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Abstract The network neutrality debate originally stems from the growing traffic asymmetry between ISPs, questioning the established peering or transit agreements. That tendency is due to popular content providers connected to the network through a single ISP, and whose traffic is not charged by distant ISPs. We propose in this paper to review the economic transit agreements between ISPs in order to determine their best strategy. We define a model with two ISPs, each providing direct connectivity to a fixed proportion of the content and competing in terms of price for end users, who select their ISP based on the price per unit of available content. We analyze and compare thanks to game-theoretic tools three different situations: the case of peering between the ISPs, the case where ISPs do not share their traffic (exclusivity arrangements), and the case where they fix a transfer price per unit of volume. Our results suggest that a minimal regulation, consisting in letting ISPs choose transit prices but imposing peering in case no agreement is reached, leads to satisfying outcomes in terms of user welfare while still leaving some decision space to ISPs, hence answering a concern they have regarding regulation in the Internet market.

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1 Introduction

The Internet has moved from an academic network connecting universities to a network used for everyday purposes and open to all. The network is now made of competitive and profit-seeking content providers (CPs) and Internet access providers (or Internet Service Providers (ISPs)). One important principle driving the current network is the *universal access* principle, meaning that all consumers are entitled to reach meaningful content, whatever the technical limitations of their service [12, 18, 31]. But because of large and bandwidth-consuming CPs (for example YouTube), some ISPs have started to wonder why distant CPs should not be charged by them, with the threat of their traffic not being delivered if they do not accept to pay [19, 30].

Our goal in this paper is to propose a model that will be solved by game-theoretic tools to better understand the relations between the three sets of players that are end users, ISPs and CPs, and to investigate from an economic point of view the relevance of a threat to not transfer the traffic coming from competitive ISPs (i.e., coming from CPs not directly connected to the considered ISP). Indeed, such threats have been used in the past (e.g., during the Cogent/Level 3 dispute in 2005); we show in this paper that even if that threat is not credible (the disconnection harms both sides), it strongly affects the result of the negotiations. In this paper, we compare the three following situations: (i) there is a peering agreement between ISPs who deliver the traffic coming from the competitive ISP at no cost (ii) there is no agreement and no traffic transfer from an ISP to another, limiting as a consequence the content offer (and therefore, potentially end-user demand) at each ISP (iii) there is a per-unit-of-volume transit price between the ISPs, which can be obtained from a negotiated agreement between the ISPs–with or without a disconnection threat–or determined by a regulator wishing to maximize user welfare. This model with the three different possibilities allows us to determine the best peering relationships between ISPs and the relevance of the threat to break the universality principle. Of course, our results have to be taken with care, since our model is a considerable simplification of the actual Internet ecosystem, whose increasing complexity since its creation is illustrated in [30].

There exists a recent flourishing literature dealing with network neutrality modeling and analysis, see among others [1, 2, 8, 11, 19–23, 25, 29] and the survey [15]. But those papers mainly discuss how revenue should be shared among providers (ISPs and CPs) or how neutrality or non-neutrality affect the providers' investment incentives, innovation at the content level, network quality, and user prices. The originality of our work relies on 1) the modeling of peering or transit traffic pricing between the ISPs, 2) the modeling of the amount of content directly connected to each ISP, and 3) the use of classical discrete choice models to define how users choose their ISP depending on price, reputation, loyalty, and available content.

With respect to the classification of approaches proposed in [15], our work is closest to [7, 9, 17], that consider several ISPs in competition (instead of a monopolistic one). However, those papers develop two-sided market models

(with users having to attract users but also CPs), while we consider the CP part as fixed and focus on the user side of the market. This enables us to develop a further analysis of the price competition, with a more realistic model than the Hotelling one used in [7,9] (where, e.g., users are forced to select one ISP) or the perfect competition among ISPs assumed in [17] (leading to ISPs making no revenues).
Note that our paper does not directly address the net neutrality problem by comparing a neutral to a non-neutral stance; rather, we intend to understand the (very closely) related issues of interconnection and peering/transit agreements among ISPs. The interconnection issues among ISPs competing to attract users are also studied in [6, 13, 16]. In [13] the decision variable as regards interconnection is the quality of the link between ISPs; in this paper we assume quality is sufficient, and the discussion is on the transit price to apply. In [6] the authors consider bargaining among ISPs deciding to interconnect or not: this is also close to our model, but no explicit competition is considered and the bargaining is on the sharing of extra revenue brought by the interconnection, no unit transit price with underlying price competition among ISPs is considered. Our work is also close to [16] where transit agreements are studied, but where the competition is among telephone providers applying usage-based pricing to users. This paper considers *flat-rate* pricing, that is the most popular one for Internet access. We however obtain some results comparable to [16], in particular the fact that it can be beneficial for ISPs to set a non-null transit price.
Finally, it is worth mentioning [14], not because the model is very similar (the authors consider a two-sided market with competition among ISPs but also among CPs), but because the issues and some conclusions are comparable. The authors study the risks of fragmentation of the Internet–where CPs having exclusivity agreements with some ISPs are not reachable for all users–and analyze the impact of net neutrality regulations (imposing null termination fees); their conclusion is that the user-welfare optimal case is the no-fragmented one, and that imposing null termination fees has ambiguous effect on user welfare while preventing exclusivity deals is an easily applicable rule that improves the outcome. Our conclusions, although in a slightly different context–interconnection transit prices among ISPs–and with a different model, are in the same vein: we advocate to let ISPs decide the transit price, the only regulation being to maintain connectivity (the counterpart of the no-exclusivity rule).

The remaining of the paper is organized as follows. Section 2 presents the basic assumptions of the model, the players, and the three scenarii we propose to analyze and compare. The user welfare is also formalized, and general formulas provided. Section 3 analyzes the game when there is peering between ISPs (i.e., with a null transit price) and end users have a full access to all CPs. Section 4 on the other hand describes the users repartition and pricing game among ISPs when they do not agree to exchange their traffic. Section 5 presents the analysis when there is a transit pricing agreement, the price being determined from a bargaining phase between ISPs, or by a regulator wishing to maximize user or social welfare. Section 6 concludes the paper by discussing the impact of the disconnection threat on the network neutrality debate, arguing that a good compromise is a minimal regulation consisting in enforcing connectivity among ISPs while letting them fix the transit price.

2 Model

2.1 Model basic components and notations

We consider two ISPs called A and B, in competition for end users. Those users are represented as a continuum of total mass assumed to be 1 without loss of generality.

We also assume that we have *content* that users may be interested to reach. We actually consider the *traffic volume* for that content, and also assume it of mass 1 without loss of generality. We call y_A (respectively, y_B) the proportion of downloaded content volume that is *directly* connected to the Internet through A (respectively, B). In other words, content providers also have to be connected to one of the two access providers A or B (or both), so that y_A represents the proportion of downloaded content attached to A. Note that the proportions y_A and y_B encompass the popularity of the contents, which weighs the computation of those proportions: y_A (respectively, y_B) represents the proportion of the total aggregated flow originating from CPs, that originates from a CP connected to ISP A (respectively, B). (We focus on downlink traffic only, since uplink traffic to content providers is limited to requests and is negligible.) Since we assume only two ISPs in this model, we have $y_A + y_B \geq 1$; the case when $y_A + y_B > 1$ corresponds to the situation where some CPs are *multihomed*, i.e., are attached to both ISPs. In that case the quantity $y_A + y_B - 1$ is the proportion of downloaded content coming from multihomed CPs. Note that some CPs do not directly contract with ISPs but rather use Content Delivery Networks (CDNs), that they operate themselves or that they pay for their service. CDN services consist in storing the data closer to the user, through cache memory servers located within (or close to) the ISPs to which users are connected. That case is also covered by our model: if the CDN has agreements with both ISPs–the most common case–then the corresponding traffic volume would count as multihomed content (a user wanting to access such content can get it with any ISP and no data transfer between ISPs is needed), while if the CDN can only directly connect to ISP A or B then the traffic volume is counted in y_A or y_B, respectively. In this paper we assume that $y_A + y_B < 2$, i.e., not all content is multihomed.

The quantities y_A and y_B are assumed fixed in our model, since we focus on the decisions that ISPs make in terms of traffic exchanges and end-user pricing. So-called *two-sided models* [3, 28] would consider y_A and y_B as the result of ISPs competing to attract content providers; we leave such approaches for future work, for sake of simplicity and because such CP-ISP interactions can be assumed to occur at a larger time scale than the one studied here.

In terms of pricing, we denote by p_A (respectively, p_B) the access price for a user to provider A (respectively, B). This access price is a flat-rate subscription fee, independent of the amount of volume that the user will download. We also assume that there might be a common price per unit of volume t for the traffic transferred between the two ISPs, defining an economic relationship between ISPs: if $t = 0$, this forms the classical peering agreement with no fee, while $t > 0$ means a transit pricing (also called "paid peering" [30]) usually adopted when there is a strong asymmetry. Note that we could consider a transit price for each traffic direction; we choose symmetric prices here for simplicity, and because in practice only one ISP pays the other, at the agreed unit price [30]. Those two situations–peering and paid transit–will be investigated in the next sections, with also the situation when the link between ISPs is broken, so that there is no possibility for users attached to an ISP to reach the content attached to the other ISP. All those relations are summarized in Figure 1.

