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► **To cite this version:**

Ye Yun, Marija Jankovic, Gül E. Okudan Kremer. Understanding the Impact of Subjective Uncertainty on Architecture Generation and Supplier Identification. *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part B: Mechanical Engineering*, 2015, Part B: Mechanical Engineering, 1 (3), 10.1115/1.4030463 . hal-01204949

**HAL Id: hal-01204949**

**<https://hal.science/hal-01204949>**

Submitted on 24 Sep 2015

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# **Understanding the Impact of Subjective Uncertainty on Architecture and Supplier Identification in Early Complex Systems Design**

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# **Understanding the Impact of Subjective Uncertainty on Architecture Generation and Supplier Identification in Early Complex Systems Design**

The Architecture & Supplier Identification Tool (ASIT) is a design support tool, which enables identification of the most suitable architectures and suppliers in early stages of complex systems design, with consideration of overall requirements satisfaction and uncertainty. During uncertainty estimation, several types of uncertainties that are essential in early design (i.e., uncertainty of modules due to new technology integration, compatibility between modules, and supplier performance uncertainty) have been considered in ASIT. However, it remains unclear whether expert estimation uncertainties should be taken into account. From one perspective, expert estimation uncertainties may significantly influence the overall uncertainty since early complex systems design greatly depends on expert estimation; while an opposing perspective argues that expert estimation uncertainties should be neglected because they are relatively much smaller in scale. In order to understand how expert estimation uncertainties (especially subjective uncertainty) influence the architecture and supplier identification results achieved through ASIT, a comprehensive study of possible modelling approaches has been discussed; and type-1 fuzzy sets and 2-tuple fuzzy linguistic representation are selected to integrate subjective uncertainties into ASIT. A powertrain design case is used to compare results between considering subjective uncertainties and not considering them. Finally, the consideration of subjective uncertainty in early conceptual design as well as other design stages is discussed.

**Keywords:** expert estimation uncertainty, subjective uncertainty, early conceptual design stage, early complex systems design, fuzzy set theory, 2-tuple fuzzy linguistic representation

## 1 INTRODUCTION

In early design phases of complex systems, the OEMs (Original Equipment Manufacturers) strive to explore possibilities, delay decisions, and at the same time control both the performance and the uncertainty of the future system (Ye, Jankovic, Kremer, & Bocquet, 2014). Nowadays, more and more OEMs tend to involve their suppliers early in design in the context of an extended enterprise (Nguyen Van, 2006), especially in airplane and automotive industries. For example, Airbus has several thousand suppliers from more than 100 countries (Airbus Group, 2014). Among these suppliers, some are called first-tier suppliers, such as Snecma and Roll Royce for engines. These first-tier suppliers usually have the full responsibility of an entire module, and become involved starting in early conceptual design phase of the system in most cases. Therefore, it is important to consider the performance and uncertainty of these suppliers when estimating performance and uncertainty of the future system. However, very few methods consider both system architecture design as well as supplier information (Ye et al., 2014). In order to fill this gap, we proposed an Architecture & Supplier Identification Tool (ASIT) (Ye et al., 2014) to support design teams when considering these issues. Given the lack of data, fuzziness and different uncertainties inherent to early design, ASIT takes into account uncertainties related to interfaces, suppliers' capability, and the capability of one subsystem/module to reach a certain performance. These data are estimated by experts using predefined linguistic terms (as shown in Tab.1 and Tab.2), and these linguistic terms are then translated into numerical scales to facilitate calculation.

We have observed that when using information coming from expert estimation, researchers use methods such as fuzzy sets (e.g., J. (Ray) Wang, 2001) and rough sets (e.g., Zhai, Khoo, & Zhong, 2009) to model the information in order to represent the

uncertainty in expert estimation. Because of new technology integration, new modules and new suppliers, ASIT also greatly relies on expert estimation. However, different from most of the existing design supporting methods, ASIT is developed for early conceptual design stage, where overall uncertainty level is much greater than in later design stages. Therefore, since the expert uncertainty is relatively small comparing to overall uncertainty, it is unclear whether using mathematical methods to model expert estimation will influence the overall results.

In this paper, we propose to analyze the influence of considering expert estimation uncertainties on ASIT results by answering the following two research questions:

1. Which uncertainty modeling method is most suitable for representing the expert estimation uncertainties within the context of ASIT?
2. Does the consideration of expert estimation uncertainties influence ASIT results?

In the following sections, we start with an overview of the ASIT, and the uncertainties caused by expert estimation in ASIT. In section three, we give a review of different uncertainty representation methods that can be used to represent subjective uncertainty. The suitability of the uncertainty representation methods is analysed based on the characteristics of the early design stage and the ASIT, making up the set of comparison criteria. In section four, the operations of fuzzy sets, the selection of suitable membership functions, and the utilization of defuzzification methods are discussed. In section five, the selected representation methods are integrated into ASIT by using a powertrain case study. In section six, the results are compared and discussed before conclusions are presented in section seven.

## **2 THE EXPERT ESTIMATION UNCERTAINTIES IN ASIT**

The ASIT is an early design support tool that aims at generating possible system architectures given a list of suppliers that are co-designing the system. First, possible architectures are generated by integrating new technologies in order to better satisfy new requirements. Then, the generated architectures are filtered by uncertainty and requirement satisfaction thresholds in order to identify the architectures with relatively high performance and low uncertainty. Three major types of uncertainties are taken into account: (1) uncertainties related to capability of one subsystem/module to reach defined system performances, (2) interface related uncertainties, and (3) uncertainty related to supplier capabilities.

The ASIT contains four phases, as shown in Fig.1. In phase one, the satisfaction of new requirements by OEM's existing products is calculated with data from existing products stored in a database. Subsequently, the un-satisfied requirements, un-satisfied functions, and responsible modules for the un-satisfaction are identified through the mapping of "requirement – function – module type". In phase two, new solutions for the responsible module types are found by experts in existing or new supplier companies. Then, all possible architectures are generated by taking one module from each module type. In phase three, uncertainty of generated architectures is calculated based on experts' estimation of three types of uncertainties; requirements satisfaction level of architectures is calculated based on experts' estimation of "function satisfaction by modules". Finally, in phase four, by using the uncertainty and requirements satisfaction thresholds (which are defined by the OEM based on the tolerance of requirements satisfaction, and the risk that they are willing to take), the generated architectures are filtered to identify potential architectures and corresponding suppliers.

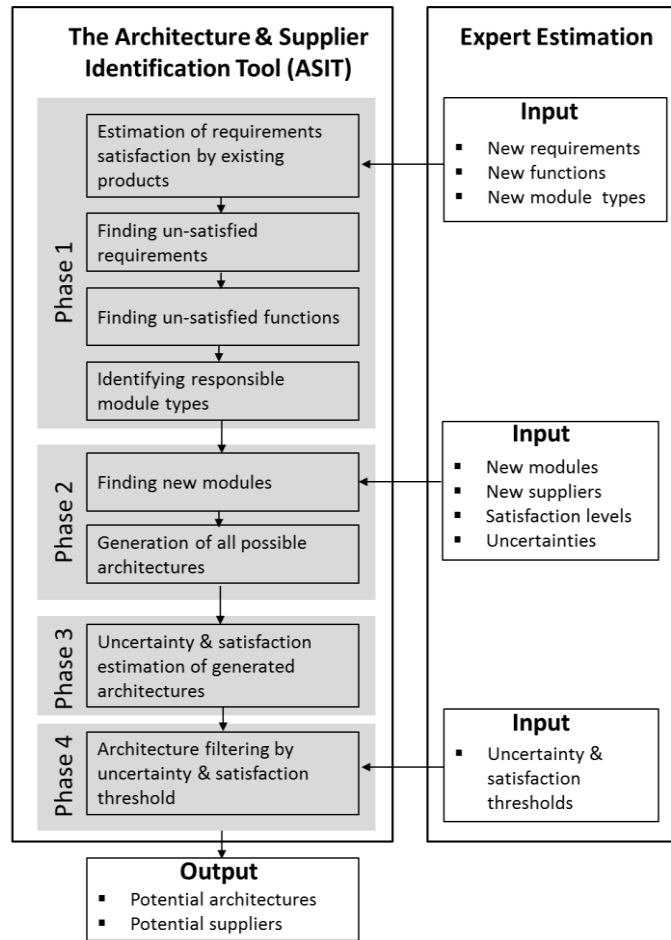


Figure 1 OVERVIEW OF ASIT

During ASIT process, experts are consulted for two types of information: satisfaction levels, and uncertainties. Although expert evaluations are often expressed in an informal way, there are many methods to make them more precise; one such mostly used method is pre-defined linguistic terms or ordinal scales. In ASIT, the satisfaction levels (Fiod-Neto & Back, 1994) and uncertainties are predefined as shown in Tab.1 and Tab.2. Here, the two forms (i.e., linguistic and numerical) are provided together since the linguistic terms provide explanatory notes while the numerical levels facilitate aggregation.

