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The analytics of the New Keynesian 3-equation Model

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Abstract: This paper aims at providing a self-contained presentation of the ideas and solution procedure of New Keynesian Macroeconomics models. Using the benchmark "3 equation model", we introduce the reader to an intuitive, static version of the model before incorporating more technical aspects associated with the dynamic nature of the model. We then discuss the relative contribution of supply, demand and policy shocks to the fluctuations of activity, inflation and interest rates, depending on the key underlying parameters of the economy.

Keywords: Dynamic IS curve, impulse response analysis, New Keynesian Macroeconomics, New Keynesian Phillips Curve, output gap, Taylor rule.

JEL codes: C63, E12, E32, E52

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Introduction

Keynesian ideas returned to the forefront of academic research in the mid 90's in a new guise to address questions related to unemployment, economic fluctuations and inflation. This followed a twenty year period that witnessed the domination of new classical ideas on questions about both monetary and real macroeconomics. Before contributing to the building of what is now considered as the workhouse of modern macroeconomics (Carlin and Soskice [2014]), the New Keynesian School proposed in the 80's a series of models aimed at providing microeconomic foundations to price and/or wage rigidity[1] and at showing that this key feature of the real world can be explained in a setting with optimizing agents with market power. An important breakthrough was about 15 years ago, with the papers of Goodfriend and King [1997] and Clarida, Gali and Gertler [1999]. These contributions introduced a framework mixing Real Business Cycle features with nominal rigidities. This setting now forms the basic analytical structure of contemporaneous macroeconomic models as exemplified by Woodford [2003] or Gali [2008].

Besides new ideas and a new modelling strategy this New Keynesian Synthesis (NKS) has adopted new solution procedures that may appear cumbersome to non-specialists. Because of their recursive structure NKS models do not admit a closed form solution but should be solved by borrowing procedures developed for the analysis of stochastic discrete time dynamics systems[2]. The aim of this paper is to provide a compact and self-contained presentation of the structure and of the standard solution procedure of the basic NKS framework known as the "three

[1] on New Keynesianism, its history, development and significance for modern economics, see for example Bludnik [2009] or Romer [1993].

[2] For an up to date exhaustive introduction to this literature see Miao [2014]

equation model". We particularly separate the main ideas conveyed by this model, using a static version of the reference framework, from the technical aspects of the solution procedure. In the presentation we emphasise the qualitative similarities between the simple graphical analysis of the static model and the Impulse Response Functions (IRFs) of the model following the occurrence of exogenous shocks. We then illustrate the key features of this model in respect of the analysis of business cycles characteristics.

The paper is organized as follows: In the first section we introduce the general structure of a benchmark NKS model that combines (the log linear versions of) a Philips' curve, an Euler equation and a monetary policy (Taylor) rule[3]. In the second section we set a simple static version of the model to obtain closed form solutions for the key macroeconomic variables and to provide the reader with a graphical analysis of the consequences of demand and supply shocks. In the third section we introduce the Blanchard-Kahn solution procedure to get IRFs and dynamic reactions of the model around a stable steady state following exogenous supply demand and policy shocks. This third section is also devoted to a discussion of business cycles characteristics of the model. Section four concludes.

1. The 3 equation new Keynesian model

The New Keynesian Synthesis (NKS) mixes the methodology of Real Business Cycles (RBC) with nominal and real rigidities to characterise short run macroeconomic developments. More particularly the NKS seeks to explain the macroeconomic short run evolution of an economy subject to real and monetary shocks and to replicate business cycle statistics. The core representation of this synthesis has given rise to what is called the "3-equation model" as the basic NKS

[3] In the appendix, we provide the micro foundations of the framework used in this paper.

setting reduces to a simple system of three equations corresponding to an AS-AD model. First, the AS curve is represented by the New Keynesian Phillips curve that relates inflation to the output gap. Second, the AD component of the model combines a dynamic IS curve (that relates the evolution of the output gap to the interest rate) and a MP (Monetary Policy) schedule (that describes how the nominal interest rate is set by the central bank following fluctuations in the output gap and in the inflation rate. This model is based on agents‘ micro founded decision rules where consumers maximize their welfare subject to an intertemporal budget constraint and where firms maximize their profit, subject to nominal rigidities, characterising the imperfect adjustment of prices on the goods market. For convenience the micro foundations of this model and the derivation of the log-linear system are presented in appendix. These equations in turn determine three main variables of interest in a closed economy, namely the output gap ($\hat{y}_t$) which is the gap between the effective output and potential output, the inflation rate ($\hat{\pi}_t$) and the nominal interest rate ($\hat{r}_t$). Formally, the model is defined as follows:

The New Keynesian Philips' Curve (PC) links current inflation ($\hat{\pi}_t$) to expected future inflation ($E_t\{\hat{\pi}_{t+1}\}$), to the current output gap ($\hat{y}_t$) and to an exogenous supply shock that takes the form of a cost push shock (ε_t^S)[4]:

$$\hat{\pi}_t = \beta E_t\{\hat{\pi}_{t+1}\} + \kappa\hat{y}_t + \varepsilon_t^S. \tag{1}$$

As shown in appendix this relationship comes from the aggregation of the supply decision of firms that have market power and can re-optimize their selling price with discontinuities (*i.e.* nominal rigidities - they cannot modify their selling price at any point in time). , Thus they set the selling price of their product depending

[4] In the paper all parameters are positive.

on three main criteria. (*i*) The first criterion is anticipated inflation: as firms cannot re-optimize their price, they take into account future inflation to set their price today. (*ii*) The second term is the output gap: when firms set their price they take into account the difference between supply and demand so that inflation reflects stresses on the goods market: firms increase their prices during periods of expansion ($\hat{y}_t > 0$) whilst they decrease it during recessions ($\hat{y}_t < 0$). (*iii*) Finally, this relation incorporates a cost push term ε_t^S (such that $\varepsilon_t^S > 0$ may indicate an increase in raw materials or energy price in the economy). Actually we assume that ε_t^S is an AR(1) process[5]: $\varepsilon_t^S = \rho^S \varepsilon_{t-1}^S + \eta_t^S$ with $\eta_t^S \sim N(0, \sigma_S^2)$ and iid. The New Keynesian Phillips' Curve is derived from the Calvo model [1983] which combines staggered price-setting by imperfectly competitive firms. As presented in the appendix, the Calvo approach assumes that in each period, only a fraction θ of firms, randomly chosen, can reset their selling prices[6]). Using this assumption, Clarida et al. [1999] show that the Phillips' curve then takes a particularly simple form in which inflation depends on the current gap between actual and equilibrium output as in the standard Phillips' curve but on expected future inflation rather than on past inflation.

