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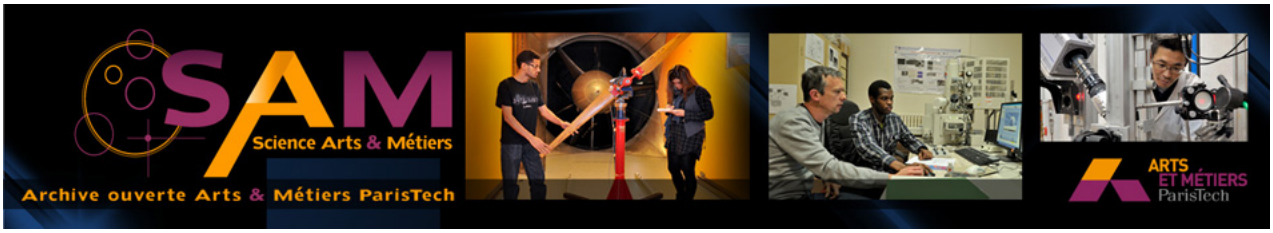

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Inertial flow in porous media: A numerical investigation on model structures

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Key words: porous media, numerical simulation, inertial effects, up-scaling

1 Introduction

The aim of this work is to study the correction to Darcy's law for inertial flow in porous media. In many situations encountered in industrial applications such as flow in column reactors, gas flow near wells for hydrocarbon recovery and CO_2 sequestration, flow in filters... , Reynolds numbers are large enough to lead to a non-linear relationship between the filtration velocity and the pressure gradient.

In this work, a numerical analysis of the non linear -inertial- correction to Darcy's law is carried out for the stationary inertial flow of a one-phase Newtonian incompressible fluid on model 2D and 3D structures. Effective properties appearing in the macroscopic model resulting from the volume averaging of the mass and momentum (Navier-Stokes) equations at the pore scale are determined using the microscopic flow fields and solving the closure problems resulting from up-scaling. From the numerical simulations, the dependence of the correction to Darcy's law on the geometrical properties of the 3D structure is studied. These properties are the shape of the solid grains which may be cubic or spherical and the degree of disorder in their arrangement in the domain. Weak disorder corresponds to a random placement of the grains of identical shape and size within each cell of a regular 3D lattice, while for strong disorder, grain size is also randomly distributed.

2 Physical model and methodology

Besides the classical Forchheimer equation based on the empirical introduction of correction coefficients into Darcy's law, a formal theoretical derivation of a more complete model has been proposed [1].

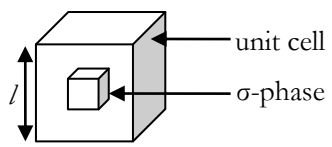

Figure 1 : representative elementary volume of a model porous structure (cubes arranged on a cubic ordered lattice the macroscopic scale of the porous medium

The macroscopic equations are obtained by averaging the microscopic mass and momentum (Navier-Stokes) equations over a representative elementary volume (REV) of a pseudo-periodic medium (see figure 1 for an example of the unit cell of a model structure) that are given by:

$$-\nabla\tilde{p}_\beta^* + \nabla^2\mathbf{v}_\beta^* = \nabla\left\langle p_\beta\right\rangle^{\beta*} + Re^*(\mathbf{v}_\beta^*.\nabla)\mathbf{v}_\beta^* \tag{1}$$

$$\nabla.\mathbf{v}_\beta^* = 0 \tag{2}$$

Where $\mathbf{v}_\beta^*$, $\tilde{p}_\beta^* = p_\beta^* - \left\langle p_\beta\right\rangle^{\beta*}$ and $\left\langle p_\beta\right\rangle^{\beta*}$ are the dimensionless velocity, pressure fluctuation and intrinsic mean pressure in the β-phase while Re^* is the Reynolds number of the flow given by $Re^* = \dfrac{\rho_\beta.l.\left|\nabla\left\langle p_\beta\right\rangle^\beta\right|}{\mu_\beta^2}$.

