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## - To cite this version:

Patrick Pujo, Ilham El Khabous, Fouzia Ounnar. Experimental assessment of the productivity improvement when using U-shaped production cells with variable takt time. International Journal of Lean Six Sigma, 2015, 6 (1), 10.1108/IJLSS-07-2013-0038 . hal-01201817

HAL Id: hal-01201817
https://hal.science/hal-01201817
Submitted on 1 Oct 2015

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## Experimental assessment of the productivity improvement when using

 U-shaped production cells with variable takt timePatrick Pujo, Ilham El Khabous, Fouzia Ounnar

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## Experimental assessment of the productivity improvement when using U-shaped production cells with variable takt time

Purpose - The aims of this research is to discuss the benefits of $U$ layout for production cell operating in variable takt time. Different experiments were conducted using benchmarks, in order to highlight the performance gap between a straight cell and a U-cell.

Design/methodology/approach - The implementation of the production cell, in U-shaped or in straight line, is optimized through linear programming based on the number of operators. The two corresponding programs, in Mosel language, use the same approach, in order to not introduce defects in the comparison of results. The study used our own datasets and well known academic benchmarks.

Findings - A comparison between the obtained takt times, with equivalent operating conditions, in U-cell and straight cell was conducted. A significant increase of the production rate was observed. This increase has often exceeded 10 per cent to reach 32 per cent. All the experiments show that, with the same number of operators, a cell in a $U$ layout is always at least as efficient, in terms of reachable production rates, than an equivalent cell in a linear layout. 96 per cent give an improvement of production rate. Moreover, the dispersion of the U-cell results is weaker, which suggests that the U-layout gives, in more robust manner, better performances.
Research limitations/implications - The results were obtained through a study of various academic benchmarks. The results must be validated on industrial situations.

Practical implications - This paper will be very useful for researchers and practitioners in order to understand lean implementations and its derived benefits. This paper will allow them evaluating and analysing the expected benefits of the implementation of the production cell in U shaped (operating in variable takt time).

Originality/value - U-Cells constitute an appropriate solution for a layout of any kind of production cells with variable structure. When facing a significant variation in the demand, the response consists in modulating the number of operators assigned to the cell. This study addresses jointly the problem of U-cells layout and the operation in variable takt time.

Keywords: U-shaped production cell, straight production line, production variable throughput, linear programming for optimization, assembly line balancing.

## 1. Introduction

Recent developments towards lean management (Holweg, 2007) reflect an increasing trend by companies to use this approach. This is often found in the literature in the form of testimonials: the effectiveness of lean management methods and tools are typically expressed through case studies (Detty and Yingling, 2000), (Motwani, 2003), (Modarress et al., 2005), (Kumar et al., 2006). For example, tools such as value stream mapping (VSM) have been the subject of evaluation 'before \& after' comparison, used alone (Singh et al., 2010) or in combination with other techniques in the case of production lines
(Alvarez et al., 2009). Based on value stream mapping and milkrun, using lean metrics, such as dock-to-dock time and lean rate, these works analyse the internal materials flow and its improvement.

Other tools, such as U-shaped cells producing or takt time variable, have not been subjected to comparison 'before \& after'.

U-shaped production cells (figure1(b)) emerged in the Western industrialized world with the introduction of the Just in Time philosophy (Monden, 1983) and are currently associated with the Lean Production approach (Shah and Ward, 2003).

The majority of the production lines not identified as falling within one of these two approaches are organized linearly (figure 1(a)). The optimization of their performance is obtained using load balancing techniques which are dealt with in numerous studies (Becker and Scholl, 2006).

Please insert here Figure 1. Straight production cell (a), U-shaped production cell (b)

U-Cells offer many advantages over traditional linear arrangements, in terms of flexibility, standardization, decrease in work in progress and the maintaining of high quality products. They also provide improvements in productivity.

In the case of high variations in production volumes, it is easy to adjust the number of operators to adapt the production rate to the instantaneous demand (takt time). Through variable takt time, the production lines enable new strategies, so that lean product realisation can become a reality for variable productions (Weston and Cui, 2008). This is common in U-Cells, but can also be achieved in straight production lines. It becomes then interesting to compare both layout approaches and evaluate their respective performances through experiments using benchmarks.

After defining the basic principles of the operation of a variable rate production line, a few fundamentals regarding U-Cells are recalled. Then, the issue of the layout optimization for such systems is addressed and linear programming models are developed aiming to obtain the best possible balancing of the operator tasks.

These models are then implemented into a solver and tested on academic benchmarks which are set to be the most impartial and the least specific possible.

Finally, the results of these experiments are analysed and discussed.

## 2. Choice of a layout architecture for a production cell

### 2.1. Cellular production: some reminders

Production lines were deployed at the beginning of the 20th century, after Henry Ford installed the first assembly line in 1913 (Sward, 1989). After being used for a century in mass manufacturing, this concept is still evolving today (Hirst and Zeitlin, 1991), being revisited (Bartholdi and Eisenstein, 1996). This history, combined with a pragmatic quest for efficiency (Magee, 2007), led the Japanese industry to develop the idea of production cells: a production cell is a small production unit which specializes in the manufacturing of a family of products or parts.

This type of organization should offer greater usage flexibility, better performances in terms of quality, cost and time, as well as a fluidization of flows combined with reduction in the volume of work in progress and greater responsiveness. In fact, a cell is a small production line whose conditions of use can vary according to needs: change of rate, of products, of the number of machines, etc. The line architecture is related either to the grouping together of all the processes related to the fabrication of a family of products, or to the grouping resulting from a frequent series of operations common to the manufacturing of different types of products.

When this approach is generalized to the whole production system, we can refer to it as cellular manufacturing (Irani, 1999): the variety of products to be manufactured, their estimated volumes and the fluctuations in demand will determine the set of cells which will need to be formed and designed. Numerous studies about cellular production focus on the characterization of this system using different approaches such as group technology (Kulak et al., 2005), integer linear programming (Arifar and Ismail, 2009), metaheuristics of genetic algorithm type (Yin and Yasuda, 2006) or ant colony optimization algorithms (Sabuncuoglu et al,. 2009).

Another emerging problem is related to the dynamics of the control of this set of cells (Wenming and Deuse, 2009). For instance, if we consider a workshop where many neighbouring cells respond to varying and desynchronized customer loads, it is possible to adjust the number of operators per cell and transfer operators within cells (communicating vessels) so that the workshop is fully controlled and maximum productivity is achieved, preserving responsiveness and dynamics whatever the variation of customer demands and providing a high potential increase in production rate. For example, a cell functioning with $n$ operators (nominal operation) can be adjusted to $n-1$ or $n+1$, but it is possible to make it function even faster, with up to $n+k$ operators, the additional operators being taken from neighbouring cells or even make it function very slowly (with only one operator) or stop it temporarily.

In any case, the search for the optimal cell productivity requires the balancing of the production loads: the key is to set up the cell so it can produce at the lowest cost possible while complying with the technical constraints.

### 2.2. Balancing of a production cell

This issue was first raised in automotive assembly lines, and is now referred to in the scientific literature as SALBP (Simple Assembly Line Balancing Problem) (Baybars,
1986). A typology of the resolution methods for this type of problems was suggested by (Boysen et al., 2007). Generally speaking, these methods consist in splitting the task as effectively as possible between the operators so that the workload is equally distributed and the production rate is optimal.

The general features of this problem are the following:

- a single operation cannot be shared between different operators;
- the operations are partially ordered;
- all operations must be executed;
- durations of operations must not depend either on the operator or on previous operations;
- an operator can carry out any operation;
- any operation can be executed by any operator;
- the products pass one by one in front of operators in a given order;
- the operators themselves are positioned in a given order;
- one single product is produced per line.

