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Evaluation of the Impact of Uncertain Advance Demand Information on Production/Inventory Systems

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Abstract: In this paper, we conduct a simulation study to evaluate the impact of using imperfect advance demand information (ADI) in a single stage production/inventory system. We mean by ADI the fact that the production system receives customer orders in advance of their due-dates. We consider four types of ADI: 1) Perfect ADI, 2) ADI with imperfect due-dates, 3) ADI with imperfect demand quantities and 4) ADI with updates. The production system is controlled by a modified base-stock policy which has two parameters that are *S*, the base-stock level, and *L*, the production release lead time*.* We intend to shed light on the impact of imperfect ADI (ADI types 2, 3, 4) on the performance of production/inventory systems. The aim of the simulation study is twofold. Firstly, the objective is to find the optimal parameters of the modified base-stock policy and secondly, to evaluate the benefits of imperfect ADI in a production/inventory system.

Keywords: supply chain, inventory management, Imperfect advance demand information, single stage production/inventory system, modified base-stock policy.

1.INTRODUCTION

Developments in supply chain concepts and information technology have improved the flow of information between companies. Hence, various information can be shared in a supply chain: sales, inventory, order status, production schedules and demand forecasts (see Lee and Whang (2000)). Such information enables to improve the performance of existing production and inventory control policies. For example, MRP controlled systems use demand forecasts to better control production orders.

The aim of our work is to evaluate the benefits of demand information sharing. Precisely, we consider a context where the customer shares demand information, i.e. orders, in advance of the order due-dates. We call this type of information, advance demand information (ADI). ADI models can be divided in two classes: Perfect and Imperfect ADI. Under perfect ADI, the supplier receives reliable information about customer demand, so orders received do not change over time, in terms of both quantity demanded and due date. Under imperfect ADI, the order information is uncertain and can change before the due-date of the order. The objectives of this paper are: (1) to find optimal ways to use imperfect ADI and (2) to evaluate the impact of imperfect ADI in production/inventory systems.

Different examples of production/inventory systems with ADI have been found in the literature. Benjaafar et al (2011) cite the example of a supplier who provides components to an aircraft manufacturer. The production process of an aircraft is very complex and passes by many stages. If we assume that the supplier provides a component which should be assembled at the last stage of production, then, the manufacturer can send ADI by indicating the status of production process and expected time before using the needed component. Another interesting example of ADI is the automotive industry: Faurecia is a supplier of automotive parts which has as customers the original equipment manufacturers (OEM). Generally, manufacturing a car passes through four workshops consecutively: Stamping, Welding, Painting and Assembly. Parts supplied by Faurecia are involved at the end of the assembly workshop. In a just in time fashion, Faurecia receives a signal of demand when a car enters the assembly stage of the customer. This signal is considered to be perfect (the customer does not change the due-date of that signal) and represents an advance order information. The order is supposed to be delivered after a fixed amount of time called "the Demand lead time". Faurecia can also receive advance demand information from the earlier manufacturing workshops (Stamping, Welding and Painting), but these signals are considered to be imperfect because the lead time variability is high at these stages.

In this study, we consider a single stage single item production/inventory system controlled by a modified basestock policy that incorporates ADI. The system is assumed to use four types of ADI: i) Perfect ADI, ii) ADI with imperfect due-dates, iii) ADI with imperfect demand quantities and iv) ADI with updates. In fact, demand information has two components: due-date and quantity demanded. So we can have diverse types of ADI, for example, when information about customer order due-dates and quantity is exact, we say that we have a system with "perfect ADI". For a system where information regarding future quantities demanded by customer is exact but there is uncertainty associate with order due-dates, we say that we have ADI with imperfect due-dates. We show

by means of a simulation model that the value of ADI decreases when information becomes imperfect. We provide interesting insights on the impact of imperfect ADI on the production system's performance.

The remainder of this article is organized as follows. In section 2, a literature review is presented. Section 3 describes the characteristics of our model and the related assumptions. Section 4 reports the simulation results and discussions. Finally, conclusions and some avenues for future research are presented in Section 5.

2.LITERATURE REVIEW

Our work is within the stream of papers that study the benefits of demand information sharing. We are essentially inspired by models introduced by Buzacott and Shanthikumar (1994), Karaesmen et al (2002) and Karaesmen et al (2004). One possible classification of research related to ADI can be based on the type of information available. Systems with perfect ADI are studied differently than systems with imperfect ADI.

