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A Metaheuristic Approach for Preference Learning in Multi-Criteria Ranking based on Reference Points

Jinyan Liu, Wassila Ouerdane, Vincent Mousseau¹

Abstract. In this paper, we are interested in a family of multi-criteria ranking methods called Ranking with Multiple reference Points (RMP). This method is based on pairwise comparisons, but instead of directly comparing any pair of alternatives, it compares rather the alternatives to a set of predefined reference points. We actually focus on a Simplified RMP model (S-RMP) in which the preference parameters include the criteria weights and the set of reference points ordered by importance. Elicitation of the parameters (from the data provided by the decision makers) leads us to the preference learning algorithms that cannot only be applied on relatively small dataset. Therefore, we propose in this work a preference learning methodology for learning S-RMP models from a large set of pairwise comparisons of alternatives. The newly proposed algorithm is a combination of an Evolutionary Algorithm and a Linear Programming approach. Empirical results and analysis are also presented.

1 Introduction

Decision makers (DMs) often face decision situations in which different conflicting viewpoints are to be considered. When modeling a real world decision problem using Multiple Criteria Decision Aid (MCDA) theories, several problematics can be considered: choice, sorting or ranking problem [18]. In this paper, we are interested in multi-criteria ranking problem, where the aim is to establish a preference order (or ranking) on the set of alternatives.

The field of MCDA offers a selection of methods and operational tools i.e. aggregation models that explicitly account for the diversity of the viewpoints considered. Each method constructs first a model of DM's preferences and then exploits this model in order to work out a recommendation. During the aggregation phase, different parameters are needed, such as weights, marginal value functions, thresholds, etc. depending on the method. Such parameters allow to elaborate models taking into account the DMs preferences. Therefore, it is clear that such preferences play a key role in the construction of the recommendations. In fact, they are meaningful and acceptable only if the DMs values are taken into account.

The process by which the DMs values or the parameters of an aggregation model are captured is called preference elicitation or disaggregation [18]. Preference elicitation aims at helping the analyst to appropriately elicit the DM's preferences to be represented in a decision model. The process consists of a dialogue with the DM, where the aim is to infer the values of the parameters from the holistic information given by the DM. We note that this is not an easy task, especially when the information provided contains inconsisten-

cies. Moreover, the DM has in general a limited knowledge on the aggregation models and can only express his preferences in a rather intuitive and ambiguous manner.

Generally speaking, there are two paradigms of preference elicitation approaches, namely direct and indirect paradigms [13]. In the direct aggregation paradigm, the parameter values are directly provided by the DM through an interactive communication with the analyst. The aggregation model is firstly constructed with these parameters and then applied to the alternative set to obtain the DM's preferences. Within such a paradigm, the DM should make enough effort to understand the meaning and the roles of these parameters and to associate appropriate values to them, which may be beyond his cognitive limitation. In the indirect disaggregation paradigm, the DM provides holistic preference information such as pairwise comparisons, assignment examples etc., from which a preference model is derived and then applied to contextual recommendation. In contrast with the direct elicitation, the parameter values are regressed from the DM [10].

Nowadays, we are facing with decision problems involving large datasets. It requires adapting and improving the algorithms and techniques to construct acceptable recommendations. Some of related challenges have been addressed in the scientific community of Preference Learning, which focus on the computational complexity rather than the decision problem itself [9]. In the perspective of preference elicitation, the emergence of applications in MCDA with the intention of dealing with large datasets provokes our research interests. In this perspective, we intend to provide an efficient algorithms to infer the parameter values of an aggregation model called S-RMP such that the DM has provided as input a large set of pairwise comparisons.

The paper is organized as follows: Section 2 introduces the RMP method and its simplified version S-RMP. We present our metaheuristic approach for S-RMP disaggregation in Section 3. The numerical analysis and the benefits of the proposed approach are provided in Section 4. At the end, we conclude the paper.

2 Ranking with Multiple reference Points

Recently, a ranking method called Ranking with Multiple reference Points (RMP) has been proposed in [16, 17]. The idea is to construct the global preference relation between two alternatives on the basis of their relative comparisons with specified reference points. This paper is concerned with learning the parameters of this method. We give in the next sections an overview of such a method as well as its simplified version named S-RMP, introduced in [23, 22]. Further work has also been presented in [2].

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2.1 General RMP method

We consider a multiple criteria ranking problem with n alternatives in the set \mathcal{A} indexed by $N = \{1, 2, \dots, i, \dots, n\}$ and m monotone criteria in the set \mathcal{F} , indexed by means of a set $M = \{1, 2, \dots, j, \dots, m\}$. The evaluations of alternatives on a criterion j take their value in the associated evaluation scale \mathcal{X}_j . The \mathcal{X} denotes the evaluation space, $\mathcal{X} = \prod_{j \in M} \mathcal{X}_j$ i.e. the Cartesian product of evaluation scales. Obviously, the evaluation of any alternative $a \in \mathcal{A}$ is a vector denoted by $a = (a_1, a_2, \dots, a_m) \in \mathcal{X}$.

The RMP method is a three-step multi-criteria paradigm for ranking alternatives. It involves k reference points such that $P = \{1, 2, \dots, h, \dots, k\}$. The evaluation of each reference point $p^h, h \in P$ on a criterion j is denoted by $p_j^h \in \mathcal{X}_j$. To establish a global preference relation between two alternatives, a and b , the method specifies the following three steps:

1. Compare each alternative $a \in \mathcal{A}$ (respectively, $b \in \mathcal{A}$) to every reference points $p^h, h \in P$ on every criterion $j, j \in M$.
2. Aggregate the results of the step 1 considering the m criteria and deduce the preference relation between two different alternatives (a, b) which is depending on the reference point p^h , also called the relative preference with respect to the reference point p^h ;
3. For each pair of alternatives (a, b) , aggregate the k preference relations into global preference relation.

The first step establishes the preference relation between each alternative and each reference point on each criterion j . In the second, we only consider the criteria for which a (respectively, b) is at least as good as p^h . This set of criteria is denoted by $C(a, p^h)$ (respectively, $C(b, p^h)$) and defined as $C(a, p^h) \in \mathcal{P}(M)$ such that $C(a, p^h) = \{j \in M \mid a_j \geq p_j^h\}$.

We define then the *importance relation among criteria with respect to the reference point*, denoted by $\blacktriangleright_{p^h}$, which means that a set of criteria is at least as important as another set of criteria as follows;

Definition 1. (*Importance relation among criteria w.r.t. reference point*)

The importance relation $\blacktriangleright_{p^h}$ is defined on $M \times M$ such that:

1. $\forall M_1 \subset M, M_1 \neq \emptyset \Rightarrow M_1 \blacktriangleright_{p^h} \emptyset, \forall h \in P$
2. $\forall M_1 \subset M \Rightarrow M \blacktriangleright_{p^h} M_1, \forall h \in P$
3. $\forall M_1, M_2 \subset M, M_1 \subset M_2 \Rightarrow M_2 \blacktriangleright_{p^h} M_1, \forall h \in P$

Thus, the (relative) preference relation \succsim_{p^h} defined in Definition 2 expresses how a pair of alternatives compare with each other with respect to the reference point p^h .

Definition 2. (*Relative preference w.r.t. reference point*)

The relative preference with respect to the reference point \succsim_{p^h} on $\mathcal{A} \times \mathcal{A}$ is defined by:

$$a \succsim_{p^h} b \Leftrightarrow C(a, p^h) \blacktriangleright_{p^h} C(b, p^h)$$

In the third step, we define firstly the importance relation \triangleright as below.

