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► **To cite this version:**

Mehdi Senouci. The endogenous direction of technological change in a discrete-time Ramsey model. 2014. hal-01206029

**HAL Id: hal-01206029**

**<https://hal.science/hal-01206029v1>**

Preprint submitted on 28 Sep 2015

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# The endogenous direction of technological change in a discrete-time Ramsey model

Mehdi Senouci\*

March 17, 2014

## Abstract

The relative price of capital (or equipment) goods with respect to consumption goods is strongly, negatively correlated with income per capita in cross-sections of countries. This stylized fact suggests that economic growth takeoffs are associated with changes in the direction of technical change. It also suggests that increases in productivity that are embodied in capital goods lead to relatively quicker growth.

The goal of this paper is to explore the message of the discrete-time Ramsey model with logarithmic utility, augmented with endogenous direction of technical change. We suppose that the representative agent, while initially at steady state, is offered the possibility to increase either labor-augmenting productivity or investment-specific productivity. We derive the marginal increase in utility from each option. We find that when the elasticity of substitution, the capital share and the rate of impatience lie within the usual ranges, investment-specific technological change is relatively undervalued, because its fruits take relatively more time to materialize.

This approach reflects some interesting ideas on the macroeconomics of structural change. However, its predictions stand at odds with cross-country evidence as well as with the early British growth experience (~1770–1913). We argue that the fixity of the production function constitutes a major obstacle for a consistent theory of the direction of technological changes on neoclassical bases.

Empirical research on economic growth strongly supports the hypothesis that there exists a variety of regimes of growth. Historical evidence on the Industrial Revolution in the UK shows clear signs of a change of regime circa the 1860's, while the US economy witnessed a switch in the early 1980's called the Great Moderation. Neoclassical theory asserts that only increases in productivity can lead to sustained economic growth. In this perspective, the direction of technological change is the prime factor to account for the variety of growth regimes.

Since the seminal study by Greenwood, Hercowitz and Krusell (1997) it has become common among economists to talk about 'investment-specific technological change' (ISTC). ISTC refers to upgrading the quality of machinery and manifests itself in a downward trend of the relative price of capital goods over consumption goods. There is wide evidence that the relative price of equipment goods and the real investment rate are strongly, positively correlated with GDP per capita in

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\*mehdi.senouci@ecp.fr, ECP and PSE. I am grateful to Daniel Cohen, Antoine d'Autume, Aurélien Eyquem and Gwenaël Moysan for their careful comments and advices.

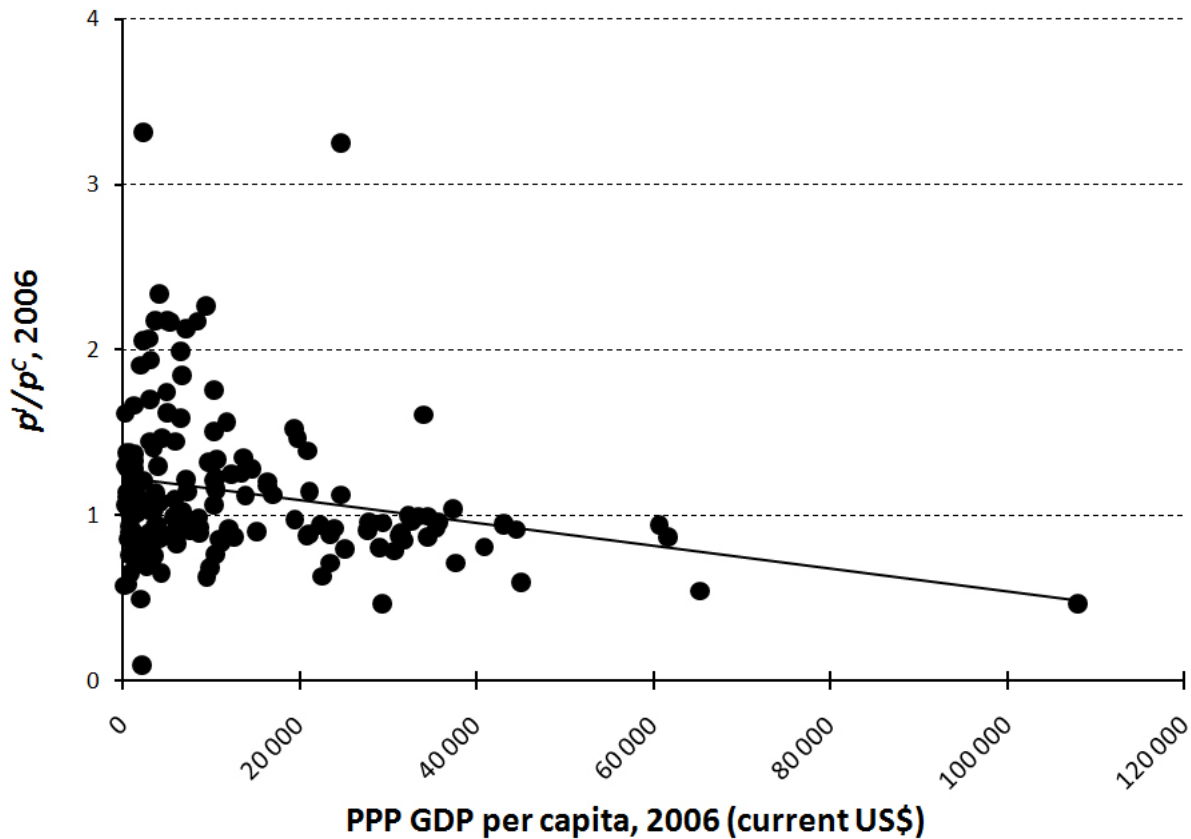


Figure 1: *The negative relationship between the relative deflator of investment over consumption and PPP GDP per capita; source: Penn World Table 8.0 ( $p^I/p^C$ ) and IMF WEO (PPP GDP per capita).*

cross-section<sup>1</sup>. We interpret this correlation as an indication of the importance of forms of technical progress linked to capital (and not only to labor) to sustain high growth rates over long periods. Although other explanations for this fact are possible (taxation, tariffs, etc.), some empirical analyses support the view that LDCs suffer from a disproportionate productivity gap in the capital sector relatively to the most advanced countries<sup>2</sup>. Figure 1 puts in regards the 2006 PPP GDP per capita and the 2006 relative deflator of investment over consumption in national accounts for 167 countries; the relationship is clearly negative.

The goal of this paper is to reconsider the message of the canonical neoclassical model of growth augmented with endogenous direction of technological change<sup>3</sup>, and to assess the validity of this

<sup>1</sup>International comparison of relative deflators of each item of national accounts (consumption, investment, exports, etc.) has been made possible by the work of Robert Summers, Irving B. Kravis and Alan Heston which gave birth to the Penn World Table (PWT) [See in particular Summers, Kravis and Heston (1980); Kravis, Heston and Summers (1981) and Summers and Heston (1988). The PWT 8 can be freely downloaded at <http://www.rug.nl/research/ggdc/data/penn-world-table>.]. All the papers quoted here make use of this database.

The seminal work of Barro (1991) famously cited the *real* investment rate were a strong predictor of GDP per capita in a cross-section of countries. When taking into account that the relative deflator of investment over consumption differs between countries – so that the nominal investment/GDP ratio is not a good proxy for capital accumulation – investment has a positive and significant effect on growth. De Long and Summers (1991, 1992) argue that this correlation indeed reflects causality, which they interpreted using an AK externality logic. Jones (1994) showed that, in a cross-section of countries, economic growth was negatively correlated with the relative price of equipment but not with the price of non-equipment investment goods, which leads him to advocate the tax rate on machinery as an important growth policy instrument. Taylor (1998) further corroborates this result using a panel of Latin American and Asian countries between 1970 and 1989. In the meta-analysis performed by Sala i Martin, Doppelhofer and Miller (2004), the relative price of investment is one of the most robust predictor of GDP per capita. Eaton and Kortum (2001) have shown that only a handful of countries are net supplier of capital goods – these countries also happen to be the most R&D intensive economies. See also Chari, Kehoe and McGrattan (1997), Restuccia and Urrutia (2001) and Armenter and Lahiri (2012).

