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Abstract: This paper proposes an active fault-tolerant control method for discrete-time linear systems with actuator faults. The main contribution of this paper lies in the capability of dealing with the actuator saturation and accepting the performance degradation. In this paper, a fault estimation observer is first designed by constructing an equivalent descriptor system model of the original plant. To accept performance degradation of the post-fault system, a degradation factor is introduced into the reference model. In the fault-tolerant control design, both the actuator saturation and performance degradation are considered, and the fault-tolerant control input and the degradation factor are simultaneously designed by solving a constrained optimization problem. Finally, an aircraft example is simulated to demonstrate the effectiveness and performance of the proposed method.

Keywords: Fault estimation, descriptor system, observer design, fault-tolerant control, performance degradation.

1. INTRODUCTION

In the past decades, fault diagnosis and fault-tolerant control techniques have been widely investigated due to their capabilities in improving safety and availability of a system. Fruitful results have been proposed in the literature, see e.g. Chen and Patton (1999), Blanke et al. (2006), Noura et al. (2009), Rodrigues et al. (2014), Rodrigues et al. (2007), Ye and Yang (2006) and the references therein.

In Lunze et al. (2003) and Steffen (2005), a new methodology called fault hiding approach has been proposed to achieve fault-tolerance. The fault hiding goal is to make the reconfigured system exhibit similar behaviour to that of the nominal plant. In this paradigm, the nominal controller is kept in the loop and a reconfiguration block is inserted between faulty plant and nominal controller to generate useful control signals so as to hide the impact of faults. As pointed out in Khosrowjerdi & Soheila Barzegary (2014), this approach provides a way for minimally-invasive alternations of the loop, which is the main advantage of the fault hiding approach. In recent years, the fault hiding methodology has been applied to piecewise affine, linear parameter-varying, and Lipschitz nonlinear systems, see in Steffen (2005), Richter (2011), Rotondo et al. (2014), Tabatabaeipour et al. (2015) and Khosrowjerdi & Soheila Barzegary (2014). In spite of wide investigation, there are still some problems to be addressed in the fault-tolerant control design based on fault hiding approach. First, many existing results do not consider the fault diagnosis problem but assume that fault has already been accurately estimated, which is obviously an impractical assumption. Second, in most of the existing fault hiding approaches, the actuator saturation is not considered. This may lead to severe problem when these approaches are applied to practical systems. Moreover, little attention has been devoted to the performance degradation in the fault-tolerant design. In practice, however, the control margin of a faulty system will degrade due to the presence of faults and actuator saturation. Therefore, the fault-tolerant control scheme should be able to accept performance degradation in order to keep the operation safety. To the best of our knowledge, only few papers (Zhang & Jiang (2003); Jiang & Zhang (2006); Theilliol et al. (2008); Qi et al. (2014)) investigated the performance degradation in fault-tolerant control design.

Considering the aforementioned problems, this paper proposes a fault-tolerant control method with consideration of the actuator saturation and performance degradation. The main contribution of this paper lies the following aspects. First, an augmented state observer by constructing a descriptor system representation is proposed to estimate

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the fault. Second, the actuator saturation is considered in the proposed fault-tolerant control. Moreover, a novel degradation factor is introduced in the fault-tolerant control design so that the proposed fault-tolerant controller has the capability of accepting performance degradation.

2. PROBLEM FORMULATION

Consider the following discrete-time linear system

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^p$ is the control input, and $y(k) \in \mathbb{R}^m$ is the output. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times p}$, and $C \in \mathbb{R}^{m\times n}$ are known constant matrices.

In this paper, only actuator faults are considered. The actuator faults are modeled as a multiplicative representation, i.e. the actuator faults change the input matrix B to B_f, which is given as

$$B_f = B\Lambda \tag{2}$$

Herein, $\Lambda \in \mathbb{R}^{p\times p}$ is a diagonal matrix which implies the impact of the actuator faults, i.e.

$$\Lambda = \begin{bmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_p \end{bmatrix} \tag{3}$$

In fact, γ_i represents the effectiveness factors of the ith actuator. $\gamma_i = 0$ means that the ith actuator is presented to be total failure, such as a stuck fault; $0 < \gamma_i < 1$ denotes the fault of the ith actuator may be a partial loss of control effectiveness; and $\gamma_i = 1$ implies the ith actuator is fault-free. For the convenience of fault estimation, the scalars λ_i, $i = 1, 2, \ldots, p$ are assumed to be constant or slow-varying.