Operations have to be split between operators so that waiting times are minimal. The waiting time of each operator is determined by the difference between the cycle time of the line (the load of the busiest operator) and the actual load (the sum of all the times of operation assumed by the operator). The solution in which the operator waiting time is the shortest provides the lowest cost.

Two cases are possible. First, if the operators are assigned to more or less versatile workstations (for example, in the case of an assembly workshop), the balancing will consist in assigning them a maximum number of operations. Then, if the operators have
to move between different workstations, the balancing will consist in assigning them various consecutive workstations, with a displacement cycle between these workstations.

Even though the mathematical formulation of these two cases is equivalent, our interest will be mainly focused on the second case. The displacement of operators between different workstations is called Chaku-Chaku (Wenming and Deuse, 2009).

Let us note that the so called "caravan system" (Spengler et al., 2005), which consists in moving k operators (out of phase by $2 \pi / \mathrm{k}$ ) along the entire cell (figure 2(a)), with an identical Chaku-Chaku, is not taken into account. We will focus on the balancing of the line between different Chaku-Chaku (one operator corresponding to one ChakuChaku), involving a manual transfer of parts between neighbouring Chaku-Chaku operators (figure 2(b)).

Please insert here Figure 2. Chaku-Chaku in 'caravan' system (a), Chaku-Chaku with manual transfer (b)

It is also important to detail the nature of tasks $T_{i}$ which refer to the operations carried out by operators $O_{k}$ on the different workstations $P_{i}$. We should distinguish between durations $D o_{i}$ of tasks $T_{i}$ and displacement times between workstations $D d_{x i}$. The latter are usually small compared to the cumulative durations of tasks performed by each workstation and they can be either neglected or integrated to the corresponding durations: $D o_{i}$ includes $D d_{x i}$. Besides the displacement time, the duration includes product loading/unloading tasks on the workstation, possible product control tasks, and either the execution of this task - in the case of a purely manual task - or the initiation of the execution of the task followed by a short period of monitoring to visually check whether the task has started correctly - in the case of an automated workstation.

This refers to the apparent time component $D_{i}$ of the operation on workstation $P_{i}$. For automated tasks, the continuation and completion of the task progress in masked time, on a period called $D m_{i}$. Note that $D o_{i}=D_{i}+D m_{i}$. To each workstation $P_{i}$ corresponds a chronogram of all the tasks of the operation (Monden, 1983), called 'Standard Operation Chart' and all the chronograms show the cyclic activity of operator $O_{k}$ in a Gantt diagram, called 'Standard Operation Combination Chart',

If $T_{k}$ is the cycle time of operator $k$, the following relationships must be fulfilled:

- For a given operator $k$, to which $n_{k}$ operations $i$ have been assigned:

$$
\begin{equation*}
\sum_{j=1}^{n_{k}} D_{i_{j}}<T_{k} \tag{1}
\end{equation*}
$$

- For each given operation $i$ :

$$
\begin{equation*}
D_{i}+D m_{i}<T_{k} \tag{2}
\end{equation*}
$$

- If $T$ is the cycle time of the cell and $n_{p}$ the number of operators, then:

$$
\begin{equation*}
\forall k \in\left[1, n_{p}\right], \max \left(T_{k}\right)=T \tag{3}
\end{equation*}
$$

These two overall variables strongly influence the resolution method, depending on whether they are fixed or to be minimized. SALBP can be expressed in 4 ways (Baybars, 1986), (Becker and Scholl, 2006):

Please insert here Figure 3. Classification of line balancing problems

SALBP-F is the feasibility study of a configuration, and not a search for optimization. SALBP-1 studies the minimization of the number of operators while respecting a given production rate. SALBP-2 aims at minimizing the cycle time so as to
achieve a maximum production rate through a fixed number of operators. SALBP-E combines those two options and maximizes the efficiency of the production line

These SALBP problems deal with the balancing of straight cells. The layout of U-shaped production cells is the subject of specific studies collectively known as SULBP (Simple U-Line Balancing Problem) (Aase et al., 2004) or UALBP (U-Assembly Line Balancing Problem) (Scholl and Klein, 1999b).

### 2.3. Balancing of a U-Cell

For U-shaped cells, the classification of the resolution methods is similar (Kriengkorakot and Pianthong, 2007). (Boysen et al., 2007) identifies a number of jobs, denoted by [|u| ].

Regarding SULBP-1, (Miltenburg, 1998) proposed first a dynamic programming approach, then an integer linear programming approach (Miltenburg, 2001b). More recently, (Sabuncuoglu et al., 2009) suggested a resolution using an ant colony metaheuristic.

Other studies relate to SULBP-E: for example, (Chiang and Urban, 2006) used a heuristic procedure; (Kara et al., 2009) applied a multi-objective fuzzy integer programming method.

Finally, a number of studies address the Mixed Model U-Line Balancing and Scheduling Problem (MMULBS) regarding the production of a variety of products in a U-Cell, simultaneously considering balancing and scheduling (Miltenburg, 2002), (Kim et al., 2006), (Kazemi et al., 2011).

### 2.4. Performance comparison between the balancing of a straight cell and a $U$ shaped cell

Numerous studies were undertaken simultaneously on both approaches (Ağpak and Gökçen, 2007, Kara et al., 2009), but focusing on the way the production cells were
organized and thus on the cell architecture generation algorithm, without comparing their performances with equivalent datasets.

As our study is performed from a perspective of variable takt time, we can compare both types of layout by changing the number of operators and comparing the respective production rates obtained. For that, we should move towards SALBP-2 and SULBP-2 approaches.

## 3. Modelling of the SxLBP-2 balancing problem for a production cell

In this section, the implemented linear programming models are described. The two models have to be homogeneous in terms of structure and design in order to avoid any methodological biases. As there is comparison, we chose to consider one of the layouts as a reference point, in terms of modelling, implementation and results analysis. Based upon the literature review, we chose to start from an existing SALBP-2 approach and extend it to a U-Cell.

### 3.1. Modelling of the SALBP-2 problem

The SALBP-2 model described here results from (Guéret et al., 2002). This model is relatively simple, gives excellent results and its implementation is already using Xpress solver. The following notations are considered:

- $n_{t}$ : number of operations,
- $D_{i}$ : duration of operation $i\left(i \in\left[1 \ldots n_{t}\right]\right)$,
- $n_{p}$ : number of operators,
- $n_{a}$ : number of arcs (sequence of operations). An arc $(u, v)$ links operation $u$ to operation $v$ if $u$ is the immediate predecessor of $v$.

In terms of modelling, $x_{i k}$ binary variables will be used to place the operations:

- $x_{i k}=1$ if and only if operation i , is assigned to operator k (see (4) \& (8)).

For all valid assignments, in accordance with (4) and (8), it is also necessary to respect precedence constraints: for any arc $(u, v)$, the rank of operation $u$ must be lower than or equal to that of operation $v$ (see (5)).

Finally, a real variable $T \geq 0$ is used (see (9)) for the cycle time of the cell, which according to (6) is equal to the cycle time of the busiest operator. The objective function consists in minimizing $T$ (see (7)).

The formulation of the SALBP-2 problem is as follow:

$$
\begin{gather*}
\forall i=1 \ldots n_{t}: \sum_{k=1}^{n_{p}} x_{i k}=1  \tag{4}\\
\forall(u, v) \in\left[1 \ldots n_{t}\right]: \sum_{k=1}^{n_{p}} k . x_{u k} \leq \sum_{k=1}^{n_{p}} k \cdot x_{v k}  \tag{5}\\
\forall k=1 \ldots n_{p}: \sum_{i=1}^{n_{t}} D_{i} \cdot x_{i k} \leq \mathrm{T}  \tag{6}\\
\operatorname{Min} z=T  \tag{7}\\
\forall i=1 \ldots n_{t}, \forall k=1 \ldots n_{p}: x_{i k} \in\{0,1\}  \tag{8}\\
T \geq 0 \tag{9}
\end{gather*}
$$

Under these conditions, the first operator handles the cell inputs while operator $n_{p}$ manages the outputs.

