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To cite this version:

Pietro Grandinetti, Federica Garin, Carlos Canudas de Wit. Towards scalable optimal traffic control. CDC 2015 - 54th IEEE Conference on Decision and Control, Dec 2015, Osaka, Japan. hal-01188811

HAL Id: hal-01188811

https://hal.science/hal-01188811

Submitted on 23 Sep 2015

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Towards scalable optimal traffic control

Pietro Grandinetti, Federica Garin and Carlos Canudas-de-Wit

Abstract— **This paper deals with scalable control of traffic lights in urban traffic networks. Optimization is done in real time, so as to take into account variable traffic demands. At each cycle of the traffic lights, the optimization concerns time instants where each traffic light starts and ends its green phase: this allows to describe both the duty-cycle and the phase shifts. First, we formulate a global optimization problem, which can be cast as a mixed-integer linear program. To overcome the complexity of this centralized approach, we also propose a decentralized suboptimal algorithm, whose simplicity allows on-line implementation. Simulations show the effectiveness of the proposed strategies.**

I. Introduction

The problem of traffic congestion has always been a crucial aspect for the design of efficient infrastructures, but it is particularly in the second half of the last century that this phenomenon has become predominant, due to the quickly increasing traffic demand and the more frequent congestions. Congestions appear when too many vehicles try to use a common transportation route, which, due to physical reasons, has limited capacity. If it happens, they lead to queueing phenomena or, even worse, to a severe degradation of the available infrastructure's usage. Hence, congestions result in reduced safety, increased pollution and excessive delays. Economic implication of such events are recently widely discussed [1, 2].

Traffic in urban scenarios is mainly regulated by the traffic lights installed at intersections of roads. Even though these devices were first conceived to guarantee avoidance of collision, with steadily increasing traffic demands it was soon realized that they may lead to more or less efficient network operation. Therefore, the idea to increase the efficiency of the infrastructure using smart traffic lights policies has constantly been employed in academic and industrial research.

Some existing strategies, called fixed–time techniques [3], have limitations due to their settings, which are based on historical, rather than real–time, data. A survey about the existing techniques can be found in [4]. More advanced schemes have been presented recently and they refer to different models for the network and for the chosen control actions, such as max pressure control [5] and cooperative green lights policies [6]. Concerning the control action, we will instead contribute focusing on a more realistic representation of traffic lights, whose mathematical abstraction often produces problems difficult to solve, due to the binary (green-red) nature of these signals. Because of that, scientific works often consider optimization that takes into account either bandwith maximization via phase shift selection [7, 8] or duty cycle design [9]. The control of both parameters is a very challenging problem still open in transportation optimization environments.

In this paper, we decided to deal with the most general and difficult problem: the control of traffic lights, in urban networks, parametrized by two degrees of freedom: such representation (i.e., their description with two variable time instants) includes the most generic scenario, being able to embody phase shift as well as duty cycle.

Our contribution consists of two strategies for traffic control: the first one is a centralized control scheme that provides an optimal solution. Even though it gives the optimal control action, it may be computationally inefficient because based on a mixed integer linear program. Therefore, we also propose a decentralized suboptimal scheme that is able to dramatically reduce the computational burden, and hence is scalable to large networks.

Furthermore, the strategies we elaborated are on–line strategies based on the receding horizon philosophy. This choice is due to the fact that in traffic scenarios demands are continuously changing and, therefore, proactive decisions are needed. Our control schemes are able to accomplish such requirement, i.e., they may improve the quality of the system in the upcoming future taking into account external demands.

The rest of the paper is organized as follows: Section II introduces the model used to describe the traffic lights behavior, the network state evolution and the metrics used to describe traffic's performance; Section III illustrates how to formulate the centralized optimal control using a mixed integer linear formulation; Section IV explains our idea to decentralize the control scheme; Section V shows the numerical results of our software simulations employing the designed algorithms; Conclusions and future works are outlined in Section VI.

II. Traffic network description

In this section we first motivate the mathematical representation of traffic lights and, then, describe the network's model and our chosen measures of traffic performance.

A. Traffic lights model

Traffic lights are electronic devices installed at the end point (w.r.t. the flow direction) of roads for the purpose to

This work has been partly funded by the EU FP7 project SPEEDD (619435).

