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# Solving Stefan problem through C-NEM and level-set approach



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X-DMS 2015 Ferrara, Italy

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### Final goal of the study

Develop numerical time domain approach able to simulate thermo-mechanical phenomena in Finite Transformations:

- Cutting/blanking processes in 3D
  - Matter splitting encountered in forming processes
- Laser drilling/cutting
  - Multi-phases problem with moving interfaces across the matter
- **Research tool** in order to be able to test new approaches and thermomechanical models

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- **Research tool** in order to be able to test new approaches and thermomechanical models

### The approach must handle:

- Large strains
- Contact
- Interfaces and discontinuities

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### Large strains

- FEM: induced **mesh distortions** are conducting to frequent re-meshing and fields projections
  - need a very efficient mesher → lack of robustness in 3D
- **Mesh Free**: only the distribution and number of nodes are to be managed
  - OK but need to simply take into account boundary conditions

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## Large strains

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  - OK but need to simply take into account boundary conditions

## Mesh Free

We have choose to use a Natural Neighbor interpolant based mesh free approach ⇒ nodal interpolation

Existing methods:

- $\alpha$ -NEM<sup>1</sup>: no geometrical description of the boundaries **but** boundaries must be quite regular
- C-Nem<sup>2,3</sup>: a geometrical model is needed for the boundaries **but** domain can be highly non convex

<sup>1</sup> E. Cueto, Int. J. Numer. Meth. in Engng, 2000

<sup>2</sup> J. Yvonnet, Int. J. Numer. Meth. in Engng, 2004

<sup>3</sup> L. Illoul, Comp. and Struc., 2011

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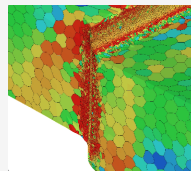
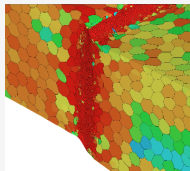
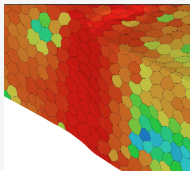
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### Interfaces and discontinuities

Example of blanking process: (C-Nem simulation<sup>1</sup>)



<sup>1</sup> L. Illoul, <http://sn-m2p.cnrs.fr>

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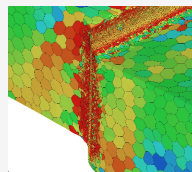
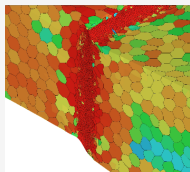
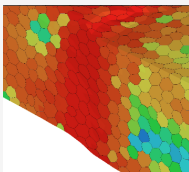
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### Interfaces and discontinuities

Example of blanking process: (C-Nem simulation<sup>1</sup>)



<sup>1</sup> L. Illoul, <http://sn-m2p.cnrs.fr>

### Interfaces modeling

- Full geometrical model: in 3D the shape evolution of the interface need a complex (and robust) surface mesher
  - Discontinuities: direct with duplication of the variables on nodes belonging to the interface<sup>2</sup>
  - **Level-set**: easy but the description is linked to the nodes distribution
    - Discontinuities: X-FEM framework → **X-NEM**<sup>3</sup>

<sup>2</sup> J. Yvonnet, Int. J. Therm., 2005

<sup>3</sup> N. Sukumar, U.S. National Congress on Comp. Mech., 2001



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### Goals of the presentation

Present a numerical method to solve problems involving discontinuities on moving internal boundaries with:

- a C-Nem approach for the interpolation (based on the natural neighbours interpolation)
- a level-set technique to represent the interface
- a local enrichment through the partition of unity concept

First results in 2D for the Stefan problem are presented

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## Few words on the C-Nem

C-Nem use a Ritz(-Galerkin) approach

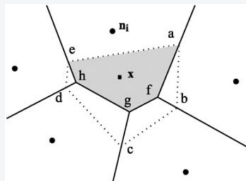
$$\mathbf{u}^h(\mathbf{x}) = \sum_{i=1}^n N_i(\mathbf{x}) \mathbf{u}_i, \quad \forall \mathbf{x} \in \Omega$$

where  $N_i(\mathbf{x})$  are **Natural Neighbour (NN) shape functions**: one shape function per node  $i$

### Natural Neighbour shape function

Based on :

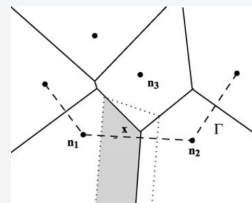
- Voronoï diagram  $\Leftrightarrow$  Delaunay tessellation
- Systematic geometric constructions (for a given set of nodes)