Table 1. SATISFACTION LEVELS

| Linguistic terms                  | Satisfaction level |
|-----------------------------------|--------------------|
| Very inadequate solution          | 1                  |
| Weak solution                     | 2                  |
| Tolerable solution                | 3                  |
| Adequate solution                 | 4                  |
| Satisfactory solution             | 5                  |
| Good solution with few drawbacks  | 6                  |
| Good solution                     | 7                  |
| Very good solution                | 8                  |
| Solution better than requirements | 9                  |
| Ideal solution                    | 10                 |

Table 2 PROPABILITIES

| Linguistic terms        | Probability |
|-------------------------|-------------|
| Impossible              | 0           |
| Nearly impossible       | 0.1         |
| Very unlikely           | 0.2         |
| Quite unlikely          | 0.3         |
| Possible                | 0.4         |
| Even chance             | 0.5         |
| Better than even chance | 0.6         |
| Quite likely            | 0.7         |
| Very likely             | 0.8         |
| Nearly certain          | 0.9         |
| Certain                 | 1           |

Expert knowledge is “what is known by qualified individuals, responding to complex, difficult (technical) questions, obtained through formal expert elicitation” (Meyer and Booker, 2001). As human beings, experts generate two categories of uncertainty: aleatory (random) and epistemic (subjective) (Medsker, Tan, & Turban, 1995). The aleatory uncertainty is also referred to as inherent uncertainty, irreducible uncertainty, and variability. It describes uncertainty due to random variation or inherent variation (Booker, Anderson, & Meyer, 2003). This type of uncertainty is considered to be rooted in the way the brain processes information (Dror & Charlton, 2006). The epistemic uncertainty is also referred to as subjective uncertainty and reducible uncertainty. The fundamental cause for this type of uncertainty is incomplete information or incomplete knowledge on system characteristics or the environment (Oberkampf, Helton, & Sentz, 2001). Aleatory and epistemic uncertainties both exist in expert estimation. However, in early conceptual design phase of complex systems, very limited amount of information exists; this causes high level of epistemic uncertainty. In comparison to epistemic



uncertainty, the aleatory uncertainty, in this phase, is much smaller in scale. Therefore when integrating the two types of uncertainties, the aleatory uncertainty can be covered by the epistemic uncertainty. That is why we propose to consider only the epistemic uncertainty in the context of this study.

One of the strategies to reduce epistemic uncertainty in complex system design is to use expert group evaluations (Medsker et al., 1995). In this study, we assume that a group of experts is used for each estimation in order to prevent from the situation that one expert does not have sufficient knowledge in a specific domain, or multiple domains.

As per the discussion above, the most important uncertainty caused by expert estimation in ASIT is the subjective (epistemic) uncertainty, which is caused by the limited information in early design. In order to test the sensitivity of the ASIT to this type of uncertainty, we need to first choose a suitable uncertainty representation method to model this type of uncertainty. In the next section, several approaches that can model the subjective uncertainty are reviewed and their appropriateness for use within ASIT is discussed.

### **3 REPRESENTING SUBJECTIVE UNCERTAINTY IN ASIT**

In order to identify the most suitable method for representing subjective uncertainty in ASIT, we have compiled uncertainty representation methods that have been the most commonly used for representing this type of uncertainty (NG & Abramson, 1990; Oberkampf et al., 2001; Booker et al., 2003):

- Subjective probability theory,
- Imprecise probability theory,

- Evidence (Dempster-Shafer) theory,
- Fuzzy sets,
- Possibility theory,
- Interval analysis theory, and
- Rough sets.

Epistemic uncertainty has been traditionally modeled as probability distributions. The probability theory has four perspectives including classical, empirical, subjective and axiomatic probabilities (Asadoorian & Kantarelis, 2005). The expert estimation can be modeled as subjective probability, which represents an individual's measure of belief that an event will occur. The information gathered for the distribution could be a mixture of limited experimental data and a person's experience, or the elicitation of multiple expert opinion (Oberkampf et al., 2001). The main concern of this method is that the "fuzziness" of information is usually lost since in probability theory, an event either occurs or not (NG & Abramson, 1990).

The imprecise probability (Walley, 1991) is a generalization of probability theory; it is used when a unique probability distribution is hard to identify. In imprecise probability, a lower probability and an upper probability are used instead of one single probability. For an uncertain event  $A$ , instead of assigning a single probability  $P(A)$ , the imprecise probability assigns an interval  $[\underline{P}(A), \overline{P}(A)]$ , with  $0 \leq \underline{P}(A) \leq \overline{P}(A) \leq 1$ , where  $\underline{P}(A)$  is the lower probability for  $A$ ,  $\overline{P}(A)$  is the upper probability for  $A$ , and  $\Delta A = \overline{P}(A) - \underline{P}(A)$  is the imprecision for event  $A$  (Coolen, 2004). Similar to the probability theory, the fuzziness of information is thought to be lost when using imprecise probability.

The evidence (Dempster-Shafer) theory (Dempster, 1967; Shafer, 1976) is a generalization of the Bayesian theory of subjective probability. It allows considering the confidence one has in the probabilities assigned to the outcomes. The evidence theory uses an interval to represent the probabilities with a lower bound called “believe” and an upper bound called “plausibility”. The “believe” is the sum of the evidence that supports the hypothesis, while the “plausibility” is 1 minus the sum of the evidence that opposes the hypothesis. According to NG & Abramson (1990), one obvious problem of the evidence theory is its implementation complexity since experts must provide all the beliefs for all subsets of possible hypotheses.

In 1965, Zadeh (1965) started a revolution in uncertainty thinking by introducing the fuzzy set theory. This theory uses a membership function to represent the degree of membership of an element to a set of objects. The degree to which an element belongs to a set is defined by a value between 0 and 1; the higher the value is the greater its belongingness, and an element can partly belong to a fuzzy set. The fuzzy set is widely used in describing linguistic information since it can effectively represent the gradual changes of people’s perception of a concept in a certain context (Dalalah & Magableh, 2008). Moreover, the fuzzy set theory also allows mathematical operations that help to provide quantitative methods to deal with qualitative data.

The possibility theory (Zadeh, 1999; Dubois & Prade, 1988; Cooman, Ruan, & Kerre, 1995) is an extension of the theory of fuzzy sets. It can be used to express the vague terms used by human experts with precision and accuracy (NG & Abramson, 1990).

Interval analysis (Moore, 1979; Kearfott & Kreinovich, 1996) is an approach that treats an interval as a new kind of number (Moore, 1979) and follows the following elementary properties (Moore & Lodwick, 2003):

$$[a, b] + [c, d] = [a + c, b + d] \quad (1)$$

$$[a, b] - [c, d] = [a - d, b - c] \quad (2)$$

$$[a, b][c, d] = [\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)] \quad (3)$$

$$[a, b] \div [c, d] = [a, b][1/d, 1/c] \text{ where, } 0 \notin [c, d] \quad (4)$$

The rough sets theory (Pawlak, 1982) uses a pair of sets to give the lower and upper approximation of the original set. This theory is used when objects are characterized by the same information and thus are indistinguishable (Pawlak, 1997). Each rough set has boundary-line elements, which cannot be properly classified using the available knowledge.

These methods have all been used in previous works to represent expert's subjective uncertainty. However, it is not clear how to choose a method according to a specific context (e.g., for ASIT). In order to identify approaches that can be seen as most suitable in the context of ASIT, we propose to consider the following four criteria:

1. ***The method is able to represent numerical levels:*** In ASIT, experts use predefined linguistic terms and related numerical levels to represent their estimation. Therefore, the chosen method should be able to represent these types of dis-continuous numerical levels.
2. ***The method requires reasonable amount of information:*** In ASIT, the estimation is provided by experts. Due to the limitations in time, budget and human capacity, the amount of information required by the mathematical representation has to be reasonable.

3. ***The method should capture the “fuzziness” of expert estimation:*** Human knowledge is imprecise. The chosen method should be able to capture this kind of imprecision.
4. ***Support multi-criteria group decision making:*** The aim of the potential supplier identification is to provide candidates for the supplier selection, which is usually a multi-criteria group decision making problem. In order to be able to use the supplier identification results directly during the supplier selection stage, the selected uncertainty representation method should be able to combine with other methods to support the multi-criteria group decision making.

With regard to the first criterion (representing the numerical levels), the *rough set theory* is not suitable since it assumes that some of the elements are characterized by the same information hence indistinguishable. However, the elements defined by the numerical levels are clearly distinguishable (e.g., numbers 1 – 10 used in Tab.1).