### 3.2. Extending the modelling of SALBP-2 to SULBP-2

The U shaped-cell layout involves treating simultaneously incoming and outgoing branches of the U-cell. Let $j$ be the branch index, with $j=1$ for the incoming branch and $j=2$ for the outgoing branch.

Modelling the SULBP-2 problem thus requires the binary variables to adjust to this notion of treating simultaneously the two branches to position the operations.

- $\quad x_{i j k}=1$ if and only if operation $i$ is assigned to operator $k$ on branch $j$ (see (10) \& (15)).

For all valid assignment, in accordance with (10) \& (15), it is also necessary to respect precedence constraints: for any arc $(u, v)$, the rank of operation $u$ must be lower than or equal to that of operation $v$. However, the calculation of this rank can differ according to the branch. In branch 1, we are back to the previous case (see (11)). In branch 2 , the precedence must be reversed (see (12)).

The principle of minimization of the busiest operator cycle time remains unchanged (see (13)), and takes into consideration the two branches, (14) \& (16)).

The formulation of the SULBP-2 problem is as follows:

$$
\begin{gather*}
\forall i=1 \ldots n_{t}: \sum_{j=1}^{2} \sum_{k=1}^{n_{p}} x_{i k j}=1  \tag{10}\\
\forall(u, v) \in\left[1 \ldots n_{t}\right]: \sum_{k=1}^{n_{p}} k \cdot x_{u k 1} \leq \sum_{k=1}^{n_{p}} k \cdot x_{v k 1}  \tag{11}\\
\forall(u, v) \in\left[1 \ldots n_{t}\right]: \sum_{k=1}^{n_{p}} k \cdot x_{v k 2} \leq \sum_{k=1}^{n_{p}} k \cdot x_{u k 2}  \tag{12}\\
\forall k=1 \ldots n_{p}: \sum_{j=1}^{2} \sum_{i=1}^{n_{t}} D_{i} \cdot x_{i k j} \leq \mathrm{T}  \tag{13}\\
\operatorname{Min} z=T  \tag{14}\\
\forall i=1 \ldots n_{t}, \forall k=1 \ldots n_{p}, \forall j=1,2: x_{i k j} \in\{0,1\}  \tag{15}\\
T \geq 0 \tag{16}
\end{gather*}
$$

In this formulation of the SULBP-2 problem, operator $n_{p}$ manages the inputs and outputs of the cell. Operator 1 is situated in the heart of the cell.

### 3.3. Implementation of the models

The implementation of these models is performed using Mosel language and Xpress solver, available under FICO optimization environment (www.fico.com).

Thereafter, only the programming of the SULBP-2 problem (figure 4) will be described, that of the SALBP-2 problem having already been described in (Guéret et al., 2002).

The binary assignment variable $x$ is represented by a 3D table $\left(n_{t}, n_{p}, 2\right)$, the third dimension being used to separately treat the two branches of the U .

There are $n_{t}$ constraints 'OneOperationPerOperator $(i)$ ' which express the fact that an operation i cannot be shared between several operators.

There are $n_{p}$ constraints 'Cycle (m)' which express the fact that the sum of operations assigned to an operator $m$ is lower than or equal to the cycle time of the U Cell.

Finally, the constraints 'Prec_branch_input(a)' and 'Prec_branch_output(a)' allow the precedence constraints between operations to be taken into consideration for each sequence of operations and whatever the assignment of an operation on one or the other of the U-Cell branches.

The number of operators is defined by the declared variation range of the variable 'OPERATORS'.

Please insert here Figure 4. SULBP-2 problem programmed in Mosel language

## 4. Experimentations with simple linear precedence graph

### 4.1. Establishment of the dataset

The dataset presented in Table 1 is an academic benchmark which was precisely elaborated for the specific purpose of controlling the variability of a number of parameters, in order to establish a performance comparison between U-shaped and straight cells.

Therefore, all the 34 tested cases were built by altering only the duration of the tasks, according to a certain number of invariants: number of operations (17 workstations), total task duration (303 Time Units), duration of the shortest task (5 TU), duration of the longest task ( 60 TU ) and purely linear flow: the output flow of operation $u$ goes directly into operation $u+1$, which induces a linear precedence graph. The above mentioned models would naturally support other values for these invariants: this is to test different configurations of task durations with a variable number of operators, on linear or U type layouts.

Please insert here Table 1. Description of the dataset: the 34 studied cases.

### 4.2. Experimentation results

Thereafter, only the result of case $\mathrm{n}^{\circ} 1$ (figure 5) will be detailed. For x operators, the distribution of the 17 operations is given, for SALBP-2 (Lx) and SULBP-2 (Ux), as well as the cycle durations for each operator and the takt time of the associated cell. Each line also indicates the Chaku-Chaku of the operator concerned.

Please insert here Figure 5. Case $n^{\circ} 1$ results

The 34 studied cases are summarized in Table 2. For each Lx \& Ux configuration, the cycle durations and the improvement percentage are given.

We observe that for one single operator, the results are the same in a straight or U-shaped cell. This is a trivial result. This experiment is thus not significant. For 2 or 3 operators, the results are equivalent in rare cases: the advantage generally is in favour of U-Cells.

Please insert here Table 2. Summary of the experimentation results (in bold: improvements greater than $10 \%$ )

Similarly, for a higher number of operators, optimization is no longer efficient. This can be easily explained since the cycle duration is bounded by the bottleneck due to the longest task assigned to the cell: Lx and Ux results converge, either for 7 operators, or starting from 6 operators. These cases are not significant. Only case 32 achieves a slight performance advantage with 7 operators, advantage which is lost with 8 operators. Between these two extreme cases (123 significant experiments), the results are always in favour of a U-shaped layout, very often with improvements in cycle time exceeding $10 \%$. Based on these significant experiments, 117 provide an improvement of the performance (95\%). This result is substantial.

### 4.3. Analysis of experimentation results

Table 3 shows a summary of the previous results. An average $8 \%$ improvement of the maximum possible production rate is obtained, with peaks up to $25 \%$. Figure 6 displays the scattering of the results.

Please insert here Table 3. Average and standard deviation of cycle durations of the various configurations, for $\operatorname{Lx} \& \mathrm{Ux}(x \in[2,6])$

Please insert here Figure 6. Distribution of the results, for $\operatorname{Lx} \& \operatorname{Ux}(x \in[2,6])$

We note that the scattering of the results is much greater for a linear layout than for a U shaped cell, which demonstrates the robustness of the U-shaped layout approach in the case of a linear precedence graph.

## 5. Experimentations with a complex precedence graph

In many manufacturing systems, flows are not always fully ordered: tasks are partially organized by precedence relations. For instance, if there are jumps between operations, or if operations can be performed in parallel or independently, these anteriority constraints between tasks can be modelled by a precedence graph, in which tasks are the nodes and precedence constraints the arcs. In the previous models, the $A N T$ table (Figure 4) makes it possible to present each precedence relationship as an elementary flow between two operations.

The whole is then a network of precedence constraints.
We will examine the constraints induced by these considerations when operating with a variable takt time.

### 5.1. Problem of implementation with a complex precedence graph

The optimization models presented in the $3^{\text {rd }}$ section for the resolution of SALBP-2 and SULBP-2 provide excellent solutions regardless of the precedence graph. Nevertheless,
they cause a major problem in the case of an operation with takt time varying with regular frequency: for instance if we wish to adapt on a daily basis the operation to the customer's demand. Indeed, when the number of operators is changed the order in which the tasks are arranged on the production line can be changed. However, it is unconceivable to move resources and workstations at each takt time variation. This of course does not occur when the precedence graph is strictly linear, as seen in paragraph 4.