<sup>2</sup>See for instance Hsieh and Klenow (2007) and Herrendorf and Valentinyi (2012).

<sup>3</sup>The framework we set up below pertains to the class of induced-innovation models. This class of models investigates the consequence of market

approach .

Namely, we use a discrete-time Ramsey structure with a logarithmic utility function. We assume that there are two goods: a capital good and a consumption good. The capital good is simply produced from the consumption good, through a linear technology. We suppose that the agent is initially at the steady state without technical change. At date 0, he is given the opportunity to undertake either some marginal labor-augmenting technical change or investment-specific technical change (LATC vs. ISTC). LATC increases the effective supply of labor, while ISTC increases the efficiency of the process through which capital is produced<sup>4</sup>.

Each choice has specific dynamic consequences and, since the agent is impatient, he does not necessarily pick the high-growth option. In particular, we show that whenever the elasticity of substitution between capital and labor is greater than the product of the capital share and the rate of preference for the present ( $\sigma > \alpha\beta$ ), the agent might choose to undertake LATC even if ISTC increases relatively more steady-state consumption – in this case, we can say that ISTC is undervalued relatively to LATC. Since a great part of the gains from LATC is immediate while a great part of ISTC comes slowly through capital accumulation, this is no surprise that (for the usual values of the parameters) the agent might have a bias towards LATC. It is also no surprise that in such a framework, the more patient the agent is, the more likely he is to choose the ISTC option.

This augmented Ramsey approach can explain why the relative price of machinery is higher in less-developed countries. But when we confront it with available macroeconomic data on the early British growth experience (~1770–1913), we find that the timing of the transition from an ISTC-led to an LATC-led growth regime circa 1860 is at odds with the theoretical predictions.

It comes out that the canonical neoclassical framework is more successful in accounting for each growth regime individually than in explaining the transition between them. We argue that, by assuming that the production function is inexorably fixed, the neoclassical model cannot depict capital-labor substitution as a technology choice, while the elasticity of substitution is the ultimate determinant of the direction of technological change.

The rest of the paper is organized as follows. Section 1 presents the augmented Ramsey model and its steady state; section 2 analyzes the endogenous direction of technical change; section 3 summarizes the conclusions and discusses the scope of validity of the approach; section 4 concludes.

## 1 The augmented Ramsey steady state

In this section, we present the model with no technological change.

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equilibrium on the direction of technical change. The prime intuition is due to Hicks (1932), who claimed that ‘a change in the relative share of factors of production is itself a spur to innovation and inventions of a particular type – directed at economizing the use of a factor which has become relatively expensive.’ Fellner (1961), Kennedy (1964), Samuelson (1965), Drandakis and Phelps (1966), Nordhaus (1967, 1973) and Acemoglu (2002, 2003) have tackled this issue. To our knowledge, what follows constitutes the first analysis of induced innovation considering investment-specific (rather than capital-augmenting) technical change.

<sup>4</sup>This distinction is consistent with the idea that one can improve on his skills or on his tools to become more productive.

Time is discrete and begins at date  $t = 0$ . The economy is populated by one eternal and impatient agent, whose preferences represented by:

$$V(C_0, C_1, \dots) = \sum_{t=0}^{\infty} \left( \frac{1}{1+\theta} \right)^t \ln C_t$$

with  $\theta > 0$ . The agent supplies  $L$  units of raw labor at each period, and labor-augmenting productivity is constant at  $h$ , so the effective labor supply at each period is at  $hL$ . The agent is also initially endowed with a stock of capital  $K_0 > 0$ . Production at date  $t$  is:

$$Y_t = F(K_t, hL) = hL f(k_t)$$

where  $K_t$  is the capital stock available at date  $t$ ,  $F$  is a constant-returns-to-scale and diminishing-marginal-returns production function,  $f$  is defined by  $f(\cdot) = F(\cdot, 1)$ , and  $k_t = K_t/hL$  stands for the effective capital-labor ratio. We denote by  $\bar{k} = K_0/hL$  the initial effective inputs ratio.

Capital is produced linearly from output: one unit of output can be turned into  $q$  units of capital –  $q$  denotes investment-specific productivity. Capital then depreciates in one period, so:

$$\forall t \geq 0, K_{t+1} = q(F(K_t, hL) - C_t)$$

or, in intensive notations:

$$\forall t \geq 0, k_{t+1} = q \left( f(k_t) - \frac{c_t}{h} \right)$$

with  $c_t = C_t/L$ . The problem of the agent seen from date  $t = 0$  is:

$$\max_{c_0, c_1, \dots} \sum_{t=0}^{\infty} \left( \frac{1}{1+\theta} \right)^t \ln c_t \quad (1)$$

$$\begin{aligned} \text{s.t.} \quad k_{t+1} &= q \left( f(k_t) - \frac{c_t}{h} \right) \\ k_0 &= \bar{k} \end{aligned} \quad (2)$$

The Lagrangian is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \left( \frac{1}{1+\theta} \right)^t \left( \ln c_t - \lambda_t \left[ k_{t+1} - q \left( f(k_t) - \frac{c_t}{h} \right) \right] \right)$$

Optimality conditions are summarized by the following equations (3), (4) and (5):

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \iff \frac{1}{c_t} = \frac{q}{h} \lambda_t \quad (3)$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = 0 \iff \lambda_t = \frac{q f'(k_{t+1})}{1+\theta} \lambda_{t+1} \quad (4)$$

$$\frac{1}{(1+\theta)^t} \frac{\lambda_t}{c_t} \xrightarrow{t \rightarrow \infty} 0 \quad (5)$$

The system in the  $(k, c)$  plane obeys to the law:

$$\begin{aligned} \frac{c_{t+1}}{c_t} &= \frac{q f'(k_{t+1})}{1+\theta} \\ k_{t+1} &= q \left( f(k_t) - \frac{c_t}{h} \right) \end{aligned}$$