In view of the second criterion (requiring reasonable amount of information), the *evidence theory* is inappropriate. According to the principles of evidence theory, experts need to provide  $2^x$  beliefs for each estimation, where  $x$  represents the number of elements. For example, when using satisfaction levels provided in Tab.1,  $x$  is equal to 10. Therefore, when estimating “how well module A satisfies function B”, experts need to provide  $2^{10}$  beliefs. With the increase of module and function counts, the number of estimation needed also increases.

The third criterion (capturing the “fuzziness” of expert estimation) makes the *subjective probability theory*, the *imprecise probability theory*, the *possibility theory* and the *interval analysis theory* inappropriate since they are not designed to capture the “fuzziness”.

With regards to the fourth criterion (support multi-criteria group decision making), many *fuzzy set* based multi-criteria group decision making methods exist, including:

- Fuzzy set theory + TOPSIS (Chen, Lin, & Huang, 2006),
- Intuitionistic fuzzy sets + TOPSIS (Boran, Genç, Kurt, & Akay, 2009),
- Fuzzy AHP (Kahraman, Cebeci, & Ulukan, 2003; Haq & Kannan, 2006; Chan, Kumar, Tiwari, Lau, & Choy, 2008),
- Fuzzy AHP + cluster analysis (Bottani & Rizzi, 2008),
- Fuzzy ANP (Vinodh, Anesh Ramiya, & Gautham, 2011),
- Fuzzy ANP + TOPSIS (Önüt, Kara, & Işık, 2009),
- Fuzzy multi-objective programming (Amid, Ghodsypour, & O'Brien, 2006),
- Fuzzy arithmetic operation (Bayrak, Çelebi, & Taşkin, 2007),
- Fuzzy SMART (Chou & Chang, 2008),
- Fuzzy QFD (Bevilacqua, Ciarapica, & Giacchetta, 2006),
- Fuzzy DEMATEL + TOPSIS (Dalalah, Hayajneh, & Batieha, 2011).

With regard to previously discussed criteria, the fuzzy set theory appears to be the most suitable mathematical representation to express subjectivity in expert estimations. In recent years, several branches of fuzzy set theory were developed; the most popular ones among these with applications in supplier identification and selection are:

- Type-1 fuzzy sets (e.g., Önüt et al., 2009),
- Interval type-2 fuzzy sets (e.g., Chen & Lee, 2010),
- Intuitionistic fuzzy sets (e.g., Boran et al., 2009),
- 2-tuple fuzzy linguistic representation (e.g., Wang, 2010).

The *type-1 fuzzy sets* is the same as the ordinary *fuzzy set theory*. The re-name is for the purpose of distinguishing from the *type-2 fuzzy sets*. This theory uses a membership function to represent the degree of membership of an element to a set. The degree to which an element belongs to a set is defined by a value between 0 and 1. Higher the value is the greater its belongingness, and an element can partly belong to a fuzzy set. The fuzzy set is widely used in describing linguistic information as it effectively represents the gradual changes of people's perception of a concept in a certain context (Dalalah & Magableh, 2008). It is also widely used for representing human uncertainty.

The *interval type-2 fuzzy sets* is a simplified form of *type-2 fuzzy sets*, which is defined by Mendel and John (2002) based on Zadeh's Extension Principle (Zadeh, 1975). The *type-2 fuzzy sets* is able to model one additional degree of uncertainty than the *type-1 fuzzy sets*. In *type-1 fuzzy sets* the membership functions are crisp, but in *type-2 fuzzy sets* the membership functions are themselves fuzzy. Therefore, a type-2 membership function is a three-dimensional membership function, which is sometimes difficult to understand and define. The *interval type-2 fuzzy sets* simplified the fuzziness of the primary membership function of *type-2 fuzzy sets* by assuming that the fuzziness of the primary membership function is equal to one. Therefore, the *interval type-2 fuzzy sets* can be seen as composed of an upper membership function and a lower membership function, which are both of type-1 membership functions. The main problem with this representation for representing expert estimation is that it might be already difficult for experts to define one membership function; defining two is even harder. Moreover, in the context of representing expert estimation, the definition of the second membership function does not provide further understanding of the problem. Consequently, the *interval type-2 fuzzy sets* does not seem to be effective to represent subjective uncertainty in expert estimation.

The *intuitionistic fuzzy sets* was proposed by Atanassov(1986) twenty years after Zadeh's fuzzy sets. The *intuitionistic fuzzy sets* use dual membership degrees in each of the sets by giving both a degree of membership and a degree of non-membership. Similar to the *interval type-2 fuzzy sets*, it might be difficult for experts to define two membership functions for each estimation; thus, the *intuitionistic fuzzy sets* do not seem convenient for representing subjective uncertainty in expert estimation either.

The *2-tuple fuzzy linguistic representation* is developed by Herrera & Martinez (2000) based on the *fuzzy set theory* and the *symbolic method*(Delgado, Verdegay, & Vila, 1993). The linguistic values (e.g., expert estimation) are usually modelled as fuzzy sets. When aggregating the linguistic values (fuzzy sets), the result may not exactly match any of the initial linguistic terms, and thus an approximate linguistic term must be found. However, the imprecision of this approximation is lost. In the *2-tuple fuzzy linguistic representation*, the linguistic information is expressed by a 2-tuple  $(s, \alpha)$ , where  $s$  represents the approximate linguistic term, and  $\alpha$  represents the imprecision of this approximation. This representation can efficiently prevent the loss of information and thus help the ranking of alternatives.

Given the discussion above, we think that the *type-1 fuzzy sets* and the *2-tuple fuzzy linguistic representation* are both suitable for representing subjectivity in expert estimations during early design. Therefore, we propose to use these two approaches to represent the subjective uncertainty and compare them to the initial results where subjectivity is not taken into account. Before integrating the two fuzzy methods into ASIT, the fuzzy techniques to be used should be defined, e.g., the operations of fuzzy sets, the selection of suitable membership functions, and the utilization of defuzzification methods. These are discussed in the next section.



#### 4 FUZZY TECHNIQUES FOR REPRESENTING SUBJECTIVE UNCERTAINTY IN ASIT

Many different fuzzy membership functions exist. It is not possible or appropriate to test all of them in this work. Therefore, it is necessary to identify the most appropriate membership function within our research context. In addition, the simplification for fuzzy number operations should also be reviewed, since operations such as the multiplication of several fuzzy members can be very tedious.

A fuzzy number is a special fuzzy set  $N = \{(x, \mu_N(x)), x \in R\}$ , where  $x$  is a real value,  $R_1: -\infty < X < +\infty$  and  $\mu_N(x)$  is a continuous mapping from  $R_1$  to  $(0,1)$  (Haq & Kannan, 2006). Operations of fuzzy numbers can be defined based on the extension principle proposed by Zadeh (1975). If  $\tilde{M}$  and  $\tilde{N}$  are fuzzy numbers, membership of  $\tilde{M}(*)\tilde{N}$  is defined as follow (Gao, Zhang, & Cao, 2009):

$$\mu_{\tilde{M}(*)\tilde{N}}(z) = \sup_{z=x*y} \min\{\mu_{\tilde{M}}(x), \mu_{\tilde{N}}(y)\} \quad (5)$$

Where  $*$  stands for any of the four algebraic operations including addition, subtraction, multiplication and division.

Fuzzification is the process of making a crisp quantity fuzzy (Ross, 2009, p. 93), which is normally the first step in using fuzzy set theory. The main objective of fuzzification is to define a membership function for each fuzzy quantity. The commonly used membership functions for linguistic terms are triangular, trapezoidal, left shoulder, right shoulder, Gaussian and Sigmoid (Garibaldi & John, 2003). Among the various shapes of membership functions, the triangular membership function has been frequently used in many fuzzy set applications (Pedrycz, 1994). Pedrycz (1994) explained the reason of the popularity of the triangular membership function: “Undoubtedly, if the semantics of

a certain linguistic term has to be specified, then the simplest form of the membership function one could think of would be to provide a modal (typical) value of the considered term along with the lower and upper bounds. The distribution of the grades of membership between these boundaries is then linear – an acceptance of any other form of relationship to bear some legitimacy may definitely call for some auxiliary information about the membership values to be furnished at the selected intermediate points distributed within these bounds.” In this paper, we propose to use the triangular fuzzy membership function mainly because of the lack of information in early conceptual design stage. As presented previously, experts use predefined linguistic terms and numerical levels to give their estimation. Therefore, we have two types of information for defining a fuzzy membership function: (1) the estimation that is given by a group of experts; (2) the predefined numerical levels, and the distance between the levels. When using the triangular membership function, we assign the expert estimation as the mode of the triangular fuzzy number, and twice the distance between two adjacent levels as the support. To the best of our knowledge, other types of membership functions all require more information than this. Therefore, we propose to use the triangular membership function to model experts’ subjective uncertainty in ASIT in early conceptual design stage.

