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Contrast re-enhancement of Total-Variation regularization jointly with the Douglas-Rachford iterations

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Abstract—Restoration of a piece-wise constant signal can be performed using anisotropic Total-Variation (TV) regularization. Anisotropic TV may capture well discontinuities but suffers from a systematic loss of contrast. This contrast can be re-enhanced in a post-processing step known as least-square refitting. We propose here to jointly estimate the refitting during the Douglas-Rachford iterations used to produce the original TV result. Numerical simulations show that our technique is more robust than the naive post-processing one.

I. INTRODUCTION

We consider the reconstruction of a 2D signal identified as a vector $u_0 \in \mathbb{R}^N$ from its noisy observation $f = \Phi u_0 + w \in \mathbb{R}^P$ with $w \in \mathbb{R}^P$ a zero-mean noise component and $\Phi \in \mathbb{R}^{P \times N}$ a linear operator accounting for a loss of information (e.g., low-pass filter). Anisotropic TV regularization writes, for $\lambda > 0$, as [1]

$$u^{\mathrm{TV}} \in \underset{u \in \mathbb{R}^N}{\operatorname{argmin}} \ \frac{1}{2} \|\Phi u - f\|^2 + \lambda \|\nabla u\|_1, \tag{1}$$

with $\nabla u \in \mathbb{R}^{2N}$ being the concatenation of vertical and horizontal components of the discrete gradient vector field of u, and $\|\nabla u\|_1 = \sum_i |(\nabla u)_i|$ being a sparsity promoting term. Anisotropic TV is known to recover piece-wise constant signals. However, even though the discontinuities can be correctly recovered in some cases, the amplitudes of u^{TV} are known to suffer from a loss of contrast compared to u_0 [2].

II. LEAST-SQUARE REFITTING PROBLEM

A simple technique to correct this effect, known as least-square refitting, consists in enhancing the amplitudes of $u^{\rm TV}$ while leaving unchanged the set of discontinuities, as

$$\tilde{u}^{\text{TV}} \in \underset{u ; \operatorname{supp}(\nabla u) \subset \operatorname{supp}(\nabla u^{\text{TV}})}{\operatorname{argmin}} \|\Phi u - f\|^2$$
(2)

where, for $x \in \mathbb{R}^{2N}$, $\operatorname{supp}(x) = \{i \in [2N]; \|x_i\| \neq 0\}$ denotes the support of x. Post-refitting identifies $\operatorname{supp}(\nabla u^{\mathrm{TV}})$ and solves (2) [3], typically with a conjugate gradient. However, u^{TV} is usually obtained thanks to a converging sequence u^k , and unfortunately, $\operatorname{supp}(\nabla u^k)$ can be far from $\operatorname{supp}(\nabla u^{\mathrm{TV}})$ even though u^k can be made arbitrarily close to u^{TV} . Such erroneous support identifications can lead to results that strongly deviates from the solution \tilde{u}^{TV} .

III. JOINT REFITTING WITH DOUGLAS-RACHFORD

To alleviate this difficulty, we build a sequence \tilde{u}^k jointly with u^k that converges towards a solution \tilde{u}^{TV} . We consider the Douglas-Rachford sequence u^k applied to the splitting TV reformulation [4] given by

$$u^{\mathrm{TV}} \in \operatorname*{argmin}_{u \in \mathbb{R}^N} \min_{z \in \mathbb{R}^{N \times 2}} \frac{1}{2} \| \Phi u - f \|^2 + \lambda \| z \|_1 + \iota_{\{z, u \ ; \ z = \nabla u\}}(z, u)$$

where ι_S is the indicator function of a set *S*. This leads to the proposed algorithm given, for $\tau > 0$ and $\beta \ge 0$, by Eq. (3) (see right column). The sequence u^k is exactly the Douglas-Rachford sequence converging towards a solution u^{TV} [5]. Regarding \tilde{u}^k , we prove the following.

Theorem 1. Let $\alpha > 0$ be the minimum non zero value of $|(\nabla u)_i|$, $i \in [2N]$. For $0 < \beta < \alpha \lambda$, \tilde{u}^k converges towards a solution \tilde{u}^{TV} .

Sketch of proof: As u^k converges towards a solution u^{TV} , for k large enough, we get after few manipulations and triangle inequalities that

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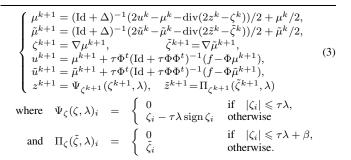




Fig. 1. Damaged image, result of TV, post-refitting and our joint-refitting.

 $\left\{ i \; ; \; |\zeta_i^k| > \tau \lambda + \beta \right\} = \sup(\nabla u^{\mathrm{TV}}) \text{ (for the given range of } \beta). \text{ As a result, for } k \text{ large enough, the sequence (3) can be rewritten by substituting } \Pi_{\zeta}(\cdot, \lambda) \text{ by the projector onto } \left\{ u \; ; \; \sup(u) \subset \sup(\nabla u^{\mathrm{TV}}) \right\} \text{ which is exactly the Douglas-Rachford sequence for the refitting problem (2) which is provably converging towards a solution } \tilde{u}^{\mathrm{TV}} \text{ [5]. } \square$

IV. RESULTS AND DISCUSSION

Figure 1 shows results on an 8bits image damaged by a Gaussian blur of 2px and white noise $\sigma = 20$. The parameter β is chosen as the smallest positive value up to machine precision. While TV reduces the contrast, refitting recovers the original amplitudes and keep unchanged the discontinuities. Post-refitting offers comparable results to ours except for suspicious oscillations due to wrong support identification.

Being computing during the Douglas-Rachford iterations, our refitting strategy is free of post-processing steps such as support identification. It is moreover easy to implement and can be used likewise for other ℓ_1 analysis penalties. Extensions of this approach for isotropic TV or block sparsity regularizations are under investigation.

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