Considering the actuator faults, the model of the faulty plant is given by

$$\begin{cases} x(k+1) = Ax(k) + B_f u(k) \\ y(k) = Cx(k) \end{cases} \tag{4}$$

Moreover, the actuator saturation is considered in this paper. That is, the control input $u(t)$ is subject to the following saturation constraint

$$u(k) \in \mathscr{U} := \{u \in \mathbb{R}^p | \underline{u}_i \leq u_i \leq \bar{u}_i\} \tag{5}$$

where $\underline{u}_i$ and $\bar{u}_i$ denote the minimum and the maximum actions of the ith actuator, respectively.

The aim of this paper is to design a fault-tolerant scheme such that the reconfigured state of the faulty plant (4) exhibits similar output to that of the following nominal model

$$\begin{cases} x_r(k+1) = Ax_r(k) + Bu_r(k) \\ y_r(k) = Cx_r(k) \end{cases} \tag{6}$$

where $x_r(k) \in \mathbb{R}^n$, $u_r(k) \in \mathbb{R}^p$ and $y_r(k) \in \mathbb{R}^m$ are the state, the control input and the output of the nominal system, respectively. Without loss of generality, it is assumed that $u_r(k)$ is designed such that the nominal output $y_r(k)$ tracks a given reference $r(k)$, i.e. $y_r(k) \to r(k)$.

Due to the presence of the actuator faults and saturation, it is reasonable to allow the faulty plant (4) to have certain performance degradation compared to the nominal model (6).

3. FAULT ESTIMATION OBSERVER DESIGN

Recently, descriptor system approach has been used to deal with fault diagnosis (Gao and Wang (2006), Gao and Ding (2007)). However, these works focus on sensor fault diagnosis. On the other hand, the descriptor system approach has not fully investigated in the actuator fault diagnosis. In view of this, this paper proposes an actuator fault estimation observer based on the descriptor system approach.

First, the actuator faults is converted into an additive representation by letting

$$f(k) = (I_p - \Lambda)u(k) \tag{7}$$

where $f(k) \in \mathbb{R}^n$ is an virtual equivalent fault, I_p is used to denote a $p \times p$ identity matrix.

By using (7), the model (4) becomes

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) - Bf(k) \\ \qquad\qquad\;\; = Ax(k) + B_f u(k) \\ y(k) = Cx(k) \end{cases} \tag{8}$$

Let

$$\bar{x}(k) = \begin{bmatrix} x(k) \\ f(k-1) \end{bmatrix} \tag{9}$$

the faulty system (8) is equivalent to

$$\begin{cases} E\bar{x}(k+1) = \bar{A}\bar{x}(k) + \bar{B}u(k) \\ y(k) = \bar{C}\bar{x}(k) \end{cases} \tag{10}$$

where

$$E = \begin{bmatrix} I_n & B \\ 0 & 0 \end{bmatrix}, \bar{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \bar{C} = [\, C \;\; 0 \,] \tag{11}$$

Using the relation in (7), the estimate of Λ can be easily obtained the additive fault term is estimated (this will be shown later). Moreover, according to the definition of $\bar{x}(k)$ in (9), the actuator fault $f(k-1)$ can be estimated if the estimation of the augmented state $\bar{x}(k)$ is obtained. Therefore, it can be concluded that fault estimation for system (4) is converted as an observer design problem for the descriptor system (10).

