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ABSTRACT

This paper deals with the gridshells built by the Navier laboratory in the last ten years. The numerical conception is developed, from the draft made by architects up to the final structure. Several numerical tasks are performed to design a gridshell. The geometry of the gridshell is first considered. Then, an important iterative step mixing geometry and mechanical considerations is carried out. In particular, it is explained how the naturally straight beams are bent together during a very quick step leading to the final shape. Thanks to this active bending, double-curvature shapes are made and offer many interests like high stiffness for a light weight structure. Lastly, the geometry of the membrane is drawn based on the numerical final geometry of the gridshell. The improvements of gridshells, including safety considerations as well as practical considerations are also developed, through the four gridshells recently built. Finally the model is improved to take into account the torsion which can have an important effect, especially when the beams have a rectangular cross section.

Keywords: *Gridshell, torsion, beam, dynamic relaxation, form-finding, active bending, prototypes, composite materials.*

1. INTRODUCTION

In the last twenty years, many applications of composite materials in the construction industry have been found. The main field of application concerns the reinforcement of concrete beams with carbon fiber plates [1] or post tension cables. More recently, a footbridge with carbon fiber stay-cable was built in Laroin (France, 2002 [2]), another footbridge, all made of glass fiber composites, was built in Aberfeldy (Scotland, 1993 [3]) as well as a movable bridge in Stonehouse (the Bonds Mill lift bridge, England, 1995 [4]). Nevertheless, applications using composite materials as structural elements remain exceptional. Although the qualities of their mechanicals properties are obvious (low density, high strength and high resistance against corrosion and fatigue), their relatively low elastic modulus is a disadvantage against steel. Indeed most slender structures in structural engineering are designed according to their stiffness and rarely to their strength. In addition, the elastic instabilities depend linearly on the Young's modulus so that, again, having a low Young's modulus is a real disadvantage when a designer tries to calculate structures based on conventional design. In order to take advantage of every characteristic of composite materials, new structural concepts have to be found.

The Structures Materials and Structures research unit of Navier laboratory (AMS) is working on the development of innovative solutions for composite material in civil engineering. Four design principles guided the conception of our structures:

- Optimal use of the mechanical characteristics of the fibers;

- Simple connection between components of the structure;

- Optimal design according to its use;

- Use of components already available in the industry with affordable material costs.

Several structures were investigated such as an innovating footbridge [5] and several experimental gridshells [6] [7] [8] [9]. The purpose of this paper is to explain the method used to design gridshells, up to the fabrication of the membrane and then to emphasis on the improvements of the building process. The following section gives a proper definition for gridshells and presents the specificity of their construction process. Then, the design aspects of the project are developed. Afterwards, the steps of construction and the improvements made through the salvo of projects are approached. Finally, the recent introduction of torsion in the model of gridshells is presented.

2. GRIDSHELLS: DEFINITION AND PROCESS OF CONSTRUCTION

The name gridshell commonly describes a structure with the shape and strength of a double-curvature shell, but made of a grid instead of a continuous surface. These structures can be made of any kind of material - steel, aluminum, wood… Generally, the metallic structures are made of short straight elements defining a cladding made of planar triangular or quadrangular elements. The complexity of this geometry requires the development of many clever and expensive assemblies. In order to avoid these complex joints, a very specific erection process was developed using the bending capability of slender components [10]: two layers of long continuous beams are first pinned together on the ground. The resulting grid has no shear rigidity, allowing large deformations in space. The grid is then elastically deformed by bending until the desired form is obtained and finally stiffened, for example with a third bracing layer of beams. With this process, the initially straight beams are bent to form a rounded stiff surface. Only few gridshells were built using this active bending method, among which the most famous are: the Mannheim Bundesgartenschau (arch: Mutschler and Partner and Frei Otto, Str. Eng: Arup, 1975 [11]), the carpenter hall of the Weald and Downland Museum (arch. E. Cullinan, Str. Eng. Buro Happold, 2002) [12] and the Japanese pavilion for the Hanover 2000 Exhibition (arch: Shigeru Ban, Str. Eng. Buro Happold) [13]. In addition, the Navier research unit has already participated to the construction of four gridshells in glass fiber reinforced polymer (GFRP), increasingly large. The gridshell for the Solidays' festival, was 300 m² large [8], 2011, and very recently in 2013, a 350 m² gridshell called "Cathedrale Ephemere de Creteil" has been built to replace the Creteil cathedral during its renovation which should last at least two years.