The basic features of triangular fuzzy numbers can be found in the work of Dubois and Prade(1978), and the basic operations can be found in the work of Chou and Chang(2008). A triangular fuzzy number can be denoted as  $N = (l, m, u)$ , its membership function  $\mu_N(x) : R \rightarrow [0,1]$  is represented as (Chang, 1996):

$$\mu_N(x) = \begin{cases} \frac{1}{m-l}x - \frac{1}{m-l}, & x \in [l, m], \\ \frac{1}{m-u}x - \frac{u}{m-u}, & x \in [m, u], \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

Where  $l \leq m \leq u$ ,  $l$ ,  $m$  and  $u$  are the lower bound of the support, the core, and the upper bound of the support of  $N$ , respectively.

Given two triangular fuzzy numbers  $\tilde{A} = (a_1, a_2, a_3)$  and  $\tilde{B} = (b_1, b_2, b_3)$ , operations of fuzzy numbers are shown below:

Addition of two fuzzy numbers  $\oplus$  :

$$\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3) \quad (7)$$

Addition of a real number  $k$  and a fuzzy number  $\oplus$  :

$$k \oplus \tilde{B} = (k + b_1, k + b_2, k + b_3) \quad (8)$$

Multiplication of a real number  $k$  and a fuzzy number  $\otimes$  :

$$k \otimes \tilde{B} = (kb_1, kb_2, kb_3) \quad (9)$$

Although the multiplication of a real number and a fuzzy triangular number is easy to calculate, a higher level operation (i.e., multiplication of two or several fuzzy numbers) is cumbersome with insurmountable computational effort (Triantaphyllou, 2000). Gao et al. (2009) demonstrated that the result from multiplication of two triangular fuzzy numbers is not a triangular fuzzy number, and the result can be obtained by using nonlinear programming method, analytical method, computer drawing method and computer simulation method. Gao et al. (2009) demonstrated that by using an analytical

method, for two triangular fuzzy numbers such as  $\tilde{A}=(a_1, a_2, a_3), \tilde{B}=(b_1, b_2, b_3)$ , the membership function of  $\tilde{N} = \tilde{A}(\times)\tilde{B}$  is:

$$\mu_{\tilde{N}}(x) = \begin{cases} \frac{-(a_1b_2 + b_1a_2 - 2a_1b_1) + \sqrt{(a_1b_2 - b_1a_2)^2 + 4(a_2 - a_3)(b_2 - b_1)x}}{2(a_2 - a_1)(b_2 - b_1)}, & a_1b_1 \leq x \leq a_2b_2 \\ \frac{-(a_3b_2 + b_3a_2 - 2a_3b_3) - \sqrt{(a_3b_2 - b_3a_2)^2 + 4(a_2 - a_3)(b_2 - b_3)x}}{2(a_2 - a_3)(b_2 - b_3)}, & a_2b_2 \leq x \leq a_3b_3 \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

One notices that the multiplication of several fuzzy numbers requires significant computational effort to obtain precise results. Therefore, in order to facilitate its application in engineering problems, a simplified formula is usually used (Chiou, Tzeng, & Cheng, 2005; Tzeng & Huang, 2011):

$$\tilde{A} \otimes \tilde{B} \cong (a_1b_1, a_2b_2, a_3b_3) \quad (11)$$

In decision making problems, a single scalar is preferred as output of a fuzzy process in order to facilitate ranking or selection. To transform fuzzy results into a scalar, defuzzification is performed. Defuzzification is defined as a mapping of fuzzy sets to elements of the universe considered significant with respect to this fuzzy set (Runkler, 1997). Widely used defuzzification methods are maximum (max) membership principle, centroid method, weighted average method, mean max membership, center of sums, and center of largest area (Ross, 2009). Within the context of this study, the triangular fuzzy membership functions are used, which are peaked output functions with their maximum equal to the “significant element”. Since in peaked output functions, the max membership principle is commonly used (Ross, 2009), we propose to use this defuzzification method in the 2-tuple linguistic representations. The max membership principle is given by the expression (Ross, 2009):

$$\mu_{\tilde{A}}(z^*) \geq \mu_{\tilde{A}}(z), \text{ for all } z \in Z \quad (12)$$

Where  $z^*$  is the defuzzified value.

In the 2-tuple linguistic representation, the single number obtained from defuzzification is transformed again to the initial expression domain (i.e., the predefined linguistic terms). The 2-tuple linguistic representation uses a 2-tuple  $(s, \alpha)$  to represent the results, where  $s$  refers to the closest linguistic term,  $\alpha$  is a numerical value expressing the value of the translation from the original result to the closest linguistic term (Herrera & Martinez, 2000).

Definition: Let  $S = \{s_0, \dots, s_g\}$  be a linguistic term set and  $\beta \in [0, g]$  a value representing the result of a symbolic aggregation operation. Then, the 2-tuple that expresses the equivalent information to  $\beta$ , which is obtained with the following function (adopted from Herrera & Martinez, 2000):

$$\Delta: [0, g] \rightarrow S \times [-0.5\delta i, 0.5\delta i] \quad (13)$$

$$\Delta(\beta) = (s_i, \alpha), \text{ with } \begin{cases} s_i, & i = \text{round}(\beta) \\ \alpha = \beta - i, & \alpha \in [-0.5\delta i, 0.5\delta i] \end{cases} \quad (14)$$

Where  $\text{round}(\cdot)$  is the usual round operation,  $s_i$  has the closest term to “ $\beta$ ”, and “ $\alpha$ ” is the value of the symbolic translation.  $\delta i$  is the gap between  $i$  and  $i-1$  for  $i \in \{1, 2, \dots, g\}$ .

Because of the lack of additional information in early conceptual design stage, the isosceles triangular membership function is used in this work. We assume that the support of the triangular fuzzy number is twice the predefined scale, which indicates the assumption that the group of experts are able to choose the correct linguistic term. In case of greater or smaller subjective uncertainty, the support of the triangular fuzzy

number can also be changed, but the principle and the reasoning of this work remains the same, and the results obtained in this work will not be greatly influenced.

In the next section, the two fuzzy methods are integrated into ASIT using a powertrain design case as the context.

## **5 THE POWER TRAIN DESIGN CASE**

This powertrain design case is used to show the initial ASIT results, and the integration of subjective uncertainties through the two selected fuzzy methods.

A powertrain is a system of mechanical parts in a vehicle that first provides energy, then converts it in order to propel the vehicle. Due to the increasing demand of lower emissions and higher fuel efficiency, the OEM plans to design a new powertrain for their motor vehicle to better satisfy market needs. Although the powertrain is normally an in-house subsystem, with only few modules outsourced (the battery and the engine for example), we assume in this case study that each module in the powertrain is planned to be outsourced to one supplier, for the purpose of illustration. The powertrain design case used in this work is very similar to the one used in Ye et al. (2014); the data used is the same except the added uncertainty information shown in Figure 6. See (Ye et al., 2014) for more information about ASIT and the powertrain design case.

### ***5.1 ASIT without Considering Subjective Uncertainty***

In ASIT phases 1 and 2, the satisfaction of new customer needs by existing products is estimated, the modules that should be improved are identified, and new modules are found. In phase 3, experts start providing estimation on satisfaction levels and uncertainties, after the generation of all possible architectures in which new modules are integrated. Since the subjective uncertainty studied in this work is caused by expert

estimation, we focus on phases 3, 4, and 5 in this case study because of space limitation. Please see Ye et al. (2014) for further information about the other phases of ASIT.