In the literature, there has been some results on observer design for discrete-time descriptor systems, see e.g. Dai (1988), Darouach et al. (2010), Wang et al. (2012). Among these results, Wang et al. (2012) proposes a new observer structure and presents a systemic design approach based on LMI technique. In this paper, the method in Wang et al. (2012) is used to address fault estimation problem. The proposed observer has the following form

$$\begin{cases} \zeta(k+1) = T\bar{A}\hat{\bar{x}}(k) + T\bar{B}u(k) + L\left[y(k) - \bar{C}\hat{\bar{x}}(k)\right] \\ \hat{\bar{x}}(k) = \zeta(k) + Ny(k) \end{cases} \tag{12}$$

where $\hat{\bar{x}}(k) \in \mathbb{R}^{(n+p)}$ is the estimate of the augmented state $\bar{x}(k)$ and $L \in \mathbb{R}^{(n+p)\times m}$ the gain matrix to be synthesized. Moreover, the matrices $T \in \mathbb{R}^{(n+p)\times(n+p)}$ and $N \in \mathbb{R}^{(n+p)\times m}$ are designed such that

$$TE + N\bar{C} = I_{n+p} \tag{13}$$

Remark 1. A necessary condition to use the observer (12) is the matrices E and $\bar{C}$ satisfy

$$\text{rank} \begin{bmatrix} E \\ \bar{C} \end{bmatrix} = n + p \tag{14}$$

With this assumption, T and N can be solved by using the Lemma 1 in Wang et al. (2015), i.e.

$$T = \Psi^{\dagger}\alpha_1 + S(I_{n+p+m} - \Psi\Psi^{\dagger})\alpha_1 \tag{15}$$

$$N = \Psi^{\dagger}\alpha_2 + S(I_{n+p+m} - \Psi\Psi^{\dagger})\alpha_2 \tag{16}$$

where $S \in \mathbb{R}^{(n+p)\times(n+p+m)}$ is an arbitrary matrix, $\Psi \in \mathbb{R}^{(n+p+m)\times(n+p)}$, $\alpha_1 \in \mathbb{R}^{(n+p+m)\times(n+p)}$, $\alpha_2 \in \mathbb{R}^{(n+p+m)\times m}$ are given by

$$\Psi = \begin{bmatrix} E \\ \bar{C} \end{bmatrix},\ \alpha_1 = \begin{bmatrix} I_{n+p} \\ 0 \end{bmatrix},\ \alpha_2 = \begin{bmatrix} 0 \\ I_m \end{bmatrix} \tag{17}$$

and $\Psi^{\dagger}$ denotes the pseudo-inverse of Ψ.

The following Theorem is presented to design matrix L.

Theorem 1. For the descriptor system (10) and a given symmetric positive definite matrix $Q \in \mathbb{R}^{(n+p)\times(n+p)}$, there is an asymptotic observer in the form of (12) if there exist a symmetric positive definite matrix $P \in \mathbb{R}^{(n+p)\times(n+p)}$ and a matrix $W \in \mathbb{R}^{(n+p)\times m}$ satisfying the following linear matrix inequality

$$\begin{bmatrix} -P-Q & (T\bar{A})^T P - \bar{C}^T W^T \\ PT\bar{A} - W\bar{C} & -P \end{bmatrix} < 0 \tag{18}$$

Moreover, the matrix L can be determined by

$$L = P^{-1}W \tag{19}$$

Proof. By using (13), the dynamic equation in (10) can be rewritten as

$$\bar{x}(k+1) = T\bar{A}\bar{x}(k) + T\bar{B}u(k) + Ny(k+1) \tag{20}$$

On the other hand, the observer equation (12) can be written as

$$\begin{aligned}\hat{\bar{x}}(k+1) &= T\bar{A}\hat{\bar{x}}(k) + T\bar{B}u(k) + Ny(k+1) \\ &\quad + L\left[y(k) - \bar{C}\hat{\bar{x}}(k)\right]\end{aligned} \tag{21}$$

Define

$$e(k) = \bar{x}(k) - \hat{\bar{x}}(k) \tag{22}$$

and subtract (21) from (20), the estimation error dynamics is obtained as

$$e(k+1) = (T\bar{A} - L\bar{C})e(k) \tag{23}$$

Take the following Lyapunov function

$$V_e(k) = e^T(k)Pe(k) \tag{24}$$

Then the difference of $V(k)$ is

$$\Delta V_e(k) = e^T(k)\left[(T\bar{A} - L\bar{C})^T P(T\bar{A} - L\bar{C}) - P\right]e(k) \tag{25}$$

It is obvious that $\Delta V_e(k) < 0$ if the following matrix inequality holds

$$(T\bar{A} - L\bar{C})^T P(T\bar{A} - L\bar{C}) - P < -Q \tag{26}$$

Using Schur complement Lemma, (26) is equivalent to

$$\begin{bmatrix} -P-Q & (T\bar{A} - L\bar{C})^T P \\ P(T\bar{A} - L\bar{C}) & -P \end{bmatrix} < 0 \tag{27}$$

By letting $W = PL$, the inequality (27) becomes (18). This completes the proof.