Definition 3. (*Importance relation among reference points*)

The importance relation \triangleright is defined on $P \times P$ such that:

1. $\forall P_1 \subset P, P_1 \neq \emptyset \Rightarrow P_1 \triangleright \emptyset$
2. $\forall P_1 \subset P \Rightarrow P \triangleright P_1$
3. $\forall P_1, P_2 \subset P, P_1 \subset P_2 \Rightarrow P_2 \triangleright P_1$

We deduce then the (global) preference relation \succsim as described in the definition 4. It means that a is at least as good as b if the coalition of reference points $P(a, b)$ which affirms that a is at least as good as b is more important than the coalition of reference points $P(b, a)$ which affirms that b is at least as good as a .

Definition 4. (*Global preference relation*)

The global preference relation on $\mathcal{A} \times \mathcal{A}$ on the basis of the relative preferences is:

$$a \succsim b \Leftrightarrow P(a, b) \triangleright P(b, a)$$

We note that there is no lack of generality to impose a dominance relation among reference points, as it was shows in [17]. There exists always an equivalent RMP model for any RMP model with k reference points such that:

$$\forall j \in M, \forall h, h' \in P, h > h' \Rightarrow p_j^h \geq p_j^{h'} \quad (1)$$

It means also that:

$$\forall a \in \mathcal{A}, \forall h, h' \in P, h > h' \Rightarrow C(a, p^h) \subseteq C(a, p^{h'}) \quad (2)$$

The two importance relations mentioned respectively in Definition 1 and Definition 3 can be rather general and built on the basis of different rules. Particularly, we present in the next section a simplified version of RMP method, namely S-RMP in which the importance relation $\blacktriangleright_{p^h}$ is defined by a concordance rule for all reference points while a lexicography of dictatorial reference points is used as the importance relation \triangleright .

2.2 Simplified RMP model

In this work, we focus on a simplified version of RMP, named S-RMP model, as considered in [23, 22].

Firstly, the importance relation $\blacktriangleright_{p^h}$ on the criteria set is re-defined based on a concordance rule by using an additive decomposition as follow:

$$C(a, p^h) \blacktriangleright_{p^h} C(b, p^h) \Leftrightarrow \sum_{j \in C(a, p^h)} \omega_j \geq \sum_{j \in C(b, p^h)} \omega_j \quad (3)$$

where ω_j represents the weight of the criterion j . Formally, the criteria weights are normalized to 1.

Secondly, as shown in [17], an important result derived from social choice theory indicates that the only importance relation, which aggregates the k relative preference relations (with respect to reference points) and leads to transitive relation on each possible set of alternatives, is obtained by a lexicographical order on the reference points. Therefore, a permutation σ on P is used lexicographically to determine the importance relations among the reference points so as to deduce globally the preference relations between alternatives. This is represented by (4).

$$\begin{aligned} a \succsim b &\Leftrightarrow \exists h^* \in P \text{ s.t. } a \succsim_{p^{\sigma(h^*)}} b \\ &\text{and } \forall h \in \{1, \dots, h^* - 1\}, a \sim_{p^{\sigma(h)}} b \\ a \sim b &\Leftrightarrow \forall h \in P, a \sim_{p^h} b \end{aligned} \quad (4)$$

In details, the first reference point is denoted by $p^{\sigma(1)}$, the second one by $p^{\sigma(2)}$ and so on. To compare a and b , we look at first the reference point $p^{\sigma(1)}$. If a is strictly preferred to b with respect to $p^{\sigma(1)}$,

then we affirm globally that a is preferred to b without even considering the other reference points and similarly vice versa. However, if a and b are considered to be indifferent with respect to $p^{\sigma(1)}$, we shall proceed with the second reference point $p^{\sigma(2)}$, etc. Once the strict relative preference between a and b is confirmed, the global preference between a and b is affirmed. If we still cannot differentiate these two alternatives until the last reference point $p^{\sigma(k)}$ has been processed then they are considered globally as tied for conclusion.

The additive decomposition of importance relation for subsets of criteria shows that the preference relation between two alternatives (a, b) can be computed by:

$$\begin{aligned}
a \succsim b &\Leftrightarrow \exists h^* \in P \text{ s.t.} \\
&\sum_{j \in C(a, p^{\sigma(h^*)})} \omega_j > \sum_{j \in C(b, p^{\sigma(h^*)})} \omega_j \\
&\text{and } \forall h \in \{1, \dots, h^* - 1\}, \\
&\sum_{j \in C(a, p^{\sigma(h)})} \omega_j = \sum_{j \in C(b, p^{\sigma(h)})} \omega_j \quad (5) \\
a \sim b &\Leftrightarrow \forall h \in P, \\
&\sum_{j \in C(a, p^h)} \omega_j = \sum_{j \in C(b, p^h)} \omega_j
\end{aligned}$$

2.3 Learning an S-RMP model

2.3.1 Problem context

We are interested in learning S-RMP models in an indirect way. To account for that, we assume that the information provided by the DM takes form of pairwise comparisons of alternatives. Learning such a model amounts to setting values for the criteria weights and the reference points which are detailed as follow:

- The normalized weights of criteria, $\omega_j, j \in M$.
- The number of reference points, denoted by k .
- The reference points, $p^h, h \in P$ where $p^h = (p_1^h, p_2^h, \dots, p_m^h)$ is a vector in the evaluation space $p^h \in \mathcal{X}$.
- The lexicographic order of the reference points, defined as a permutation σ on the index set of the reference points P . For instance, $p^{\sigma(1)}$ is the first reference point to which alternatives are compared, $\exists h \in P, \sigma(1) = h$, etc.

To better understand the problem, we provide a brief review on the latest related researches.

2.3.2 Literature review

A first attempt to elicit indirectly the preference model from the holistic information given by the DM was presented in [14] for the ELECTRE TRI method. They considered the pessimistic assignment rule, and developed a non-linear optimization formulation to infer all the parameters from a set of assignment examples. A similar approach was presented in [11] for a simplified version of ELECTRE TRI named Majority Rule Sorting Model (MR-Sort). In [11], only a few number of categories was considered and there was no veto threshold characterizing the discordance effect. Later, in [4], an extension of the previous work for ELECTRE TRI method has been presented in the group decision problems. The parameters were inferred from assignment examples, provided by multiple decision makers, based on a Mixed Integer Programming (MIP).

For the S-RMP method, [23] was the first work that proposed an MIP to infer the parameters of this models from a given set of pairwise comparisons. They assumed the existence of an S-RMP model that is able to correctly restore all the pairwise comparisons given by the DM, and provided a linearized set of constraints with binary variables. However, the algorithm proposed in [23] suffers from a limitation in the case where the pairwise comparisons provided by the DM contains inconsistent information. It cannot find any solution.

To overcome this problem, [12] proposed a new algorithm dealing with inconsistencies in the elicitation process. The proposal is also based on an MIP with binary variables. However, the algorithm searches for an optimal solution that is compatible with as many as possible pairwise comparisons provided by the DM. Besides, it is able to identify the inconsistent information.

We remark a common issue for the two algorithms developed for S-RMP models that is the high computation time due to the introduction of the binary variables. Actually, when dealing with a very limited number of pairwise comparisons, the previous two algorithms are proved to be quite efficient. Nevertheless, the computation time increases exponentially when a large quantity of pairwise comparisons are provided.