The dynamic IS curve is a log linearization of the Euler bond equation that describes the intertemporal allocation of consumption of agents in the economy:

$$\hat{y}_t = E_t\{\hat{y}_{t+1}\} - \frac{1}{\sigma}(\hat{r}_t - E_t\{\hat{\pi}_{t+1}\}) + \varepsilon_t^D \qquad (2)$$

This relation has the same function as the IS curve in the IS-LM model. As shown in the appendix it arises from the intertemporal optimization of the welfare index of a representative consumer subject to its budget constraint. Once

[5] This assumption is generally adopted in the literature to characterize exogenous shocks (see for example, Gali [2008] for a discussion)

[6] Baranowski et al. [2013] propose an endogenous mechanism.

aggregated over consumer and log-linearized around the constant state this relation can be expressed in terms of the output gap ($\hat{y}_t$). The dynamic IS curve links the current output gap to the difference between the real interest rate ($\hat{r}_t - E_t\{\hat{\pi}_{t+1}\}$), to the expected future output gap ($E_t\{\hat{y}_{t+1}\}$) and to an exogenous preference shock ε_t^D (that represents a demand shock henceforth). The demand shock is generally described by AR(1) process of the form: $\varepsilon_t^D = \rho^D \varepsilon_{t-1}^D + \eta_t^D$ with $\eta_t^D \sim N(0, \sigma_D^2)$ and is iid.

The Monetary Policy schedule (MP) is based on the Taylor rule. It links the nominal interest rate (that is controlled by monetary authorities) to the inflation rate and to the output gap[7]:

$$\hat{r}_t = \phi^\pi \hat{\pi}_t + \phi^y \hat{y}_t + \varepsilon_t^R \tag{3}$$

In this variable ε_t^R denotes a monetary policy shock that follows an AR(1) process of the form: $\varepsilon_t^R = \rho^R \varepsilon_{t-1}^R + \eta_t^R$ with $\eta_t^R \sim N(0, \sigma_R^2)$ and iid. This shock identifies monetary policy decisions which imply deviations from the standard Taylor rule such as unconventional measures or to reshape the inflation expectations in the medium run. This MP schedule aims at replacing the standard LM curve commonly found in the standard AS-AD model. It proposes an up-to-date description of the behaviour of central banks that control a short run nominal interest rate instead of a monetary aggregate (Clarida et al. [1999]).

This 3-equation model is a stylised shortcut that encompasses supply and demand relations to determine how the three main macroeconomic variables of interest (the output gap, the inflation rate and the nominal interest rate) react to exogenous supply and demand shocks. In this short presentation we ignore more recent developments associated with the introduction of financial frictions that give rise

[7] In appendices, we provide an interest rate smoothing, in this section, we neglect these features such that $\rho = 0$.

to an acceleration phenomenon (see for example Poutineau and Vermandel [2015a,b]).

2. The solution for a static version of the model

This second section simplifies the previous system (1)-(3) to convey the main ideas of the NKS model. Following Bofinger, Mayer and Wollmershäuser [2006] and Poutineau and Vermandel [2015b] we neglect the dynamic aspects of the model and we concentrate on a static version of the framework[8]. This is helpful to obtain the reduced form for the main variable of interest and to understand intuitions regarding the working of the model using tools similar to the IS-LM and AD-AS frameworks. To obtain the static version of the model we firstly assume that the monetary authorities are perfectly credible in the conduct of monetary policy so that the private sector expects that they reach the targeted inflation rate in future, namely that $E_t\{\hat{\pi}_{t+1}\} = \pi_0$, where π_0 is the long-run targeted rate of inflation. Secondly, we assume that the economy is very close to full employment so that the authorities are able to close the output gap in the future, namely that $E_t\{\hat{y}_{t+1}\} = y_0$. Thus the gap between the real interest rate and the natural interest rate disappears. In this case we can express the monetary policy rule in terms of the real interest rate. Imposing these restrictions of the simplified static framework gives:

$$\pi = \pi_0 + \kappa y + \varepsilon^S, \tag{4}$$

$$y = y_0 - \sigma r + \varepsilon^D, \tag{5}$$

[8] Dynamic aspects will be reintroduced in section 3.

$$r = \phi^{\pi}(\pi - \pi_0) + \phi^{y} y + \varepsilon^{R}. \tag{6}$$

In equilibrium the values of the output gap y^*, the inflation rate π^* and the interest rate r^* solution to the model (4)-(6) are a linear combination of exogenous shocks:

$$y^* = \frac{y_0}{\Omega} - \frac{\sigma\phi^{\pi}}{\Omega}\varepsilon^{S} - \frac{\sigma}{\Omega}\varepsilon^{R} + \frac{1}{\Omega}\varepsilon^{D},$$

$$\pi^* - \pi_0 = \frac{\kappa}{\Omega} y_0 - \frac{\sigma\kappa}{\Omega}\varepsilon^{R} + \frac{\kappa}{\Omega}\varepsilon^{D} + \frac{\Omega - \kappa\sigma\phi^{\pi}}{\Omega}\varepsilon^{S},$$

$$r^* = \frac{(\phi^{\pi}\kappa + \phi^{y})y_0}{\Omega} + \frac{\phi^{\pi}\kappa + \phi^{y}}{\Omega}\varepsilon^{D} + \frac{\phi^{\pi}}{\Omega}\varepsilon^{S} + \frac{1}{\Omega}\varepsilon^{R},$$

where $\Omega = \left(1 + \sigma(\phi^{\pi}\kappa + \phi^{y})\right)$.