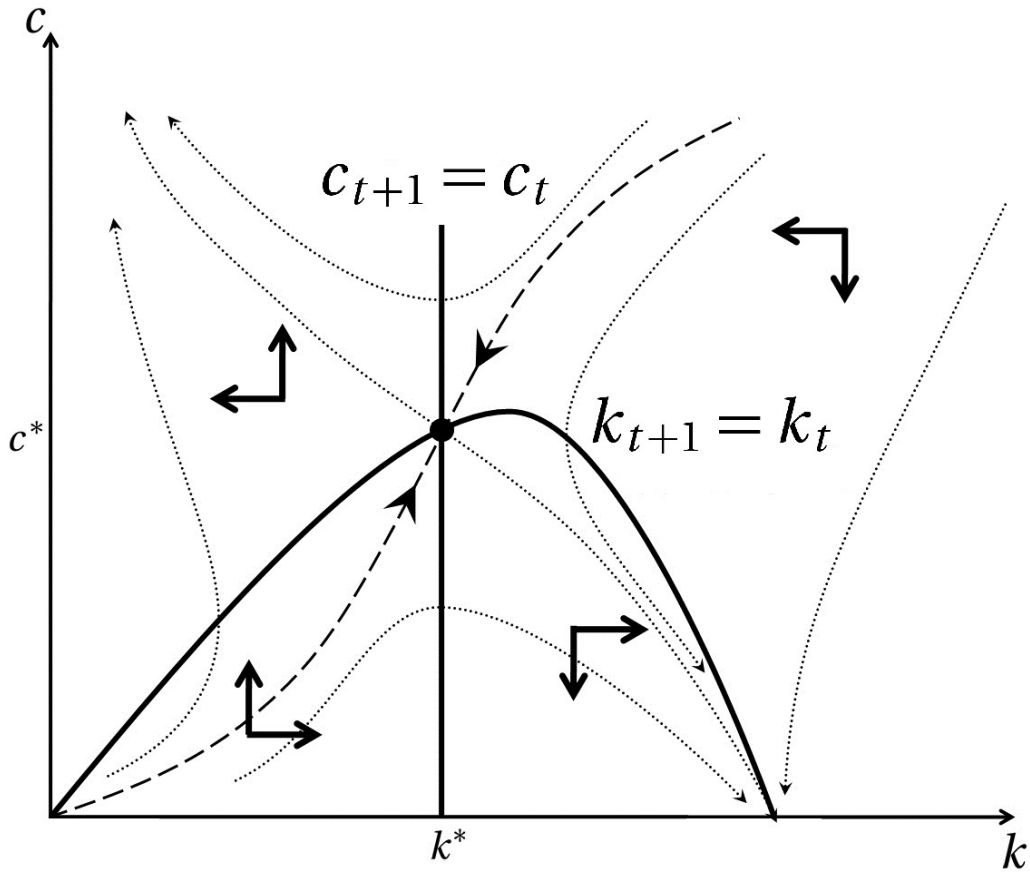


Figure 2: The phase diagram in the absence of technical change.

This version of the Ramsey model, augmented with an investment-specific productivity term, admits the traditional phase diagram represented on figure 2. The only non-trivial steady state is saddle-path stable and characterized by:

$$qf'(k^*) = 1 + \theta \quad (6)$$

$$c^* = h\left(f(k^*) - \frac{k^*}{q}\right) \quad (7)$$

Notice that the steady-state effective capital-labor ratio  $k^*$  depends only on  $q$ , not on  $h$ . Consequently:

$$\frac{h}{c^*} \frac{\partial c^*}{\partial h} = 1 \quad (8)$$

Log-differentiating (6) with respect to  $q$  yields:

$$\frac{q}{k^*} \frac{\partial k^*}{\partial q} = \frac{-f'(k^*)}{k^* f''(k^*)} \quad (9)$$

Let's denote by  $\sigma^*$  the elasticity of substitution and  $\alpha^*$  the capital share around  $k^*$ :

$$\sigma^* = -\frac{f'(k^*)(f(k^*) - k^* f'(k^*))}{k^* f(k^*) f''(k^*)}$$

$$\alpha^* = \frac{k^* f'(k^*)}{f(k^*)}$$

Through these definitions, equation (9) can be rewritten like:

$$\frac{\partial k^*}{\partial q} = \frac{\sigma^*}{1 - \alpha^*} \quad (10)$$

The steady-state increase in consumption, through equation (6) is then:

$$\begin{aligned} \frac{q}{c^*} \frac{\partial c^*}{\partial q} &= \frac{q}{f(k^*) - \frac{k^*}{q}} \frac{\partial \left( f(k^*) - \frac{k^*}{q} \right)}{\partial q} \\ &= \frac{1}{f(k^*) - \frac{k^*}{q}} \left( k^* f'(k^*) \frac{\sigma^*}{1 - \alpha^*} - \frac{k^*}{q} \left( \frac{\sigma^*}{1 - \alpha^*} - 1 \right) \right) \end{aligned}$$

With the definitions of  $\alpha^*$  and with the quasi-golden rule identity (6), we can finally write that:

$$\frac{q}{c^*} \frac{\partial c^*}{\partial q} = \frac{\alpha^* + \theta \frac{\alpha^*}{1 - \alpha^*} \sigma^*}{1 - \alpha^* + \theta} \quad (11)$$

Let  $U(\bar{k}, h, q)$  be the value function associated with the problem (1). Then, by the definition of  $U$ :

$$U(k^*(q), h, q) = \frac{1 + \theta}{\theta} \ln c^* = \frac{1 + \theta}{\theta} \left( \ln h + \ln \left( f(k^*) - \frac{k^*}{q} \right) \right)$$

### Properties of the value function

Let's now see problem (1) from a recursive angle. Bellman's principle of optimality tells us that, at optimum, for each date  $t \geq 0$ :

$$U(k_t, h, q) = \ln c_t + \frac{1}{1 + \theta} U(k_{t+1}, h, q) \quad (12)$$

Through equation (2), we can write the problem in terms of  $k_{t+1}$  only:

$$U(k_t, h, q) = \max_{k_{t+1}} \left\{ \ln \left( h \left( f(k_t) - \frac{k_{t+1}}{q} \right) \right) + \frac{1}{1 + \theta} U(k_{t+1}, h, q) \right\} \quad (13)$$

The envelope theorem applied to (13), taking  $k_t$  as a parameter, leads to the following identity valid for all  $t \geq 0$ :

$$\frac{\partial U}{\partial k} \Big|_{(k_t, h, q)} = \frac{f'(k_t)}{f(k_t) - \frac{k_{t+1}}{q}} = \frac{h f'(k_t)}{c_t} \quad (14)$$

By taking  $k_t = k^*$  and  $c_t = c^*$  in equation (14) we can derive the value of  $\frac{\partial U}{\partial k}$  at steady state:

$$\frac{\partial U}{\partial k} \Big|_{(k^*(q), h, q)} = \frac{h f'(k^*(q))}{c^*(h, q)}$$

or:

$$\frac{\partial U}{\partial k} \Big|_{(k^*(q), h, q)} = \frac{1 + \theta}{q \left( f(k^*) - \frac{k^*}{q} \right)} \quad (15)$$

Let's simply write  $\bar{k} = k$ . Via equation (14) and the capital accumulation relationship (2), equation (12) can be written in terms of  $k$  only:

$$U(k, h, q) = \ln \left( \frac{h f'(k)}{\frac{\partial U}{\partial k}} \right) + \frac{1}{1 + \theta} U \left( q \left( f(k) - \frac{f'(k)}{\frac{\partial U}{\partial k}} \right), h, q \right) \quad (16)$$

This equation characterizes the value function, but – to our knowledge – it cannot be solved analytically. In any case, this characterization will bring the results we are looking for.

## 2 Impatience and the direction of technological change

Suppose now that the agent is initially at the steady state corresponding to  $(h, q)$ . At date  $t = 0$ , the agent is offered the following choice : he can choose either to increase  $q$  marginally by a factor  $dz_q$  ( $dq = q dz_q$ ), or increase  $h$  marginally by a factor  $dz_h$  ( $dh = h dz_h$ ). In the environment of perfect certainty that we consider here, the agent shall choose the option that increases the most his indirect utility function. Let's analyze successively the two options.