To overcome this difficulty, we must linearize the complex precedence graph in order to identify the most appropriate linear precedence graph that would be able to support the different chaku-chaku configurations. It is thus necessary to identify the linear precedence graph without any back-tracking, regardless of the chaku-chaku configuration applied. So as to realize this linearization while respecting this goal, we propose a twostep process. First of all, a breakdown into chronological levels uses the 'Activity on node' approach applied in project management. Then, the total moment of From-to chart is minimized at each level.

### 5.2. Methodology for linearizing a complex precedence graph

The complex 'Activity on node' diagram is first broken down into levels (Elmaghraby, 1977). The principle is the one used in 'precedence diagramming method' approaches (Wiest, 1981), (Tavares, 1990). The first level contains activities with no antecedents. These activities are removed from the activity list, and we start the same process again with the remaining activities to get to the next level (Crandall, 1973). This allows us to structure the precedence graph with a global order.

Then, an order has to be found within activities on the same level. To do so, the notion of total moment $Z$ of From-to chart is applied (Esgin et al., 2010):

$$
\begin{equation*}
\operatorname{Min} Z=\sum_{i=1}^{N} \sum_{j=1}^{N} f_{i, j}|j-i| \tag{17}
\end{equation*}
$$

where $f_{i, j}=1$ if a precedence relation exists between the 2 activities, and $f_{i, j}=0$ otherwise.
From-to chart is the relationship flow matrix. It gives the dominance of the flows between resources and indicates the closeness of resources in a facility layout design (Mahdavi et al., 2008). From-to chart could be classified as loose and tight (Moore, 1980), depending on whether it contains respectively a few and a large number of strong precedence constraints. In our case, the From-to chart contains many entries with zero values: this corresponds to the case of flow line layout (Hassan, 1994).

To minimize this moment, we proceed to activity permutations at the same level. This allows us to find a local order which remains compatible with the previously established global order.

The linear sequence which results from this treatment can be used as a reference sequence to apply SALBP-2 and SULPB-2. Other sophisticated mathematical techniques may also be applied, but what interests us here is the possibility to compare SALBP-2 and SULBP-2 on equal and logical datasets. Thanks to this approach, we can consider that all cases with complex precedence graphs can be transformed in cases with simple linear precedence graphs. We can then use the approach mentioned in sections $3 \& 4$.

## 6. Experimentations with well-known benchmarks

More than 250 benchmark instances are presented in (Scholl and Klein, 1999a). A complete description of the datasets can be downloaded from the web at: http://alb.mansci.de/files/uploads/UALBP-1 data sets.zip

It seemed interesting to us to test some of these instances to verify our results. We considered 12 instances containing complex precedence graphs: 4 small-sized and 8 large-sized datasets.

### 6.1. Small-sized benchmarks

The first 4 instances can be considered small since the number of tasks as well as the number of required operators is around 10. A description of these instances is given in Table 4.

Please insert here Table 4. Description of small-sized instances

Table 4 gives an idea on the distribution of task durations and indicates the linear sequence equivalent to the initial complex precedence graph.

The results are shown in Table 5.

Please insert here Table 5. Results for small-sized instances

The performance is improved in 14 significant experiments among 16 (87.5\%).

### 6.2. Large-sized benchmarks

The same principle was applied to large-sized datasets. Table 6 describes the datasets.

Please insert here Table 6. Description of large-sized datasets.

The results are given in Table 7

Please insert here Table 7. Results for large-sized datasets.

90 significant experiments among 91 allow improving the performance ( $99 \%$ ).

### 6.3. Analysis of the results

This series of tests confirms the previous observations while leading to new observations.
For small-sized instances, the results confirm the performances achieved, similar to those presented in section 4 . We will observe $32 \%$ performance improvement for the Jackson instance with 3 operators.

For large-sized instances, improvements are small for low rate productions (number of operators equal to two or three). On the other hand, for higher production rates (lower takt time), improvements are substantial, easily exceeding $10 \%$. We also note that the maximum production rate is more quickly reached with a $U$ layout, since the maximum rate limit, defined by the longest task duration, is reached with fewer operators: out of the 8 cases tested, we note that for ['Heskia'], the highest rate is reached for a U layout with 11 operators while 12 would be needed in a straight line, for ['Gunther', Lutzl' \& 'Mitchell'], the maximum rate is reached with 2 fewer operators, and for 3 others ['Buxey', 'Kilbrid' \& 'Sauvey30'], with 3 fewer operators, i.e. almost $1 / 5$ of the payroll. This is a meaningful result.

## 7. Conclusion

All the experiments presented here show unequivocally that, with the same number of operators, a cell in a U layout is always at least as efficient, in terms of reachable production rates, than an equivalent cell in a linear layout. In summary, in order to produce more with similar loads, U-Cell layouts are needed.

This was shown through our experiments and also with academic benchmarks. Based on all tested cases ( 230 significant experiments), $96 \%$ of cases give an
improvement of performance. This remains of course to be validated on industrial situations.

The productivity gains observed are substantial. Furthermore, the performance robustness obtained makes it possible to validate this approach in the case of a manufacturing production subject to wide variations. Indeed, regardless of the rate requested by the downstream production, the $a d$ hoc configuration of the U layout will be the most suitable for obtaining an optimal performance. This shows the great interest of the U-shaped cell layout approach.

We conclude that $U$ cells constitute a solution for implementing any relevant and effective production system with variable structure. It is an essential tool for a lean manufacturing approach.

Acknowledgements.
The authors thank FICO, which, through its Academic Partner Program, has kindly put the FICO ${ }^{\text {TM }}$ Xpress Optimization Suite at our disposal.

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## List of figures



Figure 1. Straight production cell (a), U-shaped production cell (b)


Figure 2. Chaku-Chaku in 'caravan' system (a), Chaku-Chaku with manual transfer (b).

|  | Cycle time (rate) |  |
| :---: | :---: | :---: |
|  | Fixed | To minimize |
| Fixed | SALBP-F | SALBP-2 |
| To minimize | SALBP-1 | SALBP-E |

Figure 3. Classification of line balancing problems

```
! SULBP-2
model "U_cell balancing"
uses "mmxprs"
declarations
    OPERATORS=1..5 ! All operators : here, np=5
    OPERATIONS=1.17 ! All operations
    POS=1..2 ! POS=1 : input cell branch,
    ! POS=2 : output cell branch
    DUR: array(OPERATIONS) of integer ! Task duration of operation
    ANT: array(RA:range, 1..2) of integer ! Precedence relations between operations
    x: array(OPERATIONS,OPERATORS,POS) of mpvar ! 1 if the operator performs the operation
    ! O else
    ! Duration of the production cycle
    cycle: mpvar
end-declarations
initializations from 'U_cell balancing datal.dat'
    DUR ANT
end-initializations
! an operation performed by an operator
forall(i in OPERATIONS) AnOperationByOperator(i):= sum(m in OPERATORS, j in POS) x(i,m,j) = 1
! Sequence of operations
forall(a in RA)
    do
        Prec_branch_input(a):= sum(m in OPERATORS) m*x(ANT (a,1),m,1) <=
                                    sum(m in OPERATORS) m*x(ANT (a,2),m,1)
        Prec_branch_output(a):= sum(m in OPERATORS) m*x(ANT (a,2),m,2) <=
                                    sum(m in OPERATORS) m*x(ANT (a,1),m,2)
    end-do
! Cycle time
forall(m in OPERATORS) Cycle(m):= sum(j in POS) sum(i in OPERATIONS) DUR(i)*x(i,m,j) <= cycle
forall(i in OPERATIONS, m in OPERATORS, j in POS) x(i,m,j) is_binary
! Minimize the production cycle time
minimize(cycle)
```