For systems with perfect ADI, Milgrom and Roberts (1988) are among the first researchers that discuss the impact of ADI in production/inventory systems. They studied the issue of shifting between make-to-stock (MTS) to make-to-order (MTO) mode of production where information regarding inventories and demand are given by customers. They considered a single stage, multi-product system where demand follows a normal distribution. They found that for high level of price and large demand information, MTO strategy that uses ADI is optimal. Hariharan and Zipkin (1995) extended some existing inventory models by incorporating perfect ADI. They examined the effects of ADI on the performance of inventory systems. They concluded that the effect of using advance demand information is equivalent to reducing the supply lead time. Buzacott and Shanthikumar (1994) studied the tradeoff between safety stock and safety time in MRP Controlled Systems. Toktay and Wein (2001) studied a production/inventory system with stationary demand and dynamic forecast updates. They considered a single stage, single item, discrete-time, make-to-stock capacitated system managed by a base-stock policy. They found that a good forecast quality improves the performance of the system by reducing the expected costs and decreasing the base-stock level. They concluded also that the value of forecast information decreases when the system is saturated (high traffic). Karaesmen et al (2002) studied a discrete time make to stock queue with advance demand information. They investigated two production control policies: i) An optimal policy found using dynamic programming and ii) A heuristic policy extended from the classic base-stock policy by integrating a release lead time parameter. Karaesmen et al (2004) studied an extension of the model proposed in Karaesmen et al (2002). They considered a continue M/M/1 single stage MTS queue with ADI. They found that the system can shift from MTS strategy to MTO for sufficient horizon of visibility. Altendorfer and Minner (2014) developed an optimization model that minimizes Finished Goods Inventory and backorder costs for a multi-product production system with perfect variable due-dates.

For systems with imperfect ADI, Tan et al (2007) evaluated the impact of using imperfect ADI on inventory policies. They used a general probability framework model that integrates imperfect ADI with ordering decisions. Tan et al (2009) studied a rationing decision and inventory modeling problem using imperfect ADI. They showed that Imperfect ADI improves the performance of the system through reducing the inventory and the losing orders costs. Benjaafar et al (2011) analyzed a single stage production-inventory system where orders may become due prior to or later than the announced expected due-date or they can be canceled altogether. Gayon et al (2009) considered a MTS capacitated system with multiple customer classes where 1) order due-dates are not exactly known in advance, (2) orders can be cancelled by the customers, and (3) ADI is available only from a subset of the customers. Liberopoulos and Koukoumialos (2008) evaluated the impact of variability and uncertainty of ADI on the performance of single stage production/inventory systems. By numerical experiments, they showed that in the case of unreliable ADI with capacitated system, safety stock and safety time are interchangeable. Bernstein and DeCroix (2014) investigated the impact of ADI on the performance of a multiproduct system. They considered both perfect and imperfect mix of information.

 Through this study, we propose new models to evaluate the impact of imperfect ADI on inventory and backorder costs. To our knowledge, we are among the first authors that consider imperfect ADI in a single stage system where the customer order quantity is not a unit demand but a random variable following a Normal distribution. This work gives interesting results about the relation between base-stock levels and imperfect information.

3.MODEL DESCRIPTION AND ASSUMPTIONS

We consider a single stage, single product manufacturing system that operates in a make-to-stock environment with ADI. The system consists of one manufacturer and multiple customers (no priority rules) so we can represent them by a unique customer. Customer orders arrive in advance of their due-dates (required delivery date) and they are satisfied from the Finished Goods Inventory (FGI). We assume that there is an infinite supply of raw materials for the manufacturing stage and that the FGI is controlled by a modified base-stock policy. Additionally, we suppose that the manufacturing capacity is deterministic and that setup costs and setup times are negligible for simplification reasons.

In presence of advance demand information, the base-stock policy can be modified to include that information. In the classical base-stock policy (without ADI), the initial stock is equal to *S* (the base-stock level) and replenishment orders, i.e. production orders, are triggered at each demand arrival. If we assume that customer orders are placed in advance of their due-dates by a demand lead time *H*, then replenishment orders can be triggered *H* units of time before the customer order duedates. This approach would reduce stock outs, but it would generate high FGI levels which consequently increases the holding cost. Therefore, a control parameter *L* was proposed by Karaesmen et al (2002) to regulate the timing of

replenishment orders, it is called "the release lead time parameter". Instead of triggering replenishment orders *H* units of time before the customer order due-dates, replenishment orders are triggered *L* units of time before (where *L≤H)*, so the release is delayed by *H-L* units of time. This modified basestock policy is called the *(S,L)* policy. In this model, the basestock level *S* can be viewed as a safety stock for a MTO system.