### 2.1 An increase in $q$

If the agent chooses to increase  $q$  at date  $t = 0$ , the resulting increase in his indirect utility is<sup>5</sup>:

$$dU|_q = \frac{\partial U}{\partial q} \Big|_{(k^*(q), h, q)} dq$$

So we need to derive  $\frac{\partial U}{\partial q} \Big|_{(k^*(q), h, q)}$ . Let's differentiate equation (16) with respect to  $q$ , for any  $k > 0$ :

$$\frac{\partial U}{\partial q} \Big|_{(k, h, q)} = -\frac{\frac{\partial^2 U}{\partial q \partial k} \Big|_{(k, h, q)}}{\frac{\partial U}{\partial k} \Big|_{(k, h, q)}} + \frac{1}{1 + \theta} \left[ \left( f(k) - \frac{f'(k)}{\frac{\partial U}{\partial k} \Big|_{(k, h, q)}} + q f'(k) \frac{\frac{\partial^2 U}{\partial q \partial k} \Big|_{(k, h, q)}}{\left( \frac{\partial U}{\partial k} \Big|_{(k, h, q)} \right)^2} \right) \frac{\partial U}{\partial k} \Big|_{(k_+, h, q)} + \frac{\partial U}{\partial q} \Big|_{(k_+, h, q)} \right] \quad (17)$$

$$\text{with } k_+ = q \left( f(k) - \frac{f'(k)}{\frac{\partial U}{\partial k}} \right).$$

Let's evaluate equation (17) around steady state, i.e. by taking  $k = k^*(q) = k_+$ . Since  $q f'(k^*(q)) = 1 + \theta$ , the term in cross derivative  $\frac{\partial^2 U}{\partial q \partial k} \Big|_{(k, h, q)}$  cancels out at steady state, so we simply get:

$$\frac{\partial U}{\partial q} \Big|_{(k^*(q), h, q)} = \frac{1}{1 + \theta} \left[ f(k^*(q)) \frac{\partial U}{\partial k} \Big|_{(k^*(q), h, q)} - f'(k^*(q)) + \frac{\partial U}{\partial q} \Big|_{(k^*(q), h, q)} \right]$$

Equations (6) and (15) then yield:

$$\frac{\partial U}{\partial q} \Big|_{(k^*(q), h, q)} = \frac{1 + \theta}{\theta} \frac{1}{q} \left[ \frac{f(k^*)}{f(k^*) - \frac{k^*}{q}} - 1 \right] = \frac{1 + \theta}{\theta} \frac{1}{q} \frac{\frac{k^*}{q}}{f(k^*) - \frac{k^*}{q}}$$

Again with (6) and the definition of capital share, we can write down the final result like:

$$\frac{\partial U}{\partial q} \Big|_{(k^*(q), h, q)} = \frac{1 + \theta}{\theta} \frac{1}{q} \frac{\alpha^*}{1 - \alpha^* + \theta}$$

So a marginal increase in  $q$ , when the agent is initially at steady state, brings the following increase in indirect utility:

$$dU|_q = \frac{1 + \theta}{\theta} \frac{\alpha^*}{1 - \alpha^* + \theta} dz_q \quad (18)$$

<sup>5</sup>Remark that initial effective capital-labor ratio is unaffected, since the capital available at date  $t = 0$  has been accumulated at date  $t = -1$ , and that neither  $h$  nor  $L$  is altered.



Remark that, surprisingly enough, this formula is independent of the elasticity of substitution at steady state  $\sigma^*$ <sup>6</sup>. Thus, even when  $\sigma = 0$ , utility gains from ISTC are strictly positive, while in this case the output never increases when  $q$  gets higher. In this case, the agent takes advantage of ISTC by saving on the resources devoted to the production of capital goods and increasing the resources devoted to the production of consumption goods<sup>7</sup>.

## 2.2 An increase in $h$

Suppose now that the agent instead chooses to increase  $h$  by a factor  $dz_h$ . This choice has two immediate consequences: (i) to expand the production possibility frontier, and (ii) to make the effective capital-labor ratio decrease by  $dk = -k^* dz_h$ .

To derive  $dU|_h$ , it is useful to separate these two effects:

$$dU|_h = \left. \frac{\partial U}{\partial k} \right|_{(k^*, h, q)} dk + \left. \frac{\partial U}{\partial h} \right|_{(k^*, h, q)} dh \quad (19)$$

Via equations (15), the first part of this expression is simply:

$$\left. \frac{\partial U}{\partial k} \right|_{(k^*, h, q)} dk = -\frac{1 + \theta}{q \left( f(k^*) - \frac{k^*}{q} \right)} k^* dz_h = -\frac{k^* f'(k^*)}{f(k^*) - \frac{k^* f'(k^*)}{1 + \theta}} dz_h$$

and so:

$$\left. \frac{\partial U}{\partial k} \right|_{(k^*, h, q)} dk = -(1 + \theta) \frac{\alpha^*}{1 - \alpha^* + \theta} dz_h$$

The second term in the right-hand side of (19) is the increase in the value function when  $h$  increases but keeping the effective capital-labor ratio constant. The problem like formulated in (1) makes it clear that, when the initial effective capital-labor ratio is held constant, optimal consumption at any date increases proportionally with  $h$ ; so if  $c_t$  denotes optimal consumption at date  $t$ , then:

$$\frac{\partial c_t}{\partial h} = \frac{c_t}{h}$$

Thus, for any effective capital-labor ratio  $k$ , the value function verifies:

$$\begin{aligned} \left. \frac{\partial U}{\partial h} \right|_{(k, h, q)} &= \sum_{t=0}^{\infty} \left( \frac{1}{1 + \theta} \right)^t \frac{\partial \ln c_t}{\partial h} \\ &= \frac{1 + \theta}{\theta} \frac{1}{h} \end{aligned}$$

In particular, this is true when the economy is initially at steady state, so:

$$\left. \frac{\partial U}{\partial h} \right|_{(k^*, h, q)} dh = \frac{1 + \theta}{\theta} dz_h$$

(19) then gives the final result:

$$dU|_h = \frac{1 + \theta}{\theta} \left( 1 - \frac{\theta \alpha^*}{1 - \alpha^* + \theta} \right) dz_h \quad (20)$$

<sup>6</sup>This result is even more intriguing when we refer to equation (11), which shows that the steady-state consequences of an investment-specific technological shock do depend on the elasticity of substitution. Indeed,  $\sigma^*$  does not even appear in the derivation of  $dU|_q$ ; by the method used above the second derivative of  $f$  does not come up.

<sup>7</sup>See chapter 1.

### 2.3 Synthesis

The relative (subjective) benefits of ISTC over LATC are, then:

$$\frac{dU|_q}{dU|_h} = \frac{\frac{\alpha^*}{1-\alpha^*+\theta}}{1-\frac{\theta\alpha^*}{1-\alpha^*+\theta}} \frac{dz_q}{dz_h} = \frac{1}{1+\theta} \frac{\alpha^*}{1-\alpha^*} \frac{dz_q}{dz_h} \quad (21)$$

The agent:

- chooses to undertake ISTC if  $\frac{1}{1+\theta} \frac{\alpha^*}{1-\alpha^*} dz_q > dz_h$ ;
- chooses to undertake LATC if  $\frac{1}{1+\theta} \frac{\alpha^*}{1-\alpha^*} dz_q < dz_h$ .

So – everything else equal – the more impatient the agent is (i.e. the higher  $\theta$ ), the less likely he is to undertake ISTC.

Let's compare criterion (21) with the relative increase in steady-state consumption<sup>8</sup>. From (8) and (11), we have:

$$\frac{q \frac{\partial c^*}{\partial q}}{h \frac{\partial c^*}{\partial h}} = \frac{\alpha^* + \theta \frac{\alpha^*}{1-\alpha^*} \sigma^*}{1 - \alpha^* + \theta} \quad (23)$$

so the increase in  $q$  brings a higher increase in steady-state consumption than the increase in  $h$  if and only if:

$$\frac{\alpha^* + \theta \frac{\alpha^*}{1-\alpha^*} \sigma^*}{1 - \alpha^* + \theta} dz_q > dz_h$$

When  $\theta > 0$ , it is straightforward to see that:

$$\frac{dU|_q}{dU|_h} < \frac{q \frac{\partial c^*}{\partial q}}{h \frac{\partial c^*}{\partial h}} \iff \frac{1}{1+\theta} \frac{\alpha^*}{1-\alpha^*} < \frac{\alpha^* + \theta \frac{\alpha^*}{1-\alpha^*} \sigma^*}{1 - \alpha^* + \theta} \iff \sigma^* > \alpha^* \frac{1}{1+\theta} \quad (24)$$

This result can be stated the following way: *when the elasticity of substitution is greater than the product of the capital share and the rate of preference for the present, the agent grants relatively less value to ISTC than the relative effect of ISTC on steady-state consumption.* If we denote by  $\beta = 1/1 + \theta$  the rate of preference for the present, this criterion can be simply written “ $\sigma > \alpha\beta$ ”.