Figure 4. SULBP-2 problem programmed in Mosel language

|  | P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 | P9 | P10 | P11 | P12 | P13 | P14 | P15 | P16 | P17 | $\Sigma$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| case $\mathrm{n}^{\circ} 1$ | 5 | 23 | 12 | 10 | 15 | 5 | 30 | 8 | 12 | 10 | 20 | 32 | 9 | 24 | 13 | 60 | 15 | 303 |  |  |  |
| 1 operator |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Straight production cell | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 303 | 1 operator | L1 | 303 |
| U-shaped production cell | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 303 | 1 operator | U1 | 303 |
| 2 operators |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Straight production cell | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 150 | $1^{\text {th }}$ operator | L2 | 153 |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 153 | $2^{\text {nd }}$ operator |  |  |
| U-shaped production cell | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 151 | $1^{\text {th }}$ operator | U2 | 152 |
|  | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 152 | $2^{\text {nd }}$ operator |  |  |
| 3 operators |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Straight production cell | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 108 | $1^{\text {th }}$ operator | L3 | 108 |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 107 | $2^{\text {nd }}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 88 | $3^{\circ}$ operator |  |  |
| U-shaped production cell | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 98 | $1^{\text {th }}$ operator | U3 | 103 |
|  | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 102 | $2^{\text {nd }}$ operator |  |  |
|  | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 103 | $3^{\circ}$ operator |  |  |
| 4 operators |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Straight production cell | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 70 | $1^{\text {th }}$ operator | $\llcorner 4$ | 80 |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 80 | $2^{\text {nd }}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 78 | $3^{\circ}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 75 | $4^{\circ}$ operator |  |  |
| U-shaped production cell | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 74 | $1^{\text {th }}$ operator | U4 | 77 |
|  | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 77 | $2^{\text {nd }}$ operator |  |  |
|  | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 77 | $3^{\circ}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 75 | $4^{\circ}$ operator |  |  |
| 5 operators |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Straight production cell | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 65 | $1^{\text {th }}$ operator | L5 | 75 |
|  | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 43 | $2^{\text {nd }}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 74 | $3^{\circ}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 46 | $4{ }^{\circ}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 75 | $5^{\circ}$ operator |  |  |
| U-shaped production cell | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 62 | $1^{\text {th }}$ operator | U5 | 64 |
|  | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 64 | $2^{\text {nd }}$ operator |  |  |
|  | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 62 | $3^{\circ}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 60 | $4^{\circ}$ operator |  |  |
|  | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 55 | $5^{\circ}$ operator |  |  |
| 6 operators |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Straight production cell | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 65 | $1^{\text {th }}$ operator | L6 | 65 |
|  | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 55 | $2^{\text {nd }}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 62 | $3{ }^{\circ}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 46 | $4^{\circ}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 60 | $5^{\circ}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 15 | $6^{\circ}$ operator |  |  |
| U-shaped production cell | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 60 | $1^{\text {th }}$ operator | U6 | 60 |
|  | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 40 | $2^{\text {nd }}$ operator |  |  |
|  | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 51 | $3^{\circ}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 37 | $4^{\circ}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 60 | $5^{\circ}$ operator |  |  |
|  | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 55 | $6^{\circ}$ operator |  |  |
| 7 operators |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Straight production cell | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 50 | $1^{\text {th }}$ operator | L7 | 60 |
|  | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 58 | $2^{\text {nd }}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 42 | $3{ }^{\circ}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 41 | $4{ }^{\circ}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 37 | $5^{\circ}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 60 | $6^{\circ}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 15 | $7^{\circ}$ operator |  |  |
| U-shaped production cell | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 50 | $1^{\text {th }}$ operator | U7 | 60 |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 32 | $2^{\text {nd }}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 44 | $3^{\circ}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 15 | $4^{\circ}$ operator |  |  |
|  | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 47 | $5^{\circ}$ operator |  |  |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 60 | $6^{\circ}$ operator |  |  |
|  | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 55 | $7^{\circ}$ operator |  |  |

Figure 5. Case $\mathrm{n}^{\circ} 1$ results


Figure 6. Distribution of results, for $\operatorname{Lx} \& \operatorname{Ux}(x \in[2,6])$

Table list

| Case ${ }^{\circ}$ | P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 | P9 | P10 | P11 | P12 | P13 | P14 | P15 | P16 | P17 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 5 | 23 | 12 | 10 | 15 | 5 | 30 | 8 | 12 | 10 | 20 | 32 | 9 | 24 | 13 | 60 | 15 |
| 2 | 1 | 9 | 17 | 60 | 5 | 14 | 22 | 50 | 11 | 29 | 10 | 20 | 7 | 10 | 6 | 9 | 14 |
| 3 | 1 | 60 | 15 | 5 | 13 | 22 | 30 | 40 | 4 | 16 | 20 | 20 | 14 | 7 | 10 | 8 | 9 |
| 4 | 5 | 8 | 5 | 11 | 10 | 12 | 12 | 15 | 13 | 17 | 14 | 19 | 20 | 24 | 23 | 35 | 60 |
| 5 | 5 | 10 | 20 | 18 | 25 | 20 | 52 | 6 | 60 | 11 | 8 | 22 | 11 | 9 | 5 | 10 | 11 |
| 6 | 5 | 15 | 20 | 60 | 35 | 10 | 8 | 9 | 50 | 12 | 41 | 5 | 7 | 10 | 6 | 5 | 5 |
| 7 | 1 | 5 | 12 | 34 | 24 | 18 | 14 | 42 | 11 | 60 | 16 | 8 | 10 | 15 | 6 | 8 | 9 |
| 8 | 1 | 60 | 22 | 15 | 9 | 6 | 56 | 10 | 16 | 6 | 7 | 11 | 13 | 8 | 20 | 5 | 21 |
| 9 | 1 | 21 | 25 | 5 | 22 | 27 | 14 | 24 | 60 | 11 | 9 | 13 | 5 | 9 | 13 | 20 | 8 |
| 10 | 5 | 9 | 16 | 15 | 13 | 60 | 16 | 5 | 50 | 34 | 10 | 15 | 6 | 9 | 14 | 8 | 18 |
| 11 | 1 | 5 | 60 | 7 | 16 | 24 | 11 | 31 | 12 | 9 | 11 | 28 | 19 | 5 | 39 | 7 | 9 |
| 12 | 1 | 21 | 22 | 60 | 5 | 19 | 27 | 17 | 23 | 12 | 16 | 8 | 10 | 18 | 11 | 7 | 13 |
| 13 | 5 | 60 | 51 | 18 | 39 | 6 | 12 | 18 | 11 | 21 | 6 | 6 | 7 | 14 | 7 | 14 | 8 |
| 14 | 6 | 8 | 14 | 36 | 13 | 18 | 7 | 25 | 5 | 5 | 17 | 16 | 12 | 30 | 6 | 10 | 21 |
| 15 | 2 | 11 | 8 | 5 | 60 | 14 | 30 | 6 | 12 | 8 | 25 | 15 | 36 | 27 | 7 | 9 | 10 |
| 16 | 1 | 8 | 15 | 22 | 18 | 5 | 60 | 25 | 29 | 6 | 18 | 26 | 9 | 13 | 19 | 7 | 13 |
| 17 | 5 | 20 | 20 | 5 | 5 | 30 | 60 | 20 | 8 | 5 | 30 | 40 | 10 | 5 | 20 | 10 | 10 |
| 18 | 1 | 35 | 5 | 60 | 23 | 8 | 20 | 11 | 8 | 17 | 15 | 24 | 6 | 16 | 14 | 22 | 9 |
| 19 | 6 | 5 | 40 | 30 | 20 | 10 | 5 | 7 | 10 | 22 | 20 | 39 | 8 | 5 | 7 | 10 | 5 |
| 20 | 6 | 5 | 25 | 10 | 15 | 7 | 30 | 5 | 20 | 6 | 23 | 10 | 9 | 49 | 60 | 7 | 16 |
| 21 | 5 | 23 | 14 | 25 | 11 | 9 | 7 | 30 | 13 | 29 | 8 | 30 | 9 | 5 | 60 | 17 | 8 |
| 22 | 3 | 37 | 22 | 9 | 34 | 11 | 12 | 10 | 7 | 8 | 60 | 5 | 9 | 20 | 6 | 8 | 15 |
| 23 | 5 | 12 | 29 | 35 | 8 | 7 | 10 | 30 | 14 | 9 | 8 | 60 | 10 | 7 | 15 | 23 | 21 |
| 24 | 8 | 17 | 60 | 6 | 12 | 15 | 9 | 17 | 20 | 22 | 26 | 5 | 21 | 24 | 13 | 22 | 6 |
| 25 | 1 | 6 | 15 | 17 | 10 | 60 | 30 | 24 | 5 | 20 | 10 | 16 | 9 | 12 | 10 | 18 | 29 |
| 26 | 9 | 6 | 12 | 19 | 5 | 7 | 31 | 24 | 17 | 60 | 11 | 15 | 10 | 18 | 24 | 7 | 28 |
| 27 | 1 | 25 | 5 | 32 | 12 | 15 | 11 | 60 | 12 | 10 | 11 | 35 | 11 | 5 | 22 | 15 | 7 |
| 28 | 1 | 15 | 5 | 30 | 23 | 8 | 10 | 15 | 12 | 14 | 16 | 60 | 13 | 20 | 13 | 32 | 5 |
| 29 | 5 | 20 | 12 | 9 | 16 | 5 | 27 | 8 | 15 | 10 | 20 | 28 | 9 | 28 | 16 | 60 | 15 |
| 30 | 7 | 9 | 31 | 21 | 9 | 11 | 10 | 8 | 32 | 60 | 8 | 9 | 25 | 33 | 14 | 11 | 5 |
| 31 | 6 | 5 | 11 | 52 | 12 | 32 | 5 | 7 | 8 | 9 | 9 | 9 | 40 | 20 | 6 | 6 | 12 |
| 32 | 1 | 6 | 20 | 5 | 26 | 18 | 30 | 15 | 33 | 7 | 21 | 10 | 12 | 14 | 6 | 60 | 9 |
| 33 | 6 | 17 | 60 | 5 | 5 | 51 | 9 | 13 | 30 | 13 | 21 | 20 | 5 | 11 | 5 | 25 | 7 |
| 34 | 2 | 60 | 10 | 5 | 13 | 16 | 33 | 19 | 35 | 11 | 8 | 15 | 20 | 9 | 12 | 7 | 5 |