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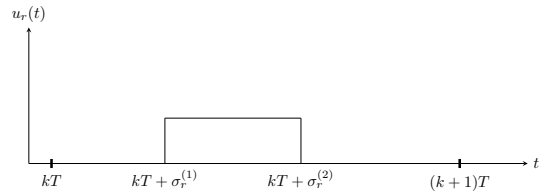


Fig. 1: Traffic lights described as a time trajectory with two degrees of freedom, represented by the timed-controlled variables $\sigma_r^{(1)}, \sigma_r^{(2)}$.

avoid any collision between vehicles. A traffic light can either allow of forbid vehicles to continue their route, therefore it can be mathematically described as a map from the set of time instants to the set $\{0, 1\}$.

In a typical urban environment a fixed cycle length T is assigned for the traffic lights. This cycle is a slot of time during which every traffic light can switch from green to red (or viceversa) at most once. Several physical reasons justify such a behavior: faster switching may, in fact, give unfavourable consequences regarding pollution generation and drivers comfort, since in this case vehicles will have to stop–and–go with higher frequency. Typical values for T are from 90 seconds up to 2 minutes.

To fully capture the above mentioned dynamics we decide to describe the time trajectory of traffic lights with a two-degree-of-freedom trajectory (see Fig. 1), given by the vector of two natural numbers $\sigma_r = \left[\sigma_r^{(1)}\ \sigma_r^{(2)}\right] \in \{T_s, 2T_s, \ldots, T\}^2$, where T_s is the sampling time used to discretize the dynamics, such that $T/T_s \in \mathbb{N}$. Hence, σ_r represents the raising and falling time instants of the signal u_r:

$$u_r(t, \sigma_r) = \begin{cases} 1 & \text{if } \sigma_r^{(1)} \leq t \leq \sigma_r^{(2)} \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

where t has to be intended modulo T.

B. Traffic dynamics on signalized networks

To describe traffic's time evolution we use the description given by a mass conservation law [10, 11] and by its discrete-time representation widely known as Cell Transmission Model (CTM) [12]. In particular, we consider an extension of the CTM for networks with FIFO policy at intersections, similar to the one we already introduced in [9]. A urban network is a collection of roads entering it ($\mathcal{R}^{\text{in}}$), internal roads ($\mathcal{R}$), and exiting roads ($\mathcal{R}^{\text{out}}$). We use the word *intersections* to identify locations where two or more roads merge. Such intersections have no capacity storage and they are signalized, in the sense that the traffic flow exiting each road $r \in \mathcal{R}^{\text{in}} \cup \mathcal{R}$ is regulated by a traffic light whose value at time instant t will be denoted by $u_r(t, \sigma_r) \in \{0, 1\}$. Whether two roads r and q are connected to the same intersection in such a way that flow can exit r and enter q we use the notation $r \rightarrowtriangle q$ (q is downstream w.r.t. r). Furthermore, to every road q is associated a value $\beta_q \in (0, 1)$ (split ratio) that expresses the percentage of the flow upstream q which actually wants to turn in q.

In urban scenarios we consider every road r as a cell of the CTM, hence the value of vehicles' density in it, indicated as ρ_r (which we consider as normalized w.r.t. the length L_r of the roads itself) depends on the flows f_r^{in} and f_r^{out}, entering and exiting r, respectively.

Inflow and outflow values are given according to the demand and supply paradigm [13]. Given a road r its demand D_r is the flow of vehicles that want to exit r; its supply S_r is the flow that r can receive according to its storage capacity.

Every road r is characterized by given parameters: the maximum speed in free flow v_r, the speed in congestion phase w_r, the maximum density allowed $\rho_r^{\max}$ and the maximum flow $\varphi_r^{\max}$. Whether the system is sampled with step size T_s, to ensure stability it must hold $T_s v_r / L_r < 1$, for every road.

Finally, external traffic demand for every road r entering the network is indicated by D_r^{in}, and external supply for every road q exiting the network is indicated by S_q^{out}.

The model is given by the following set of equations:

$$\rho_r(t + T_s) = \rho_r(t) + T_s\Big(f_r^{\text{in}}(t) - u_r(t, \sigma_r) f_r^{\text{out}}(t)\Big) \tag{2a}$$

$$f_r^{\text{in}}(t) = \begin{cases} \min\{D_r^{\text{in}}(t), S_r(t)\},\ r \in \mathcal{R}^{\text{in}} \\ \beta_r \sum_{q: q \rightarrowtriangle r} u_q(t, \sigma_q) f_q^{\text{out}},\ \text{oth.} \end{cases} \tag{2b}$$