We assume that modules that can sufficiently satisfy new requirements are found. All possible architectures with integration of these new modules are generated, as shown in Fig.2. The number “1” represents that the module belongs to the architecture, while the “0” represents the module does not belong to the architecture.

|         |                  | Possible architecture 1 | Possible architecture 2 | Possible architecture 3 | Possible architecture 4 | Possible architecture 5 | Possible architecture 6 | Possible architecture 7 | Possible architecture 8 | Possible architecture 9 | Possible architecture 10 | Possible architecture 11 | Possible architecture 12 |
|---------|------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|--------------------------|--------------------------|--------------------------|
| Modules | Engine 1         | 1                       | 0                       | 0                       | 0                       | 1                       | 0                       | 0                       | 0                       | 1                       | 0                        | 0                        | 0                        |
|         | Engine 2         | 0                       | 1                       | 0                       | 0                       | 0                       | 1                       | 0                       | 0                       | 0                       | 1                        | 0                        | 0                        |
|         | Engine 3         | 0                       | 0                       | 1                       | 0                       | 0                       | 0                       | 1                       | 0                       | 0                       | 0                        | 1                        | 0                        |
|         | Engine 4         | 0                       | 0                       | 0                       | 1                       | 0                       | 0                       | 0                       | 1                       | 0                       | 0                        | 0                        | 1                        |
|         | Battery 1        | 1                       | 1                       | 1                       | 1                       | 0                       | 0                       | 0                       | 0                       | 0                       | 0                        | 0                        | 0                        |
|         | Battery 2        | 0                       | 0                       | 0                       | 0                       | 1                       | 1                       | 1                       | 1                       | 0                       | 0                        | 0                        | 0                        |
|         | Battery 3        | 0                       | 0                       | 0                       | 0                       | 0                       | 0                       | 0                       | 0                       | 1                       | 1                        | 1                        | 1                        |
|         | Transmission 1   | 1                       | 1                       | 1                       | 1                       | 1                       | 1                       | 1                       | 1                       | 1                       | 1                        | 1                        | 1                        |
|         | Electric motor 1 | 1                       | 1                       | 1                       | 1                       | 1                       | 1                       | 1                       | 1                       | 1                       | 1                        | 1                        | 1                        |
|         | Driveshaft 1     | 1                       | 1                       | 1                       | 1                       | 1                       | 1                       | 1                       | 1                       | 1                       | 1                        | 1                        | 1                        |
|         | Final drive 1    | 1                       | 1                       | 1                       | 1                       | 1                       | 1                       | 1                       | 1                       | 1                       | 1                        | 1                        | 1                        |

Figure 2. GENERATION OF ALL POSSIBLE ARCHITECTURES

In phase 3, to calculate the overall uncertainty and satisfaction level of each architecture, estimations are provided by experts using scales defined in Tab.1 and Tab.2. Since the structure of the product is complex and the requirements vary from project to project, it is difficult for experts to estimate how well a product satisfies a requirement directly. In comparison, it is much easier to estimate how well a module satisfies a function, as shown in Fig.3. The numbers in Fig.3 represent satisfaction levels defined in Tab.1. For example, the “engine 1” is a “weak solution” for satisfying the function “respect environment”. Therefore, using information from Tab.1, the satisfaction level 2 is assigned to this estimation. Since the significant figures include all the precise digits and the first estimated digit (Serway & Jewett, 2013), we keep two significant figures

for the estimation.

| $M_{\text{fun-mod}}$ |                                | Modules  |          |          |          |           |           |           |                |                  |              |               |
|----------------------|--------------------------------|----------|----------|----------|----------|-----------|-----------|-----------|----------------|------------------|--------------|---------------|
|                      |                                | Engine 1 | Engine 2 | Engine 3 | Engine 4 | Battery 1 | Battery 2 | Battery 3 | Transmission 1 | Electric motor 1 | driveshaft 1 | Final drive 1 |
| Functions            | Respect environment            | 2.0      | 4.0      | 5.0      | 6.0      | 8.0       | 8.0       | 9.0       | 0              | 0                | 0            | 0             |
|                      | Economize fuel                 | 2.0      | 4.0      | 5.0      | 6.0      | 10.0      | 10.0      | 10.0      | 0              | 0                | 0            | 0             |
|                      | Provide power                  | 8.0      | 6.0      | 6.0      | 5.0      | 2.0       | 5.0       | 8.0       | 0              | 0                | 0            | 0             |
|                      | Transform energy to torque     | 0        | 0        | 0        | 0        | 0         | 0         | 0         | 8.0            | 6.0              | 0            | 0             |
|                      | Transmit torque to final drive | 0        | 0        | 0        | 0        | 0         | 0         | 0         | 0              | 0                | 8.0          | 0             |
|                      | Interact with surface          | 0        | 0        | 0        | 0        | 0         | 0         | 0         | 0              | 0                | 0            | 8.0           |
|                      | Accept recharge                | 0        | 0        | 0        | 0        | 1.0       | 6.0       | 8.0       | 0              | 0                | 0            | 0             |

Figure 3. FUNCTION SATISFACTION LEVEL BY MODULES

Then, using the composition of architectures (Fig.2), the “satisfaction of functions by modules” is propagated to the “satisfaction of functions by architectures”. For simplicity, we assume that how an architecture satisfies a function depends on how the modules in the architecture satisfy the function. The function satisfaction level by an architecture is defined as the average of its modules’ satisfaction levels for this function.

The “satisfaction of functions by architectures” is then propagated to the “satisfaction of requirements by architectures” by using the relations between requirements and functions (Fig.4). The requirement – function relations in Fig.4 represent the percentage that a function satisfies a requirement. For example, the requirement “CAFE standard” is satisfied 50% by the function “Economize fuel”, 50% by “provide power”.

| $M_1$        |   | Functions           |                |               |                            |                                |                       |                 |
|--------------|---|---------------------|----------------|---------------|----------------------------|--------------------------------|-----------------------|-----------------|
|              |   | Respect environment | Economize fuel | Provide power | Transform energy to torque | Transmit torque to final drive | Interact with surface | Accept recharge |
| Requirements | CAFE standard                           | 0                   | 0.5            | 0.5           | 0                          | 0                              | 0                     | 0               |
|              | Acceleration time                       | 0                   | 0              | 0.6           | 0.2                        | 0.2                            | 0                     | 0               |
|              | Cruising velocity at grade              | 0                   | 0              | 0.7           | 0                          | 0                              | 0.3                   | 0               |
|              | 0-60 mph time                           | 0                   | 0              | 1             | 0                          | 0                              | 0                     | 0               |
|              | Low greenhouse gas emission             | 1                   | 0              | 0             | 0                          | 0                              | 0                     | 0               |
|              | Rechargeable by external electric power | 0                   | 0              | 0             | 0                          | 0                              | 0                     | 1               |

Figure 4. REQUIREMENT-FUNCTION RELATIONS



For propagating the function satisfactions to requirements satisfactions, we used:

$$M_{req-arch} = M_1 \times M_{fun-arch} \quad (15)$$

Assuming equal importance of the requirements (this assumption can be changed if needed), an overall requirements satisfaction score is obtained for each of the possible architectures by calculating the average (Fig.5). This score represents how well the architecture satisfies the entire requirements set (with a 1-10 scale). We round the final result to two significant figures.

| Architecture | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  |
|--------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Satisfaction | 4.7 | 4.5 | 4.6 | 4.5 | 6.3 | 6.1 | 6.2 | 6.1 | 7.4 | 7.2 | 7.3 | 7.2 |

Figure 5. REQUIREMENTS SATISFACTION (WITHOUT CONSIDERING EXPERT UNCERTAINTY)

The overall uncertainty of an architecture is calculated based on uncertainty of modules ( $M_5$  in Fig.6), compatibility between modules ( $M_4$ ), and uncertainty of suppliers' capabilities ( $M_6$ ). The matrix  $M_3$  shows the supplier of each module. Since the uncertainties and compatibilities can all be considered as probabilities, "the overall uncertainty of an architecture" is defined as the product of all its modules' uncertainties, its suppliers' uncertainties, and the compatibilities between its modules (independence of probabilities is assumed). Experts provide their estimations in linguistic terms defined in Tab.2. These linguistic terms are then converted into numerical scales.

The overall uncertainties of possible architectures are shown in Fig.7, which represents the percentage that an architecture can be developed without any problem. The confidence that one company has on the capabilities of a given supplier is also integrated in this overall uncertainty.