Table 1. Description of the dataset: the 34 studied cases

| $\begin{gathered} \text { case } \\ n^{\circ} \end{gathered}$ | 1 operator |  |  | 2 operators |  | 3 operators |  |  |  | 4 operators |  |  | 5 operators |  |  | 6 operators |  |  | 7 operators |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | L1 | U1 | \% | L2 | U2 | \% | L3 | U3 | \% | L4 | U4 | \% | L5 | U5 | \% | L6 | U6 | \% | L7 | U7 | \% |
| 1 | 303 | 303 | 0,00\% | 153 | 152 | 0,65\% | 108 | 103 | 4,63\% | 80 | 77 | 3,75\% | 75 | 64 | 14,67\% | 65 | 60 | 7,69\% | 60 | 60 | 0,00\% |
| 2 | 303 | 303 | 0,00\% | 166 | 152 | 8,43\% | 105 | 102 | 2,86\% | 96 | 79 | 17,71\% | 79 | 62 | 21,52\% | 61 | 60 | 1,64\% | 60 | 60 | 0,00\% |
| 3 | 303 | 303 | 0,00\% | 155 | 152 | 1,94\% | 104 | 102 | 1,92\% | 85 | 78 | 8,24\% | 70 | 65 | 7,14\% | 70 | 60 | 14,29\% | 60 | 60 | 0,00\% |
| 4 | 303 | 303 | 0,00\% | 161 | 152 | 5,59\% | 108 | 102 | 5,56\% | 83 | 78 | 6,02\% | 63 | 62 | 1,59\% | 60 | 60 | 0,00\% | 60 | 60 | 0,00\% |
| 5 | 303 | 303 | 0,00\% | 153 | 152 | 0,65\% | 118 | 103 | 12,71\% | 78 | 77 | 1,28\% | 76 | 63 | 17,11\% | 60 | 60 | 0,00\% | 60 | 60 | 0,00\% |
| 6 | 303 | 303 | 0,00\% | 153 | 152 | 0,65\% | 112 | 103 | 8,04\% | 95 | 80 | 15,79\% | 79 | 62 | 21,52\% | 60 | 60 | 0,00\% | 60 | 60 | 0,00\% |
| 7 | 303 | 303 | 0,00\% | 160 | 152 | 5,00\% | 118 | 104 | 11,86\% | 86 | 76 | 11,63\% | 72 | 62 | 13,89\% | 62 | 60 | 3,23\% | 60 | 60 | 0,00\% |
| 8 | 303 | 303 | 0,00\% | 173 | 152 | 12,14\% | 107 | 102 | 4,67\% | 88 | 78 | 11,36\% | 78 | 64 | 17,95\% | 63 | 60 | 4,76\% | 60 | 60 | 0,00\% |
| 9 | 303 | 303 | 0,00\% | 155 | 152 | 1,94\% | 117 | 104 | 11,11\% | 87 | 79 | 9,20\% | 77 | 63 | 18,18\% | 63 | 60 | 4,76\% | 60 | 60 | 0,00\% |
| 10 | 303 | 303 | 0,00\% | 164 | 152 | 7,32\% | 118 | 104 | 11,86\% | 84 | 77 | 8,33\% | 71 | 64 | 9,86\% | 60 | 60 | 0,00\% | 60 | 60 | 0,00\% |
| 11 | 303 | 303 | 0,00\% | 164 | 153 | 6,71\% | 107 | 103 | 3,74\% | 82 | 78 | 4,88\% | 75 | 63 | 16,00\% | 60 | 60 | 0,00\% | 60 | 60 | 0,00\% |
| 12 | 303 | 303 | 0,00\% | 162 | 152 | 6,17\% | 117 | 103 | 11,97\% | 84 | 77 | 8,33\% | 65 | 63 | 3,08\% | 60 | 60 | 0,00\% | 60 | 60 | 0,00\% |
| 13 | 303 | 303 | 0,00\% | 169 | 155 | 8,28\% | 116 | 105 | 9,48\% | 86 | 80 | 6,98\% | 65 | 63 | 3,08\% | 63 | 60 | 4,76\% | 60 | 60 | 0,00\% |
| 14 | 303 | 303 | 0,00\% | 154 | 152 | 1,30\% | 112 | 103 | 8,04\% | 81 | 79 | 2,47\% | 67 | 63 | 5,97\% | 60 | 60 | 0,00\% | 60 | 60 | 0,00\% |
| 15 | 303 | 303 | 0,00\% | 154 | 154 | 0,00\% | 104 | 104 | 0,00\% | 96 | 79 | 17,71\% | 76 | 63 | 17,11\% | 60 | 60 | 0,00\% | 60 | 60 | 0,00\% |
| 16 | 303 | 303 | 0,00\% | 163 | 152 | 6,75\% | 114 | 104 | 8,77\% | 85 | 78 | 8,24\% | 73 | 62 | 15,07\% | 60 | 60 | 0,00\% | 60 | 60 | 0,00\% |
| 17 | 303 | 303 | 0,00\% | 158 | 153 | 3,16\% | 123 | 103 | 16,26\% | 85 | 80 | 5,88\% | 83 | 63 | 24,10\% | 63 | 60 | 4,76\% | 60 | 60 | 0,00\% |
| 18 | 303 | 303 | 0,00\% | 161 | 152 | 5,59\% | 110 | 102 | 7,27\% | 91 | 79 | 13,19\% | 67 | 62 | 7,46\% | 60 | 60 | 0,00\% | 60 | 60 | 0,00\% |
| 19 | 303 | 303 | 0,00\% | 155 | 153 | 1,29\% | 105 | 103 | 1,90\% | 90 | 79 | 12,22\% | 74 | 62 | 16,22\% | 60 | 60 | 0,00\% | 60 | 60 | 0,00\% |
| 20 | 303 | 303 | 0,00\% | 152 | 152 | 0,00\% | 117 | 103 | 11,97\% | 84 | 79 | 5,95\% | 83 | 62 | 25,30\% | 60 | 60 | 0,00\% | 60 | 60 | 0,00\% |
| 21 | 303 | 303 | 0,00\% | 166 | 152 | 8,43\% | 110 | 104 | 5,45\% | 85 | 77 | 9,41\% | 74 | 65 | 12,16\% | 67 | 60 | 10,45\% | 60 | 60 | 0,00\% |
| 22 | 303 | 303 | 0,00\% | 155 | 152 | 1,94\% | 123 | 103 | 16,26\% | 90 | 79 | 12,22\% | 67 | 63 | 5,97\% | 63 | 60 | 4,76\% | 60 | 60 | 0,00\% |
| 23 | 303 | 303 | 0,00\% | 153 | 153 | 0,00\% | 121 | 102 | 15,70\% | 81 | 77 | 4,94\% | 70 | 61 | 12,86\% | 61 | 60 | 1,64\% | 60 | 60 | 0,00\% |