$$f_r^{\text{out}}(t) = \begin{cases} \min\{D_r(t), S_r^{\text{out}}(t)\},\ r \in \mathcal{R}^{\text{out}} \\ \min\left\{D_r(t), \left\{\frac{S_q(t)}{\beta_q}\right\}_{q: r \rightarrowtriangle q}\right\},\ \text{oth.} \end{cases} \tag{2c}$$

$$D_r(t) = \min\{v_r \rho_r(t), \varphi_r^{\max}\} \tag{2d}$$

$$S_r(t) = \min\{\varphi_r^{\max}, w_r(\rho_r^{\max} - \rho_r(t))\} \tag{2e}$$

$$u_r(t, \sigma_r) = \begin{cases} 1 & \text{if } \sigma_r^{(1)} \leq t \leq \sigma_r^{(2)} \\ 0 & \text{otherwise.} \end{cases} \quad \bmod T \tag{2f}$$

C. Urban traffic performance metrics

Traffic behavior needs to be evaluated and assessed with respect to properly defined performance indices. There exist several metrics in literature to address traffic performance evaluation; in this paper we focus mainly on two features of the urban network.

1) Service of demand (SoD): A urban traffic network is an highly dynamical environment that continously receives demand from outside. This demand cannot be ignored just to favour the inner quality of the system, because the external request will end up growing with several undesired effects, due to the bigger and bigger queues arising outside.

For this reason we consider as quality of the service for a road $r \in \mathcal{R}^{\text{in}}$ the number of vehicles (users) served by that road:

$$\begin{aligned} \text{SoD}_r(t) &= f_r^{\text{in}}(t)|_{r \in \mathcal{R}^{\text{in}}} = \\ &= \min\left\{D_r^{\text{in}}(t), \varphi_r^{\max}, w_r(\rho_r^{\max} - \rho_r(t))\right\}, \end{aligned} \tag{3}$$

To improve performance we would like to maximize the sum of (3) over all roads in $\mathcal{R}^{\text{in}}$.

2) Optimization of the infrastructures usage: In urban networks there are roads preferred by the users. The civil authority would like to set traffic lights as to diminuish this

usage disparity, to guarantee a more equilibrate diffusion of vehicles, thus reducing hard congestions in main streets as well as the possibility of accidents.

A standard metric that takes into account this behavior is the Total Travel Distance [14], defined as follows:

$$\mathrm{TTD}_r(t) = \min\left\{v_r\rho_r(t), w_r(\rho_r^{\max} - \rho_r(t))\right\}. \tag{4}$$

We want to maximize the sum of (4) over all roads in $\mathcal{R} \cup \mathcal{R}^{\text{out}}$ (because boundary flows are considered by SoD).

III. Centralized optimal control

The control strategy we designed consists in solving an optimization problem at the beginning of every cycle (i.e., every $t_0 = kT$, $k \in \mathbb{N}$), in order to decide the optimal traffic lights in the upcoming cycle. Such procedure optimizes the traffic behavior using a receding horizon philosophy that predicts densities $\rho(t_0 + nT_s), n = 1, \ldots, T/T_s$. Predictions are carried out assuming that a measure of densities $\rho(t_0)$ is available for the controller.

The optimal values σ^*, i.e., the optimal activation time instants for the traffic lights, are given by:

$$\begin{aligned}\sigma^* = \arg\max_{\sigma} \sum_{n=1}^{T/T_s} \Big(a_1 \sum_{r\in\mathcal{R}\cup\mathcal{R}^{\text{out}}} \mathrm{TTD}_r(t_0 + nT_s) + \\ + a_2 \sum_{r\in\mathcal{R}^{\text{in}}} \mathrm{SoD}_r(t_0 + nT_s)\Big) \qquad (5)\\ \text{under network dynamics (2)}\\ \forall\, t = t_0 + nT_s,\ n = 1, \ldots, T/T_s,\end{aligned}$$

where $a_1, a_2 \in \mathbb{R}$ are weights for the two involved objectives.

In the rest of this section we show that equations (2) and the objective function can be reformulated using logical constraints and variables, obtaining a mixed integer linear problem (MILP). To this aim, we use the reasoning outlined in [15]. At the end of the reformulation, the problem's constraints, instead of (2), will be given by (7)–(11),(14),(16)–(18).

Such final formulation of the problem is more convenient because MILPs are extensively studied, and there exist good numerical solvers to deal with them, e.g. [16].