3.1. Perfect ADI

We consider a discrete simulation model. We assume that a customer order quantity denoted by *O(t)*arrives at the end of each period *t*. This order should be fulfilled after $H = H$ units of time, i.e. at the end of period *(t+H)*. The customer order quantity $O(t)$ is assumed to follow a normal distribution with parameters (μ, σ) . Let us denote by $X(t)$ the FGI level at the end of period t $(X(t)$ is allowed to be negative under backordering assumption). $P(t)$ is the quantity of products produced during period *t* and *D(t)* is the customer demand at the end of period *t*, we have

$$
O(t) = D(t + \overline{H}).
$$
 (1)

In other words, customer demand observed at period t corresponds to the order received at period $(t - \overline{H})$.

The evolution of $X(t)$ can be expressed by:

$$
X(t) = X(t-1) + P(t) - D(t).
$$
 (2)

The total cost in period *t*, *C(t),* is expressed in (3), where *h* and *b* are the linear FGI holding and backorder costs per item per period, respectively.

$$
C(t) = h^* X(t)^+ + b^* X(t)^-. \tag{3}
$$

Where $X(t)^{+} = Max\{0, X(t)\}\$ and $X(t)^{-} = Max\{0, -X(t)\}\$.

At the beginning of each period, the requirement $R(t)$ is calculated following the logic of a modified base-stock policy. In a classical base-stock policy, the goal of the release mechanism is to maintain inventory position at level *S*, so we have

$$
R(t) = S - X(t-1).
$$
 (4)

In a *(S,L)* policy, the goal is to keep the base-stock level *S* and take into account the requirements for future (known) demands between periods *t* and *t+L*. We thus have

$$
R(t) = S + \sum_{i=1}^{L} O(t + i - \overline{H} - 1) - X(t - 1)
$$

= $S + \sum_{i=1}^{L} D(t + i - 1) - X(t - 1)$. (5)

Being restricted by the limited manufacturing capacity *Cap*, the quantity produced during period t, $P(t)$, is equal to

$$
P(t) = Min(R(t), Cap).
$$
 (6)

We develop a simulation model to find a release policy *(S,L)* that minimizes the total average FGI holding and backorder costs. The optimization problem is

Min
$$
C(S, L) = h * E[X^+] + b * E[X^-]
$$
. (7)

 (7)

3.2. ADI with Imperfect due-dates

In the previous section, we considered a model where information about future orders is perfect. In this case, we assume that due-dates of future orders are not known exactly. At the end of each period *t*, there is a customer order arrival $O(t)$ with an estimated due-date for period $(t+H)$. However, the customer who submits the due-date information can ask for his order before or after the estimated due-date. Hence, we assume that the demand lead time *H* is a random variable which follows a normal distribution with parameters (H, σ_H) . It is important to note that the variability is on the timing of demand (i.e. *H* is a random variable) and not on the quantity demanded (i.e. *O(t)* does not change over time).

In this model, the requirement $R(t)$ is calculated based on the expected demand lead time associated with each future order, i.e. H . Hence, $R(t)$ can be expressed by:

$$
R(t) = S + \sum_{i=1}^{L} O(t + i - \overline{H} - 1) - X(t - 1).
$$
 (8)

Note that in this model, we can have the phenomenon of "Cross demands" where more than two different customer orders end with the same due-date. The FGI level $X(t)$ evolves according to (2) . Equation (1) is no longer verified, but (3) , (6) and (7) hold for this model. Equation (5) is replaced by (8).

3.3. ADI with imperfect demand quantities

In this model, we assume that the quantities associated with future orders are not known exactly, i.e. order quantities are uncertain. It is important to note that the variability is on the quantity demanded (i.e. *O(t)* is a random variable) and not on the quantity demanded (i.e. H is fixed and equal to H).

Hence, at the end of each period *t*, there is a customer order arrival with a due-date for period $t + H$. The customer who submits demand information can ask more or less than the quantity initially requested $O(t)$. Thus, we have

$$
D(t+\overline{H}) = O(t) + \varepsilon_t^q, \qquad (9)
$$

where ε_t^q is the variability associated with the quantity of products demanded at the end of period *t*. We assume that ε_t^4 follows a normal distribution with parameters $(0, \sigma_o)$.