| 24 | 303 | 303 | 0,00\% | 159 | 153 | 3,77\% | 109 | 102 | 6,42\% | 85 | 78 | 8,24\% | 74 | 65 | 12,16\% | 65 | 60 | 7,69\% | 60 | 60 | 0,00\% |
| 25 | 303 | 303 | 0,00\% | 153 | 153 | 0,00\% | 120 | 102 | 15,00\% | 90 | 79 | 12,22\% | 67 | 63 | 5,97\% | 60 | 60 | 0,00\% | 60 | 60 | 0,00\% |
| 26 | 303 | 303 | 0,00\% | 173 | 153 | 11,56\% | 112 | 102 | 8,93\% | 87 | 77 | 11,49\% | 72 | 63 | 12,50\% | 60 | 60 | 0,00\% | 60 | 60 | 0,00\% |
| 27 | 303 | 303 | 0,00\% | 175 | 152 | 13,14\% | 104 | 104 | 0,00\% | 89 | 78 | 12,36\% | 70 | 62 | 11,43\% | 68 | 60 | 11,76\% | 60 | 60 | 0,00\% |
| 28 | 303 | 303 | 0,00\% | 159 | 153 | 3,77\% | 117 | 102 | 12,82\% | 83 | 77 | 7,23\% | 73 | 62 | 15,07\% | 62 | 60 | 3,23\% | 60 | 60 | 0,00\% |
| 29 | 303 | 303 | 0,00\% | 156 | 153 | 1,92\% | 110 | 102 | 7,27\% | 81 | 77 | 4,94\% | 75 | 62 | 17,33\% | 62 | 60 | 3,23\% | 60 | 60 | 0,00\% |
| 30 | 303 | 303 | 0,00\% | 165 | 152 | 7,88\% | 105 | 103 | 1,90\% | 88 | 77 | 12,50\% | 70 | 63 | 10,00\% | 63 | 60 | 4,76\% | 60 | 60 | 0,00\% |
| 31 | 303 | 303 | 0,00\% | 163 | 152 | 6,75\% | 116 | 102 | 12,07\% | 84 | 76 | 9,52\% | 67 | 64 | 4,48\% | 64 | 60 | 6,25\% | 60 | 60 | 0,00\% |
| 32 | 303 | 303 | 0,00\% | 164 | 153 | 6,71\% | 111 | 103 | 7,21\% | 86 | 78 | 9,30\% | 69 | 66 | 4,35\% | 68 | 60 | 11,76\% | 61 | 60 | 1,64\% |
| 33 | 303 | 303 | 0,00\% | 153 | 152 | 0,65\% | 107 | 102 | 4,67\% | 83 | 77 | 7,23\% | 73 | 62 | 15,07\% | 61 | 60 | 1,64\% | 60 | 60 | 0,00\% |
| 34 | 303 | 303 | 0,00\% | 162 | 153 | 5,56\% | 113 | 103 | 8,85\% | 85 | 78 | 8,24\% | 73 | 64 | 12,33\% | 68 | 60 | 11,76\% | 60 | 60 | 0,00\% |

Table 2. Summary of the experimentation results (in bold: improvements greater than $10 \%$ )

Table 3. Average and standard deviation of cycle durations of the different

|  | 2 operators |  |  | 3 operators |  | 4 operators |  |  |  | 5 operators |  |  | 6 operators |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | L2 | U2 | \% | L3 | U3 | \% | L4 | U4 | \% | L5 | U5 | \% | L6 | U6 | \% |
| average | 160 | 153 | 5\% | 112 | 103 | 8\% | 86.1 | 78.0 | 9\% | 72.4 | 63 | 13\% | 62.7 | 60 | 7,7\% |
| standard deviation | 6.44 | 0.71 |  | 5.77 | 0.84 |  | 4.31 | 1.10 |  | 4.92 | 1.13 |  | 2.90 | 0 |  |

configurations, for $\operatorname{Lx} \& \operatorname{Ux}(x \in[2,6])$

| Instance | $$ |  | $\begin{aligned} & \approx \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { I } \\ & \text { I } \\ & \text { n } \\ & 0 \\ & 0 \\ & \text { N } \end{aligned}$ | $\begin{aligned} & \text { n} \\ & \text { N } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \text { W } \end{aligned}$ | E E 0 0 0 0 0 0 0 |  | obtained sequence |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bowman8 | 8 | 17 | 9,38 | 3 | 75 | 4,31 | C | 1,2,4,3,6,5,8,7 |
| Jackson | 11 | 7 | 4,18 | 1 | 46 | 1,94 | C | 1,2,3,4,5,7,6,8,9,10,11 |
| Jaeschke | 9 | 6 | 4,11 | 1 | 37 | 1,45 | C | 1,2,3,4,5,6,7,8,9 |
| Mansoor | 11 | 45 | 16,8 | 2 | 185 | 14,8 | C | 3,1,2,4,5,6,7,8,9,10,11 |

Table 4. Description of small-sized instances

| Instance | $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bowman8 | Lx | 75 | 42 | 28 | 22 | 20 | 17 |  |  |
|  | Ux | 75 | 38 | 26 | 21 | 18 | 17 |  |  |
|  | $\%$ | $0 \%$ | $\mathbf{1 0 \%}$ | $7 \%$ | $5 \%$ | $10 \%$ | $0 \%$ |  |  |
| Jackson | Lx | 46 | 24 | 19 | 13 | 12 | 11 | 9 | 7 |
|  | Ux | 46 | 24 | 13 | 12 | 10 | 9 | 8 | 7 |
|  | $\%$ | $0 \%$ | $0 \%$ | $\mathbf{3 2 \%}$ | $8 \%$ | $\mathbf{1 7 \%}$ | $\mathbf{1 8 \%}$ | $\mathbf{1 1 \%}$ | $0 \%$ |
| Jaeschke | Lx | 37 | 20 | 14 | 10 |  |  |  |  |
|  | Ux | 37 | 19 | 13 | 10 |  |  |  |  |
|  | $\%$ | $0 \%$ | $5 \%$ | $7 \%$ | $0 \%$ |  |  |  |  |
|  | Mansoor | Lx | 185 | 98 | 68 | 50 | 45 |  |  |
|  | Ux | 185 | 93 | 65 | 49 | 45 |  |  |  |
|  | $\%$ | $0 \%$ | $5 \%$ | $4 \%$ | $2 \%$ | $0 \%$ |  |  |  |