The main building steps are illustrated below: the grid is assembled flat on the ground (figure 1), then erected by two cranes (figure 2) and gets its final form when attached on anchorages.

Figure 1: *Cathedrale Ephemere de Creteil. Primary grid made flat.*

Figure 2: *Cathedrale Ephemere de Creteil. Primary grid deformed elastically and about to be attached to boundary conditions.*

3. COMPOSITE MATERIALS TAILOR MADE FOR ACTIVE BENDING STRUCTURES: FLEXIBILITY FOR STIFFNESS.

Most of the gridshell structures have been made of wood because it is the only traditional building material that can be elastically bent with large deformations without breaking. This flexibility generates curved shapes offering structural stiffness. However looking at other industrial fields (sport and leisure, nautical...), it can be noticed that every time high strength and high deformability are required, composite materials is replacing wood (ship masts, skis, rackets). To study accurately the question of the best material for gridshells, the authors adopted the method proposed by M. Ashby [14]. In this method, indicators characterizing the object to be designed are defined. In the case of gridshells, it is necessary to have a material with:

- High elastic limit strain in order to be able to bend the element and obtain a curved shape.

- High Young's modulus to confer to the gridshell its final stiffness after bracing.

The Ashby method drawn for these two characteristics provided several materials potentially better than wood for the gridshell application. These materials are titanium, CFRP, GFRP and technical ceramics. In addition, this study showed that steel or concrete cannot be better than wood for such an application: these materials cannot deform as much as wood. To choose between the four families of material, other aspects have to be considered. In particular, materials shall not be too brittle to be easily handled on site by workers and therefore ceramics are not suited. Because of cost limitation, titanium and CFRP cannot suit for the gridshell application.

The most valuable alternative to wood is hence glass fiber reinforced polymers (GFRP). They have higher elastic limit strain (1.5 % at best for GFRP and 0.5 % for wood) so large curvature synonymous of freedom of shape is possible. Their Young's modulus also is higher (25-30 GPa against 10 GPa for wood). This is an advantage to make a stiff structure. In addition, assuming that for a given geometry, the buckling load of a gridshell is linearly dependent on the Young's modulus, one can expect the buckling load of a gridshell in composite materials to be 2.5 to 3 times higher than one made of wood. Moreover, as composites are industrially produced, the reliability of their mechanical properties is much higher than that of natural materials like wood. Finally, while wood beams have to be made of several pieces of wood stuck together, GFRP profiles can be made continuously, as long as necessary.

Concerning costs, if one takes into account the mechanical properties and the ability of composites to be formed into efficient sections like tubes, GFRPs become very interesting challengers, especially if pultrusion production is used. Indeed, hollow sections make possible the use of light beams optimized for each application (stiffness and curvature). Moreover, the polymer chosen for the GFRP can resist to corrosion, UV and other environmental attacks, whereas wood materials need maintenance.

At this point, the composite gridshell concept is explained. The type of materials chosen is GFRP for flexibility, cost, stiffness and reproducibility reasons. The process of construction developed by the research unit can now be developed.

4. DESIGN OF A GRIDSHELL: FROM THE DRAFT TO THE FINAL SHAPE.

The approach developed at Navier laboratory is summarized here. At the beginning of the design process, the architects define a continuous shape for the gridshell. To avoid excessive stress, the shape has to be a rounded shape with curvatures as homogeneous as possible. Then, according to the range of curvatures, the engineers choose the geometric properties of the beams (mainly the outer radius as it is explained in equation 1). Given the beam properties, the engineers can draw a grid on the shape (geometrical step) and compute the resulting form of the gridshell (mechanical step). After these form-finding steps, the engineers model the third layer of beams (bracing layer) and evaluate the stresses in the structure under serviceability limit states (SLS) and ultimate limit states (ULS) loads, according to construction codes [15]. They might have to modify the mesh, or the shape of the gridshell, in agreement with the architects, to reduce the stresses. Once the form-finding has converged and the stresses are suitable, the engineers can design the membrane according to the three-dimensional shape numerically obtained. The method summarized here is developed in the following.