The associated boundary conditions are:

$$\mathbf{v}_\beta^* = \mathbf{0} \quad \text{on } A_{\beta\sigma} \tag{3}$$

$$\mathbf{v}_\beta^*(\mathbf{r}^* + \mathbf{l}_i^*) = \mathbf{v}_\beta^*(\mathbf{r}^*) \quad i = 1,2,3 \tag{4}$$

$$\tilde{p}_\beta^*(\mathbf{r}^* + \mathbf{l}_i^*) = \tilde{p}_\beta^*(\mathbf{r}^*) \quad i = 1,2,3 \tag{5}$$

By volume averaging this boundary value problem the following macroscopic model is obtained in which $\mathbf{F}.\langle \mathbf{v}_\beta \rangle$ is the Forchheimer correction vector and $\mathbf{K}$ the permeability tensor of the medium:

$$\langle \mathbf{v}_\beta \rangle = -\frac{\mathbf{K}}{\mu_\beta}.(\nabla \langle p_\beta \rangle^\beta - \rho_\beta \mathbf{g}) - \mathbf{F}.\langle \mathbf{v}_\beta \rangle \tag{6}$$

$$\nabla.\langle \mathbf{v}_\beta \rangle = 0 \tag{7}$$

In order to determine the two macroscopic tensors $\mathbf{K}$ and $\mathbf{F}$, a closure problem needs to be resolved in addition to the microscopic problem (equations (1) and (2)) to determine the microscopic flow field as it appears in the closure problem. This closure is given by:

$$-\nabla \mathbf{m}^* + \nabla^2 \mathbf{M}^* + \mathbf{I} = Re^* \mathbf{v}_\beta^*.\mathbf{M}^* \tag{8}$$

$$\nabla.\mathbf{M}^* = \mathbf{0} \tag{9}$$

$$\langle \mathbf{M}^* \rangle = \mathbf{H} \tag{10}$$

With associated boundary conditions:

$$\mathbf{M}^* = \mathbf{0} \quad \text{on } A_{\beta\sigma} \tag{11}$$

$$\mathbf{m}^*(\mathbf{r}^* + \mathbf{l}_i^*) = \mathbf{m}^*(\mathbf{r}^*) \quad i = 1,2,3 \tag{12}$$

$$\mathbf{M}^*(\mathbf{r}^* + \mathbf{l}_i^*) = \mathbf{M}^*(\mathbf{r}^*) \quad i = 1,2,3 \tag{13}$$

It must be emphasized that this closure problem has first to be solved taking Re*=0 in order to determine the permeability tensor that is given, in these circumstances, by $\langle \mathbf{M}^* \rangle = \mathbf{K}$. In a second step, the solution with a prescribed Re* provides the correction tensor $\mathbf{F}$ that is given by (see [2]):

$$\mathbf{F} = \mathbf{K}.\mathbf{H}^{-1} - \mathbf{I} \tag{14}$$

The numerical procedure described above was applied to 2D and 3D disordered structures (see figure **Erreur ! Signet non défini.** for a 3D weakly disordered medium used in our simulations). An example of a result of streamlines superimposed on the pressure deviation field obtained on a 2D strongly disordered medium for $Re^* = 1000$ and $\nabla \langle p_\beta \rangle^{\beta *} = \cos(\pi/4)\mathbf{e}_x + \sin(\pi/4)\mathbf{e}_y$ is reported in figure 3. The role of the disorder, discussed for 2D structures in [2] can hence be extended to 3D cases. Moreover, the influence of grain shape on macroscopic properties has been addressed.

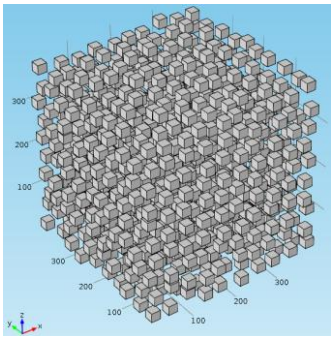

Figure 3 : 3D weakly disordered porous medium of 75% porosity formed by cubic grains

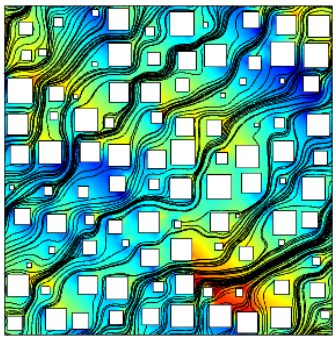

Figure 2 : Streamlines and pressure field of fluid flow in a strongly disordered porous medium of 75% porosity

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