A. Traffic lights constrained trajectory

Introducing the binary variables $\delta_r^{(1)}(t)$, $\delta_r^{(2)}(t)$, the constraint expressed by (1), for a given time instant t, is equivalent to the following:

$$\left[\delta_r^{(1)}(t) = 1\right] \longleftrightarrow \left[\sigma_r^{(1)} \le t\right] \tag{6a}$$
$$\left[\delta_r^{(2)}(t) = 1\right] \longleftrightarrow \left[t \le \sigma_r^{(2)}\right] \tag{6b}$$
$$\left[u_r(t) = 1\right] \longleftrightarrow \left[\delta_r^{(1)}(t) = 1 \wedge \delta_r^{(2)}(t) = 1\right]. \tag{6c}$$

Let $M^{(1)}$ and $m^{(1)}$ upper and lower bounds[1] such that $m^{(1)} < \sigma_r^{(1)} - t < M^{(1)}$, for every $\sigma_r^{(1)}$; then (6a) is equivalent to the following constraints:

$$\sigma_r^{(1)} - t \le M^{(1)}(1 - \delta_r^{(1)}(t)) \tag{7a}$$
$$\sigma_r^{(1)} - t > m^{(1)}\delta_r^{(1)}(t). \tag{7b}$$

Similarly, let $M^{(2)}$ and $m^{(2)}$ such that $m^{(2)} < \sigma_r^{(2)} - t < M^{(2)}$, for every $\sigma_r^{(2)}$; then (6b) is equivalent to the following constraints:

$$\sigma_r^{(2)} - t \le M^{(2)}(1 - \delta_r^{(2)}(t)) \tag{8a}$$
$$\sigma_r^{(2)} - t > m^{(2)}\delta_r^{(2)}(t). \tag{8b}$$

Finally, the constraint (6c) is equivalent to the following:

$$u_r(t) - \delta_r^{(1)}(t) \le 0 \tag{9a}$$
$$u_r(t) - \delta_r^{(2)}(t) \le 0 \tag{9b}$$
$$\delta_r^{(1)}(t) + \delta_r^{(2)}(t) - u_r(t) \le 1 \tag{9c}$$
$$\delta_r^{(1)}(t), \delta_r^{(2)}(t) \in \{0, 1\}.$$

Depending on the physics of system it may be useful to impose another constraint: we can require that the rising instant $\sigma_r^{(1)}$ and the falling one $\sigma_r^{(2)}$ are sufficiently separated in time, so ensuring that too short *green slots* are avoided. This can be obtained with the following constraints:

$$\sigma_r^{(1)} + \sigma_r^{\min} \le \sigma_r^{(2)} \tag{10a}$$
$$T_s \le \sigma_r^{(1)}, \sigma_r^{(2)} \le T, \tag{10b}$$

where $\sigma_r^{\min}$ are consistently assigned to every traffic light.

B. Collision avoidance constraints

To guarantee the safe crossing we impose an hard constraint such that at every time instant only one among roads entering the same intersection has the right of way. This condition is expressed by the following linear constraint:

$$\sum_{r:r\to q} u_r(t, \sigma_r) \le 1, \tag{11}$$

for every road q. Notice that constraint (11) let the further freedom to assign *red* to all traffic lights at the same intersection, if it is required.

C. State constraints

The dynamics defined in (2) is non linear due to the $\min$ operator and to the product between flows and traffic lights' values. Our aim is to show how such a dynamics may be reformulated with mixed integer linear constraints.

Let σ be a vector containing σ_r's for every road r, and let $\bar{f}_r^{\text{in}}(t, \sigma)$ and $\bar{f}_r^{\text{out}}(t, \sigma)$ be the following modified flows:

$$\bar{f}_r^{\text{in}}(t, \sigma) = \begin{cases} \beta_r \sum_{q:q\to r} \bar{f}_q^{\text{out}}(t, \sigma), & r \in \mathcal{R} \cup \mathcal{R}^{\text{out}} \\ f_r^{\text{in}}(t), & r \in \mathcal{R}^{\text{in}} \end{cases} \tag{12a}$$

$$\bar{f}_r^{\text{out}}(t, \sigma) = \begin{cases} u_r(t, \sigma_r) f_r^{\text{out}}(t), & r \in \mathcal{R}^{\text{in}} \cup \mathcal{R} \\ f_r^{\text{out}}(t), & r \in \mathcal{R}^{\text{out}}. \end{cases} \tag{12b}$$