$\mathbf{x}$  inside  $\Omega$

Sibson shape function

$$N_i(\mathbf{x}) = \frac{\text{Area}(\text{afghe})}{\text{Area}(\text{abcde})}$$



$\mathbf{x}$  on the boundary of  $\Omega$

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## Few words on the C-Nem

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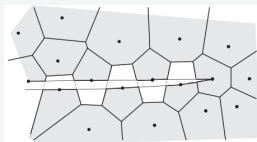
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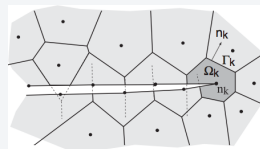
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Voronoi diagram with NN

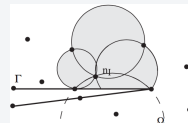


Constrained Voronoi diagram with NN

The constrained Voronov diagram (Delaunay tessellation) use a visibility criterion.  
The Delaunay tessellation is **constrained** to respect the tessellation of  $\partial\Omega$



NN supports



Constrained NN supports

## Few words on the C-Nem

### Natural Neighbour (NN) shape function – Non-convex domains

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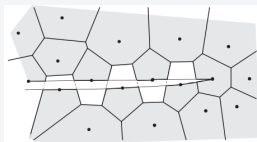
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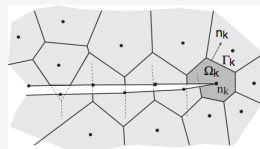
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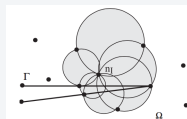


Voronoi diagram with NN

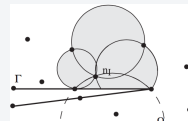


Constrained Voronoi diagram with NN

The constrained Voronov diagram (Delaunay tessellation) use a visibility criterion.  
The Delaunay tessellation is **constrained** to respect the tessellation of  $\partial\Omega$



NN supports



Constrained NN supports

## C-Nem: Constrained Natural Element Method

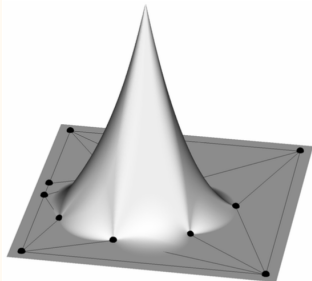
**C-NEM use the constrained NN shape functions**

## Few words on the C-Nem

### Properties of (constrained) NN interpolant

- Delta Kronecker:  
 $N_i(\mathbf{x}_j) = \delta_{ij}$
- Positivity:  
 $0 \leq N_i(\mathbf{x}) \leq 1$
- **Partition of unity:**  
 $\sum_{i=1}^n N_i(\mathbf{x}) = 1$
- **Local coordinate property:**  
 $\mathbf{x} = \sum_{i=1}^n N_i(\mathbf{x}) \mathbf{x}_i$   
 $\Rightarrow$  exact interpolation of linear fields  
 $\Rightarrow$  reproduction of large solid motions

### A Sibson shape function



### Continuity

Natural neighbor shape functions are  $C^\infty$  at any point except :

- at the nodes:  $C^0$
- on the boundary of the Delaunay circles (spheres in 3D):  $C^1$

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# Coupling C-Nem with a level-set approach

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## Main aspects

We propose to use a X-FEM like strategy by enriching the C-Nem approximation space through the **partition of unity technique**.

As for the X-FEM, the **location of the discontinuity interface** is defined by a **level-set function**. This latter being defined by the nodal values of the level-set function with the **C-Nem approximation**.

We need to define :

- the adequate enrichment function (depending on the discontinuity), based on the level-set (distance) function
- the selection of the nodes subjected to enrichment (near the interface)
- the quadrature rules for the weak forms



## Coupling C-Nem with a level-set approach

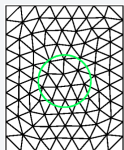
### Local enrichment of the Constrained NN interpolant

$$T^h(\mathbf{x}, t) = \sum_i N_i(\mathbf{x}) a_i(t) + \sum_{j \in I(t)} \underbrace{N_j(\mathbf{x}) \psi(\mathbf{x}, t)}_{M_j(\mathbf{x}, t)} b_j(t)$$

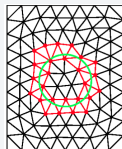
- $\psi(\mathbf{x}, t)$  is the enrichment function depending on the interface position
- $I(t)$  is the set of the nodes subjected to enrichment
- $N_i(\mathbf{x})$  are the Constrained NN shape function verifying the partition of unity. If the geometry of the domain do not evolve, these shape functions do not depend on time.