The adjustment of the output gap, the inflation rate and the nominal interest rate following alternative shocks is summarized in

Table 1. As shown in the first column a supply shock leads to a decrease in the output gap, (activity decreases below its natural level), and to an increase in the inflation rate and in the interest rate. As shown in the second column a demand shock leads to an increase in the output gap, (activity increases), in the inflation rate and the interest rate. As observed, the reactions of the variables of interest to exogenous shocks are clearly affected by the value of the parameters of the interest rate rule of the authorities ($\boldsymbol{\phi^{\pi}}$ and $\boldsymbol{\phi^{y}}$).

		Supply Shock ε^S	Demand Shock ε^D	Monetary Shock ε^R
Output gap	$\partial y/\partial\varepsilon$	$\frac{-\sigma\phi^{\pi}}{\Omega} < 0$	$\frac{1}{\Omega} > 0$	$\frac{-\sigma}{\Omega} < 0$
Inflation	$\partial\pi/\partial\varepsilon$	$\frac{1+\sigma\phi^{y}}{\Omega} > 0$	$\frac{\kappa}{\Omega} > 0$	$\frac{-\sigma\kappa}{\Omega} < 0$

Interest rate	$\partial r/\partial \varepsilon$	$\frac{\phi^{\pi}}{\Omega}>0$	$\frac{\phi^{\pi}\kappa+\phi^{y}}{\Omega}>0$	$\frac{1}{\Omega}>0$

Table 1: reduced form of the static model

To understand more clearly the reaction of the economy to supply and demand shocks we refer the reader to figures 1 and 2. Graphically the model can be represented as consisting of two panels: in the lower panel of each figure, the IS-MP block (equations (5) and (6)) presented in the (y, r) space focuses on demand side aspects and can be treated as a New Keynesian representation of the IS-LM framework; in the upper panel the AD-PC block presented in the (y, π) space determines the global equilibrium of the economy and can be treated as a New Keynesian representation of the AD-AS framework. The PC curve is shown by equation (4) and the AD curve is obtained by combining equations (5) and (6) and is defined in equation (7),

$$y = \frac{y_0}{1+\sigma\phi^{y}} - \frac{\sigma\phi^{\pi}}{1+\sigma\phi^{y}}(\pi-\pi_0) + \frac{1}{1+\sigma\phi^{y}}\varepsilon_t^{D} \qquad (7)$$

The consequences of the demand shock are presented in Figure 1. The first panel displays the adjustment of the inflation rate and the output gap. The second panel displays the adjustment of the demand side, accounting for the reaction of the central bank to the shock.