3.4. ADI with updates

This model is an extension of the models with imperfect duedates and imperfect quantities. We consider a system in which a manufacturer may receive several signals for the same order in different time horizons, where only the last signal is perfect.

To start with, we assume that there are two signals. The last signal is perfect and the former one is imperfect

Imperfect demand information

Fig. 1. The updating quantities mechanism

The updating mechanism can concern quantities or due-dates: updating quantities consists in receiving a first advance signal at the end of period *t* with an exact demand lead time $H₁$, i.e. The order has to be satisfied at the end of period *t+H1*. The quantity requested by the customer for period $t+H_1$ is $O(t)$. Then, after $(H_1 - H_2)$ units of time (i.e. at period $t + H_1 - H_2$), the manufacturer receives a second signal for the same order where the quantity to be delivered at $t+H_1$ is updated and is no more equal to *O(t)*.

Updating due-dates consists in receiving a first advance signal at the end of period *t* with an estimated demand lead time *H1*, ie. The order is estimated to be satisfied at the end of period $t+H_1$. The quantity requested by the customer for period $t+H_1$ is $O(t)$. That signal has an exact information about the quantity demanded. After $(H_1 - H_2)$ units of time (i.e. at period $t + H_1 - H_2$), the manufacturer receives a perfect information about the same customer order with a new exact demand lead time *H²* where $H_1 > H_2$.

Figure 2 depicts the updating due-dates mechanism:

Fig. 2. The updating due-dates mechanism

4.RESULTS AND DISCUSSION

In this section, we present results of the simulation study. The purpose is to find the optimal parameters *(S,L)* of the base stock-policy, and to examine the benefits of using ADI. Matlab R2012b is used to execute the simulation experiments. We run the simulation for 20000 periods and we calculate the average total cost for a list of *S* and *L* values to find which combination *(S,L)* minimizes the total cost. We assume that the capacity

Cap is constant and takes values from 50.1 to 100. The demand quantity *D(t)* follows a normal distribution *N (50, σ)* where σ can take values: 0.05, 1, 5 and 10. As for cost parameters, we take the linear holding cost $h=1$, and the linear backorder cost *b=10, 100* and *1000*. We assume that the demand lead time $\overline{H} = 24$.

4.1. Case of ADI with imperfect due-dates

Our goal is to evaluate the impact of ADI with imperfect duedates in single stage systems. We make a comparison between the performances of a system with perfect ADI and another system with different degrees of imperfect due-dates ($\sigma_H = 0$, 0.5, 1, 2, 5). The case where $\sigma_H = 0$, is equivalent to the model with perfect ADI.

It is intuitive that increasing the demand lead time uncertainty would deteriorate the performance of ADI. Figure 3 shows the impact of the error variability on the performance of a capacitated single stage system.

Fig. 3. The effect of imperfect due-dates on the average cost.

Fig. 4. The effect of the error variability on the optimal basestock level.

From Figures 3 and 4, it can be seen that for a sufficiently large demand lead time H , the average cost is decreasing linearly in *L* until a threshold level *L**, and is increasing in *L* when $L \geq L^*$. So, the cost is minimized at $L = L^*$. As the release lead time L increases from zero to L^* , the optimal base stock level *S** appears to decrease linearly with *L*.

The insight behind these results is that the optimal parameter *L** defines the desired demand lead time *H*, because any further increase in *L* beyond *L** is useless and does not improve performance in terms of average total cost reduction. Another important conclusion is that the value $S^*(L^*)$ sometimes equals to zero which means that it is optimal to operate the system in a make-to-order environment. (In a continuous review system, the value $S^*(L^*)$ is always equal to zero). We observe that increasing the error variability increases the optimal average cost, the optimal base-stock level and the optimal release lead time.