Since the agent is impatient, he does not only take into account the steady-state consequences of each choice, but rather perfectly anticipates the dynamic consequences of each choice.

When the agent chooses to increase  $h$ , he gets some immediate benefits since production increases at date  $t = 0$ ; the rest of the benefits come from capital accumulation.

If he chooses to increase  $q$ , he gets some positive benefits at date  $t = 0$  because capital is cheaper to produce, so everything else equal the agent can consume more. But most of the benefits come through capital accumulation, from  $t = 1$  on.

<sup>8</sup>The steady-state increases in consumption-value of product per capita  $y^* = hf(k^*)$  after the  $h$ - and  $q$ -shock are:

$$\begin{cases} \frac{h}{y^*} \frac{\partial y^*}{\partial h} = 1 \\ \frac{q}{y^*} \frac{\partial y^*}{\partial q} = \frac{\alpha^*}{1-\alpha^*} \sigma^* \end{cases} \quad (22)$$

We have proved that when the hypothesis  $\sigma > \alpha \frac{1}{1+\theta}$  holds, the agent prefers the consumption path resulting from an increase in  $h$  rather than the one resulting from an increase in  $q$ . With the usual values of the parameters ( $\sigma \approx 1$ ,  $\alpha \approx 1/3$ ,  $\theta > 0$ ) this inequality is fulfilled.

In the contrary, when  $\theta = 0$  the agent attempts to increase the steady-state value of consumption, without valuing transitional dynamics. In this case, (21) and (23) are both equal to  $\alpha^*/1 - \alpha^*$ .

### The Cobb-Douglas case

The case where the production function is Cobb-Douglas of parameter  $\alpha$  takes a very simple form. With  $\sigma^* = 1$ , equation (23) becomes:

$$\frac{q \frac{\partial c^*}{\partial q}}{h \frac{\partial c^*}{\partial h}} = \frac{\alpha + \theta \frac{\alpha}{1-\alpha}}{1 - \alpha + \theta} = \frac{\alpha \left(1 + \frac{\theta}{1-\alpha}\right)}{(1-\alpha) \left(1 + \frac{\theta}{1-\alpha}\right)} = \frac{\alpha}{1-\alpha}$$

Thus, while increasing  $q$  rather than  $h$  brings an increase in steady-state consumption equal to  $\frac{\alpha}{1-\alpha} dz_q$  rather than  $dz_h$ , the agent only chooses to increase  $q$  rather than  $h$  if  $\frac{1}{1+\theta} \frac{\alpha}{1-\alpha} \frac{dz_q}{dz_h} > 1$ .

## 3 How useful is the Ramsey approach to the endogenous direction of technological change?

The ‘story’ told by the framework set above might be summarized as follows: *different forms of technological change lead to qualitatively different paths of consumption, so an impatient Ramsey agent will not necessarily pick the highest-growth option. When the parameters of the model satisfy a certain criterion – ‘ $\sigma > \alpha\beta$ ’ – the consumption growth path resulting from the disembodied form of technological change (LATC) is preferred to the consumption growth path resulting from the embodied form of technological change (ISTC); in this case ISTC is undervalued relatively to LATC.*

So the Ramsey approach to the direction technological change is able to account for the negative relationship between GDP per capita and the relative price of investment in cross-section. The model depicts rich countries as rich because they have undertaken superior – albeit *longer* – forms of technological change in the past, resulting both in a lower relative price of investment goods over consumption goods and in a higher GDP per capita today.

If the approach we have presented above is correct, then the right policy to stimulate growth is to tax labor and/or subsidize capital, with the objective of increasing the relative profitability of ISTC over LATC. However, the predictions of the model lie in contrast with both cross-section evidence as well as with the early growth episode in Britain (~1770–1913). However appealing, it comes out that the Ramsey approach is a bad guide to the transition from a low-growth to a high-growth path.

### 3.1 Growth and capital shares in cross-sections of countries

The model predicts that countries with a labor share above some threshold [ $1 - \alpha > 1 - \bar{\alpha}$ ] will choose to undertake LATC and grow at rate  $dz_h$ , while countries with a labor share below this

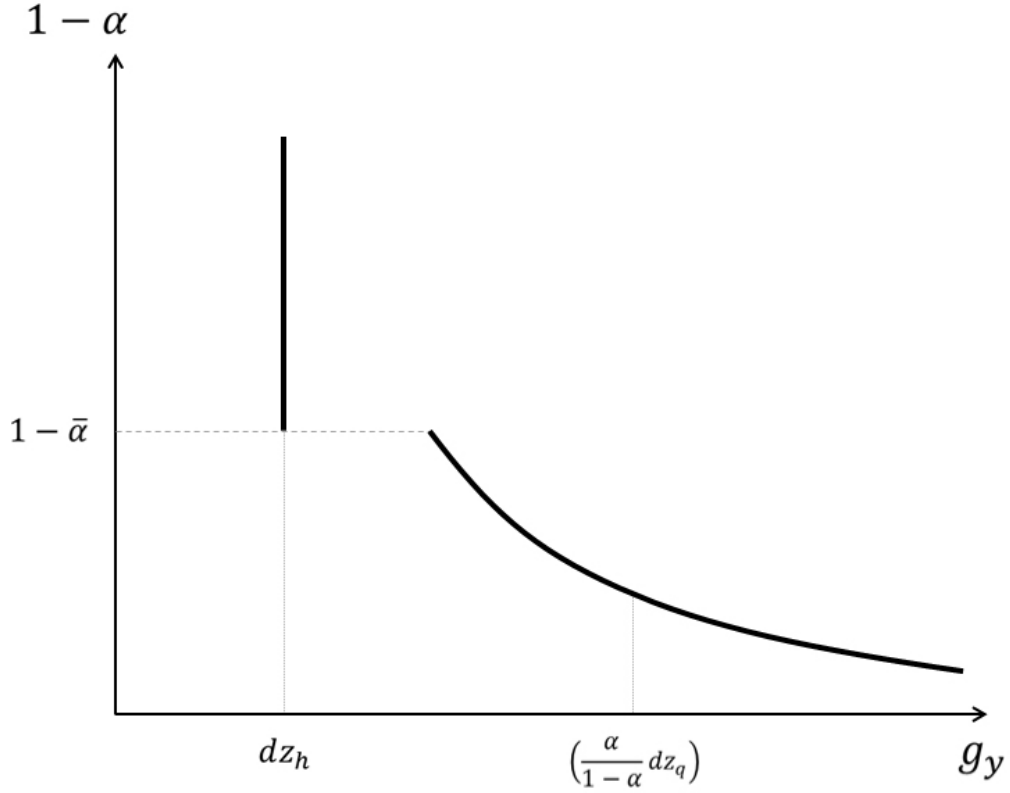


Figure 3: The model's predicted relationship between the labor share and the rate of economic growth under the assumption that  $\sigma > \alpha\beta$ .

threshold  $[1 - \alpha < 1 - \bar{\alpha}]$  will choose to undertake ISTC and grow at the rate  $\frac{\alpha}{1 - \alpha} dz_q$ . We have seen that if the parameters of the model satisfy the  $\sigma > \alpha\beta$  assumption, then the rate of economic growth is always greater under ISTC than under LATC. The situation is represented on figure 3<sup>9</sup>.

Figure 4 represents the cross-section relationship between GDP per capita and the labor share in all OECD member economies except Chile<sup>10</sup>. Data on the labor share is taken from the OECD economic database, which is accessible at <http://stats.oecd.org/>. We use the 'Labor Income Share' variable, which is available in 2006 for all thirty-three countries. As we see, there is no clear

<sup>9</sup>By restricting to the problem around steady state, we did not explore the dynamic feedback between the direction of technological change and the capital share – in this case the expected elasticity of substitution might influence the dynamic path of the economy, contrary to the criterion resting only on the capital share that we derived in equation (21).