After the estimation of the augmented state $\bar{x}_f(k)$ is obtained, the fault estimation and state estimation can be obtained as follows

$$\hat{f}(k-1) = C_f\hat{\bar{x}}(k),\ \hat{x}(k) = C_x\hat{\bar{x}}(k) \tag{28}$$

where

$$C_f = \left[\,0\ \ I_p\,\right],\ C_x = \left[\,I_n\ \ 0\,\right] \tag{29}$$

According to equation (7), we have

$$f_i(k) = (1-\lambda_i)u_i(k) \tag{30}$$

Therefore, if $\hat{f}(k-1)$ is obtained, then λ_i, $i = 1, 2, \ldots, p$ can be estimated by

$$\hat{\lambda}_i = 1 - \left[u_i(k-1)\right]^{-1}\hat{f}_i(k-1),\ i = 1, 2, \ldots, p \tag{31}$$

It should be noted that a constant Λ assumption is used here. Using (31), the estimate of B_f is obtained as

$$\hat{B}_f = B\hat{\Lambda} \tag{32}$$

where $\hat{\Lambda}$ is given by

$$\hat{\Lambda} = \begin{bmatrix} \hat{\lambda}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \hat{\lambda}_p \end{bmatrix} \tag{33}$$

Remark 2. This paper introduces a user-defined matrix Q to provide more design degrees of freedom.

Remark 3. It is noted that the equation (31) involves the inversion of $u_i(k-1)$. When $u_i(k-1)$ is zero or nearly zero, in order to avoid singularity, the estimation $\hat{\lambda}_i$ can be set to $\hat{\lambda}_i = 1$ since it is no need to implement fault-tolerant control when $u_i(k-1)$ is closed to zero.

Remark 4. It is noted that the fault detection can be easily implemented using the fault estimation results. In fact, a residual signal can be obtained as

$$\gamma(k) = \begin{bmatrix} 1-\hat{\lambda}_1 \\ 1-\hat{\lambda}_2 \\ \vdots \\ 1-\hat{\lambda}_p \end{bmatrix} \tag{34}$$

Then the fault detection scheme can be set as follows

$$\begin{cases} \|\gamma(k)\| \le \epsilon & \text{fault-free} \\ \|\gamma(k)\| > \epsilon & \text{faulty} \end{cases} \tag{35}$$

where $\epsilon > 0$ is a prescribed detection threshold. If a fault is detected, the fault-tolerant controller will be triggered to suppress the impact of the fault.

4. FAULT-TOLERANT CONTROL SCHEME

Based on the estimation results provided by the fault estimation observer, a fault-tolerant control scheme is proposed in this section.

In the fault-tolerant control design, the control input $u(k)$ is designed such that the output of the faulty system (4) is as close as possible to that of the nominal model (6). To achieve this goal, the following cost function should be minimized

$$\mathcal{J}_0 = \left[y_r(k+1) - y(k+1)\right]^T\left[y_r(k+1) - y(k+1)\right] \tag{36}$$

where

$$y_r(k+1) = Cx_r(k+1) = CAx_r(k) + CBu_r(k) \tag{37}$$

is the desired output provided by the nominal model (6), and

$$y(k+1) = Cx(k+1) = CAx(k) + CB_fu(k) \tag{38}$$

is the real output of the faulty plant (4).