We now show a scheme to rewrite (12b), i.e., $\min$ of several functions multiplied by a binary variable. For every road $r \in \mathcal{R}^{\text{in}} \cup \mathcal{R}$ let $d^{(r)}(t) \in \{0, 1\}^h$ be a vector of binary

[1] Here and in what follows the numerical values for these bounds are omitted. Notice that flows in the network are always in $[0, \varphi^{\max}]$, while time instants indicated by σ are in $[T_s, T]$. Bounds can be manually set even if some software, like Yalmip [17], is able to determine them while parsing the problem formulation.

variables, where $h = 1 + \#\{q : r \rightarrowtriangle q\}$. Then the definition of outflow (2c) is equivalent to the following constraints:

$$\left[d_0^{(\mathrm{r})}(t) = 1\right] \longrightarrow \left[f_r^{\mathrm{out}}(t) = D_r(t)\right] \tag{13a}$$

$$\left[d_q^{(\mathrm{r})}(t) = 1\right] \longrightarrow \left[f_r^{\mathrm{out}}(t) = \frac{S_q(t)}{\beta_q} \,\forall\, q : r \rightarrowtriangle q\right] \tag{13b}$$

$$\left[d_0^{(\mathrm{r})}(t) = 1\right] \longrightarrow \left[D_r(t) \leq \frac{S_q(t)}{\beta_q} \,\forall\, q : r \rightarrowtriangle q\right] \tag{13c}$$

$$\left[d_i^{(\mathrm{r})}(t) = 1\right] \longrightarrow \left[\frac{S_q(t)}{\beta_q} \leq \frac{S_i(t)}{\beta_i} \,\forall\, i \neq q\right] \tag{13d}$$

$$\sum_{i=1}^{h} d_i^{(\mathrm{r})}(t) = 1. \tag{13e}$$

Given the following upper and lower bounds: $l_0 < f_r^{\mathrm{out}}(t) - D_r(t) < L_0$, $l_q < f_r^{\mathrm{out}}(t) - S_q(t)/\beta_q < L_q$, $\psi_0 < D_r(t) < \Psi_0$, $\psi_q < S_q(t)/\beta_q < \Psi_q$, logical constraints (13a)–(13d) are equivalent to the following linear ones:

$$f_r^{\mathrm{out}}(t) - D_r(t) \geq l_0(1 - d_0^{(\mathrm{r})}(t)) \tag{14a}$$

$$f_r^{\mathrm{out}}(t) - \frac{S_q(t)}{\beta_q} \geq l_q(1 - d_q^{(\mathrm{r})}(t)) \tag{14b}$$

$$f_r^{\mathrm{out}}(t) - D_r(t) \leq L_0(1 - d_0^{(\mathrm{r})}(t)) \tag{14c}$$

$$f_r^{\mathrm{out}}(t) - \frac{S_q(t)}{\beta_q} \leq L_q(1 - d_q^{(\mathrm{r})}(t)) \tag{14d}$$

$$D_r(t) \leq \frac{S_q(t)}{\beta_q} + (\Psi_0 - \psi_q)(1 - d_q^{(\mathrm{r})}(t)) \tag{14e}$$

$$\frac{S_q(t)}{\beta_q} \leq \frac{S_i(t)}{\beta_i} + (\Psi_q - \psi_i)(1 - d_q^{(\mathrm{r})}(t)) \tag{14f}$$

$$\forall\, q : r \rightarrowtriangle q,\ \forall\, i \neq q$$

$$d^{(\mathrm{r})}(t) \in \{0,1\}^h,$$

for every road i and $j \neq i$.

Finally, given $u_r(t,\sigma) \in \{0,1\}$ setting

$$\bar{f}_r^{\mathrm{out}}(t,\sigma) = u_r(t,\sigma) f_r^{\mathrm{out}}(t) = \begin{cases} f_r^{\mathrm{out}}(t) & \text{if } u_r(t,\sigma) = 1 \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

is equivalent to

$$g(1 - u_r(t,\sigma)) + \bar{f}_r^{\mathrm{out}}(t,\sigma) \leq f_r^{\mathrm{out}}(t) \tag{16a}$$

$$-G(1 - u_r(t,\sigma)) - \bar{f}_r^{\mathrm{out}}(t,\sigma) \leq -f_r^{\mathrm{out}}(t) \tag{16b}$$

$$-G u_r(t,\sigma) + \bar{f}_r^{\mathrm{out}}(t,\sigma) \leq 0 \tag{16c}$$

$$g u_r(t,\sigma) - \bar{f}_r^{\mathrm{out}}(t,\sigma) \leq 0, \tag{16d}$$

where $g < f_r^{\mathrm{out}}(t) < G$.