Geometrical step: The method used here for determining the grid is "the compass method". This method consists in constructing a network of regular quadrangles on a surface. It was described in IL10 Gitterschalen of Frei Otto 1974 [11]. The figure 3 shows the different steps of the method on a surface which could have been three-dimensional. The task is to construct a grid using only a compass. First, two curves that intersect each other are laid down on the surface to mesh. Then, a mesh size is chosen and serves as the compass radius. The spacing of the grid is marked along each axis, from the point of intersection of the axes. The knots are determined by the intersection of two circles as shown on the figure 3. Gradually, new points are determined.

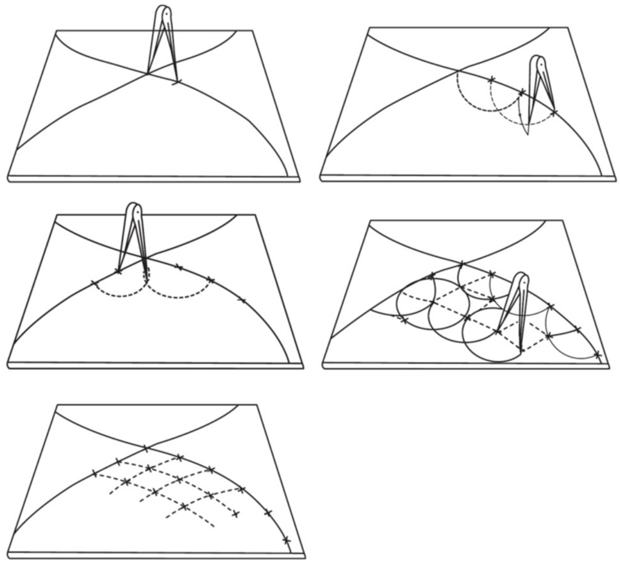

Figure 3: Construction of the grid using the compass method (Otto 1974)

So, to generate the grid of the gridshell, a 3D compass method can be performed on the surface. Obviously, the grid obtained has no mechanical meaning. The real shape of the gridshell is obtained later when the mechanical properties are considered. This method was used for the design of the gridshell for the Solidays festival (June 2011) and for the one of the “Cathédrale Ephémère de Créteil” (February 2013).

An implementation of the geometrical method has been developed at Navier laboratory, using Rhinoceros NURBS modeler. This modeler makes possible the modification of a surface through control points. This is very interesting because the compass method is also performed under the same numerical environment. Thus, modifications of the surface to mesh - but also modifications of the curves defining the mesh - are easy to do and the process of meshing is immediately auto-updated.

Let us sum up the grid drawing process: firstly, a shape for the structure is proposed by the architects. Secondly, the surface is extended and two main axes for the construction of the grid are drawn (figure 4, left). Thirdly, the mesh is generated using the compass method (figure 4, center). The mesh should cover all the functional surface and be quite homogeneous. When a satisfactory mesh is obtained, it is then trimmed to get the final mesh (figure 4, right).

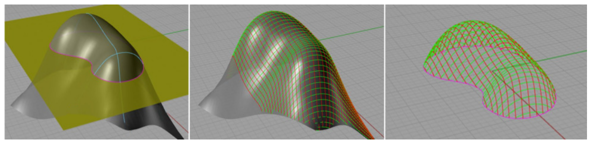

Figure 4: *meshing process of a 3D surface. Left: extension of the surface and drawing of the two curves. Centre: meshing of the extended surface. Right: cutting of the useful part of the mesh.*

Mechanical step: the actual shape of the gridshell is obtained by performing a non-linear structural analysis of the structure with real mechanical properties, as the previous step was purely geometrical. This step is the evolution from the input shape – 3D grid after the compass method – to the real shape taking mechanical considerations into account. The input shape could have been the flat grid with a slight perturbation but in this case, the form-finding would be much more costly. The non-linear algorithm used is based on dynamic relaxation algorithm [10], [16]. Many techniques are available to perform the form-finding, but dynamic relaxation is relevant in the case of gridshell. Indeed, dynamic relaxation is an explicit method which is quite easy to implement and for which the computational time is linear with the degrees of freedom, even for highly interconnected structures.

If the shape proposed by architects is suitable with the gridshell process, the geometry of the grid provided by the compass method is very similar after the geometrical step and the mechanical step. Once the real shape is found, classical structural analyses are performed with the standard loads. Obviously, the stress due to form-finding is taken into account in the structural analysis: the main source of stress is linked with the bending of the beams during the erection process. In other words, the stress σ is proportional to curvature of the beams $1/R$, (equation 1), and the curvature is mainly due to forming: even under critical loads, its shape (and so the curvature of beams, and also stress) does not change significantly. This is the main advantage of the active bending which provides high stiffness in this case.

$$\sigma = \frac{E\,y}{R} \qquad (1)$$

where E is the Young's Modulus and y the outer radius of the beam.