Table 5. Results for small-sized instances

| Instance |  |  | $\begin{aligned} & \text { ̃ } \\ & \text { E } \\ & \text { 0 } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \text { n } \\ & \text { n } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | suo!̣p.ınp syspr |  |  | obtained sequence |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Buxey | 29 | 25 | 11,17 | 1 | 324 | 6,04 | C | $\begin{aligned} & \text { 1,2,7,3,6,9,12,25,26,4,27,10,5,14,15,8,13,19,11,16,21,17,18,22, } \\ & 20,23,24,28,29 \end{aligned}$ |
| Gunther | 35 | 40 | 13,8 | 1 | 483 | 12,09 | C | $\begin{aligned} & 17,1,2,5,10,12,3,6,4,7,8,9,11,14,18,13,15,19,16,20,21,22,25,30 \\ & 23,26,31,24,32,27,33,28,34,29,35 \end{aligned}$ |
| Heskia | 28 | 108 | 36,57 | 1 | 1024 | 33,09 | C | $\begin{aligned} & \text { 2,1,3,4,5,6,8,17,19,21,22,23,24,26,7,9,20,25,27,10,18, 11, 12, 13, } \\ & 15,14,16,28 \end{aligned}$ |
| Kilbrid | 45 | 55 | 12,27 | 3 | 552 | 9,65 | C | $1,2,11,12,39,3,4,7,8,13,37,5,6,14,15,43,9,10,16,17,18,23,24,25$, $29,30,31,32,19,26,27,20,33,21,34,35,36,22,28,38,40,41,42,44,4$ 5 |
| Lutz1 | 32 | 1400 | 441,9 | 100 | 14140 | 244,9 | C | $\begin{aligned} & 3,4,2,1,5,6,9,8,7,10,11,12,13,15,14,16,17,18,19,20,21,22,23,24, \\ & 25,26,27,28,29,30,31,32 \end{aligned}$ |
| Mitchell | 21 | 13 | 5 | 1 | 105 | 2,97 | C | 1,2,3,4,21,5,6,7,8,14,9,10,11,12,13,15,16,18,19,17,20 |
| Roszieg | 25 | 13 | 5 | 1 | 125 | 2,81 | C | $1,2,3,4,5,8,6,9,10,7,11,12,13,15,17,14,16,19,20,23,18,21,22,24$, 25 |
| Sauyer30 | 30 | 25 | 10,8 | 1 | 324 | 6,07 | C | $\begin{aligned} & 1,10,3,2,4,5,11,12,16,17,6,13,18,7,14,19,8,15,20,9,21,24,22,25, \\ & 23,26,27,28,29,30 \end{aligned}$ |

Table 6. Description of large-sized datasets

| Instance | $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Buxey | Lx | 324 | 163 | 108 | 85 | 70 | 61 | 53 | 47 | 41 | 39 | 37 | 33 | 31 | 31 | 28 | 26 | 25 |
|  | Ux | 324 | 162 | 108 | 81 | 66 | 55 | 47 | 41 | 39 | 34 | 31 | 30 | 27 | 25 | 25 | 25 | 25 |
|  | \% | 0\% | 1\% | 0\% | 5\% | 6\% | 10\% | 11\% | 13\% | 5\% | 13\% | 16\% | 9\% | 13\% | 19\% | 11\% | 4\% | 0\% |
| Gunther | Lx | 483 | 244 | 168 | 124 | 104 | 91 | 77 | 71 | 67 | 60 | 54 | 48 | 46 | 45 | 40 |  |  |
|  | Ux | 483 | 242 | 161 | 121 | 98 | 82 | 73 | 65 | 59 | 51 | 48 | 45 | 40 | 40 | 40 |  |  |
|  | \% | 0\% | 1\% | 4\% | 2\% | 6\% | 10\% | 5\% | 8\% | 12\% | 15\% | 11\% | 6\% | 13\% | 11\% | 0\% |  |  |
| Heskia | Lx | 1024 | 522 | 355 | 264 | 245 | 206 | 162 | 161 | 132 | 129 | 115 | 108 |  |  |  |  |  |
|  | Ux | 1024 | 514 | 344 | 258 | 209 | 175 | 157 | 132 | 129 | 111 | 108 | 108 |  |  |  |  |  |
|  | \% | 0\% | 2\% | 3\% | 2\% | 15\% | 15\% | 3\% | 18\% | 2\% | 14\% | 6\% | 0\% |  |  |  |  |  |
| Kilbrid | Lx | 552 | 278 | 190 | 156 | 116 | 97 | 88 | 79 | 70 | 61 | 57 | 56 | 56 | 55 |  |  |  |
|  | Ux | 552 | 277 | 184 | 139 | 111 | 93 | 80 | 71 | 63 | 56 | 55 | 55 | 55 | 55 |  |  |  |
|  | \% | 0\% | 0\% | 3\% | 11\% | 4\% | 4\% | 9\% | 10\% | 10\% | 8\% | 4\% | 2\% | 2\% | 0\% |  |  |  |
| Lutz1 | Lx | 14140 | 7296 | 4920 | 3702 | 3100 | 2654 | 2284 | 1928 | 1898 | 1676 | 1628 | 1430 | 1400 |  |  |  |  |
|  | Ux | 14140 | 7078 | 4720 | 3598 | 2858 | 2392 | 2066 | 1858 | 1640 | 1514 | 1400 | 1400 | 1400 |  |  |  |  |
|  | \% | 0\% | 3\% | 4\% | 3\% | 8\% | 10\% | 10\% | 4\% | 14\% | 10\% | 14\% | 2\% |  |  |  |  |  |
| Mitchell | Lx | 105 | 56 | 37 | 28 | 23 | 21 | 18 | 16 | 16 | 15 | 13 |  |  |  |  |  |  |
|  | Ux | 105 | 53 | 35 | 27 | 22 | 18 | 16 | 15 | 13 | 13 | 13 |  |  |  |  |  |  |
|  | \% | 0\% | 5\% | 5\% | 4\% | 4\% | 14\% | 11\% | 6\% | 19\% | 13\% | 0\% |  |  |  |  |  |  |
| Roszieg | Lx | 125 | 64 | 43 | 34 | 29 | 23 | 21 | 18 | 16 |  |  |  |  |  |  |  |  |
|  | Ux | 125 | 63 | 42 | 32 | 26 | 21 | 19 | 17 | 16 |  |  |  |  |  |  |  |  |
|  | \% | 0\% | 2\% | 2\% | 6\% | 10\% | 9\% | 10\% | 6\% | 0\% |  |  |  |  |  |  |  |  |
| Sauyer30 | Lx | 324 | 166 | 111 | 87 | 73 | 57 | 53 | 48 | 44 | 38 | 35 | 33 | 31 | 28 | 26 | 26 | 25 |
|  | Ux | 324 | 162 | 109 | 82 | 66 | 55 | 48 | 42 | 38 | 35 | 31 | 29 | 27 | 25 | 25 | 25 | 25 |
|  | \% | 0\% | 2\% | 2\% | 6\% | 10\% | 4\% | 9\% | 13\% | 14\% | 8\% | 11\% | 12\% | 13\% | 11\% | 4\% | 4\% | 0\% |

Table 7. Results for large-sized datasets.