Recently, [19] presented a well-adapted algorithm for learning the parameters of MR-Sort models from large datasets. It takes the advantages of a heuristic approach combined with a Linear Programming (LP). More details are provided in [20]. We highlight the general idea of this work for sorting problems which in our work are ranking ones instead.

Preference elicitation for RMP models was also encouraged in [2]. Besides, other works are generally interested in preference disaggregation. For interested readers, please refer to [8, 6, 7].

3 A metaheuristic approach

The different approaches presented in section 2.3.2 for learning S-RMP models have formulated the learning task as a Mixed Integer linear programming (MIP) optimization problem. However, a common observation is the considerable high computation time due to the introduction of binary variables [12, 22, 23].

In the case where the decision problem implies a large dataset, suchlike MIP consumes even more computation time and usually leads to the insolvability of the problem within the limited time. This type of problems is considered as *hard* optimization problems, which is defined as optimization problems that cannot be solved to optimality ([1]).

Thus, our proposal turns to a metaheuristic algorithm with the intention of dealing efficiently with large datasets. In contrast to the MIPs, the metaheuristic will approximate as accurately as possible the parameters of the S-RMP model. We remark that multiple S-RMP models may exist. Our objective is to infer a satisfactory S-RMP model that is compatible with most (if not all of them) of the input information. An interactive model calibration process in the form of supplementary constraints on the parameters should be invoked in real decision cases.

3.1 Overview of the algorithm

The proposed metaheuristic follows the general idea of Evolutionary Algorithms (EA) and makes use of a local optimization. It starts with an initialized population of N_{model} S-RMP models rather than a single solution. *Evaluation*, *Selection*, *Mutation* and *Substitution* operations are iterated to adjust the parameters of each individual

in the operating population. The iteration will repeats until either at least one of the individuals in the population is able to restore all the input pairwise comparisons or after a certain number of times. Actually, due to the limited computation time in practice, we interrupt the algorithm before it reach the optimality more often. In our context, the limited number of iterations is set to 100. It is summarized in Algorithm 1

Algorithm 1: OVERVIEW OF THE ALGORITHM

- 1 Initialize a population of N_{model} S-RMP models
 - 2 Evaluate each individual in the population
 - 3 **repeat**
 - 4 Select the best individuals from the population with a probability ξ
 - 5 Adjust the reference points for selected individuals and yield mutants with a probability μ
 - 6 Adjust the criteria weights for mutated individuals
 - 7 Evaluate the newly adjusted individuals
 - 8 Substitute the population with a probability ζ under the constraint of the initial population size
 - 9 **until** at least one of the stopping criteria is met;
-

Actually, on the one side, the reference points are adjusted while yielding mutants. On the other side, the criteria weights are adjusted by solving an LP. The rest of this section is structured as below: Section 3.2 begins with the initialization of the reference points and the criteria weights. Section 3.3 presents how to learn the reference points. The LP for learning the criteria weights is provided in Section 3.4.

3.2 Initialization

The first step of the algorithm consists in initializing a population of N_{model} S-RMP models (in our context, $N_{model} = 10$). In meta-heuristic algorithms, it is important that the initial population spans adequately the solution space as shown in [5]. To do this, we provide a primitive method.

3.2.1 Reference points

First, we assume that the number of reference points k is fixed beforehand. From a practical perspective, the number of reference points in S-RMP models never exceeds 3, since such an S-RMP model has already a considerable capacity while restoring preference information as shown in [22].

Second, for a criterion j , k random numbers are generated from the uniform distribution in the evaluation scale \mathcal{X}_j . Depending on the preference direction of the criterion j , they are then ranked either in ascending order such that $p_j^1 \leq p_j^2 \leq \dots \leq p_j^k$ or in descending order such that $p_j^k \leq p_j^{k-1} \leq \dots \leq p_j^1$. Thus, the generated reference points guarantee that $p^h \succ p^{h-1} \succ \dots \succ p^1$, where $p^h = (p_1^h, p_2^h, \dots, p_j^h, \dots, p_m^h)$

Finally, for k reference points, there are $k!$ possible lexicographic orders. We randomly choose one of them and it is fixed once initialized.

3.2.2 Criteria weights

Concerning the weights, we use the approach of Butler [3]. Firstly, $m - 1$ random integer numbers d_i where $i \in \{1, \dots, m - 1\}$ are generated uniformly between 0 and 100. Then, they are ranked such that $d_0 = 0 < d_1 < d_2 < \dots < d_{m-1} < 100 = d_m$. The weight

of a criterion j is defined as $w_j = (d_j - d_{j-1})/100$. It is accurate to 0.01 and $0 \leq w_j \leq 1$. This ensure that the weights sum up to 1 and are uniformly distributed.

3.3 Learning the reference points

3.3.1 Evolutionary algorithm

Our algorithm starts with an initial population of a certain number of random solutions, and evolves it while yielding a new generation of population at each iteration. Conventionally, the $(t + 1)$ -th generation of solutions, denoted by \mathcal{G}_{t+1} , is obtained from the n -th generation, denoted by \mathcal{G}_t , through a procedure composed by several well-defined operations as described in [21]. [15] also proposed a review about Evolutionary Algorithms used in the context of local search.

In Algorithm 1, a proportion of the best S-RMP models in the population are selected at the beginning of each iteration. Then, they are submitted to the mutation operation. Mutations are performed on each selected model and yield mutants with a probability. The criteria weights of the mutants are adjusted (Section 3.4). After being re-evaluated, the mutants are considered as the "children", while the initial models are considered as the "parents". Finally, the substitution amounts to selecting the best models among both the parents and the children based on their evaluation. Otherwise, it also allows us to introduce newly initialized individuals to the next generation of population if necessary. The details are provided in the following sections.

3.3.2 Evaluation operation

The evaluation of the "best" individuals is done according to their *fitness* to the problem. In the context of learning an appropriate preference model from a set of information provided by the DM, several objective functions are applicable. In the case of learning S-RMP model, the fitness function corresponds to the *Ranking Accuracy* (RA) of the model, which is simply defined as the ratio of the number of pairwise comparisons restored correctly by the model to the total number of pairwise comparisons provided by the DM at the beginning of the process ((6)). Actually, this is the most straightforward measure of ranking performance.

$$RA = \frac{\text{Number of pairwise comparisons restored}}{\text{Total number of pairwise comparisons provided}} \quad (6)$$

3.3.3 Selection operation

The S-RMP model that gives the highest RA in the current population \mathcal{G}_t will be selected randomly with replacement, with a probability ξ which increases with their fitness. Thereby, we define the probability $\xi(f_s)$ associated to a given S-RMP model f_s that consists of a set of parameters s by:

$$\xi(f_s) = \frac{RA(f_s) - RA_{min}}{RA_{max} - RA_{min}} \quad (7)$$

where

$$RA_{min} = \min \{RA(f_s) \mid \forall f_s \in \mathcal{G}_t\} \quad (8)$$

$$RA_{max} = \max \{RA(f_s) \mid \forall f_s \in \mathcal{G}_t\} \quad (9)$$

3.3.4 Mutation operation

Mutation operation, which amounts to adjust the reference points, is only applied to the selected models. For each selected S-RMP model, the operation is defined in the Algorithm 2. The mutations of the reference points proceed one by one based on their lexicographic order σ and then on each criterion from $j = 1$ to m .