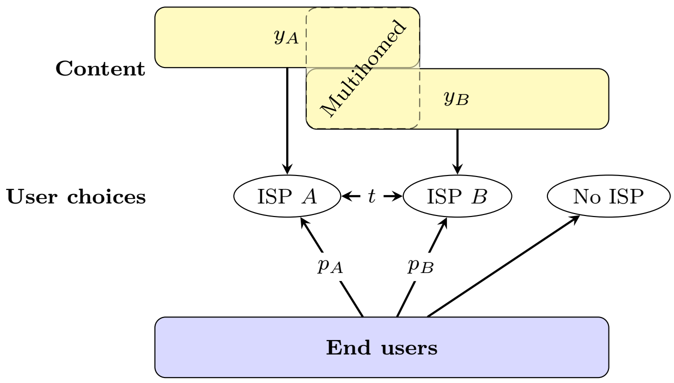

Fig. 1 Representation of relations between users, ISPs and CPs

2.2 User preferences

We assume that users select at most one ISP, their choice following a standard *discrete choice model* heavily used in travel behavior and econometrics in general [4]. We more specifically use a *logit* model that can approximate any random utility model. In a popular form of the logit model, the valuation (level of satisfaction) of a user q for an alternative or choice i is random and given by

$$V_{q,i} = v_i + \kappa_{q,i},$$

where v_i is the average valuation and $\kappa_{q,i}$ a random variable that represents some unobserved *random noise*. The $\kappa_{q,i}$ are assumed to be independent and to have a Gumbel distribution of mean 0: $\mathbb{P}[\kappa_{q,i} \leq z] = \exp(-\exp(-z - \gamma))$, where γ is Euler's constant. In the rest of this paper, since the random variables

$\kappa_{q,i}$ (and, as a result, $V_{q,i}$) are identically distributed, we will drop the index q.

The average valuation for an ISP i depends on the price p_i, but also on the amount of content available through that ISP, that we denote by x_i. For that latter measure we use the proportion of content that is likely to interest users, hence we reason on download traffic volumes: x_i is the proportion of download traffic of content reachable from ISP i. It is important to remark that the "mass" of available content x_A for a user connected through ISP A can differ from the proportion y_A of download traffic from CPs directly connected to ISP A. For instance, if ISPs exchange traffic then $x_A = x_B = 1$ (all the content is reachable from all ISPs); while if there is no transit between ISPs, $x_A = y_A$ and $x_B = y_B$.

Finally, we assume that the average valuation v_i depends on the *price per unit of available content*, p_i/x_i, through the standard logarithmic relation

$$v_i \stackrel{\text{def}}{=} \alpha \log\left(\frac{x_i}{p_i}\right), \tag{1}$$

where $\alpha > 0$ is a sensitivity parameter. This type of logarithmic functional has also recently been justified in telecommunications using the context of the Weber-Fechner Law, a key principle in psychophysics describing the general relationship between the magnitude of a physical stimulus and its perceived intensity within the human sensory system [27]. For our model, the logarithmic function implies that the disutility perceived by a price raise is a function of the *relative* price change, rather than the absolute change. The parameter α then represents how sensitive users are to a given ratio among prices per unit of content: for instance, when comparing providers A and B in the case when they exchange traffic, the difference of perception due to price is $\alpha \log(p_B/p_A)$. This difference will then affect the user choices, modulo the random part $\kappa_{q,i}$ described above. Large values of α will diminish the impact of that random part, so that users will mainly focus on prices, while on the other extreme low values of α mean that prices have no impact on user choices, since those are driven by the reputation and brand aspects contained in the variables $\kappa_{q,i}$. Note that a null price yields an infinite valuation, so that a free option will always be preferred to one with charge.

We additionally assume that there is an *outside option* labelled 0, with average valuation v_0, representing the (possibly negative) valuation for not choosing any ISP, and that will be compared with the valuation for ISPs. In accordance with (1), we also define $p_0 \stackrel{\text{def}}{=} \exp(-v_0/\alpha)$, representing the cost associated to not benefitting from any content. In our numerical illustrations, unless specified otherwise, we will take $p_0 = 1$ (or, equivalently, $v_0 = 0$).

Each user chooses the option yielding the largest valuation. Following classical discrete choice analysis, a user will choose the option $i \in \{A, B, 0\}$ with probability

$$\tilde{\sigma}_i(v_A, v_B, v_0) \stackrel{\text{def}}{=} \frac{\exp(v_i)}{\exp(v_A) + \exp(v_B) + \exp(v_0)}. \tag{2}$$

Consequently, for a given price profile (p_A, p_B, p_0) and available contents (x_A, x_B), that probability is

$$\sigma_i(p_A, p_B, p_0) \stackrel{\text{def}}{=} \frac{(x_i/p_i)^\alpha}{(x_A/p_A)^\alpha + (x_B/p_B)^\alpha + 1/p_0^\alpha}. \tag{3}$$

Several remarks can be made here:

- The probability $\sigma_i(p_A, p_B, p_0)$ is also the *proportion of users that will choose option* $i \in \{A, B, 0\}$ given that users' choice have been assumed independent and that the total mass of users is 1.
- The ISPs that set their price to zero will attract all the users. However, their revenue is null in this case.
- The distribution $(\sigma_i(p_A, p_B, p_0))_{i\in\{A,B,0\}}$ uniformly concentrates on choices whose price are minimal when α goes to ∞, and is uniform on $\{A, B, 0\}$ when it goes to zero.
- We could assume a different parameter α associated to each option α_i for $i \in \{A, B, 0\}$. As a consequence the choice probability would turn into

$$\sigma_i(p_A, p_B, p_0) \stackrel{\text{def}}{=} \frac{(x_i/p_i)^{\alpha_i}}{(x_A/p_A)^{\alpha_A} + (x_B/p_B)^{\alpha_B} + 1/p_0^{\alpha_0}}.$$

  This heterogeneous case will be discussed for some cases when it is analytically tractable. However for reasons of clarity, those discussions are given in appendices, and we focus in the paper on the homogeneous case.

2.3 ISPs' utilities

ISPs' utilities are modeled by the revenues they get, that come from the subscription of end users and the transit fees between ISPs, if any. The subscription revenues are proportional to the market share σ_i of each provider i and its price p_i. To express the transit fees, recall that for a customer of ISP A, y_A can be interpreted as the proportion of the download traffic that will go directly through ISP A, while the remaining proportion $1 - y_A$ will have to be delivered through B and then A (if possible), following Figure 1, to reach the user. That fact, then, implies some money transfer (transit costs) from ISP A to ISP B in case of transit agreements where the receiver that has asked for the service has to pay for it (the so-called *pull model*).

The total amount of traffic transferred from B to A is $(1-y_A)\sigma_A(p_A, p_B, p_0)$, paid by A to B, while it is $(1 - y_B)\sigma_B(p_A, p_B, p_0)$ from A to B, paid by B. Denote by $\Delta_{A,B}$ the differential amount of traffic that is transferred from ISP A to ISP B, i.e.

$$\Delta_{A,B} = (1 - y_B)\sigma_B(p_A, p_B, p_0) - (1 - y_A)\sigma_A(p_A, p_B, p_0). \tag{4}$$

Then the respective revenues U_A and U_B of provider A and B are

$$\begin{aligned} U_A(p_A, p_B, p_0) &= \sigma_A(p_A, p_B, p_0)p_A + t\Delta_{A,B} \\ U_B(p_A, p_B, p_0) &= \sigma_B(p_A, p_B, p_0)p_B - t\Delta_{A,B}. \end{aligned} \tag{5}$$

2.4 User welfare

The *user welfare*, or *user surplus*, is defined as the aggregated net benefit that users get from the system. We consider here as a reference outcome the one with no service, for which the (random) user value is $V_0 = v_0 + \kappa_0$. From the logit model defined in the previous subsection, the net surplus of a user is what he or she gains compared to that outside option, i.e., $Z \stackrel{\text{def}}{=} \max(0, V_A - V_0, V_B - V_0)$. Because the total user mass is 1, the user welfare, that we denote by UW, is $\mathbb{E}[Z]$. Now, remark that for $z \geq 0$,

$$\begin{aligned}
\mathbb{P}[Z \leq z] &= \mathbb{P}[(V_A - V_0 \leq z) \cap (V_B - V_0 \leq z)] \\
&= \mathbb{P}[(V_0 \geq V_A - z) \cap (V_0 \geq V_B - z)] \\
&= \frac{\exp(v_0)}{\exp(v_0) + \exp(-z)(\exp(v_A) + \exp(v_B))}.
\end{aligned}$$

The last equation is a direct consequence of (2), subtracting z to the average valuations of options A and B.

Therefore, we have

$$\begin{aligned}
\text{UW} = \mathbb{E}[Z] &= \int_{z=0}^{+\infty} \mathbb{P}[Z > z]\, dz \\
&= \int_{z=0}^{+\infty} \frac{\exp(-z)(\exp(v_A) + \exp(v_B))}{\exp(v_0) + \exp(-z)(\exp(v_A) + \exp(v_B))}\, dz \\
&= \log\left(1 + \exp(v_A - v_0) + \exp(v_B - v_0)\right) \qquad (6) \\
&= \log\left(1 + \left(x_A \frac{p_0}{p_A}\right)^{\alpha} + \left(x_B \frac{p_0}{p_B}\right)^{\alpha}\right).
\end{aligned}$$

As expected, the user welfare is always nonnegative, since users still have the possibility to select no provider as in the reference situation: they only choose to subscribe to a provider if it increases their utility.