### Selection of the nodes subjected to enrichment

In order to define the set  $I(t)$  we use the constrained Delaunay tessellation.



Constrain Delaunay Tessellation (black)  
+ Discontinuity interface (green)



Enriched nodes selection (in red)

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# Coupling C-Nem with a level-set approach

## Enrichment function $\psi$

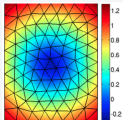
Here we have chosen an enrichment function, proposed by Moës et al<sup>1</sup>, in order to handle weak discontinuity (gradient discontinuity) :

$$\psi(\mathbf{x}, t) = \sum_{j \in I(t)} N_j(\mathbf{x}) |\Phi(\mathbf{x}_j, t)| - \left| \sum_j N_j(\mathbf{x}) \Phi(\mathbf{x}_j, t) \right|$$

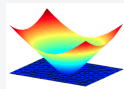
where  $\Phi(\mathbf{x}_j, t)$  is the level-set (distance) function

<sup>1</sup> N. Moës, Comput. Methods Appl. Mech. Engrg, 2003

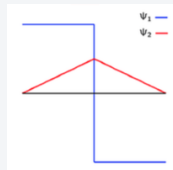
## Representation of the level-set and enrichment functions



$\Phi(\mathbf{x})$ : Level-set contours



$\Phi(\mathbf{x})$ : Level-set contours in 3D



$\psi(\mathbf{x})$ : Schematic representation

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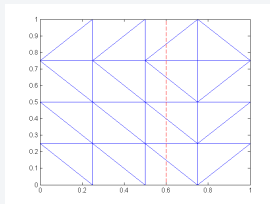
## Quadrature

- For the **integration** of the **weak forms** we use the **constrained Delaunay tessellation**
- As for the X-Fem, the triangles (tetrahedrons in 3D) intersecting the interface, are re-meshed in order to be **compatible with the interface** and to **improve the quadrature**.

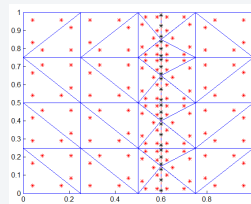


- ◆ Intersecting points
- Surface (volume in 3D) quadrature points
- Line (surface in 3D) quadrature points

## Example of quadrature points distribution



Initial Delaunay mesh



Refined Delaunay mesh – Quadrature points

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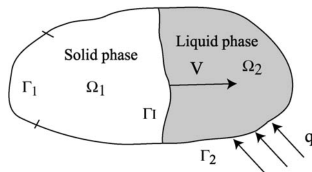
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# Stefan problem



## Strong form

- Heat equation :

$$\rho \frac{\partial}{\partial t} (c_1 T) = \nabla \cdot (k_1 \nabla T) \quad \text{in } \Omega_1(t); \quad \rho \frac{\partial}{\partial t} (c_2 T) = \nabla \cdot (k_2 \nabla T) \quad \text{in } \Omega_2(t)$$

$c_i, k_i$  : heat capacities, thermal conductivities       $\rho = \rho_1 = \rho_2$  : density

- Initial and boundary conditions :

$$\begin{cases} T(\mathbf{x}, t=0) = T_0 & \forall \mathbf{x} \in \Omega \\ T(\mathbf{x}, t) = \bar{T}(\mathbf{x}, t) & \forall \mathbf{x} \in \Gamma_1, \forall i \in [0, t_{\max}] \\ -k_i \nabla T(\mathbf{x}, t) \cdot \mathbf{n}_{12} = \bar{q}(\mathbf{x}, t) & \forall \mathbf{x} \in \Gamma_1, \forall i \in [0, t_{\max}] \end{cases}$$

- Interface velocity: depends on  $L$  the latent heat of fusion

$$\mathbf{V}(\mathbf{x} \in \Gamma_I(t)) = \frac{[q]}{L} \mathbf{n}_{12}(\mathbf{x}) \quad \text{where } [q] = (k_1 \nabla T|_{\Gamma_{12}^-(t)} - k_2 \nabla T|_{\Gamma_{12}^+(t)}) \cdot \mathbf{n}_{12}$$

- Constraint prescribed on the interface  $\Gamma_I(t)$  :

$$T(\mathbf{x}, t) = T_m \quad \forall \mathbf{x} \in \Gamma_I(t); \quad T_m: \text{melting temperature}$$