Algorithm 2: MUTATION OPERATION

- 1 Create an empty list for keeping ignorable pairwise comparisons
 - 2 **for** each reference point $p^{\sigma(h)}$ **do**
 - 3 **for** each criterion j **do**
 - 4 Generate a random variation $\pm\theta_j^{\sigma(h)}$
 - 5 Count possible impacts caused by this change
 - 6 Apply the change to the current model with a probability μ
 - 7 Append newly ignorable pairwise comparisons to the list
 - 8 Yield a mutated model
-

Firstly, both the sign and the value of the variation, denoted by $\pm\theta_j^{\sigma(h)}$, are uniformly randomized. However, it should be bounded to prevent re-degrading the quality of the models after some iterations and the boundary should depend on the current number of iterations N_{it} . For example, we take herein:

$$\theta_j^{\sigma(h)} \leq \left\lceil \frac{50}{\sqrt{N_{it}}} \right\rceil \quad (10)$$

As $\pm\theta_j^{\sigma(h)}$ will be later applied to the current model with a probability μ , we denote the changed evaluation of the reference point $p^{\sigma(h)}$ on the criterion j by $p_j^{\prime\sigma(h)}$ where

$$p_j^{\prime\sigma(h)} = p_j^{\sigma(h)} \pm \theta_j^{\sigma(h)} \quad (11)$$

Then, we should define the probability μ that indicates if the variation $\pm\theta_j^{\sigma(h)}$ should be applied. To do so, we identify at first the impacts provoked by $\pm\theta_j^{\sigma(h)}$ on the judgment of preference between alternatives, which eventually improve (or worsen) the RA, through the calculations below.

For any pairwise comparison of alternatives $a \succsim b$ provided by the DM, we define the *slack* quantity as follows:

$$s_{(a,b)}^{\sigma(h)} = \sum_{j=1}^m (\delta_{a,j}^{\sigma(h)} - \delta_{b,j}^{\sigma(h)}) \cdot \omega_j \quad (12)$$

where ω_j denotes the weight of the criterion j . $\forall (a,b) \in \mathcal{BC}, \forall h \in P$ and the lexicographic order σ , we compute $\delta_{a,j}^{\sigma(h)}$ and $\delta_{b,j}^{\sigma(h)}$ as follows:

$$\delta_{a,j}^{\sigma(h)} = \begin{cases} 1 & \text{if } a_j \geq p_j^{\sigma(h)} \\ 0 & \text{if } a_j < p_j^{\sigma(h)} \end{cases}, \quad \delta_{b,j}^{\sigma(h)} = \begin{cases} 1 & \text{if } b_j \geq p_j^{\sigma(h)} \\ 0 & \text{if } b_j < p_j^{\sigma(h)} \end{cases} \quad (13)$$

In fact, the (global) judgement of preference between (a,b) is based on the relative preference between (a,b) with respect to the reference point $p^{\sigma(h)}$, which is deduced by

$$a \succsim_{\sigma(h)} b \iff s_{(a,b)}^{\sigma(h)} \geq 0 \quad (14)$$

If $\delta_{a,j}^{\sigma(h)}$ equals to 1, it means that the criterion j is contributing to the statement $a \succsim_{\sigma(h)} b$. The criterion j is namely a contributing

criterion for a in this case. Similarly, if $\delta_{b,j}^{\sigma(h)}$ equals to 1, it means that the criterion j is weakening the statement $a \succsim_{\sigma(h)} b$. The criterion j is then namely a weakening criterion for b . Particularly, when $\delta_{a,j}^{\sigma(h)} - \delta_{b,j}^{\sigma(h)} = 0$, the criterion j is neither contributing nor weakening. We said that it is neutralized.

The variation $\pm\theta_j^{\sigma(h)}$ affects the calculation of $s_{(a,b)}^{\sigma(h)}$ through $(\delta_{a,j}^{\sigma(h)}, \delta_{b,j}^{\sigma(h)})$. All the possible impacts provoked by $\pm\theta_j^{\sigma(h)}$ can be summarized (as shown in Table 1) by the value change of $(\delta_{a,j}^{\sigma(h)}, \delta_{b,j}^{\sigma(h)})$, where the changed value can be denoted and calculated by:

$$\delta_{a,j}^{\prime\sigma(h)} = \begin{cases} 1 & \text{if } a_j \geq p_j^{\prime\sigma(h)} \\ 0 & \text{if } a_j < p_j^{\prime\sigma(h)} \end{cases} \quad (15)$$

$$\delta_{b,j}^{\prime\sigma(h)} = \begin{cases} 1 & \text{if } b_j \geq p_j^{\prime\sigma(h)} \\ 0 & \text{if } b_j < p_j^{\prime\sigma(h)} \end{cases} \quad (16)$$

Formally, the positive and negative impacts are respectively defined as follows:

$$I_{pos}(\pm\theta_j^{\sigma(h)}) = \left\{ (a,b) \in \mathcal{BC} \mid (\delta_{a,j}^{\prime\sigma(h)} - \delta_{b,j}^{\prime\sigma(h)}) - (\delta_{a,j}^{\sigma(h)} - \delta_{b,j}^{\sigma(h)}) > 0 \right\} \quad (17)$$

$$I_{neg}(\pm\theta_j^{\sigma(h)}) = \left\{ (a,b) \in \mathcal{BC} \mid (\delta_{a,j}^{\prime\sigma(h)} - \delta_{b,j}^{\prime\sigma(h)}) - (\delta_{a,j}^{\sigma(h)} - \delta_{b,j}^{\sigma(h)}) < 0 \right\} \quad (18)$$

Table 1: Impacts provoked by $\pm\theta_j^{\sigma(h)}$ on $(\delta_{a,j}^{\sigma(h)}, \delta_{b,j}^{\sigma(h)})$

	Before	After	Impact	Description
I_1	(0,1)	(0,0)	positive	weakening criterion neutralized
I_2	(0,1)	(1,1)	positive	weakening criterion neutralized
I_3	(0,0)	(1,0)	positive	criterion become contributing
I_4	(1,1)	(1,0)	positive	criterion become contributing
I_5	(1,0)	(0,0)	negative	contributing criterion neutralized
I_6	(1,0)	(1,1)	negative	contributing criterion neutralized
I_7	(0,0)	(0,1)	negative	criterion become weakening
I_8	(1,1)	(0,1)	negative	criterion become weakening
I_9	(0,0)	(1,1)	neutral	no impact on the slack
I_{10}	(1,1)	(0,0)	neutral	no impact on the slack
I_{11}	(0,0)	(0,0)	neutral	no impact on the slack
I_{12}	(1,1)	(1,1)	neutral	no impact on the slack
I_{13}	(1,0)	(1,0)	neutral	no impact on the slack
I_{14}	(0,1)	(0,1)	neutral	no impact on the slack
I_{15}	(0,1)	(1,0)	n.c.	impossible
I_{16}	(1,0)	(0,1)	n.c.	impossible

For instance, we represent, in Figure 1, a couple of examples to illustrate the adjustment of reference points. We suppose that

- The criteria (c_1, c_2, c_3 and c_4) are equal weighted, i.e. $\forall j \in \{1, 2, 3, 4\}, \omega_j = 0.25$.
- The reference points are uniformly initialized and used in sequence, i.e. first p^1 , then p^2 and p^3 .
- The DM provided $a \succsim b$ as input.

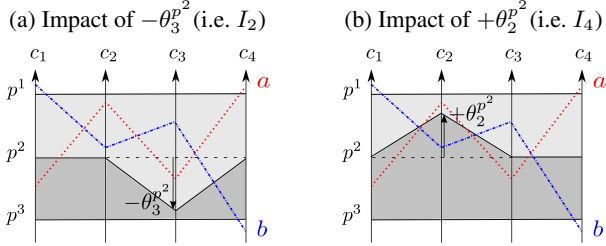
With the initialized reference points, we derive at first that $a \sim_{p^1} b$, $b \succ_{p^2} a$ and $a \succ_{p^3} b$. Since the reference points are used sequentially, we derive then $b \succ a$, which is inconsistent with the statement

of the DM. Thus, the reference points should be adjusted by the algorithm.