Unfortunately, we are unable at this stage to solve the dynamic problem of the endogenous direction of technological change that would lead to asymptotic economic growth :

$$\left\{ \begin{array}{l} \max_{(c_t, \psi_t)_{t \geq 0}} \sum_{t=0}^{\infty} \left( \frac{1}{1+\theta} \right)^t \ln c_t \\ \text{s.t.} \quad k_{t+1} = q_t \left( f(k_t) - \frac{c_t}{h} \right) \quad , \quad k_0 = \bar{k} \\ \quad \quad q_{t+1} = (1 + \bar{z}_q)^{\psi_t} q_t \quad , \quad q_0 = \bar{q} \\ \quad \quad h_{t+1} = (1 + \bar{z}_h)^{1 - \psi_t} h_t \quad , \quad h_0 = \bar{h} \\ \quad \quad 0 \leq \psi_t \leq 1 \end{array} \right. \quad (25)$$

Still, there is strong presumption that, in the dynamic framework in (25), if the elasticity of substitution is everywhere less than unity, then the relative benefits of ISTC decrease when  $q$  increases (i.e. as ISTC is undertaken), and that if the elasticity of substitution is everywhere greater than unity then the relative benefits of ISTC increase when  $q$  increases. See chapter 2.

<sup>10</sup>Countries represented on figure 4 are: Australia, Austria, Belgium, Canada, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Korea, Luxembourg, Mexico, the Netherlands, New Zealand, Norway, Poland, Portugal, the Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Turkey, the United Kingdom, and the United States.

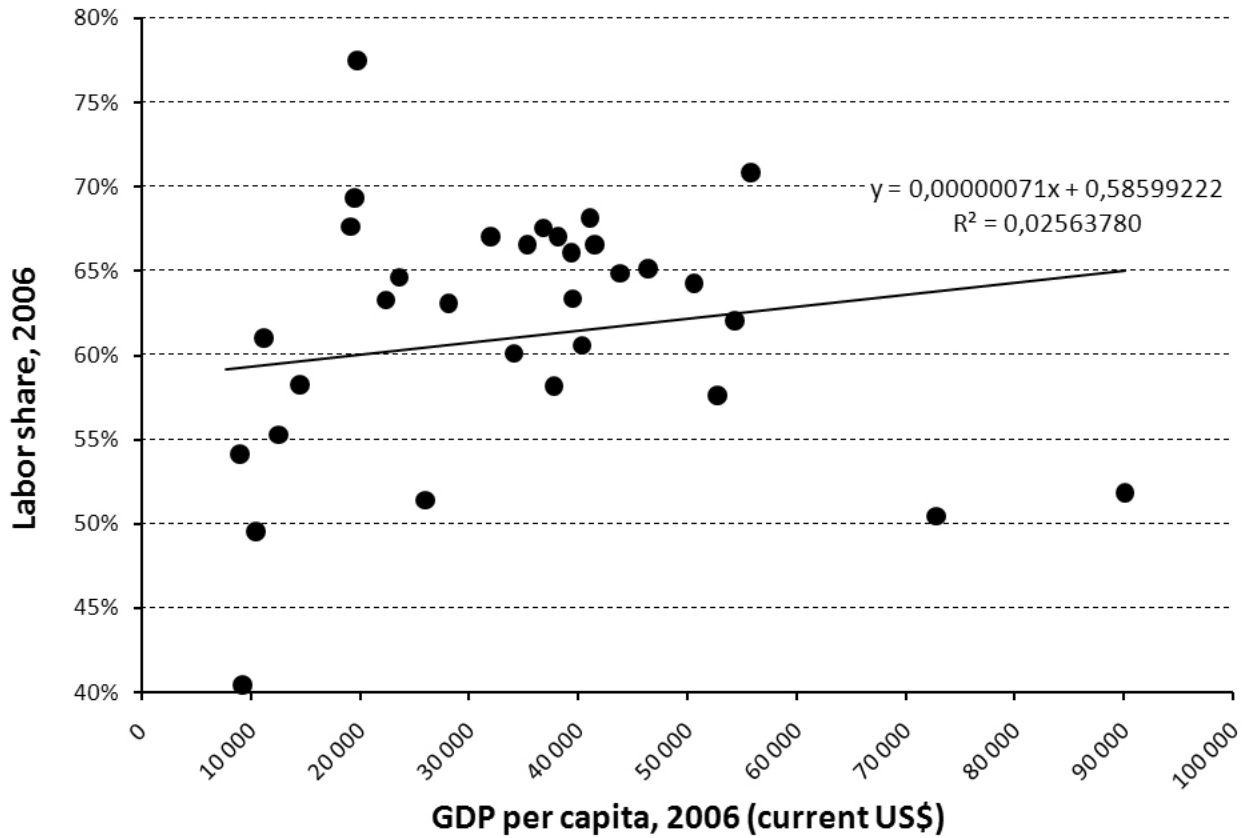


Figure 4: *The cross-section relationship between GDP per capita and the labor share in 33 OECD member countries; source: OECD StatExtracts (labor share) and IMF WEO October 2013 (GDP per capita).*

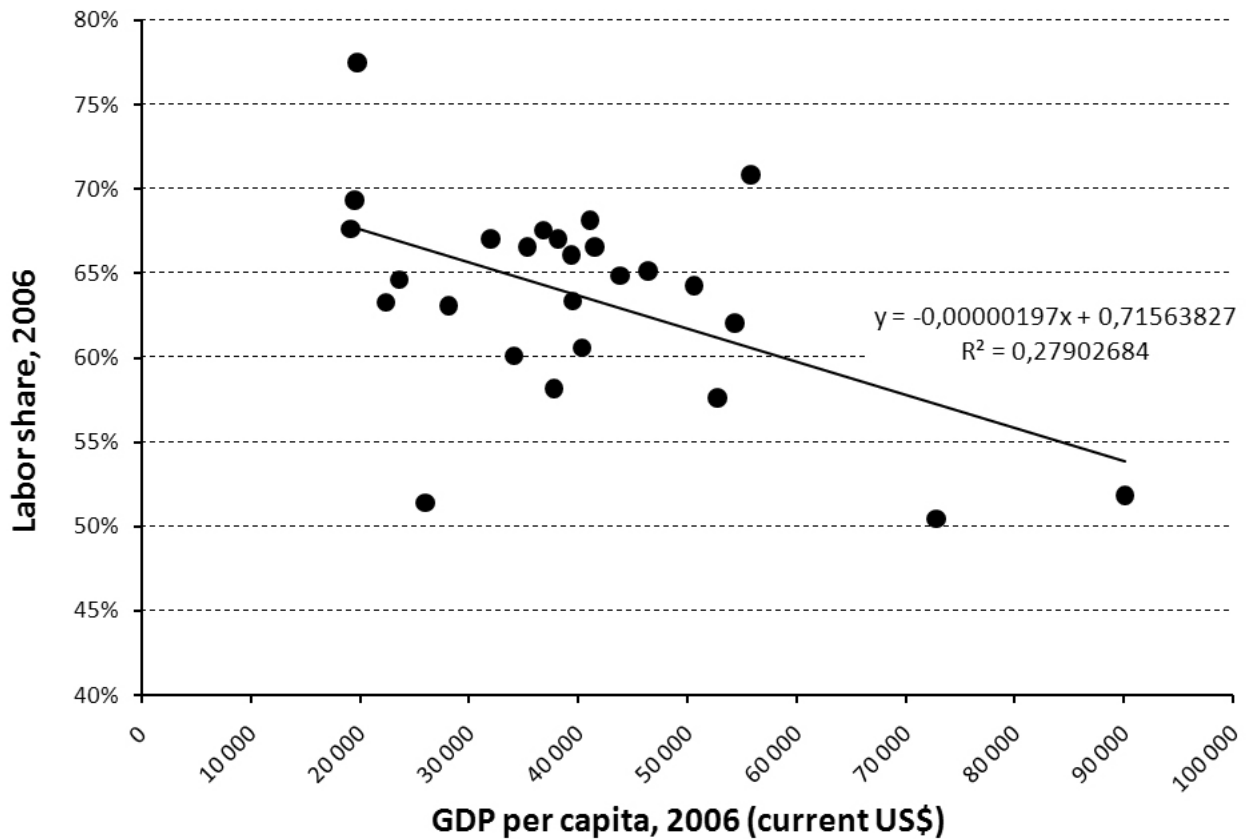


Figure 5: *Idem figure 4 for countries with GDP per capita > US\$20,000 (26 countries).*