D. Objective functions

The metrics illustrated in Section II-C are nonlinear functions of roads' density. Therefore, we use the same technique previously employed, which allows us to transform their expression as linear constraints.

1) SoD: Using the scheme illustrated by (13)–(16), expression (3) is equivalent to:

$$\pi_1(1 - b_1(t)) \leq \mathrm{SoD}_r(t) - D_r^{\mathrm{in}}(t) \leq \Pi_1(1 - b_1(t)) \tag{17a}$$

$$\pi_2(1 - b_2(t)) \leq \mathrm{SoD}_r(t) - \varphi_r^{\max} \leq \Pi_2(1 - b_2(t)) \tag{17b}$$

$$\pi_3(1 - b_3(t)) \leq \mathrm{SoD}_r(t) - w_r(\rho_r^{\max} - \rho_r(t)) \tag{17c}$$

$$\mathrm{SoD}_r(t) - w_r(\rho_r^{\max} - \rho_r(t)) \leq \Pi_3(1 - b_3(t)) \tag{17d}$$

$$D_r^{\mathrm{in}}(t) \leq \varphi_r^{\max} + (P_1 - p_2)(1 - b_1(t)) \tag{17e}$$

$$D_r^{\mathrm{in}}(t) \leq w_r(\rho_r^{\max} - \rho_r(t)) + (P_1 - p_3)(1 - b_1(t)) \tag{17f}$$

$$\varphi_r^{\max} \leq D_r^{\mathrm{in}}(t) + (P_2 - p_1)(1 - b_2(t)) \tag{17g}$$

$$\varphi_r^{\max} \leq w_r(\rho_r^{\max} - \rho_r(t)) + (P_2 - p_3)(1 - b_2(t)) \tag{17h}$$

$$w_r(\rho_r^{\max} - \rho_r(t)) \leq D_r^{\mathrm{in}}(t) + (P_3 - p_1)(1 - b_3(t)) \tag{17i}$$

$$w_r(\rho_r^{\max} - \rho_r(t)) \leq \varphi_r^{\max} + (P_3 - p_2)(1 - b_3(t)) \tag{17j}$$

$$b(t) \in \{0,1\}^3,$$

where Π, P (π, p) are upper (lower) bounds consistently chosen.

2) TTD: Similarly, expression (4) is equivalent to the following:

$$\gamma_1(1 - c_1(t)) \leq \mathrm{TTD}_r(t) - v_r \rho_r(t) \tag{18a}$$

$$\mathrm{TTD}_r(t) - v_r \rho_r(t) \leq \Gamma_1(1 - c_1(t)) \tag{18b}$$

$$\gamma_2(1 - c_2(t)) \leq \mathrm{TTD}_r(t) - w_r(\rho_r^{\max} - \rho_r(t)) \tag{18c}$$

$$\mathrm{TTD}_r(t) - w_r(\rho_r^{\max} - \rho_r(t)) \leq \Gamma_2(1 - c_2(t)) \tag{18d}$$

$$v_r \rho_r(t) \leq w_r(\rho_r^{\max} - \rho_r(t)) + (\Theta_1 - \theta_2)(1 - c_1(t)) \tag{18e}$$

$$w_r(\rho_r^{\max} - \rho_r(t)) \leq v_r \rho_r(t) + (\Theta_2 - \theta_1)(1 - c_2(t)) \tag{18f}$$

$$c(t) \in \{0,1\}^2,$$

where Γ, Θ (γ, θ) are upper (lower) bounds consistently chosen.

IV. Decentralized suboptimal control

The control strategy illustrated in the previous section, while providing the optimal values for traffic lights, requires to solve an intractable (NP–hard) problem. In this section we therefore propose a decentralized realization of such strategy, which reduces the computational burden substantially. It is based on two main ingredients: suboptimal solutions for local problems and agreement policy between local solutions.