In Figure 1(a), the criterion $j = 3$, which was originally weakening the statement $a \succsim b$ (as $\delta_{a,3}^{p^2} = 0$), is neutralized by applying the variation $-\theta_3^{p^2}$ to the reference point p^2 on the criterion $j = 3$, because $\delta'_{a,3}{}^{p^2} = \delta'_{b,3}{}^{p^2} = 1$ and $\delta'_{a,3}{}^{p^2} - \delta'_{b,3}{}^{p^2} = 0$. It is considered to be a (potential) positive impact, for the fact that, as shown in this example, we newly derive that $a \sim_{p^2} b$ and finally $a \succsim b$, which is consistent with the DM's statement. In this sense, the reference point p^2 is adjusted.

In Figure 1(b), the criterion $j = 2$, which was originally neither weakening nor contributing to statement $a \succsim b$ (as $\delta_{a,2}^{p^2} - \delta_{b,2}^{p^2} = 0$), becomes contributing by applying the variation $+\theta_2^{p^2}$ to the reference point p^2 on the criterion $j = 2$, because $\delta'_{a,2}{}^{p^2} = 1$ and $\delta'_{b,2}{}^{p^2} = 0$. It is also considered to be a (potential) positive impact, for the similar fact that we can again finally derive that $a \succsim b$ in this example with the adjusted reference point p^2 even though it was adjusted in a different way.

Figure 1: Adjustment of reference points



So far, we identify four positive possibilities as well as four negative possibilities and sum up accordingly the quantity of the positive (respectively, negative) impacts by

$$I_{pos} = \left| I_{pos} \left(\pm \theta_j^{\sigma(h)} \right) \right| = |I_1| + |I_2| + |I_3| + |I_4| \quad (19)$$

$$I_{neg} = \left| I_{neg} \left(\pm \theta_j^{\sigma(h)} \right) \right| = |I_5| + |I_6| + |I_7| + |I_8| \quad (20)$$

However, we need to make some empirical remarks such that:

1. Usually, the variation $\pm \theta_j^{\sigma(h)}$ provokes both the positive and negative impacts on different pairs of alternatives at the same time. Having more positive impacts means that we have more chance to improve the S-RMP model quality, but not necessarily.
2. Sometimes, we provoke as many negative impacts as the positive impacts. In such a case, we are more interested in the case where a large quantity of impacts provoked, as it allows us to introduce more diversity to the population.
3. When there are less positive impacts than the negative ones, or even no positive impacts provoked, we consider that it is still possible to improve the model potentially.

Based on these observations, we define accordingly the probability of applying the variation $\pm \theta_j^{\sigma(h)}$ to the current S-RMP model, denoted by $\mu(\pm \theta_j^{\sigma(h)})$ as it is associated to the variation $\pm \theta_j^{\sigma(h)}$, as below :

$$\mu \left(\pm \theta_j^{\sigma(h)} \right) = \begin{cases} \frac{I_{pos}}{I_{pos} + I_{neg}} & \text{if } I_{pos} \neq 0 \text{ and } I_{pos} > I_{neg} \\ 1 - \frac{1}{I_{pos}} & \text{if } I_{pos} \neq 0 \text{ and } I_{pos} = I_{neg} \\ 0.5 * Gaussian & \text{if } I_{pos} = 0 \text{ or } I_{pos} < I_{neg} \end{cases} \quad (21)$$

Otherwise, as we treat the reference points one by one according to the lexicographic order, once (14) is affirmed, (a, b) will be appended to the list for keeping ignorable pairwise comparisons. Actually, (a, b) is considered to be ignorable, because the adjustment for the rest of the reference points will not affect the global judgement on the preference between (a, b) and the impacts provoked by $\pm \theta_j^{\sigma(h+1)}$ on $(\delta_{a,j}^{\sigma(h+1)}, \delta_{b,j}^{\sigma(h+1)})$ (and so on, if exists) can be then ignored. In other words, once the comparison between any two alternatives (a, b) is restored by the previous reference point, no matter how they compare to the rest of the reference points, it will not be changed.

3.3.5 Substitution operation

As presented in Section 3.3.1, the mutants with the adjusted criteria weights (Section 3.4 for details) will be re-evaluated and then appended to the n -th generation of population \mathcal{G}_t . The population that contains both the "children" and the "parents" is denoted by \mathcal{G}'_t . Substitution operation decides who will survive in the $(t+1)$ -th generation of population.

Firstly, it is about selecting the best individuals from the n -th generation of population \mathcal{G}'_t . Therefore, the principle is the same as the selection operation presented in Section 3.3.3. We define the probability ζ associated to an S-RMP model f_s of being submitted to the $(t+1)$ -th generation by:

$$\zeta(f_s) = \frac{RA(f_s) - RA'_{min}}{RA'_{max} - RA'_{min}} \quad (22)$$

where,

$$RA'_{min} = \min \left\{ RA(f_s) \mid \forall f_s \in \mathcal{G}'_t \right\} \quad (23)$$

$$RA'_{max} = \max \left\{ RA(f_s) \mid \forall f_s \in \mathcal{G}'_t \right\} \quad (24)$$

Secondly, if there is not enough number of individuals that are good enough to be submitted, some newly initialized S-RMP models will be appended to the $(t+1)$ -th generation under the constraint of the initial size of the operating population.

After having defined the substitution operation, the evolutionary part of the algorithm for learning the reference points is completed. Now, we discuss how to adjust the criteria weights.

3.4 Learning the criteria weights

Assuming that the reference points are given, we consider the adjustment of criteria weights as a linear optimization problem.

We are interested in setting good values for the criteria weights ω_j , $j \in M$ to restore as many as possible pairwise comparisons provided by the DM. However, to do that we usually need to introduce binary variables into the program as it is described in [22]. In the case of a large dataset, using binary variables will considerably increase the computation time ([23, 12]). Therefore, we propose in this paper an alternative formulation without binary variables that has been proved to be quite efficient (Section 4 for the numerical test results).

We note at first that ω_j , $j \in M$ are the unknown variables to be adjusted. Considering then the *slack* defined by (12) and (13),

we remark that it is actually an indicator that shows if the (relative) preference between two alternatives can be restored by the current S-RMP model. Hopefully, for any pairwise comparison (a, b) provided by the DM, there should be at least one of the slack variables $s_{(a,b)}^{\sigma(h)}$ such that

$$\exists h^* \in P, s_{(a,b)}^{\sigma(h^*)} > 0, s_{(a,b)}^{\sigma(1)} = s_{(a,b)}^{\sigma(2)} = \dots = s_{(a,b)}^{\sigma(h^*-1)} = 0 \quad (25)$$

where σ is a specified lexicographic order of reference points.

However, we cannot translate (25) as part of the linear constraints of the problem, because it is not always true. It means that it is not always possible to find such an S-RMP model with the given reference points and the adjusted criteria weights that restores exactly all the provided pairwise comparisons.

In fact, our objective is, by adjusting the criteria weights, to make the model compatible with as many as possible pairwise comparisons provided by the DM. In this case, (25) should be integrated as part of the objective function but not as linear constraints.

