



**HAL**  
open science

## Non-Newtonian open channel flow over inclined porous bed

Guilherme Henrique Fiorot, Pascal Dupont, Fabiana de Oliveira Ferreira,  
Geraldo de Freitas Maciel

► **To cite this version:**

Guilherme Henrique Fiorot, Pascal Dupont, Fabiana de Oliveira Ferreira, Geraldo de Freitas Maciel. Non-Newtonian open channel flow over inclined porous bed. VII Brazilian Conference on Rheology, Federal University of Technology – Paraná (UTFPR), Rio de Janeiro Catholic University (PUC-Rio), Rio Grande do Sul Federal University (UFRGS), Fluminense Federal University (UFF-Rio) and Federal University of Rio de Janeiro (UFRJ), Jul 2015, Curitiba, Parana, Brazil. hal-01189050

**HAL Id: hal-01189050**

**<https://hal.science/hal-01189050>**

Submitted on 2 Sep 2015

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - NonCommercial - ShareAlike 4.0 International License

## Non-Newtonian open channel flow over inclined porous bed

**Guilherme Henrique Fiorot** (ghfiorot@gmail.com; gfiorot@insa-rennes.fr)

LGCGM, INSA de Rennes, Rennes, France & FEIS, UNESP, Ilha Solteira, SP, Brazil

**Pascal Dupont** (pdupont@insa-rennes.fr)

INSA de Rennes, Rennes, France

**Fabiana de Oliveira Ferreira** (fabiana.of@gmail.com)

**Geraldo de Freitas Maciel** (maciel@dec.feis.unesp.br)

FEIS, UNESP, Ilha Solteira, SP, Brazil

### 1. INTRODUCTION

As far as it is known, gravity-driven mass movement are yet to be completely understood by science community. The knowledge of such kind of flow is clearly necessary to enhance prevention tools for Engineering when applied to modeling natural events such as mudflows, mud floods and debris flow. However, the complexity of the problem lays basically on the proper modeling of fluid response to stress, which depends on its matrix, and even the prediction of instabilities propagation, which may render the whole scenario even more difficult to predict and control. In a simplified view, those flows can be seen as non-Newtonian fluids over natural inclined (Zanuttigh and Lamberti, 2007), neglecting cross-sectional variable (2D flow) and considering a longitudinal length  $L_0$  much greater than the vertical one  $h_0$ , equations can be summarize in the following system

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0; \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + g \sin \theta + \frac{1}{\rho} \frac{\partial \tau_b}{\partial z}; \quad (2)$$

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g \cos \theta; \quad (3)$$

where  $(x, z)$  are variables of coordinate system;  $t$  is time variable;  $(u, w)$  are components of velocity vector;  $\theta$  is channel bed slope;  $p$  is fluid pressure;  $\tau_b$  is bottom shear stress on the interface between fluid and bed;  $g$  is gravity and  $\rho$  is fluid density. For the steady and permanent case, neither velocity nor free surface height are changed and both assume constant values  $u_0$  and  $h_0$ , respectively, as shows figure 1.

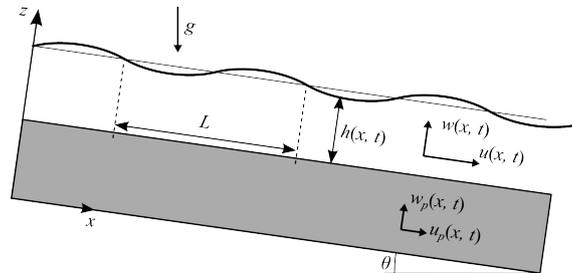


Figure 1. Schematic representation of flow. Drawing not scaled.

Such representation is limited for one main reason: rheology average fluid behavior. As size of particles in fluid matrix may vary from dust and clay ( $\approx 1 \mu\text{m}$ ) to rock and boulders ( $\approx 1 \text{m}$ ), their dynamics in fluid matrix are

completely averaged into simplified shearing laws. This fact should contribute, for example, when modeling turbulent flow where collisional particles can change turbulence dissipation thus changing flow dynamics considerably. At this point, we should focus on flows with relatively lower kinetic energy ( $Re < 2000$ ). That said, many authors (?) report that shear rate-stress relation for such fluids in simple shearing conditions is better expressed as the following non-linear relation

$$\tau = \tau_c + K_n \left( \frac{\partial u}{\partial z} \right)^n, \quad \text{for } \tau > \tau_c, \quad (4)$$

where a threshold value  $\tau_c$  (fluid yield stress),  $K_n$  is called consistency index, and  $n$  is the power-law index for fluid.

However, most of existing models for non-Newtonian fluid flow over open inclined neglect bed porosity. Of course, until recently, there was few scientific works relating yield stress fluids and porous medium both experimental and mathematically, rendering it difficult to propose any further model. In this work, a mathematical model for velocity profile is proposed for a steady and permanent Herschel-Bulkley fluid flow over an inclined porous bed. An preliminary mathematical evaluation of bed porosity is developed and some insights are given.