A. Local problems

The optimization procedure is decentralized among intersections, each of which solves a local MILP. For every local problem, i.e., over intersection A, a receding horizon approach is used and the predicted densities belong to the set

$$\mathcal{R}_{\mathrm{A}} = \left\{r \in \mathcal{R}^{\mathrm{in}} \cup \mathcal{R} \cup \mathcal{R}^{\mathrm{out}} \,:\, r \text{ entering or exiting A}\right\}, \tag{19}$$

while the optimization variables are all traffic lights in the following set:

$$\Omega_{\mathrm{A}} = \left\{\sigma_r \,:\, r \in \mathcal{R}_{\mathrm{A}}\right\} \cup \left\{\sigma_q \,:\, q \rightarrowtriangle r, r \text{ entering A}\right\}. \tag{20}$$

Furthermore, densities for roads in $\mathcal{R}^{\mathrm{in}} \cup \mathcal{R} \cup \mathcal{R}^{\mathrm{out}} \setminus \mathcal{R}_{\mathrm{A}}$ are considered constant (equal to the last measured values).

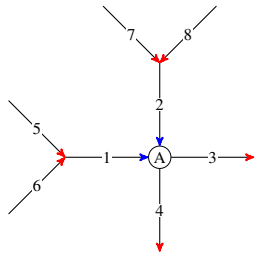


Fig. 2: Illustrative example of local subproblems.

This is the main reason of suboptimality, since the objective function maximized in every subproblem is computed over the set $\mathcal{R}_{\mathrm{A}}$, i.e.,

$$J_{\mathrm{A}} = \sum_{n=1}^{T/T_s} \Big(a_1 \sum_{r \in \mathcal{R}_{\mathrm{A}} \setminus \mathcal{R}^{\mathrm{in}}} \mathrm{TTD}_r(t_0 + nT_s) + \\ + a_2 \sum_{r \in \mathcal{R}_{\mathrm{A}} \cap \mathcal{R}^{\mathrm{in}}} \mathrm{SoD}_r(t_0 + nT_s) \Big). \tag{21}$$

As illustrative example look at Fig. 2, where we consider the optimization solved by intersection A. In such a scenario density predictions are carried out only for roads 1–4, and the problem is solved with respect to the variables $\sigma_1, \ldots, \sigma_8$ (notice that $\sigma_5, \ldots, \sigma_8$ are needed to compute inflows for roads 1 and 2).

It is worth to note that, among all σ's considered by an intersection, only a subset of them fulfills hard constraints for collision avoidance within the local optimization: referring again to Fig. 2, intersection A guarantees such fulfillment only for σ_1 and σ_2. For the other variables considered by A (marked in red in the figure) there might be some hard constraint which is not included within the local problem (in Fig. 2 this happens for σ_3 and σ_4); Therefore, the values computed by A for these variables can be interpreted only as suggestions that A would like to advice its neighbor intersections, to optimize its own objective. How such suggestions are considered is explained in the next section.

B. Agreement policy

To let local problems taking care of advices provided by neighbors we save the result of every local optmization. For instance, when A solves its subproblem, we save the variables $\sigma_{r,\mathrm{A}}$, for every r involved in such problem. Once these informations are available we can identify, for every road r, the set $\mathcal{S}_r$ of all intersections whose local problem involves σ_r. Then the average suggestion given by intersections about σ_r is computed as:

$$\hat{\sigma}_r = \frac{1}{|\mathcal{S}_r|} \sum_{\mathrm{I} \in \mathcal{S}_r} \sigma_{r,\mathrm{I}}. \tag{22}$$

We now use the values $\hat{\sigma}_r$ to modify the cost function of every local problem, as the following:

$$\hat{J}_{\mathrm{A}} = J_{\mathrm{A}} - a_3 \sum_{\sigma_r \in \Omega_{\mathrm{A}}} \|\sigma_r - \hat{\sigma}_r\|_1, \tag{23}$$

where a_3 is a real number used to weight neighbors' suggestions w.r.t. A's own objective. Notice also that the 1–norm can be turned in a mixed integer linear formulation adding two binary variables for every $\sigma_r \in \Omega_{\mathrm{A}}$. This is the computational prize to pay in order to take into account suggestions by neighbors, rather then ignoring them; the problem, however, is still significantly numerically more efficient than its centralized version. This idea can be iterated N_{it} times, in order to let suggestions spread among intersections. The scheme we implemented is the following:

0) Let N_{it} be assigned;
1) For every intersection A solve the local MILP with cost function J_{A} and save the resulting $\sigma_{r,\mathrm{A}}$;
2) If $N_{\mathrm{it}} = 0$ then stop;
3) For every signalized road r compute the average suggestions $\hat{\sigma}_r$;
4) For every intersection A solve the local MILP with cost function $\hat{J}_{\mathrm{A}}$, and use the result to update $\sigma_{r,\mathrm{A}}$;
5) If for every r and for every A there was no update in $\sigma_{r,\mathrm{A}}$ then stop;
6) Decrement N_{it} by 1, goto 2.