Hence, to account for this, we define then another two auxiliary variables $s_{(a,b)}^{\sigma(h)+}$ and $s_{(a,b)}^{\sigma(h)-}$ for $\forall h \in P, \forall (a, b) \in \mathcal{BC}$ by:

$$s_{(a,b)}^{\sigma(h)} - s_{(a,b)}^{\sigma(h)+} + s_{(a,b)}^{\sigma(h)-} \geq 0 \quad (26)$$

where both of them are positive by definition. Actually, $s_{(a,b)}^{\sigma(h)+}$ represents the positive terms in the definition of $s_{(a,b)}^h$ that contributes to the statement. Respectively, $s_{(a,b)}^{\sigma(h)-}$ represents the negative terms in the definition of $s_{(a,b)}^h$ that weaken the statement.

Moreover, in order to maximize the number of pairwise comparisons correctly restored by the model, the objective function is defined as below:

$$\max \sum_{(a,b) \in \mathcal{BC}} \sum_{h=1}^k \underline{\omega}^{\sigma(h)} \cdot \left(s_{(a,b)}^{\sigma(h)+} - \alpha \cdot s_{(a,b)}^{\sigma(h)-} \right) \quad (27)$$

where we make use of two weighting system α and $\underline{\omega}^{\sigma(h)}$.

On the one hand, $\underline{\omega}^{\sigma(h)}$ for $\forall h \in P$ weights the reference points. We remark that, by carefully choosing their values, it is approximately equivalent to using the reference points in their lexicographic order. Actually, if $k \geq 2$, whatever the value of $\underline{\omega}^{\sigma(k)}$,

$$\underline{\omega}^{\sigma(k-h)} = \frac{1}{\epsilon} \cdot \sum_{i=k-h+1}^k \underline{\omega}^{\sigma(i)} \quad (28)$$

that depends on the small value for ϵ . ϵ represent the accuracy of the criteria weights that we impose in the solution. For example, if $\epsilon = 10^{-3}$ and we take $\underline{\omega}^{\sigma(3)} = 1$ for an S-RMP model with 3 reference points, then we can deduce that $\underline{\omega}^{\sigma(2)} = 10^3$ and $\underline{\omega}^{\sigma(1)} = 10^6 + 10^3$.

On the other hand, α balances the compensatory behavior between pairwise comparisons. We note that, to avoid using binary variables, such an objective function cannot guarantee that the maximal number of pairwise comparisons can be correctly restored. However, by setting a big enough value for α (for example, $\alpha = 10^3$), it is able to reduce the compensatory effects efficiently.

We summarize the linear program as below:

$$\begin{aligned} \max \quad & \sum_{(a,b) \in \mathcal{BC}} \sum_{h=1}^k \underline{\omega}^{\sigma(h)} \cdot \left(s_{(a,b)}^{\sigma(h)+} - \alpha \cdot s_{(a,b)}^{\sigma(h)-} \right) \\ \text{s.t.} \quad & \sum_{j=1}^m \omega_j = 1 \\ & \forall (a, b) \in \mathcal{BC}, \forall h \in P, \\ & s_{(a,b)}^{\sigma(h)} = \sum_{j=1}^m \left(\delta_{a,j}^{\sigma(h)} - \delta_{b,j}^{\sigma(h)} \right) \cdot \omega_j \\ & s_{(a,b)}^{\sigma(h)} - s_{(a,b)}^{\sigma(h)+} + s_{(a,b)}^{\sigma(h)-} \geq 0 \\ & s_{(a,b)}^{\sigma(h)+} \geq 0, s_{(a,b)}^{\sigma(h)-} \geq 0 \end{aligned}$$

4 Numerical analysis

In this section, the proposed algorithm is further investigated numerically based on a large set of artificially generated data. Statistical techniques were used to demonstrate the advantages of the metaheuristic comparing with the conventional disaggregation methods for outranking models based on LP.

We are firstly concerned with the quality of the solution. For a suchlike evolutionary approach, it is represented by the "best" individual in the operating population. Secondly, there is usually a compromise between the optimality of the solution and the number of iterations it takes to reach a satisfactory result as we mentioned in Section 3.1. By simulating the different decision circumstances with a large quantity of randomly generated data, we investigate the improvement curves produced in each circumstance to better understand the behavior of the algorithm. Otherwise, we examine also the runtime characteristics of the proposed metaheuristic, since one of the interest of this work is to overcome the insolvability of the MIP-based disaggregation methods in dealing with large datasets (Section 2.3.2).

The different decision circumstances that we studied are discussed in the next section.

4.1 Experiments

In the experiments, we consider 1000 alternatives and a varying numbers of criteria (4, 6, 8 or 10). We generate randomly an initial S-RMP model to test the algorithm. It is denoted by f_β , as it is defined by a set of parameters β . It simulates the preference model of a fictitious DM. Then, 500 reference pairwise comparisons among the alternatives are derived from the initial model f_β based on their randomly generated evaluations on the criteria. We consider also different numbers of reference points in the initial model f_β (Ini. NRP, for Initial Number of Reference Points) to simulate different complexity levels of the DM's preference system.

The experiments are divided into two groups according to the different preset numbers of reference points in the inferred model (Inf. NRP): In Group A, we infer S-RMP models with only 1 reference point whatever the initial number of reference points in β . While, in Group B, we infer S-RMP models with exactly their initial number of reference points, so as to compare these two groups of experiment and better understand the importance of carefully setting a corresponding number of reference points in S-RMP disaggregation. The different cases that we considered are distinguished by:

- Number of criteria = 4, 6, 8 or 10
- Ini. NRP = 1,2 or 3
- Inf. NRP = 1 or the initial number

The experiments are then numbered by a trio-index as shown in Table 2 and Table 3. For instance, "4.1.A" means that, we consider 4 criteria and the initial model is generated with 1 reference point and it is in the group "A". For each experiment, all the provided results are based on the average of 100 repeated trials under the same testing conditions ².

4.2 Empirical results

4.2.1 Quality of the solution

The inferred model is denoted by $f_{\hat{\beta}^*}$ with some parameters $\hat{\beta}^*$. As shown in Table 2 and Table 3, the "closeness" between the initial model f_{β} and the inferred model $f_{\hat{\beta}^*}$ is measured by the RA. Since we generated the input information without introducing any inconsistency, the measured value of RA is always expected to be as close as possible to 1.00. The "Starting RA" represents the quality of the "best" individual in the initialized population. The "Final RA" represents the quality of the final inferred model $f_{\hat{\beta}^*}$, which is the "best" individual in the population when the algorithm terminated. However, we remind that multiple non-identical individuals may give the same value of RA, as the solution is not unique (as presented in Section 2.3.1).

In the group A of experiments, we generated the initial models with 1, 2 or 3 reference points and inferred S-RMP models involving only 1 reference point (Table 2). We notice that the best final RA, 0.997, is observed in the experiment 4.1.A, while the worst is observed in the experiment 4.3.A. It shows, in general, the capability of the proposed metaheuristic while inferring an S-RMP model as simply as possible.

However, exceptions are observed in the experiment 10.3.A. The final RA is instead even greater than 10.1.A on average. The randomized starting models are also better than in the other two cases (10.1.A and 10.2.A). This will be further discussed (Section 4.3).

