T = \left( \sum_{i \in \text{inner rotor}} \left( \sum_{j \in \text{outer rotor}} F_t(i, j) \right) \right) \cdot r_{in} \quad (3)$$

where  $T$  is the total torque of the coupling,  $F_t(i, j)$  is the tangential force exerted by the magnet  $j$  of the outer rotor on the magnet  $i$  of the inner rotor ( $F_y$  or  $F_z$ ) and  $r_{in}$  is the average radius of the magnets of the inner rotor. So

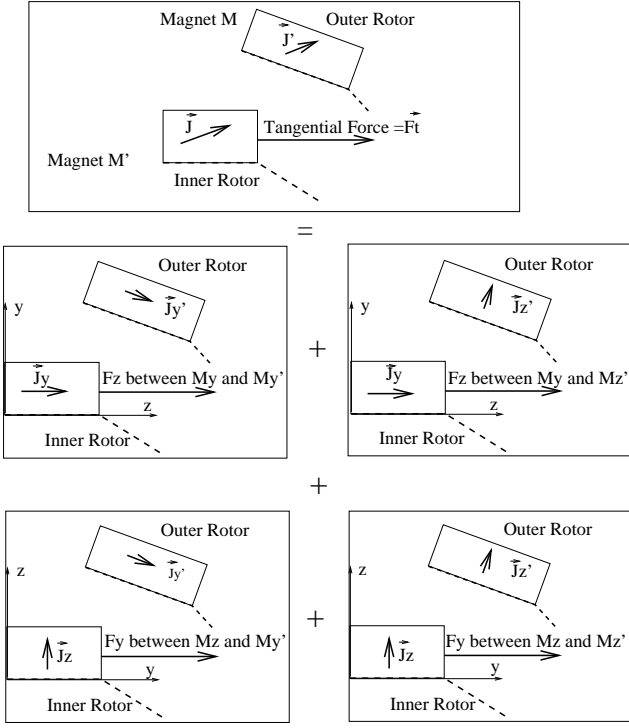


Fig. 8. decomposition of the tangential component of the force between an inner and an outer magnet

the torque of a given coupling can be easily computed as a function of the main dimensions of the coupling and of the angular relative position of the rotors by the use of simple geometry considerations which allow to determine the positions, the orientations and the magnetization direction of each magnet.

$$Torque = F(n_p, N, L, R_a, Th_i, Th_o, a, \alpha) \quad (4)$$

where  $\alpha$  is the angular relative position of the rotors. Notice that this calculation method allows to compute exactly the torque of such a coupling with 3D hypothesis. As an example the torque of a 20 poles couplings with 5 magnets by pole has been computed as a function of the angular relative position of the rotors  $\alpha$ . In this example values of the average radius of the air gap, of the magnet thickness and of the length of device are respectively  $R_a = 100mm$ ,  $Th_i = Th_o = 10mm$  and  $L = 1m$ . In Fig. 9 the torque versus  $\alpha$  for this example is shown. We can notice that the waveform of this torque is very near of a sinusoid. This way of computing allows very small calculation time. the CPU time necessary to calculate the torque for a position in the previous example is only 1.36 seconds in a Pentium II 333 Mhz under Linux Operating System.

#### IV. INFLUENCE OF THE NUMBER OF MAGNETS BY POLE

In this part of the study we consider a fixed number of pole pairs and fixed set of dimensions of the couplings

