Supplemental Material: Effects of inertia on the steady-shear rheology of disordered solids

Alexandre Nicolas[§], Jean-Louis Barrat
LIPhy, Université Grenoble-Alpes & CNRS, F-38000 Grenoble, France

Jörg Rottler[§] ([§]: equal contribution)
Department of Physics and Astronomy, The University of British Columbia, 6224 Agricultural Road, Vancouver, British Columbia V6T 1Z4, Canada

I. VELOCITY AUTOCORRELATION FUNCTIONS

Figure 1 presents the transverse velocity autocorrelation functions for different damping strengths, in the quiescent system.

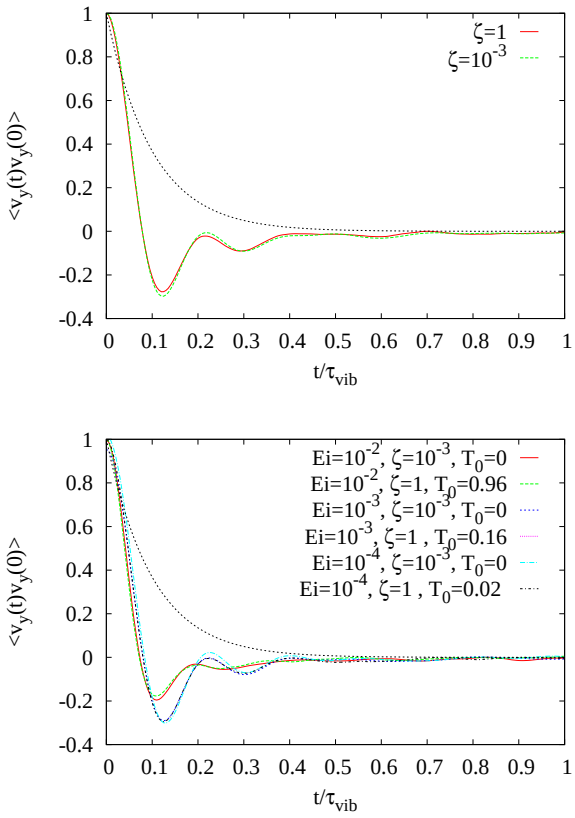

FIG. 1. Transverse velocity autocorrelation functions for particle mass $m = 1$ and different damping magnitudes ζ in (top) quiescent systems, *i.e.*, Ei $= 0$, at $T_0 = 0.16$ and (bottom) sheared systems, with Ei and T_0 as indicated in the legend. Note that the values of T_0 used for the flows at $\zeta = 1$ were chosen to match the kinetic temperatures of the strongly underdamped flows ($\zeta = 10^{-3}$) at the same rescaled shear rate Ei. The dashed black line is an envelope fit with the function $\exp(\frac{-t}{0.1\tau_{\rm vib}})$

II. KINETIC ENERGY DISTRIBUTIONS

Figure 2 presents the distribution of the kinetic energies among the particles in the system.

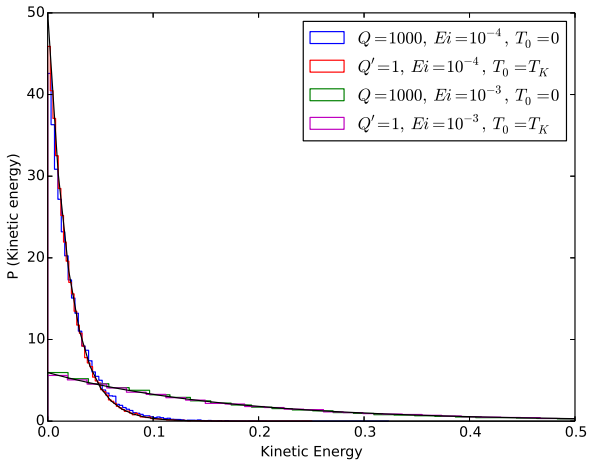

FIG. 2. Distribution of the kinetic energies $e_K = \frac{1}{2}mv_i^2$ of the individual particles in the athermal and thermostatted systems (as indicated in the legend). Solid black lines are fits to Boltzmann distributions $\propto e^{-e_K/T_K}$