Designing a gridshell is a difficult task. As a guideline, the designer should check that:

- The curvature in each bar is not too high, to avoid the beams breaking, even with relaxation and fatigue phenomena. In practice, according to Eurocomp [15], the maximal stress in the bar must not exceed 30% of the strength of the beam. This limit stress gives a limit curvature under which the risk of break is low enough to be acceptable (equation 1).

- The entire surface is meshed

- The mesh does not get too concentrated locally

If the grid is too weak to support the external loads, the designers have to reinforce it by reducing the size of the mesh and/or modifying the geometry of the cross section of the beams. If the outer radius is increased, the stress due to the form-finding gets higher as the maximal stress in a beam is proportional to both the curvature and the outer radius of the cross section of the beam

Taking wind and snow loads into account presupposes that the gridshell is covered by a membrane. The membrane is made according to the geometry obtained after the mechanical step. Given the geometry, the surface is partitioned in planar surfaces (with a tolerance depending on the material of the membrane). Then the pieces are sewed to form the three-dimensional membrane. Figure 5, some pieces of membrane (the yellow ones) can easily be identified thanks to colouring. The membranes are PVC coated sheeting.

Figure 5: *Long term erected composite gridshell. Some planar parts (yellow) of the membrane can be seen.*

Once the iterative numerical design process is achieved, the building phase can begin.

5. PROTOTYPES

First prototypes: to demonstrate the feasibility of composite gridshells, four full scale prototypes of composite material gridshells have been built. The two first ones were built on the campus of the Université Paris-Est. The first prototype was a purely experimental structure which was tested under several loading conditions in order to investigate the behavior of gridshell structures and to compare it with the numerical models (figures 6 and 7). Detailed results of these tests can be found in [9]. The behavior of the prototype is very close to numerically performed simulations, with the dynamic relaxation algorithm presented in [7], [16]. This gridshell had been used for five years as a shed for equipment. It has been dismantled last year but it retains great importance since it is the one which have stayed erected for the longest time. Now the beams have been retrieved and are investigated to understand long term evolutions (mainly creep and fatigue).

Figure 6: *First experimental gridshell under testing.*

Prototypes sheltering people: as previously written, two gridshells built to house people have been recently made. The first one for the Solidays festival (June 2011) and the last one built to temporarily replace the Creteil Cathedral (February 2013 for at least 2 years of use). More details about the context of this gridshell and about the project can be found in [17]. Compared to the two first experimental gridshells, these two are larger and had to take many new aspects into consideration for public safety.

These two last gridshells, built in collaboration with the engineering company T/E/S/S, have got several improvements. First, their size were so large that most of the tubes of the structure had to be built from several tubes joined together longitudinally. Second, the gridshells had to obtain a clearance from the authorities to house people for a specified period. This clearance was given after a committee had validated both the design and the construction process.

The shape of the Solidays Gridshell was looking like a half peanut (two domes) while the Ephemeral Cathedral looks more like a stretched dome structure. The dimensions of these structures are quite similar: around 7 m high, 25 m long and 15 m wide. They are constituted of about 2 kilometres of pultruded unidirectional tubes from Topglass (polyester resin from DSM + Owens Corning glass fibres) with a Young's modulus of 25 GPa and a limit stress of 400 MPa. The available length and diameter of the tubes are respectively 13.4 m and 41.7 mm; the wall thickness of the tubes is 3.5 mm.

Given the short period of time for the projects, the geometries of the membrane of these two last gridshells were drawn according to the shapes obtained numerically.

Computation: the computation has been performed for different mesh sizes. It appears that a mesh size of one meter was acceptable to withstand the loads studied (dead weight, wind, snow). Under these loads, it is important to check that the stress remains acceptable in all the elements of the structure, but since the stress in the bars is mainly due to the form-finding, if the structure has been cautiously designed the stress might not reach too high values. In this case, the stresses are very close to the ones obtained for the gridshell without any load applied (figure 7).