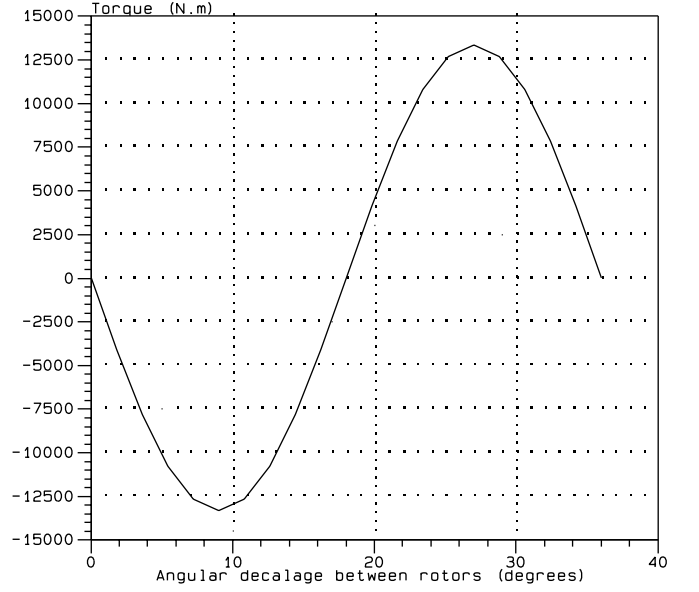


Fig. 9. waveform of the torque for a 20 poles couplings with 5 magnets by pole

( $a, w_i = w_o, R_a, L$ ). The corresponding values for the main dimensions of the couplings are :

- $R_a = 100mm$
- $a = 2mm$
- $Th_o = Th_i = 10mm$
- $L$  is fixed to  $1 m$  to minimize the influence of the length of the structures in the third dimension.

The Fig. 10 shows the evolution of the pull-out torque versus the number of magnets by pole for 10 pole pairs, 15 pole pairs and 20 pole pairs. We can notice that the value of the pull out torque tend to an asymptotic value when the number of pole pairs increases. This asymptotic value correspond to a continuous progressive magnetization in the magnet area. We can notice that this asymptotic value can be 2.6 more important than the value of the pull-out torque obtained for the classical radial magnetization case which corresponds to the 1 magnet by pole case. We can also remark that this asymptotic value depends on the number of pole pairs for a given radius a given air gap and a given thickness of the magnets.

#### V. INFLUENCE OF THE NUMBER OF POLE PAIRS

In this part of the study we want to see the influence of the number of pole pairs on the pull-out torque of a progressive magnetization coupling. To see this influence we calculate the pull-out torque for a given set of characteristic dimensions ( $R_a, a, Th_o = Th_i, L$ ) and for different number of magnet by poles when the number of pole pairs is vary-

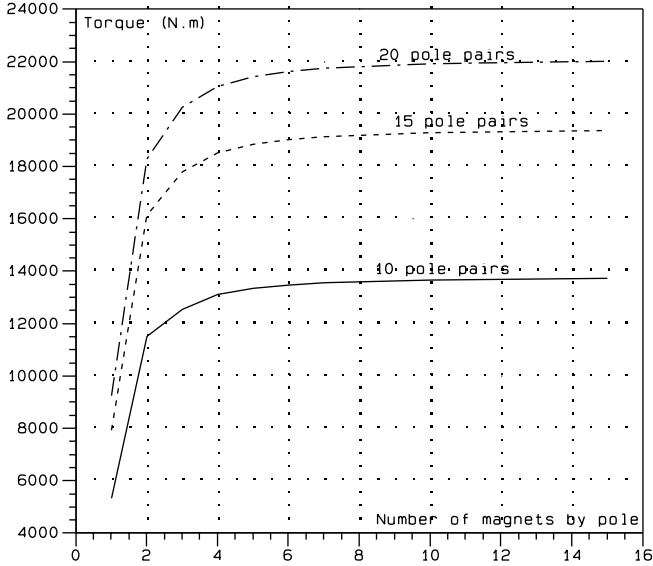


Fig. 10. Evolution of the pull-out torque versus the number of magnet by pole for 10, 15 and 20 pole pairs

ing. The fixed set of dimensions is the same than in the paragraph 5. Fig. 11 presents the evolution of the pull-out torque of such a progressive magnetization coupling for 1 magnet by pole, 2 magnets by pole, 3 magnets by pole, 4 magnets by pole, and 5 magnets by pole couplings versus the number of pole pairs. We can notice that all the curves present a maximum. This result is well known in the classical radial couplings [9]. The maximal value of pull-out torque is obtained for the same number of pole pairs for all the different number of magnets by poles. That means that the optimal number of pole pairs does not depend on the number of magnet by pole and depends only on the mains dimensions of the coupling ( $R_a$ ,  $a$ ,  $Th_o = Th_i$ ,  $L$ ).