III. ROBUSTNESS OF OUR FINDINGS: TEST OF A DIFFERENT THERMOSTAT

To test the robustness of our results, we subsitute a Langevin thermostat for the Dissipative Particle Dynamics scheme used in the main text. Figure 3 proves that, for low enough damping strength, the athermal Langevin flow curve also becomes nonmonotonic, with a rate-weakening region. More importantly, the thermostatting procedure described in the main text is also operational here, as evidenced in the figure. This confirms the connection between the underdamped rheology and the extent of kinetic heating.

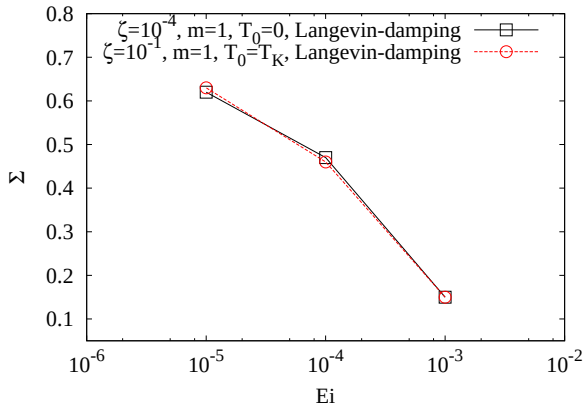

FIG. 3. Flow curves obtained with a Langevin thermostat (*black*) in an underdamped athermal system and (*red*) in the associated (less underdamped) thermostatted system.

IV. QUALITATIVE EXPLANATION FOR THE NONMONOTONIC FLOW CURVES IN THE HIGHLY UNDERDAMPED REGIME

As can be seen in Fig. 2 of the paper, the flow curves at large enough Q are nonmonotonic and thus display a minimum $(\mathrm{Ei}_m, \Sigma_m)$, where Ei is the rescaled shear rate. At this point, using the chain rule,

$$0 = \frac{d\Sigma}{d\mathrm{Ei}}(\mathrm{Ei}_m) = \frac{\partial\Sigma}{\partial T_K}\Big|_{\mathrm{Ei}_m} \cdot \frac{dT_K}{d\mathrm{Ei}} + \frac{\partial\Sigma}{\partial \mathrm{Ei}}\Big|_{T_K = T_K(\mathrm{Ei}_m)},$$

(As a technical detail, note that, in practice, the second partial derivative - at fixed kinetic temperature - can only be calculated in a system with stronger damping thermostatted to $T_0 = T_K(\mathrm{Ei}_m)$). Thus, the presence of a minimum results from a competition between two (antagonistic) effects:

(i) an increase of the sample temperature with the shear-rate, which bends the flow curve downwards

(ii) an intrinsic rate effect at large shear rates, whereby the driving gets so strong that particles no longer remain at the bottom of the energy wells (collisions, etc.).

When Q increases, *i.e.*, when the damping weakens, the temperature effect (i) gets stronger, so that the minimum is expected to be deeper, which is indeed confirmed by Fig. 2 of the main text.

V. DEVIATIONS FROM THE PREDICTED BEHAVIORS OBSERVED AT HIGH KINETIC TEMPERATURES

At high kinetic temperatures T_K(*i.e.*, in particular at large shear rates and/or large values of Q), Figs. 2 and 3 of the main text show that deviations appear from the Chattoraj-Lemaitre formula for the temperature-dependence of the stress (Eq. 3), and, to a much lower extent, from the scaling prediction represented in Fig. 3.

This is not surprising: Chattoraj and Lemaitre explicitly derived their for formula in the low-temperature limit, and our scaling prediction was based on the hypothesis that regions yield (undergo a plastic event) at a local stress Σ_0, whereas, owing to thermal activation, the local stress at yield may be significantly lower at high T_K.