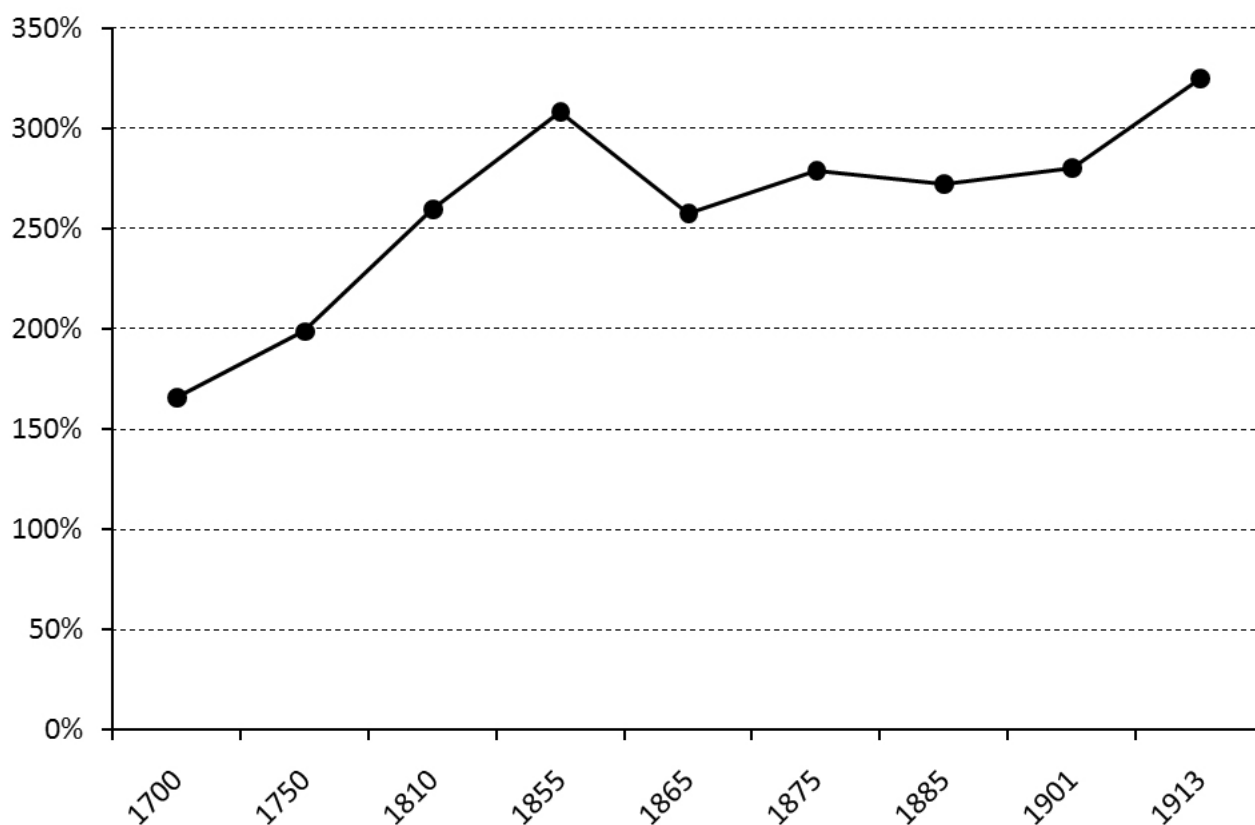


Figure 6: *The estimated ratio of non-land, non-housing domestic capital to GDP, UK, 1700–1913; source: Piketty and Zucman (2014).*

relationship between income per capita and the labor share for the whole sample. Contrary to what was predicted by theory, this relationship is rather positive than negative and the least advanced countries of our sample display on average *higher* capital shares.

Nevertheless, the right part of figure 3 is clearly observable. If we remove the seven countries with a GDP per capita less than US\$20,000<sup>11</sup>, the relationship becomes clearly negative and significant, like shown on figure 5. Thus, when we restrict our attention to the most advanced countries only, there is a positive link between the capital share and GDP per capita, as predicted by the neoclassical growth framework under ISTC<sup>12</sup>.

### 3.2 The Industrial Revolution in Great Britain: ~1770–1913

We have seen in chapter 3 that the first phase of the Industrial Revolution, from the late eighteenth to the mid-nineteenth century, was best described as fueled by ISTC, coupled with capital/labor gross substitutability. From the point of view of the distribution of income, the capital share increased at the expense of the labor and of the land share<sup>13</sup>. During this period, the investment rate increased<sup>14</sup>, while according to recent estimates by Piketty and Zucman (2014) the ratio of non-housing and non-land domestic capital to GDP increased from around 200 percent in 1750 to around 300 percent in 1855 (figure 6), as one would expect in a growth regime led by ISTC coupled

<sup>11</sup>These countries are the Czech Republic, Estonia, Hungary, Mexico, Poland, the Slovak Republic and Turkey.

<sup>12</sup>The same pattern emerges when we plot PPP GDP per capita instead of GDP per capita.

<sup>13</sup>See Allen (2009). Indeed, real wages did not increase at all before the mid-nineteenth century.

<sup>14</sup>Crafts (1985) estimates that the investment rate grew from 6.0 percent in 1760 to 11.7 percent in 1831.

with capital-labor gross substitutability ( $\sigma > 1$ ).

From the mid-nineteenth century to the First World War, human capital clearly become the prime engine of growth. Literacy rate soars and the investment rate stabilizes<sup>15</sup>, while according to Piketty and Zucman (2014) the ratio of non-housing/non-land domestic capital stabilizes as well (figure 6).

This is beyond dispute that the early economic growth episode in Britain has witnessed two distinct phases, the first one being dominated by physical capital and the second one by human capital. The neoclassical framework does a good job at explaining each economic growth regime. But how well does our augmented Ramsey framework perform in accounting for the transition between the first to the second one?

Poorly, indeed. First, if we accept that the Industrial Revolution began somewhere in the eighteenth century, then according to Allen's (2009) data we must recognize that it happened at a moment when the labor share was relatively high. In the contrary, the transition to a human capital-led path by the mid-nineteenth century happened when the labor share was relatively low – exactly the opposite of what is expected from theory. Secondly, we do not see any growth collapse with the transition to the human capital regime; instead, as is well-known the rate of economic growth seems to have been higher in the latter part of the nineteenth century than during the period from around 1770 to around 1850<sup>16</sup>.

So our approach does a poor job at accounting for the switch of economic growth regime by the mid-nineteenth century in the UK.

## 4 Conclusion

The analysis of economic growth through the variety of directions of technological change is appealing because it might bring insights into the diversity of growth regimes without resorting to any externality or non-linearity.

In this paper, we have analyzed the problem of an impatient Ramsey agent (with logarithmic utility) facing the choice between improving marginally either on labor-augmenting productivity or on investment-specific productivity while initially at steady state. We have found that this choice was governed by a very simple criterion involving only the ratio of the capital share to the labor share and the impatient rate. The prime message is that the benefits from LATC depend on the labor share – i.e. on the elasticity of output with respect to labor – while the benefits from ISTC depend on the capital share – i.e. on the elasticity of output with respect to capital.