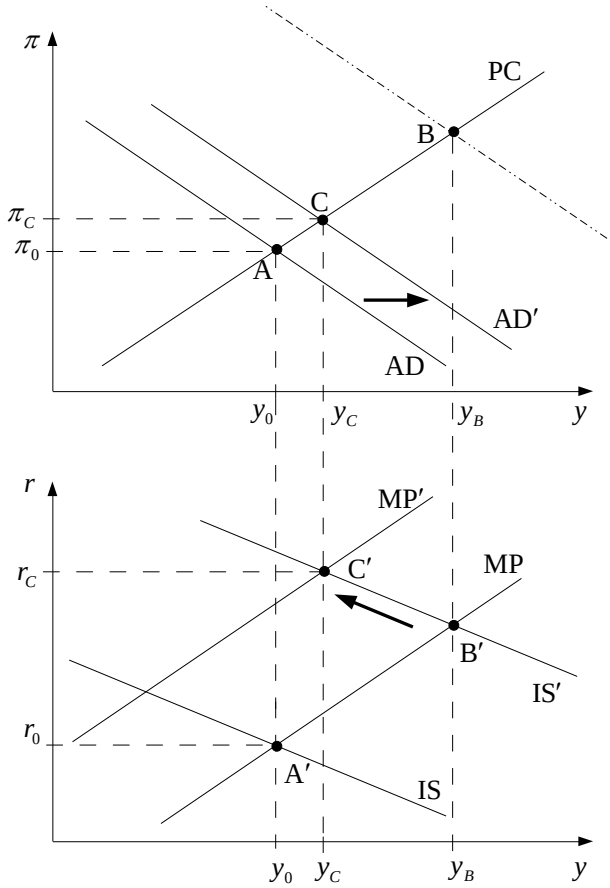

Figure 1: Demand shock

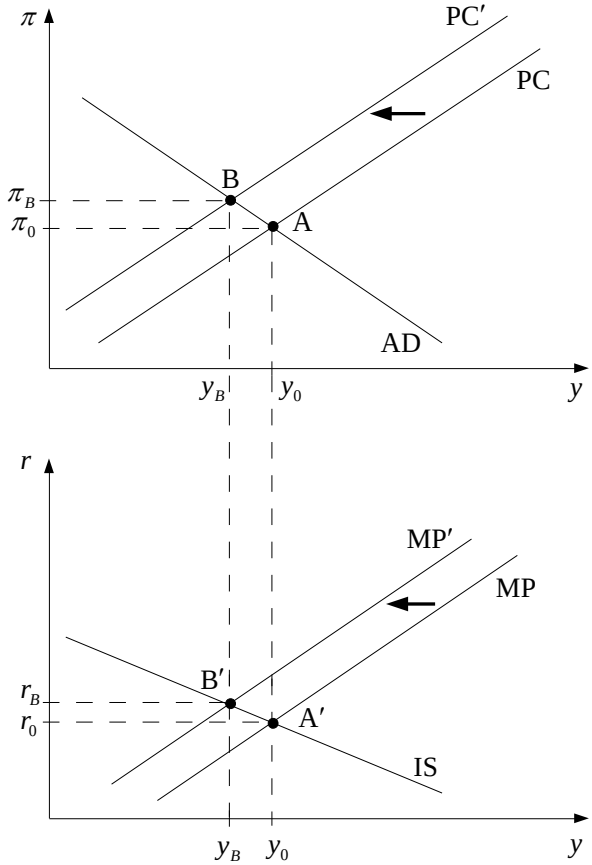

Figure 2: Supply shock

To understand the main differences between the two panels one has just to remember that the IS curve (5) moves one for one with a demand shock whilst it moves by less than one for the demand curve (7). Thus, taking point A as the initial equilibrium of the model a positive demand shock moves the IS curve from IS to IS' in the lower panel, which in turn, ignoring the reaction of the central bank, moves the demand schedule to the dotted line. As the temporarily equilibrium B implies an increase in the inflation rate the central bank reacts by increasing the interest rate for any value of the output gap. Thus, the MP curve in the lower panel moves left from MP to MP'. This, in turn, leads the aggregate demand curve to move to the left from the dotted line to AD'. In the final equilibrium C, the evolution of aggregate demand from AD to AD' – combines

both the initial demand shock and the monetary reaction – is less than proportional to the demand shock. Furthermore, with the reaction of the central bank the increase of inflation is dampened. Finally the positive demand shock leads to an increase in the output gap, an increase in inflation and a rise in interest rates, as summarized in Table 1.

The consequences of the supply shock are presented in Figure 2. In this example the supply shock is a positive inflation shock (that corresponds to a decrease in the supply of goods). Following this supply shock the Phillips' curve moves upwards to the left in the (y, π) space. This shock leads to an increase in the rate of inflation and the central bank reacts by raising the interest rate. Graphically the reaction of the central bank means increasing the interest rate for any value of the output gap so that the MP curve moves left to MP' in the lower panel of Figure 2. Once all the adjustments have been implemented the final equilibrium lies at point B which is characterized by a negative output gap (namely activity falls below its natural value), an increase of the inflation rate over its targeted value and at point B' an increase in the interest rate (needed to dampen part of the inflation consequences of the supply shock).

Finally, the balance between the consequences of the shocks on activity and inflation depends on the slope of the demand curve which, in turn, is affected by the reaction of the central bank to inflation rate and output gap developments. A more conservative central bank (namely a central bank that puts a higher weight on inflation and a lower weight on the output gap) makes the slope of the demand curve of the economy flatter in the upper panel of figures 1 and 2, which translates into lower fluctuations in the interest rate but to a higher variability of the output gap. Conversely if the stance of the central bank reaction is more sensitive to the output gap and less sensitive to inflation the MP and AD curves become steeper and shocks have a lower impact on activity and a higher impact on inflation.

3. The Fully-Fledged Model

In the dynamic version of the model (1)-(3), each period t corresponds to a quarter. As the fully fledged model does not allow a closed form solution it must be simulated around a stable steady state. The solution procedure, based on the Blanchard-Kahn [1980] approach[9], requires the choice of numerical values for the parameters of the model in order to compute Impulse Response Functions (IRFs hereafter) and the corresponding variance decomposition of the three variables of interest of the model.

a. The Solution Procedure

The solution procedure introduced by Blanchard and Kahn [1980] is based on matrix calculus and is aimed at selecting a unique stable dynamic path to describe the reaction of the variables following the occurrence of exogenous shocks. The Blanchard-Kahn condition defines a necessary criterion to get this result through the equality between the number of forward variables and the number of unstable eigenvalues. Practically the problem of the eigenvalues translates into the problem of appropriate values of the structural parameters of the model or their combinations. To be solved the model has first to be written in a state-space representation. For our linear model (1)-(3), defining $\Xi = \left(\sigma + \phi^{y} + \kappa\phi^{\pi}\right)^{-1}$, this representation is:

$$\begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \end{bmatrix} = \frac{\Xi}{\beta\sigma}\begin{bmatrix} \sigma & 1-\beta\phi^{\pi} \\ \sigma\kappa & \kappa+\beta(\sigma+\phi^{y}) \end{bmatrix}\begin{bmatrix} E_t\{\hat{y}_{t+1}\} \\ E_t\{\hat{\pi}_{t+1}\} \end{bmatrix} \cdot + \frac{\Xi}{\beta\sigma}\begin{bmatrix} -\sigma\varepsilon_t^{S} \\ \sigma\beta\varepsilon_t^{D} + \beta\varepsilon_t^{S} - \beta\varepsilon_t^{R} \end{bmatrix}, \tag{8}$$

The Blanchard-Kahn condition states that there are as many eigenvalues of the

[9] In this paper we adopted the Blanchard-Kahn approach for solving the model, given its anteriority and popularity in literature. However, the reader should be aware of the existence of other methods introduced by Klein (2000) and Sims (2000). Miao (2014) offers a nice comparison between these three approaches.

matrix $\mathbf{Z}_T = \begin{bmatrix} \sigma & 1-\beta\phi^{\pi} \\ \sigma\kappa & \kappa+\beta(\sigma+\phi^{y}) \end{bmatrix}$ greater than one in modulus, as there are non-predetermined variables. Since there are two forward-looking variables in the model (1)-(3) ($\hat{y}_t$ and $\hat{\pi}_t$), we know that there should be exactly two eigenvalues outside the unit circle to get one unique stable trajectory of each of the model's variable around the steady state. Given the form of the matrix $\mathbf{Z}_T$, the Blanchard-Kahn condition for the model (1)-(3) reduces to the following relation: $\kappa(\phi^{\pi}-1)+(1-\beta)\phi^{y}>0$.