| M <sub>5</sub> |          |          |          |           |           |           |                |                  |              | M <sub>6</sub> |                  |            |            |            |            |            |            |            |            |   |
|----------------|----------|----------|----------|-----------|-----------|-----------|----------------|------------------|--------------|----------------|------------------|------------|------------|------------|------------|------------|------------|------------|------------|---|
| 1              | 1        | 0.80     | 0.70     | 1         | 0.90      | 0.50      | 1              | 1                | 1            | 1              | Uncertainty      | 1          | 0.70       | 0.90       | 1          | 0.50       | 1          | 1          | 1          | 1 |
| Engine 1       | Engine 2 | Engine 3 | Engine 4 | Battery 1 | Battery 2 | Battery 3 | Transmission 1 | Electric motor 1 | driveshaft 1 | Final drive 1  | Supplier 1       | Supplier 2 | Supplier 3 | Supplier 4 | Supplier 5 | Supplier 6 | Supplier 7 | Supplier 8 | Supplier 9 |   |
|                |          |          |          | 0.20      | 0.10      | 0.30      | 1              | 0.30             | 1            | 1              | Engine 1         | 1          | 0          | 0          | 0          | 0          | 0          | 0          | 0          | 0 |
|                |          |          |          | 1         | 0.70      | 0.50      | 1              | 1                | 1            | 1              | Engine 2         | 1          | 0          | 0          | 0          | 0          | 0          | 0          | 0          | 0 |
|                |          |          |          | 0.50      | 0.90      | 0.80      | 0.90           | 0.80             | 0.90         | 0.90           | Engine 3         | 0          | 1          | 0          | 0          | 0          | 0          | 0          | 0          | 0 |
|                |          |          |          | 0.50      | 0.80      | 0.90      | 0.90           | 0.90             | 0.80         | 0.90           | Engine 4         | 0          | 0          | 1          | 0          | 0          | 0          | 0          | 0          | 0 |
| 0.20           | 1        | 0.50     | 0.50     |           |           |           |                | 1                | 1            | 1              | 1                | Battery 1  | 0          | 0          | 0          | 1          | 0          | 0          | 0          | 0 |
| 0.10           | 0.70     | 0.90     | 0.80     |           |           |           |                | 0.80             | 0.90         | 0.90           | 0.90             | Battery 2  | 0          | 0          | 0          | 1          | 0          | 0          | 0          | 0 |
| 0.30           | 0.50     | 0.80     | 0.90     |           |           |           |                | 0.90             | 0.90         | 0.90           | 0.90             | Battery 3  | 0          | 0          | 0          | 0          | 1          | 0          | 0          | 0 |
| 1              | 1        | 0.90     | 0.90     | 1         | 0.80      | 0.90      |                | 1                | 1            | 1              | Transmission 1   | 0          | 0          | 0          | 0          | 0          | 1          | 0          | 0          |   |
| 0.30           | 1        | 0.80     | 0.90     | 1         | 0.90      | 0.90      | 1              | 1                | 1            | 1              | Electric motor 1 | 0          | 0          | 0          | 0          | 0          | 0          | 1          | 0          |   |
| 1              | 1        | 0.90     | 0.80     | 1         | 0.90      | 0.90      | 1              | 1                | 1            | 1              | Driveshaft 1     | 0          | 0          | 0          | 0          | 0          | 0          | 0          | 1          |   |
| 1              | 1        | 0.90     | 0.90     | 1         | 0.90      | 0.90      | 1              | 1                | 1            | 1              | Final drive 1    | 0          | 0          | 0          | 0          | 0          | 0          | 0          | 0          |   |

Figure 6. UNCERTAINTY INFORMATION

| Architecture | 1     | 2 | 3    | 4    | 5     | 6    | 7    | 8    | 9     | 10    | 11    | 12    |
|--------------|-------|---|------|------|-------|------|------|------|-------|-------|-------|-------|
| Uncertainty  | 0.060 | 1 | 0.16 | 0.18 | 0.020 | 0.37 | 0.15 | 0.15 | 0.010 | 0.080 | 0.040 | 0.050 |

Figure 7. UNCERTAINTY (WITHOUT CONSIDERING EXPERT UNCERTAINTY)

## 5.2 Taking into Account Subjective Uncertainty in ASIT

### 5.2.1 Using Type-1 Fuzzy Sets

Using this method, each expert's estimation of satisfaction levels (using Tab.1) is converted to a  $\tilde{1}-\tilde{10}$  fuzzy number scale. The isosceles triangular membership functions are used as shown in Fig. 8. These fuzzy numbers are then aggregated by using fuzzy operations defined in section 4.

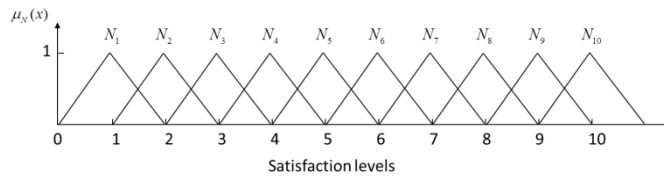


Figure 8. FUZZY NUMBERS FOR SATISFACTION LEVELS

Let us take the satisfaction of requirement “CAFE standard” for architecture 6 as an example. In order to calculate how the architecture 6 satisfies the requirement “CAFE standard”, the functions “Economize fuel” and “Provide power” need to be considered

since they each satisfy 50% of this requirement (Fig.4).The possible architecture 6 is composed of modules “engine 2”, “battery 2”, “transmission 1”, “electric motor 1”, “driveshaft 1”, and “final drive 1”. In order to calculate how the architecture satisfies the function “Economize fuel”, we need to calculate the average of how the engine 2 and the battery 2 satisfy this function. Therefore:

$$\begin{aligned}
 FSL_2 &= (0.5 \times 4) \oplus (0.5 \otimes (9,10,11)) \\
 &= 2 \oplus (4.5, 5, 5.5) = (6.5, 7, 7.5)
 \end{aligned}$$

Here, the satisfaction level “4” is not converted to a fuzzy number, since the engine 2 is an existing module. Therefore, the information of engine 2 comes from previous projects, instead of expert estimation, so that the subjective uncertainty is not considered. Using the same principle, satisfaction of function “Provide power” can be calculated. Then, the satisfaction of requirement “CAFE standard” can be calculated by:

$$\begin{aligned}
 RSL_1 &= (0.5 \otimes (6.5, 7, 7.5)) \oplus (0.5 \otimes (5, 5.5, 6)) \\
 &= (3.25, 3.5, 3.75) \oplus (2.5, 2.75, 3) \\
 &= (5.75, 6.25, 6.75)
 \end{aligned}$$

Assuming equal importance of the requirements (which can be changed if necessary), an overall requirements satisfaction score is obtained by calculating the average for each possible architectures (shown in Fig.9).

|                     |               |               |               |               |
|---------------------|---------------|---------------|---------------|---------------|
| <b>Architecture</b> | 1             | 2             | 3             | 4             |
| <b>Satisfaction</b> | 4.7           | 4.5           | (4.3,4.6,5.0) | (4.2,4.5,4.9) |
| <b>Architecture</b> | 5             | 6             | 7             | 8             |
| <b>Satisfaction</b> | (5.7,6.3,6.8) | (5.5,6.1,6.6) | (5.3,6.2,7.1) | (5.2,6.1,7.0) |
| <b>Architecture</b> | 9             | 10            | 11            | 12            |
| <b>Satisfaction</b> | (6.9,7.4,7.9) | (6.6,7.2,7.7) | (6.4,7.3,8.2) | (6.3,7.2,8.1) |

Figure 9. REQUIREMENTS SATISFACTION BY USING TYPE-1 FUZZY SETS (VALUES)

The requirements satisfactions obtained in Fig.9 can be illustrated using triangular fuzzy numbers as shown in Fig.10, with the threshold set at 6.

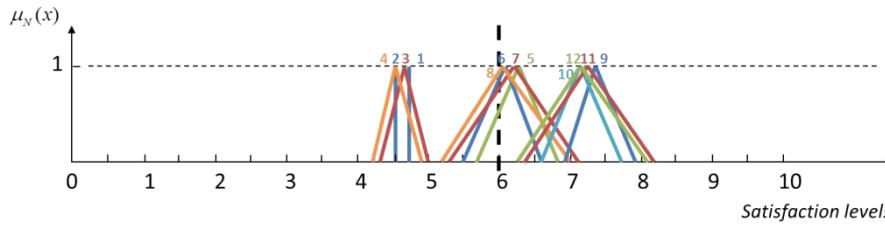


Figure 10. REQUIREMENTS SATISFACTION BY USING TYPE-1 FUZZY SETS (MEMBERSHIP FUNCTIONS)

In this case, it is obvious that architectures 1, 2, 3, and 4 are below the satisfaction threshold (since the entire fuzzy number is below the threshold), while architectures 9, 10, 11, and 12 are above the threshold. The situation for architectures 5, 6, 7, and 8 is more complicated, because they are partly above and partly below the threshold. We discuss this kind of situations later in this chapter.

When estimating uncertainty of modules ( $M_5$ ), compatibility between modules ( $M_4$ ), and uncertainty of suppliers' capabilities ( $M_6$ ), probability levels are needed from Tab.2. In order to integrate subjective uncertainty, we convert each numerical level in Tab.2 into a triangular fuzzy number. The membership function of the fuzzy number set is shown in Fig.11. Since "0" represents "impossible" and "1" represents "certain", we have low fuzziness level, thus we keep them as crisp values during fuzzification.

The overall uncertainty of an architecture is defined as the product of all its modules' uncertainties, its suppliers' uncertainties and the compatibilities between the modules. Therefore, the multiplication of several triangular fuzzy numbers is needed. Simplified as described in section 4, the result achieved is shown in Fig.12.