## 2. MATHEMATICAL DEVELOPMENT

### 2.1 Boundary Condition

It is necessary to find the proper relation between pressure drop on porous medium and Darcian velocity ( $u_p, w_p$ ) to properly assume the boundary conditions on the interface bed-fluid. A large review on the subject is presented on Chhabra *et al.* (2001) where dynamics of non-Newtonian fluids (power-law fluids) on many porous media are related. There they made clear that fluid properties greatly change its dynamics inside tortuous paths affecting the macro behavior, which is summarized by a Darcy's Law type relation. Following previous works from Beavers and Joseph (1967) and Rao and Mishra (2004), we shall assume continuity of shear rate on the interface through the following boundary condition

$$\left. \frac{\partial u}{\partial z} \right|_{z=0} = \frac{\chi}{k^{\frac{1}{n+1}}} (u - u_p). \quad (5)$$

The shearing conditions exhibit by equation 5 can be assumed as the non-Newtonian effect from yield stress is comprised by means of longitudinal Darcian velocity  $u_p$ . The other boundary condition is related to the normal velocity in the porous medium which, on the interface, should not vary once it is inside the boundary layer so  $w_p = w$  for  $z = 0$ . Recalling the recent work from (Chevalier *et al.*, 2013), the authors present an adaptation of Darcy's Law to Herschel-Bulkley fluids where they claim, based on experimental data, that the relation between pressure drop and Darcian velocity should be on the form

$$-\frac{\partial p_p}{\partial x} = \frac{\tau_c}{k_I} + \frac{K_n}{k_K} u_p^n \quad (6)$$

where  $k_I$  and  $k_K$  are porous medium parameters relating shape, dispersion and voids inside porous medium: the first what we call here inertial form factor, that represents how fluid preferable paths act interacts with fluid no shearing inertial state of fluid; the second one we call kinematic form factor, which falls into the usual permeability for porous medium. It's worth reminding that this relation is only valid for fluids with particles in suspensions that have mean diameter much smaller than pore space, so that no percolation effect is considered. Modifying Darcy's law for a Herschel-Bulkley fluid,

$$u_p = - \left( \frac{k_K}{K_n} \right)^{1/n} \left[ \left( \frac{\partial p_p}{\partial x} + \frac{\tau_c}{k_I} \right)^{1/n} - \rho g \sin \theta \right] \quad (7)$$

$$\frac{\partial u_p}{\partial x} + \frac{\partial w_p}{\partial z} = 0 \quad (8)$$

$$\frac{\partial p_p}{\partial z} = -\rho g \cos \theta \quad (9)$$

where  $p_p$  represent total pressure in the porous medium and  $p_p = p$  for  $z = 0$ . Hydrostatic pressure is assumed following initial equations of flow to estimate scales between flow over and inside the bed. Then, pressure gradient inside porous medium is the same as in the flow thus leading to the pressure scale

$$P \sim K_n \left( \frac{u_0}{h_0} \right)^n \frac{L_0}{h_0} \quad (10)$$

From equation 8, we reach a scale to the longitudinal velocity in the porous medium to be:

$$U_p \sim \left( k_K \frac{u_0^n}{h_0^{n+1}} - \Omega_f \right)^{\frac{1}{n}} \quad (11)$$

where  $\Omega_f = \frac{k_K}{k_I} \frac{\tau_c}{K_n}$  is the ratio between an inertial threshold and a kinematic resistance.

Rescaling, then, the boundary conditions for porous medium (on the interface between fluid flow and porous bed) assuming such scales, they are simplified into

$$\left. \frac{\partial u}{\partial z} \right|_{z=0} = \frac{\chi}{k_K^{\frac{1}{n+1}}} u; \quad w|_{z=0} = 0. \quad (12)$$

## 2.2 The velocity profile $u(z)$

For hydrostatic pressure distribution, one can reach that shear rate is given as shows equation 13.

$$\frac{\partial u}{\partial z} = \left( \frac{\rho g (h_0 - z) \sin \theta - \tau_c}{K_n} \right)^{\frac{1}{n}}. \quad (13)$$

Then, solving equation 13 for  $u$  and using the parameter  $\gamma = k_K^{\frac{1}{n+1}} / \chi z_0$ , the velocity is finally written as:

$$u(z) = \left( \frac{n}{n+1} \right) \left( \frac{\rho g \sin \theta}{K_n} z_0^{n+1} \right)^{\frac{1}{n}} \left[ 1 - \left( 1 - \frac{z}{z_0} \right)^{\frac{n+1}{n}} + \left( \frac{n+1}{n} \right) \gamma \right]. \quad (14)$$

On the plug region ( $z_0 < z < h_0$ ), the velocity is constant equal to  $u(z_0)$  (equation 15).

$$u(z_0) = \left( \frac{n}{n+1} \right) \left( \frac{\rho g \sin \theta}{K_n} z_0^{n+1} \right)^{\frac{1}{n}} \left[ 1 + \left( \frac{n+1}{n} \right) \gamma \right], \quad (15)$$

where  $z_0 = h_0 - \tau_c / (\rho g \sin \theta)$ .