2.5 Scenarios and game analysis

We recall that in the next sections, we will analyze the pricing game between ISPs (and the transit pricing agreement) in three different scenarios:

1. In the first one, users can access all the content, independently of the chosen ISP, because there is a peering agreement between ISPs ($t = 0$ and $x_A = x_B = 1$ in our model).
2. In the second one, the link between ISPs is broken because they fail to agree on traffic exchange. Therefore, users can only access the content associated with the ISP they have chosen.
3. Finally, in the third scenario, the ISPs define a price per unit of volume they charge each other for the traffic downloaded from the CPs associated to their competitor and transmitted to their customers.

We can notice a hierarchy in the game analysis: at the highest time scale (only for the third scenario), the transit price is chosen (by a bargaining phase or by a regulator). At the intermediate level, ISPs compete on prices to attract customers and maximize their revenue in a classical non-cooperative game. Finally, at the smallest time scale, customers choose their ISP based on available content and price. Remark that those levels are solved by *backward induction*, anticipating the result at the time scale below.

3 Scenario 1: Peering between ISPs

In this scenario, ISPs have a peering agreement, and the users can access all the available content. Consequently $x_A = x_B = 1$ and $t = 0$ in our model. For given subscription fees p_A and p_B at providers A and B, the proportion of users choosing option $i \in \{A, B, 0\}$ is

$$\sigma_i(p_A, p_B, p_0) = \frac{(1/p_i)^\alpha}{(1/p_A)^\alpha + (1/p_B)^\alpha + 1/p_0^\alpha}. \tag{7}$$

The revenue of $i \in \{A, B\}$ can be expressed (with $t = 0$) as

$$\begin{aligned} U_i^{\text{peer}}(p_A, p_B, p_0) &= \sigma_A(p_A, p_B, p_0) p_i \\ &= p_i \frac{(1/p_i)^\alpha}{(1/p_A)^\alpha + (1/p_B)^\alpha + 1/p_0^\alpha}. \end{aligned}$$

Knowing what will be the user repartition for a price profile (p_A, p_B) (with p_0 fixed), ISPs try non-cooperatively to maximize their revenue. The equilibrium notion is that of Nash equilibrium, that would be a price profile from which no ISP can increase its revenue by unilaterally changing its price [26]. Formally, it is a price profile $(p_A^{\text{peer,NE}}, p_B^{\text{peer,NE}})$ such that $\forall p_A, p_B$, $U_A^{\text{peer}}(p_A^{\text{peer,NE}}, p_B^{\text{peer,NE}}, p_0) \geq U_A(p_A, p_B^{\text{peer,NE}}, p_0)$ and $U_B^{\text{peer}}(p_A^{\text{peer,NE}}, p_B^{\text{peer,NE}}, p_0) \geq U_B(p_A^{\text{peer,NE}}, p_B, p_0)$.

From the definition, $(0, 0)$ is a Nash equilibrium because a unilateral increase in price from an ISP will drive all demand to the other ISP (recall that a free option is always preferred from our discrete choice model, the associated valuation being infinite), hence a revenue still at 0. Yet, the choice for an ISP to deliver the service for free is a *strictly dominated strategy* unless the other ISP is doing the same (in which case it is only weakly dominated, since all strategies lead to a null revenue). Because non-zero price equilibria Pareto-dominate the price profile $(0, 0)$, in the rest of the paper we assume that ISPs choose strictly positive prices when such a Nash equilibrium exists.

The following proposition establishes the existence and uniqueness of a Nash equilibrium in the peering case.

Proposition 1 *If $1 < \alpha \leq 2$, there exists a unique Nash equilibrium different from (therefore not considered) $(0, 0)$, with equilibrium prices*

$$p_A^{\text{peer,NE}} = p_B^{\text{peer,NE}} = \left(\frac{2-\alpha}{\alpha-1}\right)^{1/\alpha} p_0 \stackrel{\text{def}}{=} p^{\text{peer,NE}}.$$

The ISPs' revenues are then

$$U_A^{\text{peer,NE}} = U_B^{\text{peer,NE}} = \frac{\alpha - 1}{\alpha} p^{\text{peer,NE}}$$
$$= (2-\alpha)^{1/\alpha} \frac{(\alpha-1)^{1-1/\alpha}}{\alpha} p_0.$$

The case when $\alpha \leq 1$ results in infinite prices, and $\alpha \geq 2$ leads to a price war, i.e., providers decreasing their prices to 0, so that $(0,0)$ is the unique equilibrium.

Proof We consider the derivatives of ISPs' revenues with respect to their own prices. For $i \in \{A, B\}$, we obtain

$$\frac{\partial U_i^{\text{peer}}}{\partial p_i} = \sigma_i(1 - \alpha(1 - \sigma_i)), \tag{8}$$

with σ_i given in (7).

When $\alpha \leq 1$, the derivatives are always strictly positive, and prices tend to infinity.

We now consider the case $\alpha > 1$. Remark that σ_i is a strictly decreasing function of p_i on $\mathbb{R}_+$, going from 1 to 0, when the competitor sets a strictly positive price. Therefore, from (8), for each value $p_j > 0$ of its competitor's price, provider i has a unique best-reply price, that is strictly positive and such that $\alpha(1 - \sigma_i) = 1$, *i.e.* $\sigma_i = 1 - \frac{1}{\alpha}$. Therefore, when $\alpha > 1$ there can be only one Nash equilibrium $(p_A^{\text{peer,NE}}, p_B^{\text{peer,NE}})$ with positive prices, that is such that $\sigma_A = \sigma_B = 1 - \frac{1}{\alpha}$. We immediately notice that such an equilibrium can only exist if $\alpha \leq 2$, because otherwise we would have $\sigma_A + \sigma_B > 1$. In that case, solving $\sigma_A = \sigma_B = 1 - \frac{1}{\alpha}$ based on (7) easily leads to the Nash equilibrium prices provided in the proposition.

To analyze the case when $\alpha > 2$, we explicitly express the best-reply price function $\text{BR}_i(p_j)$ of provider i when its competitor sets a strictly positive price p_j. That price is the value p_i such that $\sigma_i = 1 - \frac{1}{\alpha}$, i.e., $\text{BR}_i(p_j) = p_i = \left((\alpha - 1)(p_j^{-\alpha} + p_0^{-\alpha})\right)^{-1/\alpha}$. Remark that when $\alpha \geq 2$, since $p_0 > 0$ we have $(\alpha-1)(p_j^{-\alpha} + p_0^{-\alpha}) > p_j^{-\alpha}$ and therefore $\text{BR}_i(p_j) < p_j$. In other terms, when a provider sets a strictly positive price p_j, the best reply of its competitor is strictly below that price. That situation leads to a price decrease until providers do not make any revenue. □

Notice that the result for $\alpha \leq 1$ corresponds to users with low price sensitivity: when a provider i increases its price p_i, the demand σ_i decreases slowly, and the product $p_i\sigma_i$ increases. In economic terms, this corresponds to a price elasticity E_i of demand for provider i (when the other providers keeps its price constant) being small in absolute value, i.e.

$$E_i \overset{\text{def}}{=} \frac{p_i}{\sigma_i} \frac{\partial \sigma_i}{\partial p_i} = -\alpha(1 - \sigma_i) \in (-1, 0).$$

In our model, such small price elasticities incentivize ISPs to set infinite prices (as seen previously). Since such an outcome is not realistic, we omit that case an from now on assume that $\alpha > 1$. The opposite situation of prices decreasing to 0 when $\alpha \geq 2$, to attract customers of the competitor, is called *price war*. It comes from a large price-sensitivity of users.

Let us now analyze the outcome that ISPs can achieve by cooperating, i.e., by setting prices so as to maximize the sum $U_A^{\text{peer}} + U_B^{\text{peer}}$ of their utilities.

Proposition 2 *The sum of ISPs utilities is maximized by setting prices p_A and p_B to*

$$p_{\max}^{\text{peer}} = \left(\frac{2}{\alpha - 1}\right)^{1/\alpha} p_0, \tag{9}$$

and is equal to $U_{\max}^{\text{peer}} = \dfrac{\alpha-1}{\alpha} p_{\max}^{\text{peer}} = 2^{1/\alpha} \dfrac{(\alpha-1)^{1-1/\alpha}}{\alpha} p_0.$

Proof First note that if at least one price is zero, then the total revenue of ISPs is null, hence such a price profile cannot be optimal. Thus, either the optimal price of a provider is ∞, or it nullifies the derivative. Remark that the derivative of $U_A^{\text{peer}} + U_B^{\text{peer}}$ with respect to p_i with $i \in \{A, B\}$ (and with $j \neq i$) is

$$\frac{\partial (U_A^{\text{peer}} + U_B^{\text{peer}})}{\partial p_i} = \frac{\left(p_i^{1-\alpha} + \alpha p_j^{1-\alpha} - (\alpha-1) p_i \left(p_j^{-\alpha} + p_0^{-\alpha}\right)\right)}{p_i^{1+\alpha} (p_i^{-\alpha} + p_j^{-\alpha} + p_0^{-\alpha})^2}.$$

Recall that $\alpha > 1$, and note that the derivative with respect to p_i ($i \in \{A, B\}$) is strictly negative if p_i is sufficiently large. The optimal pricing is when the two above derivatives are null, *i.e.* when

$$p_A^{1-\alpha} + \alpha p_B^{1-\alpha} - (\alpha - 1) p_A \left(p_B^{-\alpha} + p_0^{-\alpha}\right) = 0$$
$$p_B^{1-\alpha} + \alpha p_A^{1-\alpha} - (\alpha - 1) p_B \left(p_A^{-\alpha} + p_0^{-\alpha}\right) = 0.$$

The difference between those equations leads to $(p_A - p_B)(p_A^{-\alpha} + p_B^{-\alpha} + p_0^{-\alpha}) = 0$ which implies $p_A = p_B$. Then their common value $p_{\max}^{\text{peer}}$ is obtained from any of those equations, yielding the unique solution given in (9). □

We now compute the cost of competition for ISPs, through the ratio of their total revenues without cooperation (i.e., at the Nash equilibrium) and with collaboration. That ratio equals

$$\frac{U_A^{\text{peer,NE}} + U_B^{\text{peer,NE}}}{U_{\max}^{\text{peer}}} = 2\left(1 - \frac{\alpha}{2}\right)^{1/\alpha}.$$

Notice that it does not depend on p_0. We remark that the larger the user sensitivity to prices, the more ISPs lose by not cooperating.