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## Weak form

Find  $T \in H^1(\Omega)$  with  $T = \bar{T}$  on  $\Gamma_1$  such that

$$\int_{\Omega} \rho c \frac{\partial T}{\partial t} \delta T \, d\Omega + \int_{\Omega} k \nabla T \cdot \nabla \delta T \, d\Omega = \int_{\Gamma_I} \alpha (T - T_m) \delta T \, d\Gamma + \int_{\Gamma_I} [\mathbf{q} \cdot \mathbf{n}_{12}] \delta T \, d\Gamma$$

(Simplify form :  $\bar{q}(t) = 0$ )

## Time discretization using implicit scheme 1

The implicit backward Euler integration scheme between  $t^{n-1}$  and  $t^n$  gives:

$$\int_{\Omega} \rho c \frac{T^n - T^{n-1}}{dt} \delta T^n \, d\Omega + \int_{\Omega} k \nabla T^n \cdot \nabla \delta T^n \, d\Omega = \int_{\Gamma_I} \alpha (T^n - T_m) \delta T^n \, d\Gamma + \int_{\Gamma_I} [[\mathbf{q}^n \cdot \mathbf{n}_{12}^n]] \delta T^n \, d\Gamma$$

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## Time discretization using implicit scheme 1

$$\int_{\Omega} \rho c \frac{T^n - T^{n-1}}{dt} \delta T^n d\Omega + \int_{\Omega} k \nabla T^n \cdot \nabla \delta T^n d\Omega = \int_{\Gamma_I} \alpha (T^n - T_m) \delta T^n d\Gamma + \int_{\Gamma_I} [[\mathbf{q}^n \cdot \mathbf{n}_{12}^n]] \delta T^n d\Gamma$$

## Time discretization using implicit scheme 2

$$\int_{\Omega} \rho c T^n \delta T^n d\Omega + dt \int_{\Omega} k \nabla T^n \cdot \nabla \delta T^n d\Omega = \int_{\Omega} \rho c T^{n-1} \delta T^n d\Omega + dt \int_{\Gamma_I} \alpha (T^n - T_m) \delta T^n d\Gamma + dt \int_{\Gamma_I} (k_1 - k_2) (\nabla T^n \cdot \mathbf{n}_{12}^n) \delta T^n d\Gamma$$

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## Time discretization using implicit scheme 2

$$\int_{\Omega} \rho c T^n \delta T^n d\Omega + dt \int_{\Omega} k \nabla T^n \cdot \nabla \delta T^n d\Omega = \int_{\Omega} \rho c T^{n-1} \delta T^n d\Omega$$

$$+ dt \int_{\Gamma_I} \alpha (T^n - T_m) \delta T^n d\Gamma + dt \int_{\Gamma_I} (k_1 - k_2) (\nabla T^n \cdot \mathbf{n}_{12}^n) \delta T^n d\Gamma$$

## Matrix Form

$$(\mathbf{C} + dt\mathbf{K}) \mathbf{T}^n = \mathbf{F}$$

with

$$\mathbf{C} = \int_{\Omega} \rho c \mathbf{N}^{nT} \mathbf{N}^n d\Omega$$

$$\mathbf{K} = \int_{\Omega} k \mathbf{B}^{nT} \mathbf{B}^n d\Omega - \int_{\Gamma_I} \alpha \mathbf{N}^{nT} \mathbf{N}^n d\Gamma + \int_{\Gamma_I} (k_2 - k_1) \mathbf{N}^{nT} (\mathbf{B}^n \cdot \mathbf{n}_{12}^n) d\Gamma$$

$$\mathbf{F} = \int_{\Omega} \rho c \mathbf{N}^{nT} (\mathbf{N}^{n-1} \mathbf{T}^{n-1}) d\Omega + dt \int_{\Gamma_I} (\alpha T_m) \mathbf{N}^{nT} d\Gamma$$



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## Interface convection<sup>1</sup>

### Velocity extension

$$\text{sign}(\Phi) \nabla F \cdot \nabla \Phi = 0 \quad \text{with } F = \mathbf{V} \cdot \mathbf{n}_{12} \text{ on } \Gamma_I$$

### Level-set updating

$$\frac{\partial \Phi}{\partial t} + \mathbf{V} \cdot \nabla \Phi = 0$$

<sup>1</sup> J. Chessa, Int. J. Numer. Meth. Engng, 2002

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## Pseudo-code

Let  $\mathbf{T}^{n-1}$  and  $\Phi^{n-1}$  be known.