Parameter	Value	Description
β	0.99	discount factor
σ	1	relative risk aversion
ϵ	6	elasticity of substitution amongst goods
φ	1	elasticity of marginal disutility with respect to labour
ϕ^{π}	1.5	influence of inflation rate in the interest rate rule
ϕ^{y}	0.5/4	influence of output gap in the interest rate rule
ρ^{S}	0.90	persistency of supply shock
ρ^{D}	0.90	persistency of demand shock
ρ^{R}	0.40	persistency of monetary policy shock
θ	3/4	probability of retaining old price

Table 2: Calibration of parameters
Source: Authors' synthesis

This condition reduces to the choice of appropriate values for the parameters of the model. A sufficiently relevant condition for the previous one is that the monetary authorities should respond more than proportionally to inflation developments (namely, $\phi^{\pi}>1$) according to the Taylor principle. In this case a rise in inflation leads to a more than proportional rise in nominal interest leading to an increase in real interest rates that affects agents' economic decisions and thus the real macroeconomic equilibrium of the model. The choice of parameters

is therefore a main feature of the analysis as it must both represent economic features and contribute to the Blanchard-Kahn condition. As presented in Table 2, following Galí [2008], we use a calibration of the model parameters that is normally selected in the literature. The intra-temporal elasticity between intermediate goods is set at 6 which implies a steady state mark-up of 20 % in the goods' market corresponding to what is observed in main developed economies. The sensitivity of the inflation rate to changes in the marginal cost (κ) is equal to 0.13 roughly. The value of the discount factor set at 0.99 implies that the steady state quarterly interest rate r equals one and the steady state real return on financial assets of about 4 percent per year. Average price duration amounts to three quarters which is consistent with empirical evidence[10]. The values of coefficients in the interest rate rule (3) are consistent with variations observed in the data on inflation and the interest rate given in the annual rates[11]. Because in our model periods are interpreted as quarters the output gap coefficient has to be divided by 4.

b. Impulse-response analysis

The mechanisms by which random innovations change into endogenous variables fluctuations may be illustrated by impulse response functions (IRFs). Each IRF isolates the impact of a particular shock throughout the economy. To document the response of activity, inflation and nominal interest we sequentially describe the consequences of a supply, demand and interest rate shock.

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[10] Galí, Gertler, López-Salido [2001] and Sbordone [2002] provide estimations based on aggregate data. Galí [2007] points also to some micro evidence.

[11] These values were originally proposed by Taylor [1999] as a good approximation of the monetary policy conducted by the Federal Reserve in years 1986-1999 when the head of the USA central banking system was Alan Greenspan. His monetary policy decisions largely followed standard Taylor rule criteria.

The demand shock: Figure 3 documents the consequences of a 1% positive demand shock. As observed the increase in goods' demand leads to an increase in activity so that the output gap becomes positive. However, as production overshoots its natural value this rise in activity increases the inflation rate. Since both the output gap and inflation rate increase the central bank should react by raising the nominal interest rate.

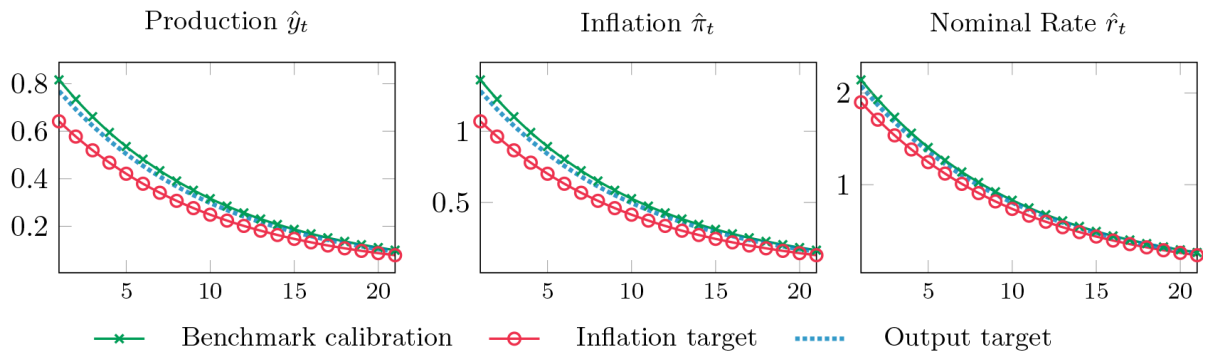

Figure 3: Effects of a 1% demand shock

Note: Benchmark regime is: $\phi^{\pi} = 1.5$, $\phi^{y} = 0.5/4$, inflation target regime: $\phi^{\pi} = 1.7$, output gap regime: $\phi^{y} = 0.8/4$.

According to the Taylor principle the nominal interest rate increases more than proportionally to inflation developments to affect real exchange rates. This policy however is not sufficient to close the positive output gap immediately or to dampen the inflation rate. The effect of monetary policy should be assessed over time on the output gap (activity goes back to its natural value as time passes) and on the rate of inflation (that converges towards its natural value). The adjustment time path is affected by the parameter value of the Taylor rule. As presented in Figure 3 a higher concern for inflation or output gap reduces the volatility of both activity and inflation. Thus, stricter monetary policy leads to more moderate responses of variables to the demand shock.

The supply shock: In Figure 4, the IRFs describing the consequences of a 1% increase in inflation (i.e. the negative supply shock acts as an increase in the price

of raw materials or energy that increases the real marginal cost of production) can be assessed as follows: This shock has a direct impact on inflation that rises and overshoots its targeted value. As a consequence monetary authorities should react according to the Taylor principle by raising the interest rate. Since the increase in the nominal interest rate is higher than the rate of inflation the real rate rises. This, in turn, negatively affects output that decreases under its natural value. However, as time passes, the increase in the interest rate dampens inflation. Finally, the output gap goes back to its steady state value whilst the inflation rate reaches its targeted value. As previously for the demand shock the time path of variables is affected by the parameter values of the Taylor rule. A higher concern for output gap (as represented with 'inflation target' IRF) leads to weaker responses of real variables and stronger responses of nominal variables. Inversely a higher concern for inflation leads to stronger responses of real variables and weaker responses of inflation and nominal interest rate.