Whenever the algorithm stops, the value assigned to every traffic light is

$$\sigma_r^* = \sigma_{r,\mathrm{A}}, \text{ where } r \text{ enters A}, \tag{24}$$

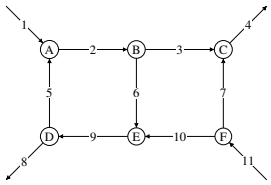
since every intersection guarantees the fulfillment of collision avoidance (hard) constraints only over roads entering it.

Notice that if $N_{\mathrm{it}} = 0$ is given, this means that suggestions from neighbors are ignored. An important feature of our scheme is that the subproblems' complexity does *not* depend on the network size; therefore the algorithm complexity is linear in the number of intersections and in the chosen number of iteration, which is a tunable parameter. Our numerical results, presented in the next section, show that the algorithm stops already after three iterations as no changes appear in the solution.

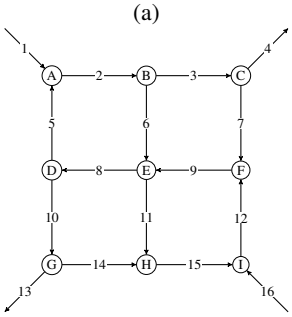
Another benefit of this decentralized strategy is that step 1 (the same applies to step 4), requires to solve a set of optimization problems that are completely indipendent of each other. Therefore the procedure can be implemented even more efficiently in a parallel architecture, where there is a controller at every intersection that exchanges σ's values with the others.

V. Simulations and comparisons

The strategies illustrated in the previous sections have been tested via software simulations in MatLab environment, using software [16, 17] to solve the mixed integer linear programs. The simulated scenario is the following: all roads in the network have same physical properties (v,w,$\varphi^{\max}$,$\rho^{\max}$). Each simulation is run for a virtual time of 45 minutes, when traffic lights' cycle is 90 seconds and sampling time is 10 seconds. As explained in Section III, the optimization problem is solved at the beginning of each cycle, therefore in this set-up the control values are computed 30 times. The constant $\sigma^{\min}$ is set equal to 2 sample steps, so the minimum green slot is 20 seconds. Outside the network, time-varying demands and supplies are randomly uniformly generated in the interval $[0.5, 1]\varphi^{\max}$ for the entire simulated time; by



(a)



(b)

Fig. 3: Networks used for simulations and comparisons.

TABLE I: Normalized performance comparison between the centralized strategy (MILP), considered as benchmark, and the decentralized one (Dec–MILP).

	Network 3a		Network 3b	
	MILP	Dec–MILP	MILP	Dec–MILP
TTD	1	0.82	1	0.83
SoD	1	0.86	1	0.85
cpu time	1	0.6	1	0.03

doing so, the network's state changes during the simulation (from overall free to overall congested, and viceversa), and the controllers are then tested in different circumstances. The numerical results here presented are obtained as mean values over the entire simulation time.

We would like to stress the fact that the centralized strategy we propose is able to solve a very general problem, despite the computational inefficiency. The proposed traffic lights' representation, along with the numerical optimization, guarantees optimal behavior by means of the chosen objective index, as it may cleverly choose phase shifts between lights as well as green time for each of them. Therefore, the results obtained applying this techinique are considered as benchmark (upper bounds) to evaluate performances of the decentralized strategy, from traffic and computational performance point of view. We generate the afore mentioned scenario for two sample networks, shown in Fig. 3.

Representative results of the simulations are reported in Table I. Our numerical results are encouraging:

- The decentralized strategy obtains performance around 80-85% of the ones obtained by the centralized optimization;
- Computational time is dramatically reduced, especially with growing network's size (as expected from the NP–hardness of the problem). Notice also that in these simulation we did not make use of possible multithread implementation.

VI. Conclusions and future works

In this paper we presented an optimal control scheme for urban signalized traffic networks that schedules traffic lights taking into account upcoming external demands. This strategy, on one hand, ensures optimal performances, on the other, requires to solve a mixed integer linear program. To overcome the hardness of this problem, we proposed a decentralized realization of the same control scheme, that allows parallel computation. Our software simulations have shown that the computational load is extremely reduced by the decentralized scheme, which moreover achieves quite good performances from traffic point of view.

Future research will aim to investigate the scalability properties of the depicted techniques, combined with improvements on the gap with the upper bound provided by the centralized algorithm.

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