For $1 < \alpha < 2$, it can be readily checked that a larger sensitivity to price (i.e., a larger α) yields a larger user welfare and smaller ISPs revenues, and also that a price competition among ISPs is better for users, this improvement increasing with α because the price sensitivity exacerbates competition as highlighted before.

4 Scenario 2: No traffic exchanged between ISPs

Because of the potential traffic asymmetry between ISPs, the peering agreement of previous section may not seem satisfactory for one ISP. Two alternatives are considered in this paper: to break the connection between ISPs so that subscribers of a provider have access to the content of that provider only, or to set a transit price such that an ISP has to pay for the content hosted by the competitor and accessed by its own customers (this last scheme is the topic of the next section). We aim in this section to study what happens if there is no traffic exchanged between ISPs, and if we can identify a loser and/or a winner. Breaking the transit possibility is not necessarily beneficial for a provider since users have access to less content and may then prefer not to subscribe to any provider. It is interesting to note that breaking the connection between ISPs has been implemented in the past; this was for instance the case in 2005 during a dispute between the ISPs Cogent and Level 3, with as a consequence undelivered emails and unreachable web sites for some customers [10].

In our model, we make the simplifying assumption that the CP-ISP agreements will remain unchanged, while it is likely that some CPs may want to switch ISPs or to be multihomed. Such dynamics are not considered here.

From our model, if the communication link between ISPs is cut, then $x_A = y_A$ and $x_B = y_B$, and from (3), the ISP revenues are

$$U_A^{\mathrm{cut}}(p_A, p_B, p_0) = p_A \frac{(y_A/p_A)^\alpha}{(y_A/p_A)^\alpha + (y_B/p_B)^\alpha + 1/p_0^\alpha}$$
$$U_B^{\mathrm{cut}}(p_A, p_B, p_0) = p_B \frac{(y_B/p_B)^\alpha}{(y_A/p_A)^\alpha + (y_B/p_B)^\alpha + 1/p_0^\alpha}.$$

The next proposition characterizes the outcome of the competition between ISPs in this case. Here too, $(p_A, p_B) = (0, 0)$ is a Nash equilibrium but the free strategy is a *strictly dominated strategy* unless the other ISP is doing the same so that we will again ignore this strategy when a Nash equilibrium different from $(0, 0)$ exists.

Proposition 3 *If $1 < \alpha \leq 2$ and $0 < y_i < 1, i = 1, 2$, there exists a unique Nash equilibrium different from $(0, 0)$, with equilibrium prices $p_A^{\mathrm{cut,NE}} = y_A p^{\mathrm{cut,NE}}$ and $p_B^{\mathrm{cut,NE}} = y_B p^{\mathrm{cut,NE}}$, where*

$$p^{\mathrm{cut,NE}} = p^{\mathrm{peer,NE}} = \left(\frac{2-\alpha}{\alpha-1}\right)^{1/\alpha} p_0.$$

The ISPs' revenues are then $U_A^{\mathrm{cut,NE}} = y_A U^{\mathrm{cut,NE}}$ and $U_B^{\mathrm{cut,NE}} = y_B U^{\mathrm{cut,NE}}$, where

$$U^{\mathrm{cut,NE}} = \frac{\alpha-1}{\alpha} p^{\mathrm{cut,NE}} = (2-\alpha)^{1/\alpha} \frac{(\alpha-1)^{1-1/\alpha}}{\alpha} p_0.$$

The case $\alpha \geq 2$ leads to a price war with $(0, 0)$ as the unique equilibrium.

Proof The proof mimics the one of Proposition 1, since we again have for each provider i

$$\frac{\partial U_i^{\text{cut}}}{\partial p_i}(p_A, p_B, p_0) = \sigma_i(1 - \alpha(1 - \sigma_i))$$

with $\sigma_i = \frac{(y_i/p_i)^\alpha}{(y_A/p_A)^\alpha + (y_B/p_B)^\alpha + 1/p_0^\alpha}$. □

At a Nash equilibrium, ISPs' price (resp., revenue) is equal to the price (resp., revenue) they set (resp., get) in the peering scenario of Section 3, multiplied by the proportion of content they control. As an important conclusion, *no* ISP i has an interest in breaking the connection, whatever the content y_i it controls, because its revenue will be reduced (or the same if $y_i = 1$).

Surprisingly, if one ISP controls the whole set of contents (for example if $y_A = 1$, i.e., all CPs are attached to ISP A), then it gets exactly the same revenue as in the first scenario, whatever the content hosted by the competitor.

In the previous section we have studied the case where ISPs were cooperating (and peering) in order to maximize the sum of their revenues. Though we have not looked at how they split the total revenue among themselves. A question we would like to answer now is: is there a situation such that *both ISPs* have an interest in cooperating for a given repartition of the total revenue with respect to a price competition and no traffic exchange? If the answer is positive, what is the interval such that the bargaining is satisfying?

Let us consider a revenue sharing agreement among ISPs, when cooperating and peering, such that provider A gets a proportion π_A of the total revenue $U_{\max}^{\text{peer}}$, and provider B obtains the rest. ISP A would then prefer that agreement over a pure competitive situation with no traffic exchange, if $\pi_A U_{\max}^{\text{peer}} \geq U_A^{\text{cut,NE}} = y_A U^{\text{cut,NE}}$. From Propositions 2 and 3, we obtain that this holds if and only if $\pi_A \geq y_A(1 - \alpha/2)^{1/\alpha}$. Similarly, such an agreement is acceptable for ISP B if and only if $\pi_B = 1 - \pi_A \geq y_B(1 - \alpha/2)^{1/\alpha}$, so that the agreement is stable when

$$y_A(1 - \alpha/2)^{1/\alpha} \leq \pi_A \leq 1 - y_B(1 - \alpha/2)^{1/\alpha}.$$

In particular, we remark that the width of the stable area increases with $\alpha \in (1, 2)$. For example, when α tends to 2, any revenue sharing of cooperatively obtained revenue is acceptable by ISPs, because competition with such price-sensitive users would lead to a price war and null revenues. On the other opposite, when α tends to 1 then the acceptable sharing set is reduced to $\pi_A \in [y_A/2, 1 - y_B/2]$. Figure 2 displays that acceptable region for π_A in terms of $\alpha \in (1, 2)$, with $y_A = 0.8$ and $y_B = 0.5$.

Finally, Figure 3 shows the utilities of ISPs and user welfare as a function of the parameter α. The shape of the curves is the same as in Scenario 1, even if the utilities of ISPs are about two times smaller.

5 Scenario 3: transit pricing

We now address the case where the traffic transferred from an ISP to the other is compensated for by some payment. We first determine if a Nash equilibrium

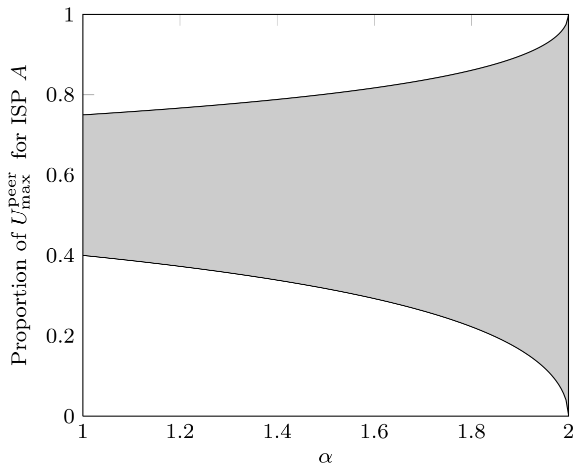

Fig. 2 In gray, the set of proportions of $U_{\max}^{\text{peer}}$ that ISP A can receive so that both ISPs accept to cooperate with respect to a broken connection threat. Here $y_A = 0.8$ and $y_B = 0.5$.