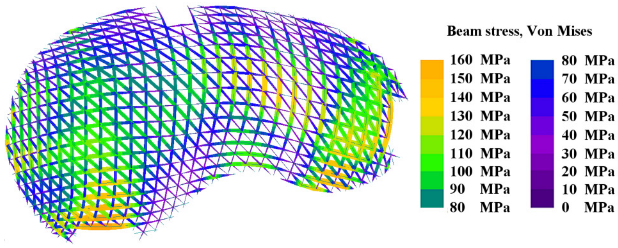

Figure 7: *Stress resulting from forming in the Solidays gridshell.*

Fabrication: once the form of the structure was defined, the coordinates of the extremities were picked up and precisely reported by geometers on site and stakes were positioned. Then, the grid was assembled flat on the ground: tubes were cut to the right dimensions with hacksaws and connected to the others with standard swivel scaffolding elements (figure 8, left). These scaffolding elements allow rotation around their axis. They have been chosen for their low cost due to industrial production.

Then the grid was deformed and shifted by two cranes that hook up the grid in several places (figure 2). The final form was reached when the extremities of the beams were fixed on the anchorages. The erection phase required only a few hours' work for about ten people whereas the preparation of the grid can take many days.

The following structural step was the bracing. This step is essential as, before bracing, the grid still holds its shear degree of freedom. The bracing transforms every deformable quadrangle into two rigid triangles, in order to obtain a shell behavior. The third direction of beam was installed as shown in figures 8 (center and right) with the use of new scaffolding elements. Once the bracing is installed, the gridshell gets its full mechanical properties and its stiffness becomes about twenty times the stiffness of the grid before bracing [8]. The bracing step does not visually change the form of the gridshell, but since the bracing cannot be done everywhere at the same time, it may modify the shape a little. This step is the most fastidious one because the third layer of beams has to be set up in the deployed geometry. Thus the operators have to adjust each connector and tighten the beam inside it, generally in a basket, a few meters over the ground.

Figure 8: *Left: Joint detail. Centre: Mesh before bracing. Right: Mesh after bracing*

Once the structural part finished, the positioning of the membrane could be started. First the PVC coated membrane was pulled above the gridshell (figure 9, up). In order to fix the canvas, a girder following the contour of the gridshell was set up 10 cm above the soil. For the Creteil gridshell, this girder was a pultruded rod able to support a large amount of shear stress as well as high curvatures (here hollow cross-sections are not suited and the outer radius has to be smaller than for structural beams). The canvas was then positioned and stretched. This step was supposed to be critical as polypropylene-PVC coated canvas is almost not stretchable, and was manufactured according to the geometry of the numerical model. So a mistake during the numerical design or during the building phase could have led to a situation where the canvas would not really fit to the structure. As the gridshells were accurately set up, the canvas fitted to their shape. No wrinkle was observed (figure 9, down). The membrane might play a part in the structural behavior of the grid but given the high dependence to modelling (in particular to friction between beams and membrane and also between connectors and membrane), it is very difficult to evaluate accurately the real stiffening effect of the membrane.

Figure 9: *Up: Positioning of the PVC coated membrane (Creteil). Down: Membrane without wrinkle (Solidays gridshell).*

Improvements: to deal with the fact that these last gridshells are made to shelter people many improvements have been added to the previous prototypes. In particular, fire, waterproofing, lightning and thermal considerations have been added to the primary mechanical considerations. Nevertheless mechanical properties have been considered with much more attention to ensure public safety. The reference construction guide - named Eurocomp, for composite materials – guided the construction. In particular, the way of production of beams, their constitutive materials as well as the characteristic time of solicitations acting on the structure were taken into account. Moreover, many assays were performed to get the real properties of the beams (mean strength, variation coefficient).

In addition, a robustness study has been performed on the Solidays gridshell [18]. This study showed that the gridshell can undergo accidental situation such as vandalism without risking collapse. Indeed, thanks to redundancy, the stress from a break would spread largely and the stress in the neighboring beams would not get too high. At the same time, large displacement of broken beams would be visible and the evacuation of public could be launched. This kind of ductility is named pseudo-ductility in this case when fragile materials are mainly used. This study has also showed that the buckling of the gridshell has to be avoided at any cost: if buckling starts, the curvature of some beams would highly increase and the stress in these beams would increase at the same time. Then, if the load increases a little more the stresses can increase highly due to buckling, which can lead to damage and to the ruin of the structure.