## VI. INFLUENCE OF THE MEAN RADIUS OF THE AIR GAP

To study the influence of the mean radius of the air gap we determine the evolution of the optimal number of pole pairs which corresponds to the maximal pull-out torque and of the corresponding value of the pull out torque, when the radius is varying. This value is evaluated for each radius by the same way as in the paragraph 5. The study is done with a fixed set of dimensions ( $a = 2mm$ ,  $Th_o = Th_i = 10mm$ ,  $L = 1m$ ) and with 4 magnets by pole structures. This value of 4 magnets by pole corresponds to a good compromise between the number of magnets by poles and the value of the pull-out torque. In fact, as shown in Fig. 10, with 4 magnets by pole the pull-out torque is near to the asymptotic value obtained when the number of magnet by pole increases. The

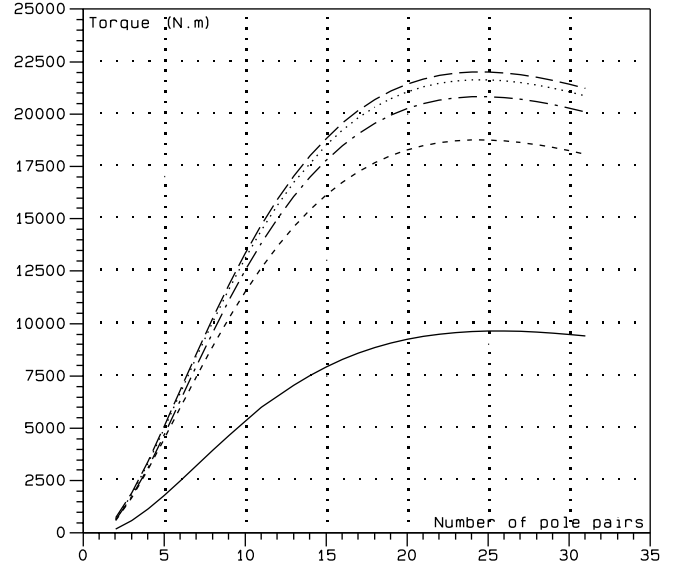


Fig. 11. Evolution of the pull-out torque versus the number of pole pairs for 1, 2, 3, 4 and 5 magnets by pole

Fig. 12 shows the evolution of the optimal number of pole pairs which corresponds to the maximal value of pull-out torque, when the mean radius of the air gap is varying. We can notice that the optimal number of pole pairs is proportional to the mean radius of the air gap. That means that for the optimal configurations for a fixed magnet width and for a fixed air gap one pole occupies the same length in the circumference of the air gap. That means that this length,  $L_p = \frac{Ra * \Pi}{np}$ , is the same for all the optimal configurations. In the Fig. 13 the curve in solid line represents the evolution of the value of the maximal pull-out torque obtained for the optimal number of pole pairs versus the mean radius of the air gap. The curve in dotted line is the interpolation of the first curve by the following function

$$f(R_a) = k.(R_a)^2 \text{ where } k = 2.3.10^6 \quad (5)$$

We can notice that these two curves are nearly identical. That means that the maximal value of the pull-out torque, obtained for the optimal number of poles pairs, appears to be proportional to the square of the mean radius of the air-gap. This assumption seems logical : the force between two adjacent poles of the inner and outer rotors is approximately the same for all the optimal configurations because the poles occupy the same length in the circumference of the air gap. So the torque appears to be proportional to the number of poles pairs and to the radius. That means that the torque appears to be proportional to the square of the radius.

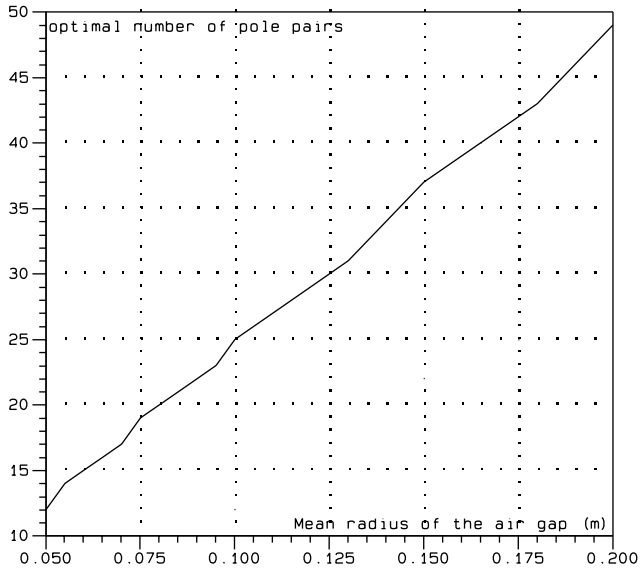


Fig. 12. Evolution of the optimal number of pole pairs versus the mean radius of the air gap