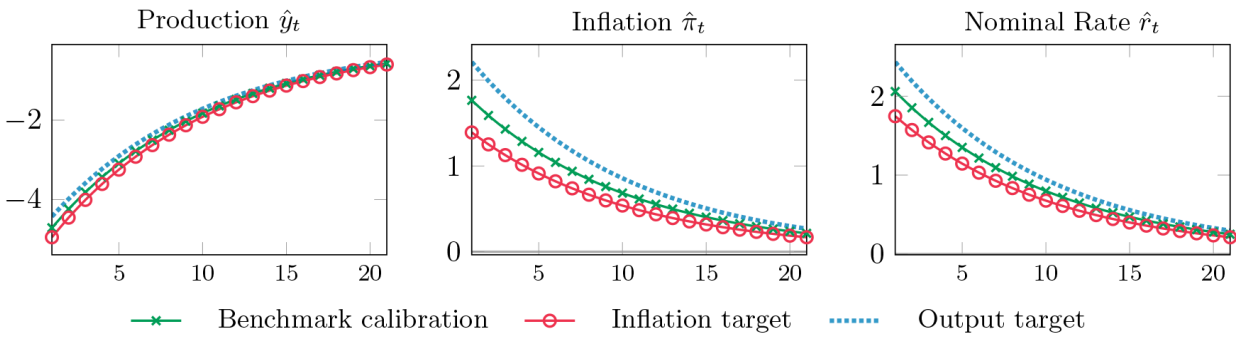

Figure 4: Effects of a 1% supply shock

Note: Benchmark regime is: $\phi^{\pi} = 1.5$, $\phi^{y} = 0.5/4$, inflation target regime: $\phi^{\pi} = 1.7$, output gap regime: $\phi^{y} = 0.8/4$.

The monetary policy shock: In Figure 5, the IRFs describing the consequences of a 1% increase in the nominal interest rate (corresponding to a 25 basis point increase in the exogenous shock measured in quarterly terms as presented in the figure) can be interpreted as follows: Because of sticky prices the initial increase

in the nominal interest rate implies a corresponding increase in the real interest rate at the initial period. This depresses demand in the economy as it leads households to delay their consumption through intertemporal consumption smoothing as reported in the Euler condition. Since activity is demand determined firms' production decreases. In the meanwhile the drop in demand generates deflation. The economy recovers overtime, since, according to the Taylor rule, a decrease in both activity and in the inflation rate leads to a reduction in the nominal interest rate after the initial period.

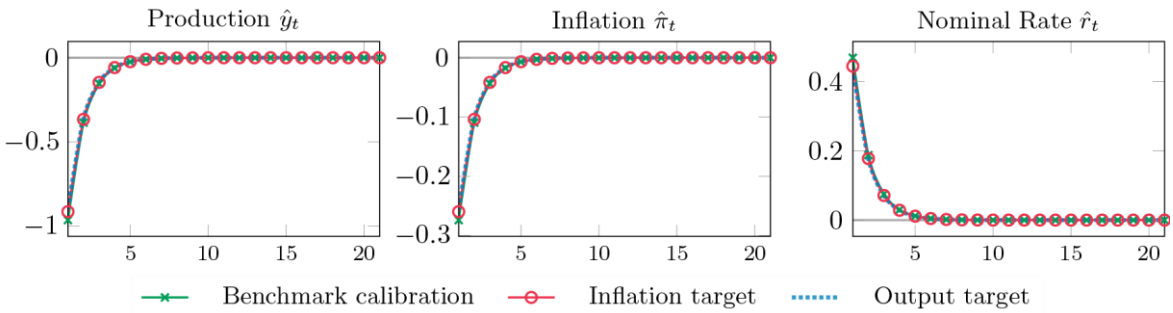

Figure 5: Effects of 1% a monetary policy shock

Note: Benchmark regime is: $\phi^{\pi} = 1.5$, $\phi^{y} = 0.5/4$, inflation target regime: $\phi^{\pi} = 1.7$, output gap regime: $\phi^{y} = 0.8/4$.

c. Business Cycle Statistics

IRF analysis aims at isolating the effect of a particular shock on the dynamics of endogenous variables. However, in real life situations, shocks occur both randomly and jointly to affect the macroeconomic equilibrium. The combined effect of supply and demand shocks over time is captured by historical variance analysis. The aim of this exercise is both to evaluate the relative contribution of each type of shock on the motion of macroeconomic variables over time and to appreciate how a particular design for economic policy may dampen the effect of one particular type of shock. Table 3 shows the variance decomposition of activity, inflation and the nominal interest rate under the benchmark calibration

of table 2 and evaluates the sensitivity of the benchmark results to alternative values of key behavioural and policy parameters of the model.

		Supply ε_t^S	Demand ε_t^D	Monetary Policy ε_t^R
1- Benchmark				
Production	$\hat{y}_t$	95.93 %	3.16 %	0.91 %
Inflation	$\hat{\pi}_t$	48.13 %	51.31 %	0.56 %
Interest rate	$\hat{r}_t$	63.00 %	36.65 %	0.34 %
2-Sticky economy $\boldsymbol{\theta = 0.95}$				
Production	$\hat{y}_t$	96.72 %	3.16 %	0.09 %
Inflation	$\hat{\pi}_t$	99.07 %	0.76 %	0.17 %
Interest rate	$\hat{r}_t$	99.02 %	0.08 %	0.00 %
3-Quasi-flexible economy $\boldsymbol{\theta = 0.01}$				
Production	$\hat{y}_t$	90.84 %	2.99 %	6.17 %
Inflation	$\hat{\pi}_t$	0.00 %	99.53 %	0.47 %
Interest rate	$\hat{r}_t$	0.00 %	93.72 %	6.28 %
4-Aggressive Monetary Policy $\boldsymbol{\phi^{\pi} = 2.5}$				
Production	$\hat{y}_t$	99.09 %	0.46 %	0.45 %
Inflation	$\hat{\pi}_t$	39.81 %	59.36 %	0.83 %
Interest rate	$\hat{r}_t$	62.49 %	36.36 %	1.15 %
5-Output-oriented monetary policy $\boldsymbol{\phi^{y} = 1}$				
Production	$\hat{y}_t$	96.02 %	3.16 %	0.82 %
Inflation	$\hat{\pi}_t$	89.10 %	10.85 %	0.06 %
Interest rate	$\hat{r}_t$	97.60 %	2.38 %	0.02 %

Table 3: Variance decomposition

In the first panel of Table 3 (Benchmark calibration), supply side shocks (namely price mark- up shocks) explain most of the output variability leaving only a marginal contribution (around 4%) to demand and interest rate shocks. In contrast the variability of the inflation rate is mainly explained by demand and monetary policy innovations. Finally, around 2/3 of interest rate variability is explained by real supply side shocks.