Since the $x(k)$ is not available, (36) cannot be used to design $u(k)$. Herein, the estimated output $\hat{y}(k+1)$ is used to substitute $y(k+1)$. Based on $\hat{x}(k)$ and $\hat{B}_f$, the estimated output $\hat{y}(k+1)$ is obtained as follows

$$\hat{y}(k+1) = Cx(k+1) = CA\hat{x}(k) + C\hat{B}_f u(k) \tag{39}$$

For the purpose of fault-tolerance, the estimated output $\hat{y}(k+1)$ is expected to be as close as possible to the desired output $y_r(k+1)$. Consequently, the following cost function should be minimized

$$\mathcal{J}_1(u) = [y_r(k+1) - \hat{y}(k+1)]^T [y_r(k+1) - \hat{y}(k+1)] \tag{40}$$

Theoretically, the fault-tolerant control input $u(k)$ can be obtained by solving the following problem

$$u^*(k) = \arg\min_u \mathcal{J}_1(u) \tag{41}$$

Nevertheless, in certain situations, it is impossible to perfectly reach the desirable performance due to the presence of faults and actuator saturation. In view of this, it is necessary to adjust the reference to keep the system safety. To this end, this paper proposes a reference adjustment method by modifying the nominal model (6) as

$$\begin{cases} x_r(k+1) = Ax_r(k) + Bu_r(k) \\ \tilde{y}_r(k) = Cx_r(k) - \rho r(k) \end{cases} \tag{42}$$

where

$$0 \leq \rho \leq 1 \tag{43}$$

is a scalar to be designed.

As mentioned before, the control aim is to make the output $y_r(k)$ track a reference $r(k)$. Unfortunately, it is necessary to accept performance degradation in some fault cases. To this end, the scalar ρ is introduced to modify the reference signal from $r(k)$ to $(1-\rho)r(k)$ so that the output is able to track it. In fact, the scalar ρ can be considered as a degradation factor. That is, if $\rho = 0$, the faulty plant with the fault-tolerant controller can achieve perfect performance recovery. The fault-tolerant controller can partially recover the desired performance if $0 < \rho < 1$. Finally, $\rho = 1$ implies that the post-fault system lose tracking capability and only can keep a safety operation by setting $\hat{y}(k+1) = 0$.

Now, the task of the fault-tolerant control design is to find the optimal values of $u(k)$ and ρ such that the estimated output $\hat{y}_f(k+1)$ is as close as possible to the adjusted output $\tilde{y}_c(k+1)$. To this end, the following cost function is presented

$$\mathcal{J}_2(u,\rho) = [\tilde{y}_r(k+1) - \hat{y}(k+1)]^T [\tilde{y}_r(k+1) - \hat{y}(k+1)] \tag{44}$$

Considering the actuator saturation (5) and the constraint (43), the fault-tolerant control input $u(k)$ and the degradation factor ρ should be obtained by solving the following constrained optimization problem

$$\begin{aligned} &[u^*(k), \rho^*] = \arg\min_{u,\,\rho} \mathcal{J}_2(u,\,\rho) \\ &\text{s.t.} \begin{cases} \underline{u}_i \leq u_i \leq \bar{u}_i, \ i = 1, 2, \cdots, p \\ 0 \leq \rho \leq 1 \end{cases} \end{aligned} \tag{45}$$

Remark 5. The proposed fault-tolerant control $u(k)$ can be seen as a model predictive control law. However, $u(k)$ does not have an explicit form since it is obtained by solving the constrained optimization problem (45). Only numerical algorithms are available for solving the constrained optimization problem. In this paper, the constrained optimization problem (45) is solved by the Matlab function *fmincon*.

5. SIMULATIONS

In this section, a simulation example is used to show the effectiveness of the proposed method. The considered example is a VTOL aircraft model taken from Tan & Edwards (2003). The continuous-time model has the following form

$$\begin{cases} \dot{x}(t) = A_c x(t) + B_c u(t) \\ y(t) = Cx(t) \end{cases} \tag{46}$$

where

$$A_c = \begin{bmatrix} -9.9477 & -0.7476 & 0.2632 & 5.0337 \\ 52.1659 & 2.7452 & 5.5532 & -24.4221 \\ 26.0922 & 2.6361 & -4.1975 & -19.2774 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix}$$

$$B_c = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.5200 & 4.4900 \\ 0 & 0 \end{bmatrix} \quad C_c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Using the Euler approximation method with a sampling period $t_s = 0.02$s, the continuous system (46) can be discretized into the form of (4) with the following parameters