Other improvements were done, for the connection of the grid with the soil (figure 10, left) and also for the assembly of 13 m pipes to form long beams up to 35 m (figure 10, right). The engineering company T/E/S/S highly participated in the development of these assembly devices. The difficulty is to make connections able to transmit stress in a way that the assembled beams keeps the mechanical properties of the primary GFRP beams. In particular, the joining up of two beams have to:

- transmit normal stress for structural stiffness

- have similar bending stiffness for the continuity of the global shape

Thus, to combine these requirements, the system presented on the figure 10, right, was chosen. Each beam was assembled with a slightly larger sleeve using three pins. This assembly can undergo axial forces up to 30 kN. This theoretical value (obtained with the help of [15]) has been experimentally validated thanks to a salvo of assays. In addition, some extra glue was put inside the assembly to prevent from relative movements and thus fatigue phenomena. Then, between the sleeves of the two connected beams, a M20 threaded rod is screwed inside bolts welded to the sleeves. This threaded rod was designed to behave like the structural beams, under bending stress.

Figure 10: *Connections devised jointly by the Navier laboratory and the company T/E/S/S.*
Left: Pin anchorage for beams.
Right: Assembly used to join two beams.

For theses prototypes, the numerical model only took axial stress and bending moment into account. This numerical model, based on dynamic relaxation, was quite simple since the inputs were only the positions of control nodes [9]. The limitation of such a model is that torsion in the elements is neglected. For beams with isotropic cross sections, this model provides good results in term of shape, as the bending/torsion coupling is very low. An isotropic section is generally a circular or square section. The bending/torsion coupling exists because of the eccentricities of the connections. Indeed, the axial forces of a beam might create moments in the other beam linked by the connection. Yet it has been shown [19] that a combination of bending and torsion can lead to fracture for pultruded profiles, even for low torsion rates. In addition, such a model is not relevant for beams with anisotropic cross sections, which is an important limitation. This is the reason why a new model has been developed.

6. DEVELOPMENT OF A MODEL DEALING WITH TORSION

A new model taking torsion into account has been developed at Navier laboratory. This model is based on the works of Bergou [20] and Audoly [21]. We adapted these works to dynamic relaxation. The model is enriched with a new input: besides the position of the neutral axis of the beam, $\boldsymbol{x}$, a new field of angles θ makes possible the knowledge of the orientation of the sections. This field of angles is the angle of rotation between a field of reference frames and the field of material frames $(\boldsymbol{d_1}, \boldsymbol{d_2}, \boldsymbol{t})$, where $\boldsymbol{t}$ represents the tangent. The chosen reference frames are the Bishop frames [22], [23].

Thus, two beams having identical neutral axis can be differentiated, as illustrated in the figure 11.

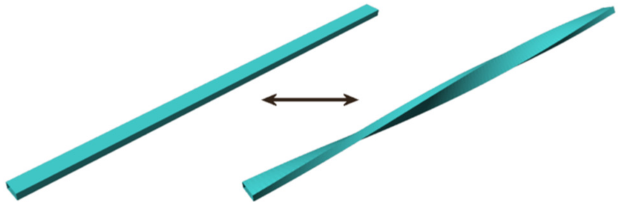

Figure 11: *Rate of torsion of a rectilinear beam*

Given the positions $\boldsymbol{x}$, the angles θ, and their equivalent $\bar{\boldsymbol{x}}$ and $\bar{\theta}$ in the rest configuration, it is possible to compute the linear internal forces and moments acting on the beam, by differentiating the elastic energy .

For a clamped beam, the linear internal force $\boldsymbol{f}$ and the linear internal torsional moment m are respectively given by the equations 2 and 3:

$$\begin{aligned}\boldsymbol{f}(s) &= -\left[(\boldsymbol{\omega}-\overline{\boldsymbol{\omega}})^T B \begin{pmatrix} \boldsymbol{d_1}^T \\ \boldsymbol{d_2}^T \end{pmatrix}\right]''(\boldsymbol{s}) \\ &+ \left[\left(\int_s^L (\boldsymbol{\omega}-\overline{\boldsymbol{\omega}})^T B(s)\ R_{\pi/2}\,\boldsymbol{\omega}\,dt\right)(\kappa\boldsymbol{b})^T\right]'(s) \\ &+ (GJ)(L)\left(\theta'-\overline{\theta'}\right)(L)\,(\kappa\boldsymbol{b})'(s)\end{aligned} \tag{2}$$

$$m(s) = \left[GJ\left(\theta'-\overline{\theta'}\right)\right]'(s) + \left(\boldsymbol{\omega}(s) - \overline{\boldsymbol{\omega}}(s)\right)^T B(s)\ R_{\pi/2}\,\boldsymbol{\omega}(s) \tag{3}$$

Where $\boldsymbol{\omega} = \begin{bmatrix} \boldsymbol{x}''.\boldsymbol{d_1} \\ \boldsymbol{x}''.\boldsymbol{d_2} \end{bmatrix}$, B is the bending stiffness matrix, $\kappa\boldsymbol{b}$ the curvature binormal, $R_{\pi/2}$ is the rotation matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, G is the shear modulus and J the constant of torsion.

