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# Brief Announcement: Anonymous Obstruction-free $(n, k)$ -Set Agreement with $n - k + 1$ Atomic Read/Write Registers

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**Abstract.** This paper presents an obstruction-free solution to the  $(n, k)$ -set agreement problem in an asynchronous anonymous read/write system using solely  $(n - k + 1)$  registers. We then extend this algorithm into (i) a space-optimal solution for the repeated version of  $(n, k)$ -set agreement, and (ii) an  $x$ -obstruction-free solution using  $(n - k + x)$  atomic registers (with  $1 \leq x \leq k < n$ ).

## 1 Context & motivation

Due to failures, concurrent processes have to deal not only with finite asynchrony, i.e., finite but arbitrary process speed, but also with infinite asynchrony. In this context, mutex-based synchronization is useless, and pioneering works in *fault-tolerant* distributed computing, such as [7], have instead promoted the design of concurrent algorithms.

**A first challenge: multi-writer registers.** When processes communicate with *Single-Writer Multi-Reader* (SWMR) atomic registers, a concurrent algorithm usually associates each process with a register. In the case where processes communicate with *Multi-Writer Multi-Reader* (MWMR) atomic registers, as any process can write any register, the previous association is no longer granted for free. To still benefit from existing SWMR registers-based solutions, a classical reduction consists in emulating SWMR registers on top of MWMR registers. In a system of  $n$  processes,  $n$  MWMR atomic registers are needed when the simulation is non-blocking [4]. Hence, if the underlying system provides less than  $n$  MWMR registers, the simulation approach is irrelevant and novel techniques must be found.

**A second challenge: anonymity.** Some algorithms based on MWMR registers require processes to write control values that include their identities. On the contrary, in an *anonymous* system, processes have no identity, the same code, and the same initialization of their local variables. Hence, they are in a strong sense identical. In such a context, the core question that interests us is the following: “Is it possible to solve a given problem with MWMR registers and anonymous processes, and if the answer is “yes”, how many registers do we need ?”

**Consensus and  $k$ -set agreement.** This paper focuses on the  $k$ -set agreement problem in a system of  $n$  processes. This problem introduced in [3], and denoted  $(n, k)$ -set agreement in the following, is a generalization of consensus, which corresponds to the

case where  $k = 1$ . Assuming that each participating process proposes a value, every non-faulty process must decide a value (termination), which was proposed by some process (validity), and at most  $k$  different values are decided (agreement).

**Impossibility results and the case of obstruction-freedom.** When  $k$  or more processes may fail, there is no deterministic wait-free read/write solution to  $(n, k)$ -set agreement [2]. To sidestep this impossibility result, we consider a progress property weaker than wait-freedom, namely *obstruction-freedom*. This property states for  $(n, k)$ -set agreement that a process decides a value only if it executes solo during a “long enough period” without interruption. The notion of  $x$ -obstruction-freedom [8] generalizes this idea to any group of at most  $x$  processes.

## 2 Contributions of the paper

This paper details a *genuine obstruction-free* algorithm solving the  $(n, k)$ -set agreement problem in an *asynchronous anonymous read/write* system where any number of processes may crash. Our algorithm makes use of  $(n - k + 1)$  MWMR registers, i.e., exactly  $n$  registers for consensus. In anonymous systems,  $(n, k)$ -set agreement requires  $\Omega(\sqrt{\frac{n}{k}} - 2)$  MWMR registers [6]. On another hand, the best obstruction-free  $(n, k)$ -set agreement algorithm known so far requires  $2(n - k) + 1$  registers [5]. Hence, our algorithm provides a gain of  $(n - k)$  MWMR registers.

In the *repeated* version of the  $(n, k)$ -set agreement problem, processes participate in a sequence of  $(n, k)$ -set agreement instances. It was recently proved [6] that  $(n - k + 1)$  atomic registers are necessary to solve repeated  $(n, k)$ -set agreement. This paper shows that a simple modification of our base construction solves *repeated*  $(n, k)$ -set agreement without additional atomic registers, being consequently optimal.

Our base algorithm, its extension to solve repeated  $(n, k)$ -set agreement, as well as an  $x$ -obstruction-free variation that uses  $n - k + x$  MWMR registers are all detailed in our companion technical report [1].

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