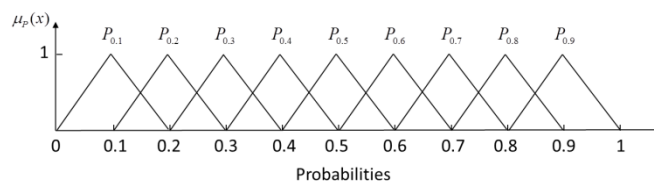


Figure 11. FUZZY MEMBERSHIP FUNCTION FOR POSSIBILITIES

|              |                      |                    |                     |                     |
|--------------|----------------------|--------------------|---------------------|---------------------|
| Architecture | 1                    | 2                  | 3                   | 4                   |
| Uncertainty  | 0.060                | 1                  | (0.060,0.16,0.39)   | (0.069,0.18,0.43)   |
| Architecture | 5                    | 6                  | 7                   | 8                   |
| Uncertainty  | (0,0.016,0.054)      | (0.17,0.37,0.72)   | (0.035,0.15,0.58)   | (0.035,0.15,0.58)   |
| Architecture | 9                    | 10                 | 11                  | 12                  |
| Uncertainty  | (0.0039,0.015,0.043) | (0.026,0.082,0.22) | (0.0069,0.043,0.21) | (0.0090,0.054,0.26) |

Figure 12. UNCERTAINTY USING TYPE-1 FUZZY SETS (VALUES)

The illustration of overall uncertainties is shown in Fig.13, with the threshold set at 0.1.

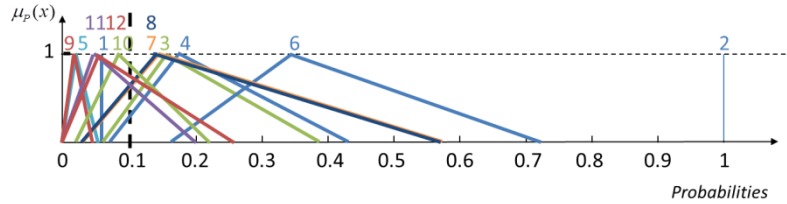


Figure 13. UNCERTAINTY USING TYPE-1 FUZZY SETS (MEMBERSHIP FUNCTIONS)

In this case, it is quite obvious that architectures 1, 5, and 9 are below the threshold, while the architectures 2, 6, 3, 4, and 7 are above it. However, it is more difficult to define the situation of architectures 10, 11, and 12. We focus on the fuzzy results that are partly above and partly below the threshold for both satisfaction levels and uncertainties, as shown in Fig.14.

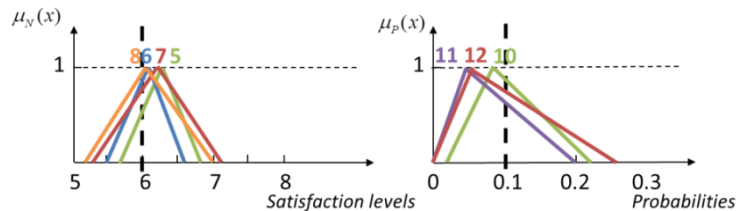


Figure 14FUZZY RESULTS THAT ARE PARTLY ABOVE THE THRESHOLD

The utilization of fuzzy methods is to the purpose of considering fuzziness in human estimation, which is also represented in the results shown in Fig.14. Therefore, when considering whether these results pass the threshold, the fuzziness should also be considered, which means that the belongingness of these results to the set that passes the threshold is also fuzzy. Therefore, we think that each of these results should have a degree of “passing the threshold”, and whether the architecture or supplier belongs to

the identified architecture or supplier depends on decision maker’s tolerance about the result.

There are many ways to represent decision makers’ tolerance level. In this work, we use the  $\alpha$ -cut to represent the tolerance, and define that the result passes the threshold if the maximum value of the fuzzy number after the  $\alpha$ -cut passes the threshold. With this definition, the decision totally depends on the value of  $\alpha$ — which represents the tolerance level of the decision maker (bigger  $\alpha$  is, smaller the tolerance is). For example, if we set  $\alpha$  at 0.7 for the overall uncertainty as shown in Fig.15, the architectures 10 and 12 will pass the threshold and the architecture 11 does not pass the threshold.

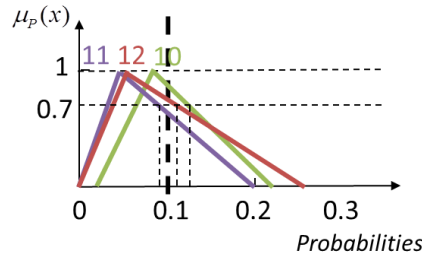


Figure 15 USING  $\alpha$ -CUT TO REPRESENT TOLERANCE LEVEL

However, the purpose of using the  $\alpha$ -cut is only to show that these kinds of fuzzy results should be considered fuzzy regarding their belonging to the identified candidates. They can be below or above the threshold regarding decision makers’ tolerance. Therefore, in architecture and supplier identification results, we represent this kind of candidates as “possible candidates depend on tolerance level”.

### 5.2.2 Using 2-Tuple Fuzzy Linguistic Representation

The 2-tuple fuzzy linguistic representation is exactly the same as the type-1 fuzzy sets from fuzzification until obtaining fuzzy results (shown in Fig.9 and Fig.12). After obtaining fuzzy results, the 2-tuple fuzzy linguistic representation defuzzifies the results, and converts the defuzzification result back to the closest initial linguistic terms, and the

distance between the original results to the closest linguistic term.

According to the reasoning in section 4, the max membership principle is used for defuzzification, and the results are shown in Fig.16 and Fig.17.

|              |     |     |     |     |     |     |     |     |     |     |     |     |
|--------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Architecture | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  |
| Satisfaction | 4.7 | 4.5 | 4.6 | 4.5 | 6.3 | 6.1 | 6.2 | 6.1 | 7.4 | 7.2 | 7.3 | 7.2 |

Figure 16 REQUIREMENTS SATISFACTION AFTER DEFUZZIFICATION

|              |       |   |      |      |       |      |      |      |       |       |       |       |
|--------------|-------|---|------|------|-------|------|------|------|-------|-------|-------|-------|
| Architecture | 1     | 2 | 3    | 4    | 5     | 6    | 7    | 8    | 9     | 10    | 11    | 12    |
| Uncertainty  | 0.060 | 1 | 0.16 | 0.18 | 0.020 | 0.37 | 0.15 | 0.15 | 0.010 | 0.080 | 0.040 | 0.050 |

Figure 17 UNCERTAINTY AFTER DEFUZZIFICATION

When using the 2-tuple fuzzy linguistic representation, a 2-tuple  $(s, \alpha)$  is used, where  $s$  represents the closest linguistic term, and  $\alpha$  represents the distance. For example, the overall requirements satisfaction level of architecture 1 is equal to 4.7 (see Fig.16), its closest linguistic terms is represented by the level 5, which is “a satisfactory solution”. The distance between 4.7 and 5 is equal to -0.3. That is why in 2-tuple fuzzy linguistic representation, the overall requirements satisfaction level of architecture 1 is  $(N_5, -0.3)$ . The result of requirement satisfaction and uncertainty when using the 2-tuple fuzzy linguistic representation are shown in Fig.18 and Fig.19, respectively.

|              |               |               |               |               |              |              |
|--------------|---------------|---------------|---------------|---------------|--------------|--------------|
| Architecture | 1             | 2             | 3             | 4             | 5            | 6            |
| Satisfaction | $(N_5, -0.3)$ | $(N_5, -0.5)$ | $(N_5, -0.4)$ | $(N_5, -0.5)$ | $(N_6, 0.3)$ | $(N_6, 0.1)$ |
| Architecture | 7             | 8             | 9             | 10            | 11           | 12           |
| Satisfaction | $(N_6, 0.2)$  | $(N_6, 0.1)$  | $(N_7, 0.4)$  | $(N_7, 0.2)$  | $(N_7, 0.3)$ | $(N_7, 0.2)$ |

Figure 18. REQUIREMENTS SATISFACTION USING 2-TUPLE LINGUISTIC REPRESENTATION

|              |                     |                    |                    |                     |              |                     |
|--------------|---------------------|--------------------|--------------------|---------------------|--------------|---------------------|
| Architecture | 1                   | 2                  | 3                  | 4                   | 5            | 6                   |
| Uncertainty  | $(P_{0.1}, -0.040)$ | $(1, 0)$           | $(P_{0.2}, -0.04)$ | $(P_{0.2}, -0.02)$  | $(0, 0.016)$ | $(P_{0.4}, -0.03)$  |
| Architecture | 7                   | 8                  | 9                  | 10                  | 11           | 12                  |
| Uncertainty  | $(P_{0.2}, -0.05)$  | $(P_{0.2}, -0.05)$ | $(0, 0.015)$       | $(P_{0.1}, -0.018)$ | $(0, 0.043)$ | $(P_{0.1}, -0.046)$ |