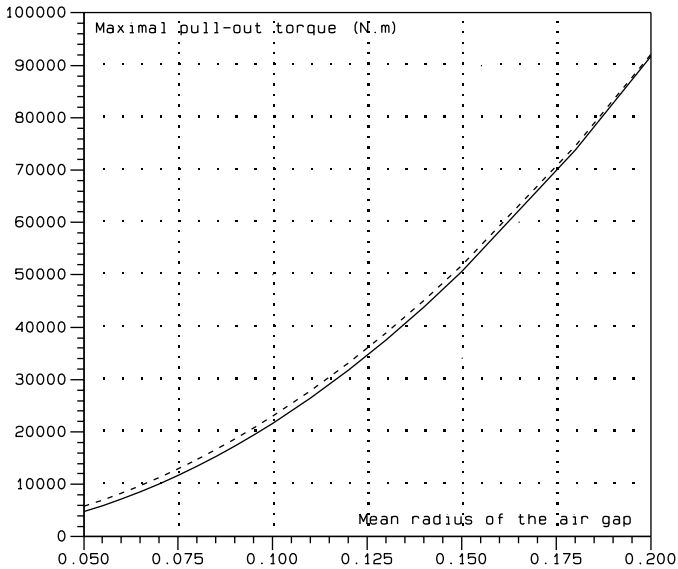


Fig. 13. Evolution of the maximal pull-out torque versus the mean radius of the air gap

## VII. INFLUENCE OF THE MAGNET THICKNESS

In this part of the study we want to evaluate the influence of the variation of the magnet thickness in the per-

formances of the couplings. For a fixed set of dimensions ( $a = 2mm$ ,  $R_a = 100mm$ ,  $L = 1m$ ) the optimal number of pole pairs and the corresponding value of the pull-out torque is calculated, in the same way as in the paragraph 5, when the magnet thickness is varying. Fig. 14 shows the evolution of the optimal number of pole pairs versus the magnet thickness. Fig. 15 shows the evolution of the corresponding value of maximal the pull-out torque versus the magnet thickness. We can see that the value for the optimal number of poles pairs does not depend of the number of magnet by pole. This number of pole pairs converges toward and asymptotic value when the magnet thickness increases. In Fig. 15 we notice that the value of the optimal pull-out torque reaches a maximum and then decreases when the magnet thickness increases. The value of the thickness which corresponds to this maximum does not depend on the number of magnets by pole. That means that an optimal value for the magnet thickness exists and corresponds to an optimal value for the magnet volume. This can be explained by the fact that when the magnet thickness increases the width of the parallelepipedic magnet of the inner rotor is limited by the minimal radius of the inner rotor as shown in Fig. 16. Then if the thickness increases too much, this width become too small to obtain a good value for the pull-out torque of the coupling.

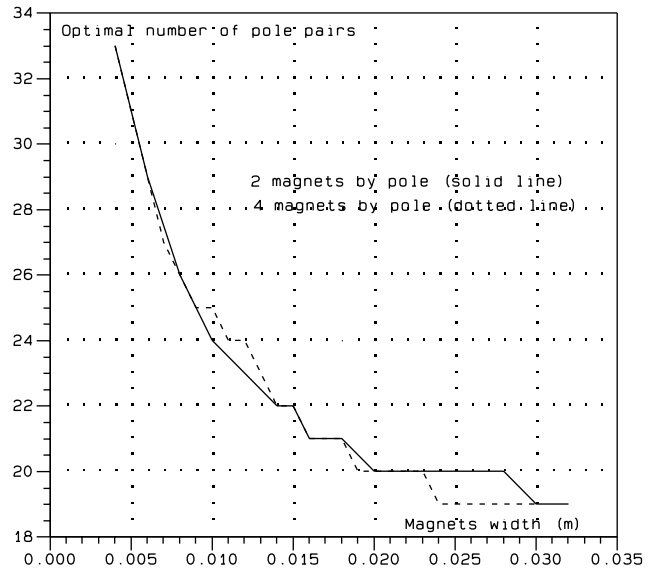


Fig. 14. Evolution of the optimal number of pole pairs versus the magnet width for 2 magnets by pole and 4 magnets by poles structures