- Compute the velocity of the interface  $\mathbf{V}^{n-1}$  on  $\Gamma_I$ 

$$\mathbf{V}^{n-1} = \frac{[\mathbf{q}]}{L} \mathbf{n}_{12}^{n-1}$$
- Extend this velocity to the whole domain  $\Omega$  solving
 
$$\text{sign}(\Phi) \nabla F \cdot \nabla \Phi = 0 \quad \text{with } F = \mathbf{V}^{n-1} \cdot \mathbf{n}_{12} \text{ on } \Gamma_I$$
- Determine  $\Phi^n$  by updating the level-set function through
 
$$\frac{\partial \Phi}{\partial t} + F |\nabla \Phi| = 0$$
- Localize integration points by dividing the elements cut by  $\Gamma_I$  into sub-elements matching  $\Gamma_I$  using  $\Phi^n$  only
- Build matrices  $\mathbf{C}$  &  $\mathbf{K}$  and vector  $\mathbf{F}$
- Compute  $\mathbf{T}^n$  by solving the heat equation
 
$$(\mathbf{C} + dt\mathbf{K}) \mathbf{T}^n = \mathbf{F}$$

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## Outline

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- 2 Few words on the C-Nem
- 3 Coupling C-Nem with a level-set approach
- 4 Stefan problem
- 5 First results**
  - Interface motion
  - Temperature errors
- 6 Conclusion

Introduction

C-Nem

C-Nem +  
Level-set

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Interface motion

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## First results

Introduction

C-Nem

C-Nem +  
Level-set

Stefan problem

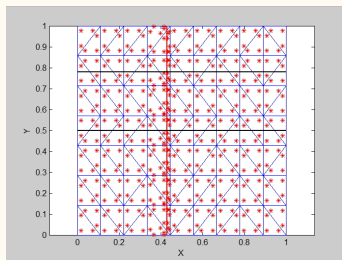
First results

Interface motion

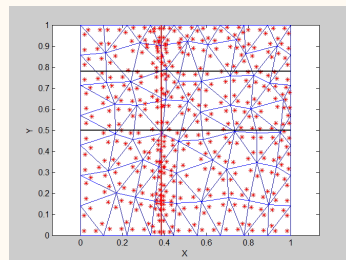
Temperature errors

Conclusion

### Motion on the interface across the mesh



Regular grid



Irregular grid

# First results

Introduction

C-Nem

C-Nem +  
Level-set

Stefan problem

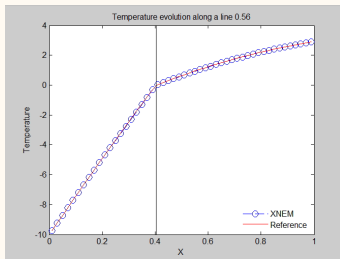
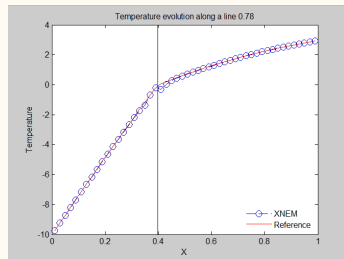
First results

Interface motion

Temperature errors

Conclusion

## Motion of the interface at two $Y$ levels

Level  $Y = 0.56$ Level  $Y = 0.78$

# First results

Introduction

C-Nem

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Level-set

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First results

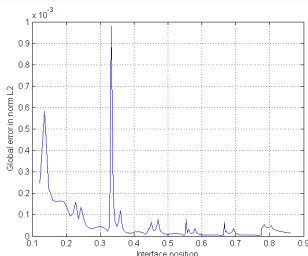
Interface motion

Temperature errors

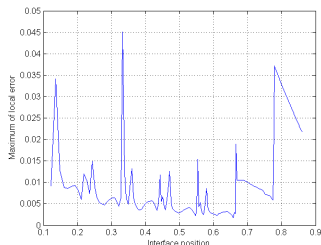
Conclusion

## Global and local errors on temperature

Global error



Local error



$$e^2(t) = \int_{\Omega(t)} \frac{[T_{Num}(\mathbf{x}, t) - T_{Sol}(\mathbf{x}, t)]^2}{[T_{Sol}(\mathbf{x}, t)]^2} dS$$

$$e(t) = \sup_{\mathbf{x} \in \Omega} \frac{|T_{Num}(\mathbf{x}, t) - T_{Sol}(\mathbf{x}, t)|}{|T_{right} - T_{left}|}$$

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## Conclusion

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### Weak form

- First results are encouraging
- Partition of unity technique seems to work as well with the C-Nem than with the FEM
- It is a first approach in 2D, investigation must be done on more complex geometries and in 3D
- Main errors are observed in the "enriched zones" where partition of unity not exactly respected are observed (error  $\approx 10^{-2}$ )

Work still in progress ...



***Thanks for your attention***

***Any questions ?***

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