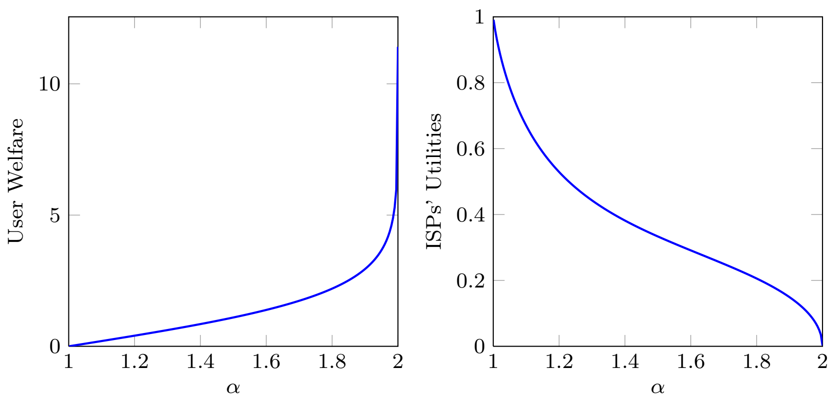

Fig. 3 User welfare *(left)* and sum of ISP's utilities *(right)* at Nash equilibrium, with no exchanged traffic, with $x = 0.8$.

exists in that context when such a transit unit price $t > 0$ is determined, and we characterize it. Then, we discuss how the transit price can be determined on top of the pricing game for customers, by a regulator or through a bargaining phase between ISPs. Remark that, here again, the transit price is chosen first, but by backward induction, anticipating the equilibrium of the (pricing) game played afterwards.

We begin by studying the game given by the revenue functions (5), with a fixed transit price $t > 0$, the case $t = 0$ corresponding to our Scenario 1. A first difference with the previous scenarios is that the pricing $(0, 0)$ is no longer a Nash equilibrium. Indeed, when an ISP sets its price to zero, it gets no revenue from the users, but only from the transferred traffic charged to the other ISP, while it has to pay for its client accessing the content attached to the competitor ISP. Then two cases appear:

- If the amount of traffic transferred between each ISP is exactly the same, i.e., $\Delta_{A,B} = 0$ (see (4)), then both ISPs earn zero revenue. But since at least one ISP has some content associated to it - say, w.l.o.g., ISP A, i.e., $y_A > 0$ -, that one could get a strictly positive revenue by setting any strictly positive price $p_A > 0$: the market shares would be $\theta_A = 0$ and $\theta_B = 1$, leading to a revenue $U_A = ty_A > 0$.
- If $\Delta_{A,B} \neq 0$, then one ISP (ISP A if $\Delta_{A,B} < 0$, ISP B otherwise) gets a strictly negative revenue, whereas it can ensure a nonnegative one by setting a strictly positive price and having no subscribers, hence only collecting revenue from transit traffic payments.

In order to determine the Nash equilibrium of the game, we first analyze the best-response functions of each ISP. The derivative of ISP i's revenue is (with $i, j \in \{A, B\}$, $j \neq i$):

$$\frac{\partial U_i}{\partial p_i} = \frac{K(p_j)p_i^{\alpha} + L_i(p_j)p_i^{\alpha-1} + 1}{p_i^{2\alpha}\left(p_i^{-\alpha} + p_j^{-\alpha} + p_0^{-\alpha}\right)^2}, \tag{10}$$

where $K(z) \stackrel{\text{def}}{=} (1-\alpha)(z^{-\alpha}+p_0^{-\alpha})$ and $L_i(z) = \alpha t\left((2 - y_A - y_B)z^{-\alpha} + (1 - y_i)p_0^{-\alpha}\right)$. Notice that $L_i \geq 0$. Furthermore, $K(z) < 0$ since $\alpha > 1$.

We do not reach an analytical expression of the best-response prices of each ISP. Nevertheless, several useful properties are listed in the next proposition, whose proof is given in Appendix C. For convenience, we denote by $\mathrm{BR}_A^t(p_B)$ and $\mathrm{BR}_B^t(p_A)$ the best-response correspondences for, respectively, ISP A and B with transit price t.

Lemma 1 *For every $y_A, y_B \in [0, 1]^2$, the best-response price correspondence of each ISP i to the competitor (ISP $j \neq i$) pricing strategy satisfies the following properties:*

(i) it is single-valued,
(ii) it is continuous,
(iii) it is uniformly bounded with strictly positive bounds. Furthermore, each ISP can ensure a strictly positive net revenue (that includes user subscriptions and transit prices).
(iv) it increases with the transit price t, i.e. $\forall p_j, t, r$,

$$t > r \Leftrightarrow BR_i^t(p_j) > BR_i^r(p_j),$$

(v) $BR_i^t(p_j)$ is strictly increasing (resp., strictly decreasing, constant) in the competitor's price if $\frac{(\alpha(2-y_A-y_B)t/p_0)^{\alpha}}{(\alpha-1)^{\alpha-1}}\left(\frac{1-y_j}{2-y_A-y_B}\right)$ is lower than (resp., greater than, equal to) one.

Those properties can be used to prove the existence of a Nash equilibrium for the pricing game played among ISPs, in the situation of paid transit.

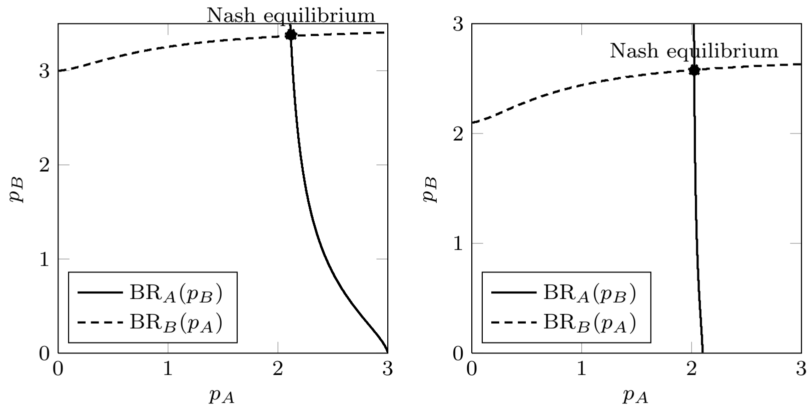

Fig. 4 ISP's best response functions, with $y_A = 0.8$, $\alpha = 1.5$, $t = 1$, for $y_B = 0.2$ *(no CP multihoming, left)* and $y_B = 0.5$ *(30% of multihomed content, right).*

Proposition 4 *Consider the game where the ISPs compete on their prices p_A and p_B to maximize their revenue given in (5). Then there exists a Nash equilibrium $(p_A^{\text{trans,NE}}, p_B^{\text{trans,NE}})$ with $p_A^{\text{trans,NE}} > 0$ and $p_B^{\text{trans,NE}} > 0$, resulting in strictly positive revenues.*

Proof From Lemma 1, the best-response function of each ISP i is continuous, and bounded by strictly positive values. Let us denote by m_i (resp., M_i) the lower (resp., upper) bound of BR_i^t. Then consider the application

$$\begin{aligned} g : [m_A, M_A] \times [m_B, M_B] &\mapsto [m_A, M_A] \times [m_B, M_B] \\ (p_A, p_B) &\to (\mathrm{BR}_A^t(p_B), \mathrm{BR}_B^t(p_A)). \end{aligned}$$

Since g is continuous and $[m_A, M_A] \times [m_B, M_B]$ is a compact convex subset of $\mathbb{R}^2$, from Brouwer's fixed point theorem, it has a fixed point that constitutes a Nash equilibrium with strictly positive prices $(p_A^{\text{trans,NE}}, p_B^{\text{trans,NE}}) \in [m_A, M_A] \times [m_B, M_B]$. The strict positivity of the revenues comes from the fact that, from item *(iii)* of Lemma 1, each ISP can always ensure a strictly positive revenue, whatever the other ISP price. □

Note that, unlike in the previous scenarios, there is no price war when $\alpha \geq 2$ as soon as $t > 0$, meaning that both providers benefit from the transit pricing because they then reach a strictly positive revenue. In the following, we show that it is at the expense of the user welfare.

Figure 4 shows a numerical approximation of best-response functions. Their form suggests that the Nash equilibrium is unique; however, we did not manage to provide theoretical evidence of that result, and can only conjecture the Nash equilibrium is unique.

We know that, for every price transit t, there exists a Nash equilibrium. We are now interested in the way the transit price is determined anticipating the fact that ISP will select some Nash equilibrium prices afterwards. The next proposition states that a regulator seeking to optimize the user welfare should impose a null transit pricing.

Proposition 5 *The unit transit price maximizing user welfare is* $t = 0$*, which corresponds to the peering situation between ISPs (Scenario 1).*

The proof is given in Appendix D.

A regulator seeking to maximize user welfare will set the transit price to zero. But if the aim is to maximize the ISPs (sum of) utilities, it is no longer the case, as illustrated on Figure 5, where the value of t that maximizes the total ISPs' utility is about 0.8. Figure 6 illustrates the evolution of ISPs' utility at the Nash equilibrium when the transit price t varies. For small values of t, the utilities of both ISPs increase. One can notice that the utility of the ISP that owns the least content starts to decrease first. We also see that the point that maximizes the sum of utilities is very close to the maximal possible revenue for ISPs if they cooperate, which corresponds to $U_{\max}^{\text{peer}}$ given in the peering scenario.

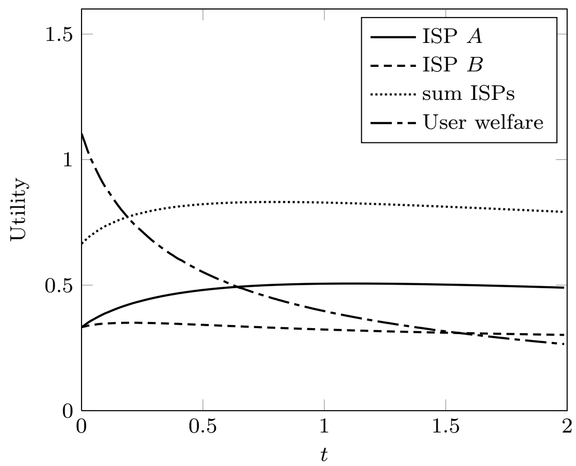

Fig. 5 Utilities of the stakeholders at Nash equilibrium prices, as a function of t, with $y_A = 0.8$, $y_B = 0.2$, $\alpha = 1.5$.