In panel 2 (sticky economy) and panel 3 (quasi flexible economy) we evaluate the sensitivity of the benchmark results to alternative assumptions regarding nominal rigidities. In the sticky economy only 5% of the total number of firms can reset their price each period. Whilst in the quasi flexible situation 99% of the total number of firms reset their prices each quarter. The main consequences can be assessed with regard to the contribution of supply side shocks to inflation and interest rates. Remarkably supply side shocks have no effect on either inflation or interest rates when prices are flexible. In contrast the fluctuations of the output gap are more sensitive to interest rate shocks whilst the effect of demand shocks on activity is almost unaffected.

In panel 4 and 5 we evaluate the sensitivity of the benchmark results to alternative assumption regarding the conduct of monetary policy. A monetary policy which is more aggressive in terms of inflation (panel 4) dampens the effect of demand shocks on activity (and in contrast makes output development more sensitive to supply shocks) and reinforces the impact of demand shocks on inflation (whilst, conversely, it dampens the impact of supply side shocks on this variable). Finally, this policy has almost no noticeable effect on the relative contribution of shocks on interest rate developments. In panel 5 an output oriented monetary policy increases the effect of supply shocks on inflation and interest rate whilst leaving the relative contribution of shocks on activity almost unchanged. The results obtained in these last two panels may serve as simple guideline to determine the

nature of monetary policy depending on both its objective and the origin of shocks. If an economy is mainly affected by price mark-up shocks monetary policy should be more closely oriented towards output developments. As this policy is able to dampen the effect of supply shocks on inflation, whilst having no noticeable effect on activity, monetary authorities are able to stabilise prices more easily. In contrast if the economy is affected by demand shocks the authorities have to use arbitrage as a more aggressive policy to fight inflation which dampens the impact of demand shocks on activity whilst it increases the impact of demand shocks on inflation.

Conclusions

In this paper we have described in a concise way the main ideas conveyed by the 3 equation New Keynesian model and the main elements of the solution procedure required to analyse the dynamics of the model. To introduce the reader to these types of models we have presented a simple static version of the model that gives both direct reduced forms and provides the basis for a simple graphical analysis of the macroeconomic equilibrium. We have then introduced the Blanchard-Kahn solution procedure and report IRFs to describe the dynamic adjustment of the economy over periods. Finally we have used the historical variance analysis to evaluate how a modification of the value of key parameters of the model affect the relative contribution of supply side and demand side shocks. Our aim was not to provide the reader with a comprehensive and up to date catalogue of all the results obtained by this New Keynesian literature but rather to offer a clear and simple presentation of the basic ideas and the required technical tools needed to solve this type of model that have become the conventional workhorse of today's macroeconomics.

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Appendices

A - Micro-foundations

A.1. Households

There is a continuum of households $j \in [0;1]$ with a utility function $U(C_t, H_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\varphi}}{1+\varphi}$, the representative household maximizes its welfare, defined as the expected stream of utilities discounted by $\beta \in (0,1)$:

$$\max_{C_t(j), H_t(j), B_t(j)} E_t \sum_{\tau=0}^{+\infty} \beta^\tau U(C_{t+\tau}(j), H_{t+\tau}(j)) \tag{A.8}$$

Under the budget constraint:

$$P_t C_t(j) + e^{\sigma \varepsilon_t^D} B_t(j) = R_{t-1} B_{t-1}(j) + W_t H_t(j), \tag{A.9}$$

where $\sigma > 0$ and $\varphi > 0$ are shape parameters of the utility function with respect to consumption and to labour supply whilst χ is the a shift parameter which scales the steady state labour supply to realistic values. As in Smets and Wouters (2005) we introduce an AR(1) demand shock process in the budget constraint of the representative household denoted ε_t^D.

After replacing the Lagrange multiplier the first order conditions are defined by the Euler bond condition:

$$\left(\frac{C_{t+1}(j)}{C_t(j)}\right)^\sigma = \frac{\beta}{e^{\varepsilon_t^D}} E_t \frac{R_t}{\pi_{t+1}}, \tag{A.10}$$

where $\pi_{t+1} = P_{t+1}/P_t$ is the inflation rate and the labour supply equation is determined by:

$$\chi C_t(j)^{\sigma} H_t(j)^{\varphi} = \frac{W_t}{P_t} \tag{A.11}$$

These equations define the optimal paths of labour and consumption and maximize the welfare index of the representative household.

A.2. Firms

The representative firm i maximizes its profits:

$$\max_{H_t(i), Y_t(i)} P_t(i) Y_t(i) - W_t H_t(i), \tag{A.12}$$

under the supply constraint:

$$Y_t(i) = H_t(i). \tag{A.13}$$

We suppose that firms solve a two-stage problem. In the first stage, firms choose labour demand in a perfectly competitive market. The first order condition is:

$$MC_t(i) = MC_t = \frac{W_t}{P_t} \tag{A.14}$$

where MC_t denotes the nominal marginal cost of producing one unit of goods.