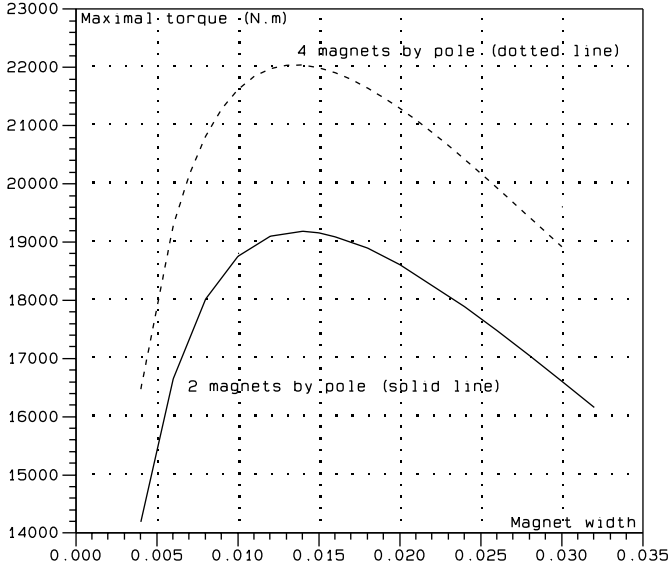


Fig. 15. Evolution of the maximal pull-out torque versus the magnet width for 2 magnets by pole and 4 magnets by poles structures

### VIII. INFLUENCE OF THE LENGTH OF THE DEVICE IN THE THIRD DIMENSION

To study the influence of the length of the couplings in the third dimensions, we have studied the behavior of a coupling for a given set of values for the number of pole pairs, the average radius of the air-gap, and the magnet thickness, when the length of the device is varying. These values are  $n_p = 24$ ,  $R_a = 100mm$ ,  $Th_i = Th_o = 10mm$ , respectively. The Fig. 17 shows the evolution of the pull out torque of the air gap versus the number of pole pairs for structures with 1 and 4 magnets by pole. The curves in dashed line presents an evaluation of the pull-out torque of a coupling by linear interpolation of a very long structure with a bidimensional hypothesis. We can notice that the difference between the linear approximation (2D hypothesis), and the calculated value (3D hypothesis) remains quasi-constant in absolute value, when the length of the device varies. That can be explained because the difference between the 3D and 2D case is due to an end effect in the two ends of the structure. In this two ends the magnetic flux due to the rotor magnets is not contained entirely in the air gap of the coupling, so the torque is more important with 2D calculation hypothesis than in the 3D calculation case. In other hand, this effect is localized in space in the two ends of the structures, so the value of the difference between 2D and 3D calculation does not vary very much with the length of the structure. That means that all the assumptions, which have established in