We now compare several policies for choosing the transit price.

- A first policy consists in maximizing the user welfare. From the previous proposition, that amounts to setting $t = 0$.
- A second policy consists in maximizing the sum of ISP utilities (for instance it could be applied by a regulator).
- And finally, we compare those two policies with the one obtained by a non-cooperative bargaining process between the ISPs two possibilities are considered.

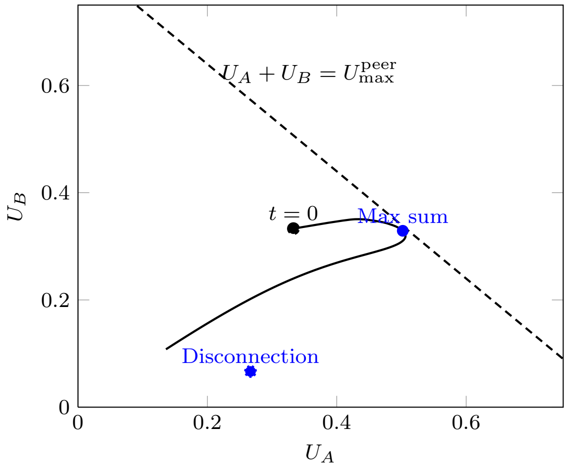

Fig. 6 ISP utilities at Nash equilibrium prices, when the transit price t varies. The threat point (*i.e.*, the utilities corresponding to the Nash equilibrium when ISPs are disconnected), the point optimizing the sum of utilities as well as the maximal revenue by cooperating on prices (dashed line) are drawn. Parameter values: $\alpha = 1.5$, $y_A = 0.8$, $y_B = 0.2$.

Let us detail the bargaining process. Here we use the bargaining, or negotiation game proposed in [24]: each ISP independently chooses a set of acceptable transit prices t (prices that ensure a chosen amount of revenue), and if the intersection of those sets is non-empty, the transit price is arbitrarily taken in the intersection, otherwise the threat is executed. This negotiation scheme has several equilibria, but in [24] the most likely to be played is the one maximizing the product of the utilities minus the utility at the threat. In other words, that equilibrium transit price maximizes

$$\max(0, U_A^{\text{trans,NE}}(t) - U_A^{\text{threat}}) \cdot \max(0, U_B^{\text{trans,NE}}(t) - U_B^{\text{threat}}),$$

where U_i^{threat} represents the utility that ISP i obtains if the negotiation fails (two cases will be considered in the following). Remark that this solution is the classical axiomatic Nash bargaining solution [26].

Our numerical results suggest that *the peering threat favors the small ISP against the big one, when compared to the disconnection threat.* More specifically, Figure 7 compares the utility of each ISP obtained with the bargaining procedure when the threat corresponds to the disconnection (i.e., $U_i^{\text{threat}} = U_i^{\text{cut,NE}}$), to the utility achieved when the threat (because of the legislation) is that ISPs are forced to maintain a connection (i.e., $U_i^{\text{threat}} = U_i^{\text{peer,NE}}$). We observe that the ISP with the most content (the big ISP) monotonically (in terms of proportion of contents) benefits from the disconnection threat, while the opposite is true for the other ISP (the small ISP). In the case with enforced peering threat, the small ISP still loses some revenue when its weight decreases, but the revenue of the big ISP is no longer monotonic. We observe that the enforced peering threat is a better rule than the disconnection threat for the small ISP, while it is the opposite for the big ISP. An interpretation comes from the comparison of both threat situations from the ISPs' point of view: as seen in Sections 3 and 4, and also illustrated in Figure 6, both ISPs have the same revenue in the peering case, while the big ISP has a larger revenue than the small one in the disconnection case. As a result, in the case of a disconnection threat (with respect to the peering threat) the small ISP has less bargaining power than the big one, since it has more to lose if no agreement is reached. This benefits the big ISP in the negotiation. With the peering threat the effect is the opposite: the small ISP would get the same revenues as the big one if no agreement is found, while with positive transit prices it obtains less than the big one as can be seen on Figure 6.

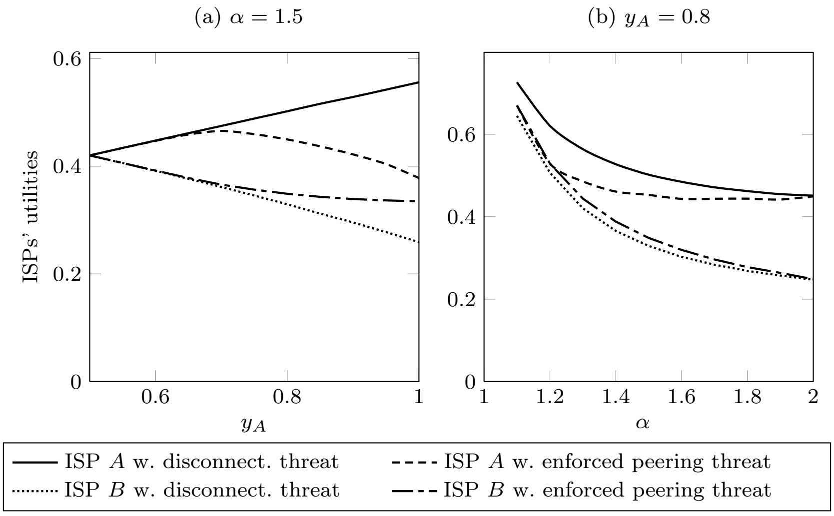

Fig. 7 Comparison of each ISP's utility after bargaining process with disconnection and enforced peering threats, when y_A and α vary, with $y_B = 1 - y_A$ (no content multihoming).

It also appears that *the enforced peering threat (with respect to the disconnection threat) favors users against ISPs.* The intuition behind this observation stems from our previous reasoning: enforced peering gives more bargaining power to the small ISP, who will therefore obtain a smaller transit price during the negotiation, hence leading to a situation closer to the peering scenario (that maximizes user welfare). In Figure 8, we show how the utilities of ISPs and of the users are impacted by the chosen policy, when the user sensitivity α and the proportion y_A of download traffic of content attached to ISP A vary, in the case when no content is multihomed ($y_B = 1 - y_A$). We first remark that the bargaining with disconnection threat leads to ISP revenues and user welfare very similar to the policy maximizing the sum of ISPs' utilities (the curves of total ISP revenues superimpose). This means that the non-cooperative (bargaining) choice for t with the disconnection threat leads to a nearly optimal

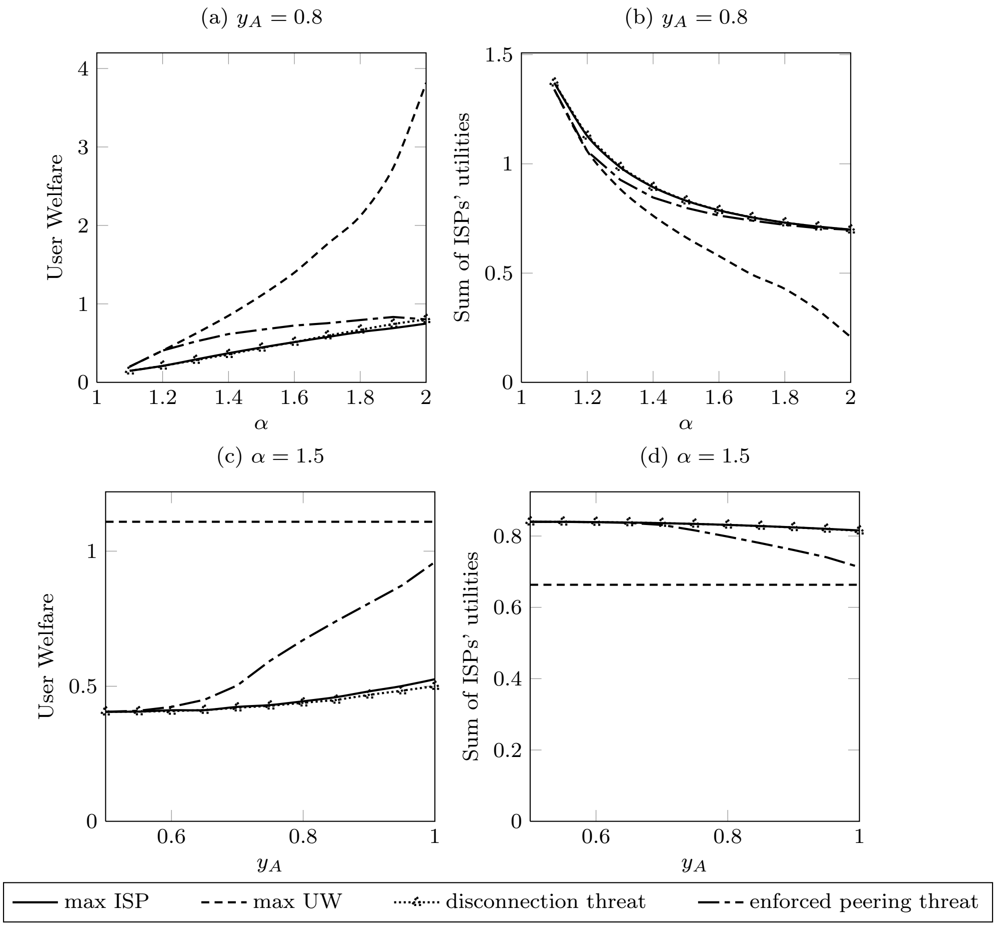

Fig. 8 ISPs' utilities and user welfare for different policies for choosing t, when α and y_A vary, with $y_B = 1 - y_A$ (no content multihoming).

point (that takes into account the competition between ISP afterwards) for ISPs. Nevertheless, as one can see, when y_A is close to 1 or α close to 2 the user welfare is not exactly the same in both cases (see Figure 8 (a) and (c)), hence this is not a general property. One can also observe in Figure 8 (c) and (d) that the utilities are not very sensitive to the repartition of content among ISPs (the parameter y_A), except for the bargaining with enforced peering threat. In particular, the solution maximizing user welfare does not depend on y_A, hence utilities are constant in that case. Finally, the bargaining solution obtained with the enforced peering threat is almost optimal for users (resp., ISPs) when their sensitivity α is low (resp., high). The difference of content owned by each ISP also decreases the utility of ISPs in this case, while the achieved utility is optimal when contents are equally hosted by the ISPs.