In the second stage problem the firms cannot optimally set prices. There is a fraction of firms θ that are not allowed to reset prices , prices then evolve according to $P_t(i) = P_{t-1}(i)$ whilst for the remaining share of firms $1 - \theta$, they are able to set their selling price such that $P_t(i) = P_t^*(i)$, where $P_t^*(i)$ denotes the optimal price set by the representative firm given the nominal rigidity. The maximization programme is thus defined as:

$$\max_{P_t^*(i)} E_t \sum_{\tau=0}^{+\infty} \frac{\lambda_{t+\tau}^c}{\lambda_t^c} (\beta\theta)^\tau \left[P_t^*(i) - MC_{t+\tau}(i)\right] Y_{t+\tau}(i), \tag{A.15}$$

under the downward sloping constraint from goods' packers:

$$Y_{t+\tau}(i) = \left(\frac{P_t^*(i)}{P_{t+\tau}}\right)^{-\frac{\mu_{t+\tau}}{\mu_{t+\tau}-1}} Y_{t+\tau}, \qquad \tau > 0, \tag{A.16}$$

where $\mu_t = \frac{\epsilon}{\epsilon-1} e^{\gamma\varepsilon_t^S}$ is the time-varying mark-up, ϵ denotes the imperfect substitutability between different goods varieties, ε_t^S denotes the mark-up shock and γ is a shift parameter that normalizes the shock to unity in the log-linear form of the model as in Smets and Wouters [2005]. Since firms are owned by households they discount the expected profits using the same discount factor as households ($\beta^\tau \lambda_{t+\tau}^c / \lambda_t^c$). The first order condition is thus:

$$E_t \sum_{\tau=0}^{+\infty} \frac{\lambda_{t+\tau}^c}{\lambda_t^c} \frac{(\beta\theta)^\tau}{\mu_{t+\tau} - 1} \left[P_t^*(i) - \mu_{t+\tau} MC_{t+\tau}(i)\right] Y_{t+\tau}(i) = 0. \tag{A.17}$$

A.3. Authorities

To close the model the monetary policy authority sets its interest rate according to a standard Taylor Rule:

$$\frac{R_t}{\bar{R}} = \left(\frac{R_t}{\bar{R}}\right)^\rho \left(\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi^\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi^y}\right)^{1-\rho} e^{\varepsilon_t^R}, \tag{A.18}$$

where R_t is the nominal interest rate, π_t is the inflation rate, Y_t is the level of output and ε_t^R is an AR(1) monetary policy shock. Finally, parameters $\bar{R}$, $\bar{\pi}$ and

$\bar{Y}$ are steady state values for the interest rate, the inflation rate and GDP[12]. The central bank reacts to the deviation of the inflation rate and the GDP from their steady state values in a proportion of ϕ^{π} and ϕ^{y}, the central bank also smoothes its rate at one degree ρ.

A.4. Equilibrium conditions

After aggregating all the varieties supplied by firms the resource constraint for the economy is defined by:

$$Y_t = C_t \tag{A.19}$$

Whilst the aggregation between constrained firms and non-constrained firms leads to the following equation for aggregate prices:

$$(P_t)^{\frac{1}{1-\mu_t}} = \theta(P_{t-1})^{\frac{1}{1-\mu_t}} + (1-\theta)(P_t^*)^{\frac{1}{1-\mu_t}} \tag{A.20}$$

B - Linearization

To obtain the steady state of the model, we normalize prices $P = 1$ whilst we assume that households work one third of their time $H = 1/3$. Then we find:

$$\bar{C} = \bar{Y} = \bar{H},$$
$$\overline{W} = \overline{MC} = \frac{1}{\mu},$$
$$\chi = \bar{W}\bar{C}^{-\sigma}\bar{H}^{-\varphi}.$$

First, combining the Euler bond equation (A.10) and the resources constraint (A.19), *i.e.* $\hat{y}_t = \hat{c}_t$, then production is determined by:

[12]Under a credible central bank, $\bar{\pi}$ and $\bar{Y}$ also can be interpreted as the targets of the central bank in terms of inflation rate and GDP.

$$\hat{y}_t = \hat{y}_{t+1} - \frac{1}{\sigma}(\hat{r}_t - E_t\hat{\pi}_{t+1}) + \varepsilon_t^D. \tag{A.21}$$

The labour supply equation (A.11) in log-deviation is:

$$\widehat{w}_t = \sigma\hat{c}_t + \varphi\hat{h}_t, \tag{A.22}$$

where $\widehat{w}_t$ denotes the variations of the real wage. Up to a first order approximation of the firm price optimization solution (A.17) and the aggregate price equation (A.20), the linearized new Keynesian Phillips curve is:

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta}\widehat{mc}_t + \varepsilon_t^S. \tag{A.23}$$

Thus the real marginal cost is: $\widehat{mc}_t = \widehat{w}_t$ and the production function: $\hat{y}_t = \hat{h}_t$, then from the labour supply equation, the marginal cost can be simplified as: $\widehat{mc}_t = (\sigma + \varphi)\hat{y}_t$. Then the Philips' curve is:

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta}(\sigma + \varphi)\hat{y}_t + \varepsilon_t^S. \tag{A.24}$$

Finally, the monetary policy is determined by:

$$\hat{r}_t = \rho\hat{r}_{t-1} + (1-\rho)(\phi^\pi\hat{\pi}_t + \phi^y\hat{y}_t) + \varepsilon_t^R. \tag{A.25}$$

To summarize, our model is determined by the following set of three equations:

$$\begin{cases} \hat{y}_t = \hat{y}_{t+1} - \frac{1}{\sigma}(\hat{r}_t - E_t\hat{\pi}_{t+1}) + \varepsilon_t^D \\ \hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta}(\sigma + \varphi)\hat{y}_t + \varepsilon_t^S \\ \hat{r}_t = \rho\hat{r}_{t-1} + (1-\rho)(\phi^\pi\hat{\pi}_t + \phi^y\hat{y}_t) + \phi^{\Delta y}(\hat{y}_t - \hat{y}_{t-1}) + \varepsilon_t^R \end{cases}$$

Where shock processes are determined by:

$$\varepsilon_t^i = \rho^i \varepsilon_{t-1}^i + \eta_t^i, i = D, S, R \tag{A.26}$$