Table 2: Group A of experiments

Exp.	Num. Cri.	Ini. NRP	Inf. NRP	Starting RA	Final RA
4.1.A	4	1	1	0.807	0.997
4.2.A	4	2	1	0.746	0.904
4.3.A	4	3	1	0.735	0.880
6.1.A	6	1	1	0.813	0.987
6.2.A	6	2	1	0.761	0.934
6.3.A	6	3	1	0.741	0.912
8.1.A	8	1	1	0.810	0.969
8.2.A	8	2	1	0.760	0.940
8.3.A	8	3	1	0.746	0.925
10.1.A	10	1	1	0.805	0.960
10.2.A	10	2	1	0.761	0.931
10.3.A	10	3	1	0.810	0.969

In the group B of experiments, we assume that we can correctly set the number of reference points in the S-RMP models to be inferred (Table 3). At this time, the worst final RA is observed in the experiment 10.3.B. It is reasonable, since it is the most complicated case that has been considered in our analysis. However, 0.950,

which means 95% of the 500 input pairwise comparisons could be restored correctly in the inferred S-RMP model, is already satisfactory enough for the disaggregation.

Table 3: Group B of experiments

Exp.	Num. Cri.	Ini. NRP	Inf. NRP	Starting RA	Final RA
4.1.B	4	1	1	0.807	0.997
4.2.B	4	2	2	0.810	0.983
4.3.B	4	3	3	0.819	0.973
6.1.B	6	1	1	0.813	0.987
6.2.B	6	2	2	0.798	0.966
6.3.B	6	3	3	0.796	0.968
8.1.B	8	1	1	0.810	0.969
8.2.B	8	2	2	0.780	0.961
8.3.B	8	3	3	0.784	0.961
10.1.B	10	1	1	0.805	0.964
10.2.B	10	2	2	0.777	0.952
10.3.B	10	3	3	0.775	0.950

By comparing with the group A, we observe that the starting RAs as well as the final RAs are significantly enhanced in the group B of experiments by presetting correctly the number of reference points, especially in the experiments where the initial models are generated with more than one reference points (except for 10.3.A and 10.3.B).

To better understand the behavior of the proposed algorithm, the improvement curves are drawn and investigated below.

4.2.2 Behavior of the algorithm

The improvement curves show not only the starting and the final point, but the evolution of RA in function of the number of iterations during the whole process. As shown in Figure 2 (for Group A) and Figure 3 (for Group B), the RA is improved and converges progressively. Without any surprise, the exceptional phenomenon that we pointed out in Section 4.2.1 for 10.3.A is also observed in Figure 2(d). The dotted curve that represents the case involving 3 reference points is instead placed above the curve that represents the case involving 1 reference point (Section 4.3 for further discussions).

For a fixed number of criteria, by comparing each of the subfigures in Group A with the ones in Group B (for example, Figure 2(a) with Figure 3(a)), we can firstly visualize the intensively reduced level of RA due to the improperly set number of reference points. Moreover, we notice that the reduced level is more significant when the model involves fewer criteria.

Otherwise, we observe that, in Figure 3, the dashed line for "2p" and the dotted line for "3p" almost coincide with each other. It means that, comparing with the single reference point cases, the improving RA is reduced quasi-equally in the multiple reference points cases regardless of the number of reference points involved.

Actually, the improvement curves can also be grouped by number of reference points (as shown in Figure 4 and Figure 5) instead of by criteria number. In Figure 4(c), we inferred S-RMP models that involve only 1 reference point from the information derived from the initial models that involve 3 reference points. We notice that, not similar to the other figures, the curves (in Figure 4(c)) that represent the cases where more criteria are involved are above the others that represent the cases where fewer criteria are involved. Related discussions are provided in Section 4.3.

Moreover, the improvement curves can be divided into two stages. The first stage is the rising section, while the second stage is the flat section. The flat section shows the final RA that we can reach,

² Intel Core i3-2120 3.30 GHz CPU, 4 GB RAM, Ubuntu 14.04 LTS, Eclipse IDE Kepler SR2, Cplex 12.6

Figure 2: Group A of experiments (num. of criteria)

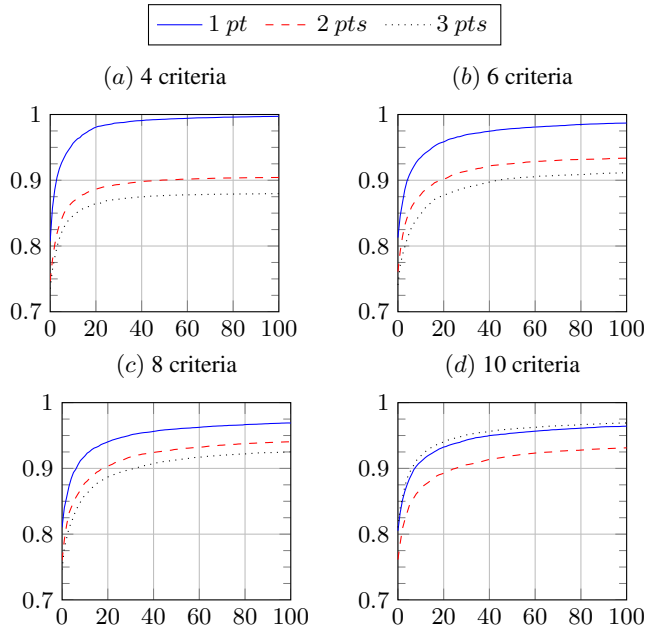


Figure 3: Group B of experiments (num. of criteria)

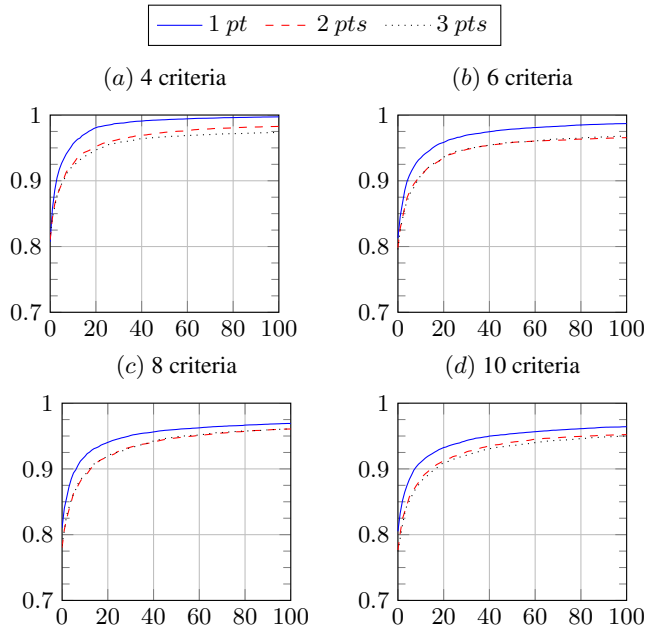


Figure 4: Group A of experiments (num. of ref. pts)