Arbitrating between user welfare and ISP revenues is generally done using the aggregate *social welfare*, that estimates the overall value of the system. However, our user welfare represents the aggregate *perceived* utility of users, that is not expressed in monetary units because of our choice of a logarithmic perception of prices, motivated by psychophysics studies and recent experiences in the context of telecommunications. On the other hand, the ISP welfare is in monetary units, so summing them directly would be artificial. One may define a conversion constant and define social welfare a weighted sum of ISP revenues and user welfare, however choosing non-arbitrarily that constant is beyond the scope of this paper, and since it has a strong influence on the resulting social welfare we cannot draw conclusions on social welfare here.

6 Conclusions

The results of our paper suggest that the scenario where no transit is performed by ISPs should never be chosen, since no stakeholder benefits from it. In accordance with previous works on interconnection of competing networks, we also observed that under price competition, paid transit can be preferred to peering by both ISPs. This especially holds when users are highly sensitive to price: paid transit is then the only agreement under which ISPs can ensure strictly positive revenues, avoiding a price war, for any strictly positive value of the transit unit price. Paid transit therefore appears as the best solution to ensure ISP rentability in a highly competitive context like the current Internet ecosystem, where customers are very volatile and frequently switch providers.

Moreover, the transit price can also be used by a regulatory entity to drive the ISP price to a desirable direction, be it in terms of global ISP revenues (the optimal revenue when ISP collaborate can be approached without collaboration through a proper choice of the transit price) or of user welfare (that can be favored if a sufficiently low transit price is imposed).

Finally, if the transit price is fixed among ISPs through a negotiation, our study suggests that a limited regulation consisting in imposing transit agreements (i.e., imposing that transit be performed to ensure a global connectivity, but at a price chosen by the ISPs) benefits to users, who eventually perceive lower prices, and a higher welfare. Such a regulation indeed reduces the bargaining power of the ISPs controlling the most content, and hence favors the emergence and survivability of new ISPs with less content. Without this regulation, our numerical results suggest that bargaining (with a disconnection threat) leads to an outcome very close to the one where ISPs cooperate to maximize their total revenue.

Coming back to the Network Neutrality debate, and the request from ISPs to be rewarded for transit, we find that our results corroborate their claim and concerns, since null transit prices may lead to ISPs making no revenue despite their infrastructure investments. The results in this paper therefore support the transit price scenario, that is also likely to be chosen by ISPs if the choice is theirs, so that the need for regulation is not obvious. However, this paper also suggests that a limited regulation (enforcing global connectivity) does prevent incumbent ISPs from having a dominant position in the bargaining, and favors competition and users. Given the results presented here, we would advocate that such a minimal regulation be imposed, and that other choices be left to the stakeholders (here, the ISPs), remaining consistent with the freedom spirit that prevailed at the beginnings of the Internet. From a practical point of view, such a policy also has the advantage of being extremely simple to implement, since connectivity among ISPs is very easy to verify. The regulator would just need to declare disconnection illegal, and to advertise that rule in order to affect the negotiations among ISPs.

As a direction for future work, we would like to investigate the case where the transit ISPs are paid for transferring the external content to their customers (the *push* model), instead of being charged in this paper (the *pull* model). This would correspond to another interpretation of the service offered by ISP, where ISPs would sell to content providers the access to their networks, while in this paper they sell to their users the access to all content. That new interpretation may also raise the necessity of considering a two-sided market, where ISPs compete to attract end-users but also to attract content providers.

Also, our current research efforts aim at understanding the influence of other types of actors in the content distribution chain, namely Content Delivery Networks (CDNs), on the setting described in this paper. CDNs dramatically affect the volumes of data exchanges and may re-balance the forces; here also, a careful economic analysis of the strategical behaviors of all actors is necessary.

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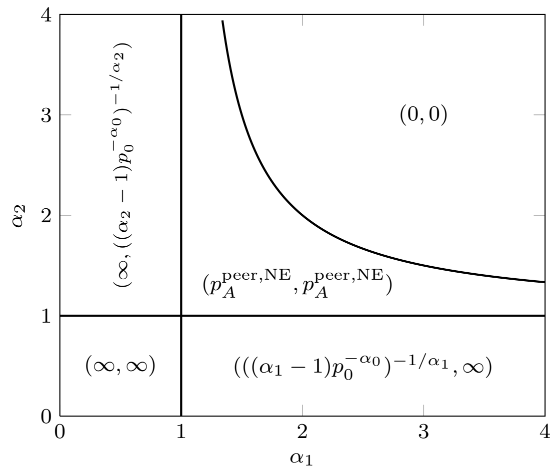

Fig. 9 Nash equilibrium in the peering case, in terms of the heterogeneous sensitivies (α_1, α_2).

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A The peering scenario with heterogeneous sensitivity parameters

A.1 Equilibrium prices

In the heterogeneous case with different αs, proceeding as in the proof of Proposition 1 and using the fact that $\frac{\partial U_i^{\text{peer}}}{\partial p_i} = \sigma_i(1-\alpha_i(1-\sigma_i))$, it can be shown that, at equilibrium:

- If there is one $i \in \{A, B\}$ such that $\alpha_i \leq 1$, then $p_i^{\text{peer,NE}} = \infty$, because the utility is increasing, independently of the parameters α_j for $j \neq i$. The opponent $j \neq i$ chooses $p_j^{\text{peer,NE}} = \infty$ for the same reason if $\alpha_j \leq 1$, and $p_j^{\text{peer,NE}}$ such that $\sigma_j = 1 - 1/\alpha_j$, i.e., $p_j^{\text{peer,NE}} = ((\alpha_j - 1)p_0^{-\alpha_0})^{(-1/\alpha_j)}$ otherwise.
- If $\alpha_A, \alpha_B > 1$,
  - If $(1/\alpha_A) + (1/\alpha_B) \geq 1$, there is a unique solution to the system of equations with null derivatives, giving

$$p_A^{\text{peer,NE}} = \left(\frac{\alpha_B\alpha_A - \alpha_B - \alpha_A}{\alpha_B(1-\alpha_A)}\right)^{1/\alpha_A} p_0^{\alpha_0/\alpha_A}$$
$$p_B^{\text{peer,NE}} = \left(\frac{\alpha_A\alpha_B - \alpha_A - \alpha_B}{\alpha_A(1-\alpha_B)}\right)^{1/\alpha_B} p_0^{\alpha_0/\alpha_B}.$$

  - If $(1/\alpha_A) + (1/\alpha_B) \leq 1$, then we again have a price war with, at equilibrium, $p_A^{\text{peer,NE}} = p_B^{\text{peer,NE}} = 0$.

The equilibrium cases in terms of (α_1, α_2) are summarized in Figure 9.

A.2 The cost of competition to ISPs

In the heterogeneous case, the derivatives of the sum of providers utilities $U_A^{\text{peer}} + U_B^{\text{peer}}$ are

$$\frac{\partial(U_A^{\text{peer}}+U_B^{\text{peer}})}{\partial p_A} = \frac{p_A^{1-\alpha_A}+(1-\alpha_A)p_A(p_B^{-\alpha_B}+p_0^{-\alpha_0})+\alpha_A p_B^{1-\alpha_B}}{p_A^{1+\alpha_A}(p_A^{-\alpha_A}+p_B^{-\alpha_B}+p_0^{-\alpha_0})^2}$$
$$\frac{\partial(U_A^{\text{peer}}+U_B^{\text{peer}})}{\partial p_B} = \frac{p_B^{1-\alpha_B}+(1-\alpha_B)p_B(p_A^{-\alpha_A}+p_0^{-\alpha_0})+\alpha_B p_A^{1-\alpha_A}}{p_B^{1+\alpha_B}(p_A^{-\alpha_A}+p_B^{-\alpha_B}+p_0^{-\alpha_0})^2}.$$

Here again, if there is a $i \in \{A, B\}$ such that $\alpha_i \leq 1$, the derivative with respect to p_i is always positive, and setting $p_i = \infty$ is the optimal strategy. Then the total revenue is infinite whatever the value of p_j for $j \neq i$. Now, if $\alpha_A, \alpha_B > 1$, the system

$$p_A^{1-\alpha_A} + \alpha_A p_B^{1-\alpha_B} - (\alpha_A - 1)p_A\left(p_B^{-\alpha_B} + p_0^{-\alpha_0}\right) = 0$$
$$p_B^{1-\alpha_B} + \alpha_B p_A^{1-\alpha_A} - (\alpha_B - 1)p_B\left(p_A^{-\alpha_A} + p_0^{-\alpha_0}\right) = 0$$

leads to $p_B = \dfrac{\alpha_B(\alpha_A - 1)}{\alpha_A(\alpha_B - 1)}p_A$. Indeed, α_B times the first equation minus α_A times the second one gives $(\alpha_B(\alpha_A - 1)p_A - \alpha_A(\alpha_B - 1)p_B)\,(p_A^{-\alpha_A} + p_B^{-\alpha_B} + p_0^{-\alpha_0}) = 0$. However, we did not reach an analytical expression for the optimal value of p_A, that can be computed numerically.