## 3. RESULTS AND DISCUSSIONS

The velocity profile  $u(z)$  is studied to respect the plug velocity  $u(z_0)$  which corresponds to the freestream velocity (free surface velocity), and the vertical coordinate  $z$  to respect  $z_0$  thus scaling the problem, as shows equation 16.

$$U(Z) = \frac{u(z)}{u(z_0)} = \left[ 1 - \frac{(1-Z)^{\frac{n+1}{n}}}{1 + \gamma \left( \frac{n+1}{n} \right)} \right], \quad (16)$$

where  $Z = z/z_0$ . It's also valuable to define the free surface height which should be equal to  $H_0 = h_0/z_0 = 1/(1 - C^*)$  where  $C^* = \tau_c / (\rho g h_0 \sin \theta)$ . Mean flow velocity  $U_m$  is then calculated as

$$U_m = \int_0^1 U(Z) DZ + \int_1^{H_0} DZ. \quad (17)$$

The effect of the porous bed over the velocity can be summarized as the promotion of a slip velocity on the interface between the bed and the flow. Figure 2(a) shows that for a solid bed ( $\gamma = 0$ ), interface velocity  $U|_{Z=0} = 0$ . However, the smallest increase in bed porosity leads to a non-zero velocity at the interface. For shear thinning fluids ( $n < 1$ ), the interface velocity increases more rapidly with porosity.

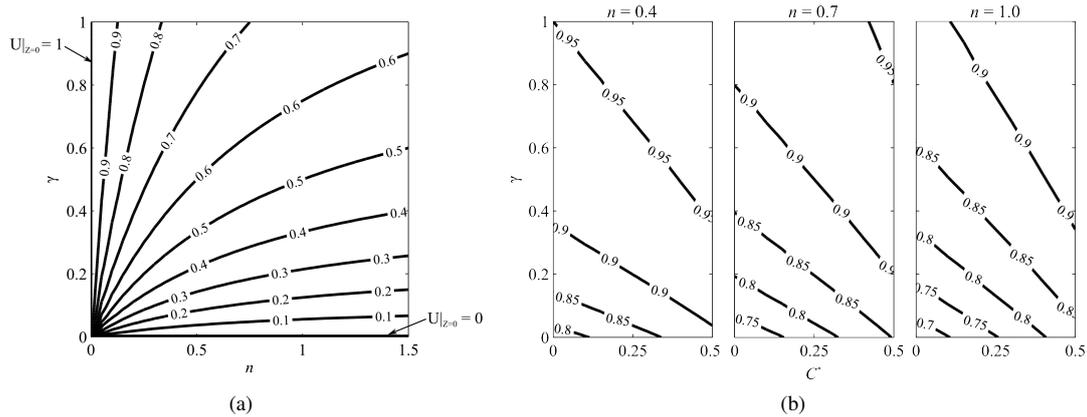


Figure 2. Analysis of rheological properties and porous medium on flow velocity. (a): Velocity at the interface  $U|_{z=0}$  as function  $n$  and  $\gamma$ , for  $C^* = 0$ ; (b): Mean flow velocity  $U_m$  as function  $C^*$  and  $\gamma$  for different values of  $n$ .

When in presence of a yield stress, the same behavior is noticed. This effect lead to a mean flow velocity  $U_m$  which is closer to freestream velocity the higher yield stress parameter  $C^*$ , as shows figure 2(b). This observation is in agreement with the plug zone which is greater as the yield stress increases, thus leading to a quasi-constant velocity distribution over the vertical coordinate. Numerical and, most importantly, experimental validation of the velocity profile should be employed to assure hypothesis adopted in this work. When dealing with fluid composed by sediment particles, percolation effect can hardly be avoided, thus obstructing the proper use of such mathematical development. However, for homogeneous fluids such as emulsions and gels, these assumptions should be reasonable and could be easily verified.

#### 4. ACKNOWLEDGEMENTS

We'd like to thank CNPq (Conselho Nacional de Desenvolvimento Tecnológico e Científico do Brasil) for scholarship grant and internship fundings.

#### 5. REFERENCES

- Beavers, G.S. and Joseph, D.D., 1967. "Boundary conditions at a naturally permeable wall". *Journal of Fluid Mechanics*, Vol. 30, No. 1, pp. 197–207.
- Chevalier, T., Chevalier, C., Clain, X., Dupla, J.C., Canou, J., Rodts, S. and Coussot, P., 2013. "Darcy's law for yield stress fluid flowing through a porous medium". *Journal of Non-Newtonian Fluid Mechanics*, Vol. 195, pp. 57–66.
- Chhabra, R.P., Comiti, J. and Machac, I., 2001. "Flow of non-Newtonian Fluids in fixed and fluidised beds". *Chemical Engineering Science*, Vol. 56, pp. 1–27.
- Rao, A.R. and Mishra, M., 2004. "Peristaltic transport of a power-law fluid in a porous tube". *Journal of Non-Newtonian Fluid Mechanics*, Vol. 121, No. 2-3, pp. 163–174.
- Zanuttigh, B. and Lamberti, A., 2007. "Instability and surge development in debris flows". *Reviews of Geophysics*, , No. 45, pp. 1–45.