The main benefit of the Ramsey approach to the direction of technological change is that it can be brought to data. As we have seen in section 3, it seems that this approach performs well in explaining the behavior of economies inside each regime, but does a bad job in accounting for the transition between different growth regimes led by different forms of technical change.

Indeed, the evidence we have exhibited here is completely opposed to the conclusions of the model: poor countries seem to display higher capital shares than rich countries, while the early British growth experience shows a change of regime towards ISTC by the end of the eighteenth

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<sup>15</sup>See the review of literature in Galor (2005).

<sup>16</sup>Crafts and Harley (1992).

century – when the labor share seem to have been relatively high – and eventually transited to an LATC-led path by the mid-nineteenth century, when the labor share was relatively low.

To our knowledge, the only piece of theory able to account for this fact is found in Habakkuk (1962) – who argue that the dearness of labor in the USA was itself an incentive for designing labor-economizing machines. The ‘Habakkuk thesis’ is not neoclassical in nature, since it assumes that wages are determined independently from the marginal productivity of labor – namely, by the option value of working in agriculture.

We believe that endogenizing the elasticity of substitution  $\sigma$  and thereby, the production function, would constitute a bridge between the theory that explains well the shape of the growth regimes independently (the neoclassical growth theory) and the theory that explains the switches between regimes (the Habakkuk thesis).

## References

- [1] D. ACEMOGLU (2002) “Directed Technical Change,” *Review of Economic Studies*, Vol. 69, No. 4 (Oct.), pp. 781–809.
- [2] D. ACEMOGLU (2003) “Labor- and Capital-Augmenting Technical Change,” *Journal of the European Economic Association*, Vol. 1, No. 1 (Mar.), pp. 1–37.
- [3] R. C. ALLEN (2009) “Engels’ pause: Technical change, capital accumulation, and inequality in the british industrial revolution,” *Explorations in Economic History*, Vol. 46, pp. 418–435.
- [4] R. ARMENTER and A. LAHIRI (2012) “Accounting for Development through Investment Prices,” *Journal of Monetary Economics*, Vol. 59, No. 6 (Oct.), pp. 550–564.
- [5] R. J. BARRO (1991) “Economic Growth in a Cross Section of Countries,” *Quarterly Journal of Economics*, Vol. 106, No. 2 (May), pp. 407–443.
- [6] V. V. CHARI, P. J. KEHOE and E. R. MCGRATTAN (1997) “The Poverty of Nations: A Quantitative Exploration,” *Federal Reserve Bank of Minneapolis Staff Report #204*.
- [7] N. F. R. CRAFTS (1985) *British Economic Growth during the Industrial Revolution*, Clarendon Press, Oxford.
- [8] N. F. R. CRAFTS and C. K. HARLEY (1992) “Output Growth and the British Industrial Revolution: a Restatement of the Crafts-Harley View,” *Economic History Review*, Vol. 45, No. 4 (Nov.), pp. 703–730.
- [9] J. B. DELONG and L. H. SUMMERS (1991) “Equipment Investment and Economic Growth” *Quarterly Journal of Economics*, Vol. 106, No. 2 (May), pp. 445–502.
- [10] J. B. DELONG and L. H. SUMMERS (1992) “Equipment Investment and Economic Growth: How Strong Is the Nexus?” *Brookings Papers on Economic Activity*, Vol. 23, No. 2, pp. 157–211.
- [11] E. M. DRANDAKIS and E. S. PHELPS (1966) “A Model of Induced Innovation, Growth and Distribution,” *Economic Journal*, Vol. 76, No. 304 (Dec.), pp. 823–840.



- [12] J. EATON and S. KORTUM (2001) "Trade in Capital Goods," *European Economic Review*, Vol. 45, No. 7 (Jun.), pp. 1195–1235.
- [13] W. FELLNER (1961) "Two Propositions in Theory of Induced Innovation," *Economic Journal*, Vol. 71, No. 282 (Jun.), pp. 305–308.
- [14] O. GALOR (2005) "From Stagnation to Growth: Unified Growth Theory," in *Handbook of Economic Growth, vol. 1A* (P. Aghion and S. N. Durlauf, eds.), Elsevier, Amsterdam: chapter 4, pp. 171–293.
- [15] J. GREENWOOD, Z. HERCOWITZ and P. KRUSELL (1997) "Long-Run Implications of Investment-Specific Technological Change," *American Economic Review*, Vol. 87, No. 3 (Jun.), pp. 342–362.
- [16] H. J. HABAKKUK (1962) *American and British Technology in the Nineteenth Century*, Cambridge University Press, Cambridge.
- [17] B. HERRENDORF and Á. VALENTINYI (2012) "Which Sectors Make Poor Countries so Unproductive?," *Journal of the European Economic Association*, Vol. 10, No. 2 (Apr.), pp. 323–341.
- [18] J. R. HICKS (1932, [2<sup>nd</sup> edition 1963]) *The Theory of Wages*, Macmillan, London.
- [19] C. KENNEDY (1964) "Induced Bias in Innovation and the Theory of Distribution," *Economic Journal*, Vol. 74, No. 295 (Sep.), pp. 541–547.
- [20] C.-T. HSIEH and P. J. KLENOW (2007) "Relative Prices and Relative Prosperity," *American Economic Review*, Vol. 97, No. 3 (Jun.), pp. 562–585.
- [21] C. I. JONES (1994) "Economic Growth and the Relative Price of Capital," *Journal of Monetary Economics*, Vol. 34, No. 3 (Dec.), pp. 359–382.
- [22] I. B. KRAVIS, A. HESTON and R. SUMMERS (1981) "New Insights into the Structure of the World Economy," *Review of Income and Wealth*, Series 27, pp. 339–55.
- [23] W. D. NORDHAUS (1967) "The Optimal Rate and Direction of Technical Change," in *Essays on the Theory of Optimal Economic Growth* (K. Shell, ed.), MIT Press, Cambridge (MA).
- [24] W. D. NORDHAUS (1973) "Some Skeptical Thoughts on the Theory of Induced Innovation," *Quarterly Journal of Economics*, Vol. 87, No. 2 (May), pp. 208–219.
- [25] T. PIKETTY and G. ZUCMAN (2014) "Capital is Back: Wealth-Income Ratios in Rich Countries 1700–2012," *Quarterly Journal of Economics*, forthcoming.
- [26] D. RESTUCCIA and C. URRUTIA (2001) "Relative Prices and Investment Rates," *Journal of Monetary Economics*, Vol. 47, No. 1 (Feb.), pp. 93–121.
- [27] X. SALA I MARTIN, G. DOPPELHOFFER and R. I. MILLER (2004) "Determinants of Long-Term Growth: A Bayesian Averaging of Classical Estimates (BACE) Approach," *American Economic Review*, Vol. 94, No. 4 (Sep.), pp. 813–835.
- [28] P. A. SAMUELSON (1965) "A Theory of Induced Innovation Along Kennedy-WeisÃd'cker Lines," *Review of Economics and Statistics*, Vol. 47, No. 4 (Nov.), pp. 343–356.

- [29] R. SUMMERS and A. HESTON (1988) "A new set of international comparisons of real product and price levels estimates for 130 countries, 1950–1985," *Review of Income and Wealth*, Series 34, pp. 1–25.
- [30] R. SUMMERS, I. B. KRAVIS and A. HESTON (1980) "International Comparisons of Real Product and Its Composition: 1950–77," *Review of Income and Wealth*, Series 26, pp. 19–66.
- [31] A. M. TAYLOR (1998) "On the Costs of Inward-Looking Development: Price Distortions, Growth, and Divergence in Latin America," *Journal of Economic History*, Vol. 58, No. 1 (Mar.), pp. 1–28.