$$A = I_4 + A_c t_s = \begin{bmatrix} 0.8010 & -0.0150 & 0.0053 & 0.1007 \\ 1.0433 & 1.0549 & 0.1111 & -0.4884 \\ 0.5218 & 0.0527 & 0.9161 & -0.3855 \\ 0 & 0 & 0.0200 & 1.0000 \end{bmatrix}$$

$$B = B_c t_s = \begin{bmatrix} 0.0088 & 0.0035 \\ 0.0709 & -0.1518 \\ -0.1104 & 0.0898 \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, the proposed method can be easily applied to the considered system. By simply choosing the matrix S in (15) and (16) as a null matrix, the matrices T and N are obtained as

$$T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 6.5287 & 0.7428 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 \\ 95.3441 & 2.2115 & 0 & 0 & 0 & 0 \\ 44.5137 & -5.5532 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$N = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -6.5287 & -0.7428 & 0 \\ 0 & 0 & 0.5 \\ -95.3441 & -2.2115 & 0 \\ -44.5137 & 5.5532 & 0 \end{bmatrix}$$

Then, choosing $Q = 0.1I_6$ and using Theorem 1 gives

$$L = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 6.5266 & 0.7387 & 4.3816 \\ 0 & 0 & 0.5817 \\ 78.6823 & 0.9073 & 11.7589 \\ 29.8637 & -6.5237 & 5.5359 \end{bmatrix}$$

In addition, $u(k)$ will be determined on-line based on the estimation result.

In the simulation, the initial conditions are set as $x(0) = [\,0\ 0\ 0\ 0\,]^T$ and $x_f(k) = [\,0.018\ {-0.024}\ 0.011\ {-0.02}\,]^T$ In addition, the nominal control input $u_c(k)$ is given by

$$u_r(k) = K_y y_r(k) + K_r r(k)$$

where

$$K_y = \begin{bmatrix} 10.3300 & 10.8076 \\ 9.2095 & 11.1475 \\ 0.5836 & -2.8857 \end{bmatrix}^T, \; K_r = \begin{bmatrix} 6.5412 & 3.9476 \\ -7.4861 & -9.9664 \\ 0.1903 & 0.3607 \end{bmatrix}^T$$

and $r(k) = [\,1\ 1\ 0\,]^T$ is a given signal. Moreover, the minimum and the maximum of the control input are chosen as $\underline{u}_i = -40$ and $\bar{u}_i = 40$, respectively.

In the following, two simulations are given to demonstrate the performance of the proposed method. First, the second actuator is assumed to lose its 70% effectiveness from the instant $k = 200$. In this situation, the fault estimation result is depicted in Fig. 1. Herein, λ_1 and λ_2 are depicted in real line while the estimates are shown in dashed one. Fig. 1 shows that the proposed fault estimation observer provides accurate fault estimate.

In this situation, the fault tolerant control result is depicted in Fig. 2, and the control inputs and the degradation factor are illustrated in Fig. 3. From Fig. 2, it can be seen that the proposed fault-tolerant control approach achieves perfect performance recovery. This result can also be verified by the degradation factor in Fig. 3 (note that $\rho(k) = 0$ implies that there is no performance degradation).

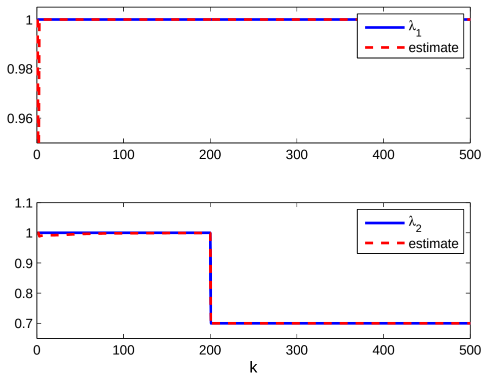

Fig. 1. The fault estimation result in a partial loss of effectiveness fault scenario

In some cases, it is impossible to achieve fault-tolerance without performance degradation. To illustrate this, a total loss of effectiveness fault is simulated here. It is assumed that the first actuator loses its total effectiveness from the instant $k = 300$. In this situation, the fault estimation result is depicted in Fig. 4. the fault tolerant control result is depicted in Fig. 5, and the control inputs and the degradation factor are illustrated in Fig. 6. From the degradation factor in Fig. 6, it can be concluded that the available control margin is not able to track the desirable reference. In this situation, the fault-tolerant controller drive the output of the faulty system to zero in order to guarantee the system safety.