The $'$ notation corresponds to the partial derivative with respect to the curvilinear abscissa s.

Knowing the internal and external forces acting on the beam and the boundary conditions, the equilibrium configuration can be computed thanks to the dynamic relaxation method. This model can deal with various boundary conditions, such as hinge and built-in. As an output, the model provides the geometry of the beam and the stresses, as illustrated in the figure 12 where the moment of torsion is reported.

The model has been implemented in Python language in the software Grasshopper. This software presents the advantage of being coupled with the drawing software Rhinoceros, which provides high interactivity between model and geometry. Several comparisons have been made with the Finite Element software Abaqus. Abaqus was run with B31 Timoshenko beam elements, in a non linear static analysis. Some results are presented in the figure 12. For identical mesh refinement these two methods have been compared. The accuracy of the model developed is very good: a relative difference smaller than 1% has been obtained between the stresses and displacements given by our model and the ones given by Abaqus. This accuracy remains even in case of strong bending/torsion coupling (figure 12 for example).

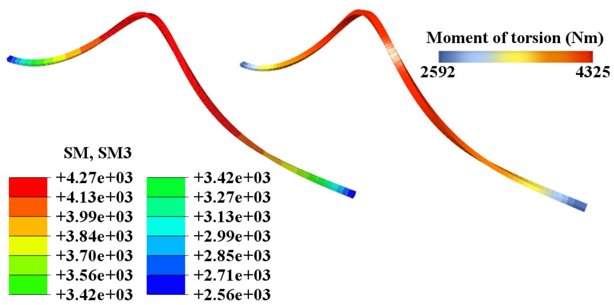

Figure 12: *Comparison between the FE method (Abaqus, left) and the model implemented (right)*

A model for links has also been implemented, enabling the modelling of structures involving linked beams, such as gridshells. It becomes possible to compare the behavior of gridshells composed of square section beams (figure 13, left) or composed of oblong section beams (figure 13, right). The influence of the beams section on the shape is thus underlined. As a result, in order to create beam structures with original shapes, the architect and the engineer could use beams with oblong section.

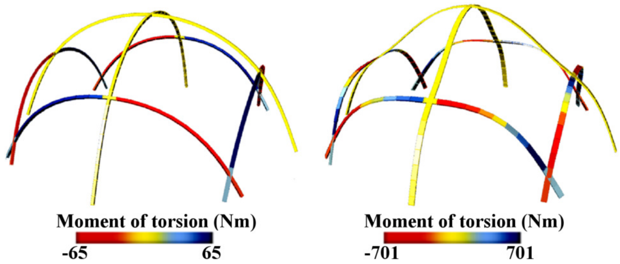

Figure 13: *Equilibrium shapes of two gridshells identical at the flat stage. On the left, the beams have square-sections, on the right the sections are oblong. The shapes are rather different and the moments of torsion really different.*

This study enriches the dynamic relaxation tool used for modelling gridshells, by adding the influence of torsion. It enables the designer to use beams with anisotropic cross sections, and gives him access to the stresses induced by torsion. It should thus help him to prevent the fractures due to bending/torsion coupling shown in [19] for beams with isotropic cross sections.

7 CONCLUSION

The design, fabrication and improvements of the gridshells are developed in this paper. The geometrical and mechanical design steps are explained, as part of an iterative design process. Several gridshells have been constructed, with progressive improvements. A major difficulty consists in ordering the membrane according to the numerically obtained shape. The PVC coated membrane being almost non-stretchable, the conception and the fabrication have to be absolutely mastered. Finally, a major and recent improvement is presented. It consists in taking torsion into account, in the dynamic relaxation algorithm. The model developed is far more general since it can deal with beams in which torsion is not negligible, as oblong beams. The new model, which can deal with any structure made of beams, has been compared with finite element method with good results in accuracy and computational time.

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