Table 1. The impact of capacity, for *h***=1,** *b***=10, Demand~N(50,10), imperfect due-dates (0,2)**

| Cap | $C^*(L=0)$ | $S^*(L=0)$ | \mathcal{C}^* | L^* |
|------|------------|------------|-----------------|-------|
| 50,1 | 382,89 | 621 | 374,61 | 12 |
| 50,2 | 321,83 | 475 | 308,68 | 9 |
| 50,5 | 186,01 | 333 | 177,00 | 7 |
| 51 | 138,78 | 265 | 133,98 | 5 |
| 52 | 125,18 | 229 | 119,66 | 4 |
| 55 | 116,89 | 193 | 113,14 | 4 |
| 60 | 112,69 | 170 | 109,59 | 3 |
| 65 | 109,38 | 156 | 106,82 | 3 |
| 70 | 106,94 | 148 | 104,88 | 3 |
| 80 | 103,67 | 137 | 102,66 | 2 |
| 90 | 101,47 | 128 | 101.05 | 2 |

[Table 1](#page-5-0) shows the impact of capacity increment on system performance when due-dates are imperfect. *S*(L=0)* represents the optimal base-stock level for a system operating in a pure make-to-stock environment without ADI. *C*(L=0)* represents the average total cost for the release policy $(S^*(L=0), L=0)$. C^* is the cost of operating the system in a pure make-to-order environment. It can be seen that the optimal release lead time *L** decreases with capacity. It means that a smaller demand lead time *H* is required to shift from a make-to-stock to a make-to-order system. We also observe that the optimal base-stock level decreases with capacity increment. In a system with perfect ADI, both relative and absolute values of gain is increasing in capacity and take their maximum values for higher amount of capacity. However, this is not the case when ADI is imperfect.

4.2. Case of ADI with imperfect quantities

Figure 5 shows the impact of forecast quantity on the performance of ADI. We consider five values of error variability. The results indicate that the error variability increment increases the optimal average total cost, the optimal base-stock level and the optimal release lead time.

Additionally, we observe that the effect of imperfect quantities on system performance is less significant than imperfect duedates.

Fig. 5. The effect of imperfect quantities on the average cost.

4.3. ADI with updates

We show results for a system which receives one update on quantities. We assume that the first signal of customer order arrives in advance with $H_1=24$ units of time and contains imperfect information about demand quantity. The second signal is perfect and arrives in advance with $H_2 = 18$ units of time. We make a comparison between the costs of managing the system with the two signals. It can be seen that the first signal corresponds to the case of ADI with imperfect quantities, while using the second signal corresponds to a model with perfect ADI.

Fig. 6. Variations of average cost in system with updates.

[Fig. s](#page-5-1)hows the variations of a system which uses both perfect and imperfect signals. It can be observed that there is a slight difference between the average total costs of the two signals. In fact, it is difficult to decide which signal to choose, especially, when the demand lead time of the second signal *H²* is sufficiently long to attain the value of the optimal release

lead time *L**. In such a situation, we recommend to run simulations for the two signals and decide whether to choose the first or the second signal.

5.CONCLUSION AND FUTURE WORK

In this paper, we propose simple control policies that incorporate perfect and imperfect advance demand information. We consider a periodic review capacitated single stage system with normally distributed demand and a fixed processing capacity. We evaluate the impact of four types of ADI: i) perfect ADI, ii) ADI with imperfect due-dates, iii) ADI with imperfect demand quantities and iv) ADI with updates in quantity. We found that the imperfectness of demand information reduces the benefits of ADI. We also found that imperfect due-dates deteriorate the system's performance more than imperfect demand quantities. This result is quite intuitive because of the phenomenon of "Cross demands". Simulation results show that in system with updates, we can use only imperfect signals if the demand lead time related to the perfect signal is not large enough to achieve the optimal release lead time *L*.* Simulation results confirm earlier insights of Milgrom and Roberts (1988) and Karaesmen et al (2002): advance demand information and inventory are substitutes. For sufficiently large demand lead times, information can replace the base-stock which allows the system to shift from a make-to-stock to a make-to-order setting. In terms of relative gains, imperfect demand information has not a significant impact on cost reductions, especially, for systems with small capacities. In fact, systems with limited capacities cannot respond to information about future customer orders.

There are several avenues for future research in ADI. According to the literature review, it would be of interest to study systems with multiple products and imperfect ADI. For example, we can consider systems with multiple customer classes having different priorities. In each period, every class of customers send advance orders for multiple products. The objective is to find a release mechanism that minimizes the average holding and backorder costs. Almost all research papers used the base-stock policy to evaluate the benefits of ADI (the base-stock policy was proved to be optimal) but industrials use another practical policies as MRP and Kanban. We propose to find new methods to incorporate ADI in those most used policies. We can consider hybrid basestock/Kanban policies with imperfect ADI in multi-stage production/inventory systems.

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