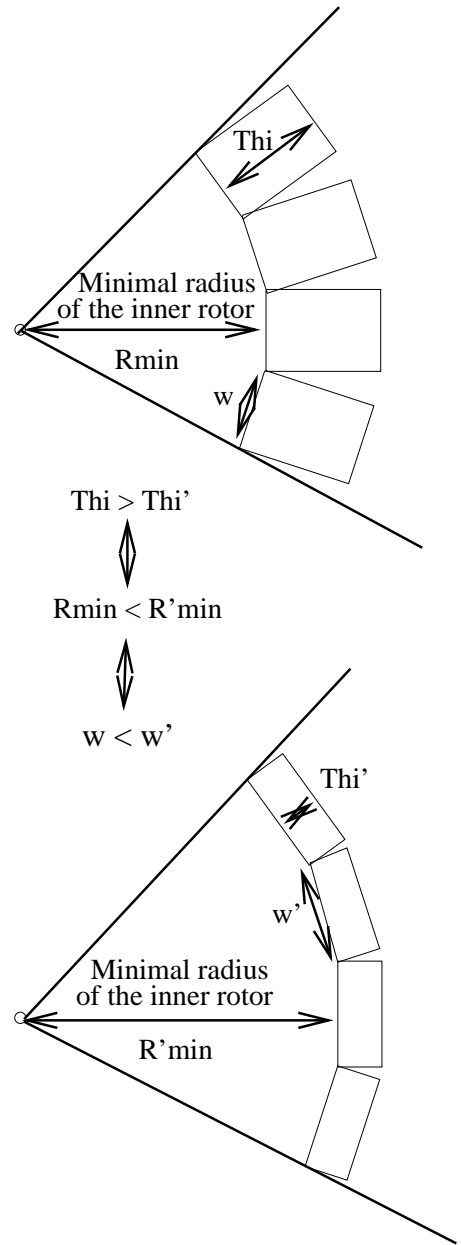


Fig. 16. Influence of magnets thickness on the width of the magnet of inner rotor

the precedent paragraphs, do not depends on the length of the structure.

### IX. CONCLUSION

In this paper Permanent Magnet couplings with progressive magnetization have been studied using an analytical method based on forces formulas between magnets. The study shows that progressive magnetisation PM couplings allows to obtain better pull-out torque than the traditional radial ones and have a lower inertia.

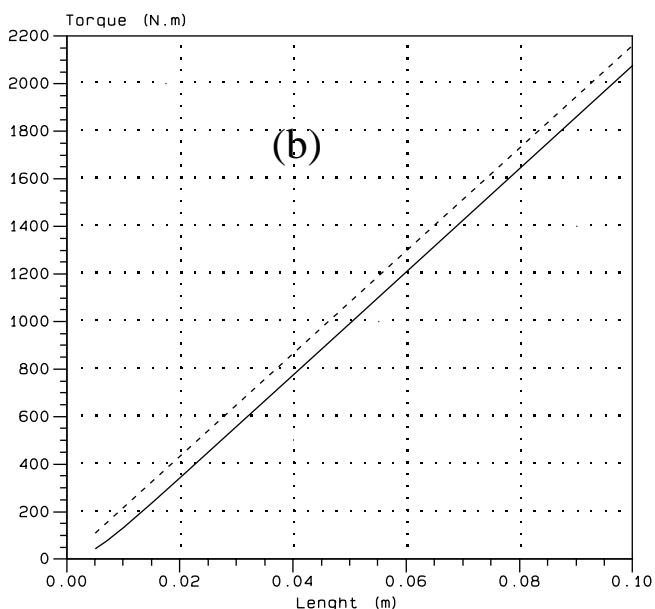
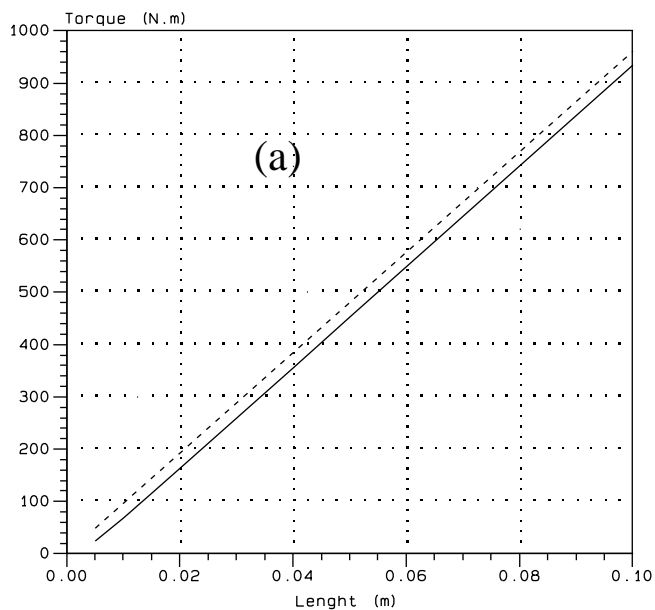


Fig. 17. Torque versus the device length for 1 and 4 magnets by pole (respectively (a) and (b)) : 2D case in dashed line, 3D case in solid line

The systematic study of the performances of these progressive magnetization couplings when the main dimensions are varying allows us to determine some general rules of design.

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