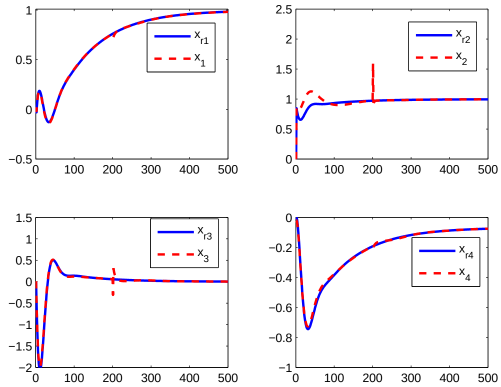

Fig. 2. The fault-tolerant result in a partial loss of effectiveness fault scenario

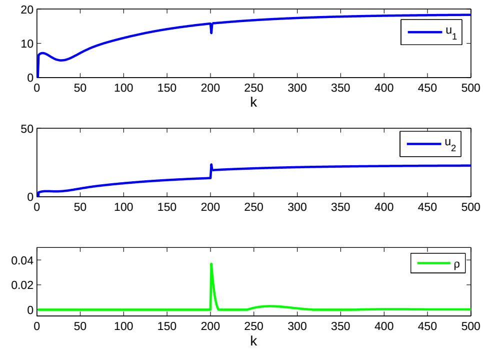

Fig. 3. The control inputs and the degradation factor in a partial loss of effectiveness fault scenario

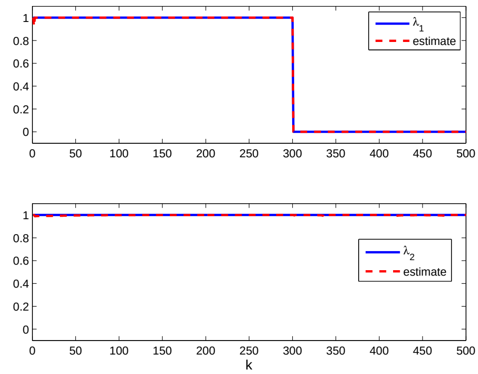

Fig. 4. The fault estimation result in a total loss of effectiveness fault scenario

6. CONCLUSION

This paper proposes a new actuator fault-tolerant control strategy by using fault estimation observer design and constrained optimization algorithm. By transforming the faulty system into a descriptor system representation, an augmented observer which is able to simultaneously estimate the faults and states can be designed. Based on the estimation result, a fault-tolerant control design method with a reference adjustment paradigm is proposed

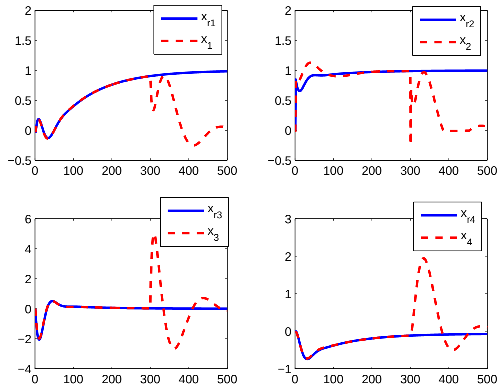

Fig. 5. The fault-tolerant result in a total loss of effectiveness fault scenario

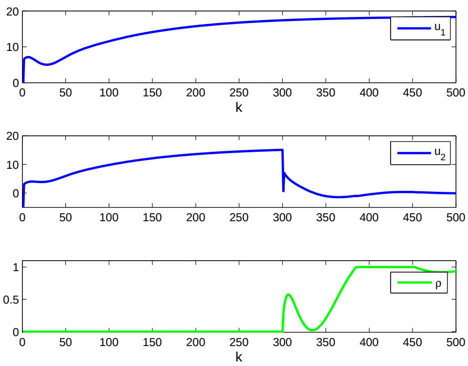

Fig. 6. The control inputs and the degradation factor in a total loss of effectiveness fault scenario

and formulated into a constrained optimization problem. The novelty of the proposed fault-tolerant control method lies in its capability of dealing with actuator saturation and accepting performance degradation, which is of significant importance for practical systems.

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