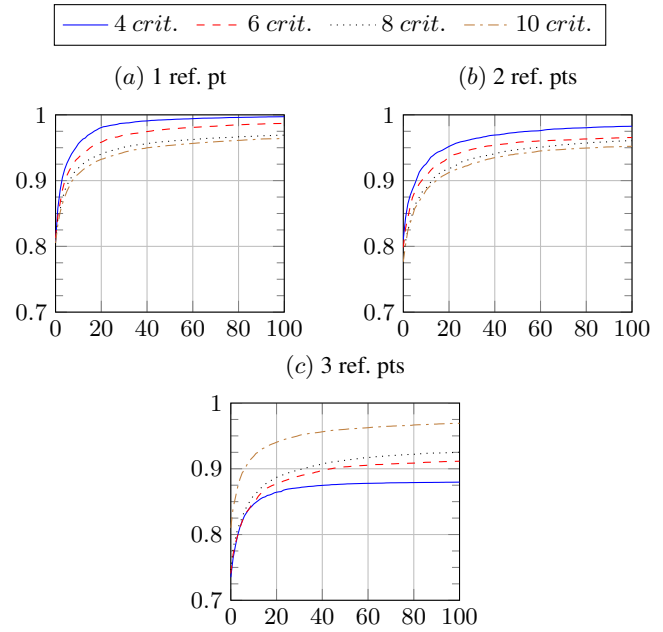
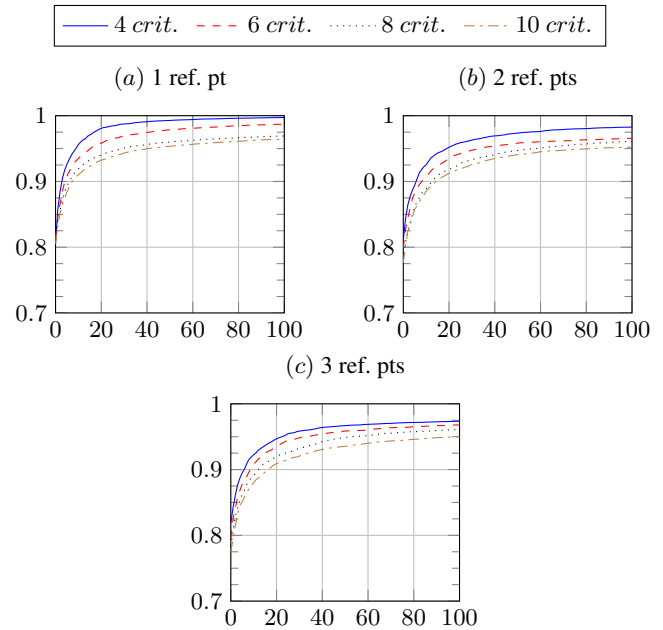


Figure 5: Group B of experiments (num. of ref. pts)



while the rising section also reveals some important advantages of the proposed metaheuristic.

We measure the first stage of the improvement curves by the rapidity of convergence. It is defined as the necessary number of iterations needed to reach a quasi-satisfactory RA. To be precise, we define the quasi-satisfactory RA as the 97% of the final RA. It shows the efficiency of the metaheuristic algorithm. The results are presented in Table 4.

We can observe that the metaheuristic approach is generally quite efficient. Table 4 shows that the inferred models can reach a quasi-satisfactory state within around the first 30 iterations and it depends on the complexity of the preference model of the DM.

4.2.3 Computation time

The average computation time is provided in Table 5. We observed that, even for the case involving 3 reference points and 10 criteria, the computation time remains within around 3 minutes. These results should be further compared to the computation times of algorithms using MIP formulations.

Besides, not only should the runtime value be measured, but also the evolution of computation time as a function of the criteria number or the number of the reference points should be investigated. It is interesting while comparing with other MIP-based algorithms, as it demonstrates the significant runtime advantage of the metaheuristic algorithm.

Table 4: Rapidity of convergence (group B, in number of iterations)

Num. Cri.	1 ref. pt	2 ref. pts	3 ref. pts
4	14	21	19
6	20	21	22
8	20	29	31
10	23	29	32

Table 5: Computation time (group B, in seconds)

Num. Cri.	1 ref. pt	2 ref. pts	3 ref. pts
4	44.55	69.58	77.98
6	85.78	101.48	110.20
8	119.67	136.46	146.35
10	148.28	169.75	182.06

By drawing the runtime curves, as shown in Figure 6(a), we notice that the computation time is proportional with the criteria number. We remind that it increases exponentially in the MIP-based S-RMP disaggregation methods (Section 2.3.2). It endues the metaheuristic approach with the capacity of dealing with the decision instances that involve a large quantity of evaluating criteria.

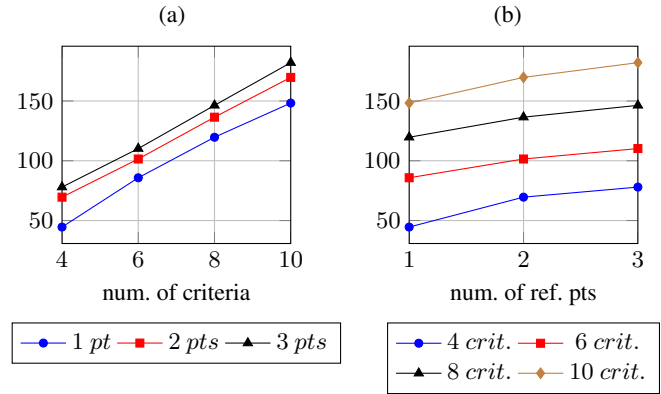
Besides, we also notice that we consume more computation time when dealing with more reference points, but not as much as for the LP-based methods. To be clear, we draw once again the runtime curves in function of the number of reference points (as shown in Figure 6(b)). At this point of view, the obtained curves are all sub-linear regardless of the criteria number while in the LP-based S-RMP disaggregation methods, they are usually exponential vs. the number of reference points.

4.3 Discussion

By checking the result of the numerical tests, we notice that the preset number of reference points should be carefully adjusted to derive a better solution, since the final RA is intensively reduced (except for 10.3.A) in Group A (i.e. learning S-RMP models with single reference point) of experiments when the initial number of reference points increases. Nevertheless, starting from an S-RMP model with single reference point shows greater interpretability to the problem. It actually expresses a very natural rule based on the distinction of two classes of evaluation on each criterion. Besides, it helps us to estimate the complexity of S-RMP models and to fix the number of reference points. It means that we should consider increasing the number of reference points when the final RA is not as satisfactory as we expected.

Moreover, we also notice that the number of criteria should be limited. On one hand, we observe that, when the number of reference points is fixed, the final RA is descending progressively when the criteria number increases. On the other hand, the exceptional phenomenon that we observed (10.3.A) shows that, when there are more criteria, it is easier to derive an S-RMP model that involves only one reference point and gives a better RA. In other words, it is easier to adjust one reference point by calibrating its valuation and the weights of criteria than to calibrating multiple reference points at the same time.

The study about the rapidity of convergence of the algorithm could help us to adjust when to interrupt the algorithm when dealing with real datasets, and to economize the total computation time in different application circumstances. For instance, in the online web ap-

Figure 6: Computation time (in seconds)

plications, the response time should usually be much more valuable than in the offline recommendations. To obtain the most accurate result, it is possible to run the program during a very long time, even day and night, in the offline cases. However, an acceptable solution should be worked out as quickly as possible in the online cases and then be adjusted step by step by a follow-up interactive process.

5 Conclusion

This paper presents an efficient metaheuristic approach to infer S-RMP models from a large set of pairwise comparisons provided by the DM. The proposed algorithm was tested with a large quantity of artificially generated data that simulates a variety of different decision circumstances. Firstly, the metaheuristic is able to deal with instances involving as many as 500 pairwise comparisons. Suchlike instances cannot be solved using MIP formulations. Secondly, the computation time is proportional with the number of criteria involved and sub-linearly increases with the number of reference points. Finally, even if the metaheuristic is not able to learn an S-RMP model which to restore all pairwise comparisons, it infers S-RMP models which restore up to at least 95% of the input information within a reasonable computation time. We remark that it constitutes a good trade-off of quality of result vs. computation time.

One of the interesting questions emerged from this work is the adjustment of the number of reference points. Moreover, the performance of the proposed metaheuristic should also be explored when we are in presence of inconsistencies, for example, in the case of group decision problems or in the case of real world applications.

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