Figure 19. UNCERTAINTY USING 2-TUPLE FUZZY LINGUISTIC REPRESENTATION

## 6 COMPARISON OF RESULTS AND DISCUSSION

In section 5, we obtained the requirements satisfaction levels and uncertainties for each of the architectures. The threshold is set at 6 or  $N_6$  for satisfaction levels, and 0.1 or

P0.1 for uncertainty to filter the generated architectures. The potential supplier identification results obtained without considering subjective uncertainty, considering subjective uncertainty using type-1 fuzzy sets and 2-tuple linguistic representation are compared in Fig.20.

| <b>Without Integrating Expert Uncertainty</b>               |              |             |             |
|---|--------------|-------------|-------------|
| (Clustering: satisfaction $\geq$ 6, uncertainty $\geq$ 0.1) |              |             |             |
| Architecture  | Satisfaction | Uncertainty | Supplier    |
| 6   | 6.1          | 0.37        | 1,4,6,7,8,9 |
| 7   | 6.2          | 0.15        | 2,4,6,7,8,9 |
| 8   | 6.1          | 0.15        | 3,4,6,7,8,9 |

| <b>Integrating Expert Uncertainty with type-1 fuzzy sets</b> |               |                     |             |
|--|---------------|---------------------|-------------|
| (Clustering: satisfaction $\geq$ 6, uncertainty $\geq$ 0.1)  |               |                     |             |
| Architecture   | Satisfaction  | Uncertainty         | Supplier    |
| 6  | (5.5,6.1,6.6) | (0.17,0.37,0.72)    | 1,4,6,7,8,9 |
| 7  | (5.3,6.2,8.1) | (0.035,0.15,0.58)   | 2,4,6,7,8,9 |
| 8  | (5.2,6.1,7.0) | (0.035,0.15,0.58)   | 3,4,6,7,8,9 |
| 10   | (6.6,7.2,7.7) | (0.026,0.082,0.22)  | 1,5,6,7,8,9 |
| 11   | (6.4,7.3,8.2) | (0.0069,0.043,0.21) | 2,5,6,7,8,9 |
| 12   | (6.3,7.2,8.1) | (0.0090,0.054,0.26) | 3,5,6,7,8,9 |

| <b>Integrating Expert Uncertainty with 2-tuple fuzzy linguistic representation</b>      |                       |                            |             |
|---|-----------------------|----------------------------|-------------|
| (Clustering: satisfaction $\geq$ N <sub>6</sub> , uncertainty $\geq$ P <sub>0.1</sub> ) |                       |                            |             |
| Architecture  | Satisfaction          | Uncertainty                | Supplier    |
| 6   | (N <sub>6</sub> ,0.1) | (P <sub>0.4</sub> ,-0.03)  | 1,4,6,7,8,9 |
| 7   | (N <sub>6</sub> ,0.2) | (P <sub>0.2</sub> ,-0.05)  | 2,4,6,7,8,9 |
| 8   | (N <sub>6</sub> ,0.1) | (P <sub>0.2</sub> ,-0.05)  | 3,4,6,7,8,9 |
| 10  | (N <sub>7</sub> ,0.2) | (P <sub>0.1</sub> ,-0.018) | 1,5,6,7,8,9 |
| 12  | (N <sub>7</sub> ,0.2) | (P <sub>0.1</sub> ,-0.046) | 3,5,6,7,8,9 |

Figure 20. COMPARISON OF SUPPLIER IDENTIFICATION RESULTS

Upon comparison of the results, we see that without considering subjective uncertainty, ASIT identifies three architectures (6, 7 and 8), which are also identified with consideration of subjective uncertainty using the two fuzzy methods.

When using type-1 fuzzy sets, three “possible architectures depend on tolerance level” are also identified (numbers 10, 11 and 12). Whether these three architectures belong to the identified architectures depends on the decision makers’ tolerance level.

When using the 2-tuple fuzzy linguistic representation, aside from architectures 6, 7 and 8, two additional architectures (10, 12) are identified. This expansion of the scope in results is because that the 2-tuple fuzzy linguistic representation converts the



defuzzification result to the initial linguistic terms, which rounds up the results, thus increases the tolerance level.

From the comparison one can observe that (1) the architectures and suppliers identified without considering subjective uncertainty are also identified with consideration of subjective uncertainty by using the two fuzzy methods; (2) considering subjective uncertainty enlarges the result's scope mainly because the consideration of subjective uncertainty increases the level of tolerance (i.e. more tolerant) when filtering candidates. However, the tolerance level can also be changed without considering subjective uncertainty, and by simply changing the thresholds. As shown in Fig.21, by changing the uncertainty threshold to 0.05, the same scope for results can be obtained as integrating subjective uncertainty using 2-tuple linguistic representation. By changing the uncertainty threshold to 0.04, same results can be obtained as using type-1 fuzzy sets to model subjective uncertainty.

| Without Integrating Expert Uncertainty |                       |                         |                     |
|--|-----------------------|-------------------------|---------------------|
| Satisfaction threshold                 | Uncertainty threshold | Identified architecture | Identified supplier |
| 6                                      | 0.1                   | 6                       | 1,4,6,7,8,9         |
|  |                       | 7                       | 2,4,6,7,8,9         |
|  |                       | 8                       | 3,4,6,7,8,9         |
| 6                                      | 0.05                  | 6                       | 1,4,6,7,8,9         |
|  |                       | 7                       | 2,4,6,7,8,9         |
|  |                       | 8                       | 3,4,6,7,8,9         |
|  |                       | 10                      | 1,5,6,7,8,9         |
|  |                       | 12                      | 3,5,6,7,8,9         |
| 6                                      | 0.04                  | 6                       | 1,4,6,7,8,9         |
|  |                       | 7                       | 2,4,6,7,8,9         |
|  |                       | 8                       | 3,4,6,7,8,9         |
|  |                       | 10                      | 1,5,6,7,8,9         |
|  |                       | 11                      | 2,5,6,7,8,9         |
|  |                       | 12                      | 3,5,6,7,8,9         |

Figure 21 Changing thresholds without considering subjective uncertainty

According to the analysis above, we found that (1) the result of ASIT without considering subjective uncertainty is reliable because the identified architectures are also within the result's scope with consideration of subjective uncertainty; (2) same

results as considering subjective uncertainty can be obtained without considering subjective uncertainty by simply changing the value of thresholds. Therefore, we conclude that the consideration of subjective uncertainty does not considerably influence ASIT results, so that it is not necessary to consider the subjective uncertainty in ASIT in early conceptual design stage. However, it is very important to note that this conclusion is obtained under the situation of using triangular membership functions and which is due to the lack of information in early conceptual design stage. In other design stages, this conclusion may not be valid since when more information is available, other types of fuzzy membership functions can be used, which may lead to different results. This result also points out that under certain situation, it is not useful to consider expert estimation uncertainty. Considering expert estimation uncertainty without analyzing the situation may result in wasted effort.

There are several limitations in this work, however. Firstly, although the max membership principle is demonstrated as a proper choice for defuzzification, it is still interesting to test other defuzzification methods, which may lead to different outcomes. Secondly, we assumed that using a group of experts is able to correct the cognitive bias of each expert. However, the cognitive bias is potentially a fruitful issue that is worth investigating further under the topic of expert uncertainty. In addition, this work is carried out specifically in the context of ASIT. It may benefit from a generalization to a broader context.

## **7 CONCLUSIONS**

Due to innovation integration in early design phases, aside from previous project data, expert estimations are often used. In this work, we investigated how subjective uncertainty resulting from expert estimations influences the result of ASIT; ASIT is an

early design support tool proposed in our previous work. It is important to understand to what extent this approach can be used, and can yield robust results in an industrial context, and whether it is necessary to consider subjective uncertainty in ASIT.

After analysing different uncertainty representation methods for subjective uncertainty, both Type-1 and 2-tuples fuzzy sets were found suitable for representing this type of uncertainty in ASIT. A powertrain design case was used to compare the results of the original ASIT and the results with integration of subjective uncertainties. The comparison shows that considering subjective uncertainty enlarges the result set (i.e., more architectures and suppliers are identified) because the consideration of subjective uncertainty increases the level of tolerance (i.e., more tolerant) when filtering candidates. However, the initial set without considering subjective uncertainty was found to match the results in part; note that with the uncertainty integration the set of results expanded. Therefore, considering subjective uncertainty in ASIT will not have a considerable impact on the overall ASIT results, so that it is unnecessary to consider subjective uncertainty in ASIT in early conceptual design stage.

The result of this work is valid only in the context of early conceptual design, where information is lacking, so that only triangular membership function can be used. The results also pointed out that it is not always necessary to consider expert estimation uncertainty. Before considering this type of uncertainty, the context should be analysed to prevent the waste of effort.

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