A.3 User Welfare in the heterogeneous case

We also immediately get from (6)

$$\mathrm{UW}(p_A, p_B, p_0) = \log\left(1 + p_0^{\alpha_0}\left(\left(\frac{x_A}{p_A}\right)^{\alpha_A} + \left(\frac{x_B}{p_B}\right)^{\alpha_B}\right)\right).$$

B The disconnection scenario with heterogeneous αs

In the heterogeneous case with different αs, the results can again be obtain similarly to the homogeneous case, using that $\frac{\partial U_i^{\text{cut}}}{\partial p_i} = \sigma_i(1 - \alpha_i(1 - \sigma_i))$. It can be shown that, at equilibrium:

- If there is one $i \in \{A, B\}$ such that $\alpha_i \leq 1$, then again $p_i^{\text{cut,NE}} = \infty$. The opponent $j \neq i$ chooses $p_j^{\text{cut,NE}} = \infty$ for the same reason if $\alpha_j \leq 1$, and $p_j^{\text{peer,NE}} = y_j((\alpha_j - 1)p_0^{-\alpha_0})^{(-1/\alpha_j)}$ otherwise.
- If $\alpha_A, \alpha_B > 1$,
  - If $(1/\alpha_A) + (1/\alpha_B) \geq 1$, there is a unique solution to the system of equations with null derivatives, giving

$$p_A^{\text{cut,NE}} = y_A\left(\frac{\alpha_B\alpha_A - \alpha_B - \alpha_A}{\alpha_B(1 - \alpha_A)}\right)^{1/\alpha_A} p_0^{\alpha_0/\alpha_A}$$
$$p_B^{\text{cut,NE}} = y_B\left(\frac{\alpha_A\alpha_B - \alpha_A - \alpha_B}{\alpha_A(1 - \alpha_B)}\right)^{1/\alpha_B} p_0^{\alpha_0/\alpha_B}.$$

  - If $(1/\alpha_A) + (1/\alpha_B) \leq 1$, then we again have a price war with, at equilibrium, $p_A^{\text{cut,NE}} = p_B^{\text{cut,NE}} = 0$.

C Proof of Lemma 1

Proof We provide here the proof for ISP A only, as it is symmetric for ISP B. Recall that, by definition, $K(v) \stackrel{\text{def}}{=} (1-\alpha)(v^{-\alpha}+p_0^{-\alpha})$, and $L_A(v) \stackrel{\text{def}}{=} \alpha t\left((2-y_A-y_B)v^{-\alpha}+(1-y_A)p_0^{-\alpha}\right)$, so that $K<0$ if $\alpha>1$, and $L_A \geq 0$.

(i) The derivative (10) of the revenue function of ISP A is strictly positive for p_A small enough, and strictly negative for p_A large enough. It follows that a best response must satisfy $\frac{\partial U_A}{\partial p_A}=0$. We did not reach any closed form solution to this equation, thus we are naturally led to studying the sign of the derivative of U_A. Notice that the sign and zeros of this derivative are the same as those of the function

$$S(u,v) \stackrel{\text{def}}{=} K(v)u^{\alpha}+L_A(v)u^{\alpha-1}+1, \tag{11}$$

with $u=p_A$ and $v=p_B$. Since $\alpha>1$, $K(v)<0$. Looking at the derivative $\frac{\partial S}{\partial u}$ shows that S is increasing if $u \leq \dfrac{1-\alpha}{\alpha}\dfrac{L_A(v)}{K(v)}$, and strictly decreasing otherwise. Now, remark that for every v, $S(0,v)>0$ and $\lim_{u\to\infty} S(u,v)=-\infty$. Since S is continuous, it follows that the equation $S(\cdot,v)=0$ has a unique solution, so that the best-response function is single valued. Let us denote by $\bar{u}(v)$ the unique solution to $S(\cdot,v)=0$, which is also the best-reponse to $p_B=v$. One can remark that $S(u,v)$ is positive if and only if $u \leq \bar{u}(v)$. This gives us a criterion to compare a value u with the root $\bar{u}(v)$, that we will frequently use in the rest of the proof.

(ii) The continuity is a consequence of Berge's maximum theorem [5]. The hypotheses of the proposition are valid here, so that the best-response price correspondence is upper hemicontinuous. Since that correspondence is single-valued, it is a continuous function.

(iii) From the previous analysis, we have for all v

$$\bar{u}(v) > \frac{1-\alpha}{\alpha}\frac{L_A(v)}{K(v)} = \frac{t(2-y_A-y_B)(v^{-\alpha}+(1-y_A)p_0^{-\alpha})}{v^{-\alpha}+p_0^{-\alpha}} \geq t(1-y_A), \tag{12}$$

hence a uniform lower bound for $\bar{u}(v)$ that is strictly positive as soon as $y_A<1$. In the case $y_A=1$, our bound goes to zero when $v\to\infty$. However, we can directly see from (11) that we have $\lim_{v\to\infty}\bar{u}(v)=\dfrac{p_0}{(\alpha-1)^{1/\alpha}}$, which is strictly positive. By the continuity of the best-response, there exists $v_0>0$ such that

$$v \geq v_0 \Rightarrow \bar{u}(v) \geq \frac{p_0}{2(\alpha-1)^{1/\alpha}} \stackrel{\text{def}}{=} C_1.$$

On the other hand, from (12) we have

$$v \leq v_0 \Rightarrow \bar{u}(v) \geq \frac{t}{1+p_0^{-\alpha}v_0^{\alpha}} \stackrel{\text{def}}{=} C_2,$$

and therefore, $\bar{u}(\cdot)$ is also uniformly bounded on $\mathbb{R}^+$ by $\min(C_1,C_2)$, that is a strictly positive constant when $x=1$.

For the uniform upper bound, we claim that

$$\bar{u}(v) \leq u_0 \stackrel{\text{def}}{=} \max\left(1, \frac{(2-y_A-y_B)\alpha t}{\alpha-1}, \frac{p_0^{\alpha}+\alpha t(1-y_A)}{\alpha-1}\right).$$

To check that, it suffices to show that $S(u_0,v)<0$ for all $v>0$. We first have

$$\begin{aligned} u_0 &\geq \max\left(\frac{(2-y_A-y_B)\alpha t}{\alpha-1}, \frac{p_0^{\alpha}+\alpha t(1-y_A)}{\alpha-1}\right) \\ &\geq \frac{1+\alpha t((2-y_A-y_B)v^{-\alpha}+(1-y_A)p_0^{-\alpha})}{(\alpha-1)(v^{-\alpha}+p_0^{-\alpha})} = \frac{1+L_A(v)}{-K(v)}, \end{aligned}$$

where the second inequality comes from the fact that we take a weighted sum (with positive weights) of the two terms above. Since $u_0 \geq 1$, it follows $u_0^{\alpha-1}(-K(v)u_0 - L_A(v)) \geq 1$, and $S(u_0, v) = K(v)u_0^{\alpha} + L_A(v)u_0^{\alpha-1} + 1 \leq 0$.
We claim that playing a best-response always yields a strictly positive revenue for the ISP. Indeed, it can ensure a revenue larger than zero by setting its price to infinity: that way it gets no customers (hence no subscription revenues), and therefore does not pay any transit fee. However, that cannot be a best-response, since it should be bounded. Hence the best-response is strictly better, which results in a strictly positive revenue.
(iv) Let t and r be two transit prices, with $t > r$. Assume that the price v of ISP B is fixed, and let $\bar{u}_r$ denote the best-response of ISP A to v under transit pricing r. Then

$$(1-\alpha)(v^{-\alpha} + p_0^{-\alpha})\bar{u}_r^{\alpha} + \alpha r((2 - y_A - y_B)v^{-\alpha} + (1 - y_A)p_0^{-\alpha})\bar{u}_r^{\alpha-1} + 1 = 0. \tag{13}$$

To show that the best-response under transit pricing t is greater than $\bar{u}_r$, it suffices to establish that

$$(1-\alpha)(v^{-\alpha} + p_0^{-\alpha})\bar{u}_r^{\alpha} + \alpha t((2 - y_A - y_B)v^{-\alpha} + (1 - y_A)p_0^{-\alpha})\bar{u}_r^{\alpha-1} + 1 > 0. \tag{14}$$

because, from the proof of (i), $S(u, v) > 0$ only if $u < \bar{u}(v)$, and thus $S(\bar{u}_r, v) > 0$ means $\bar{u}_r < \bar{u}_t = \bar{u}(v)$. But (14) comes directly from (13) and $t > r$.
(v) Let $v > w$. We seek to compare $\bar{u}(v)$ and $\bar{u}(w)$. By definition, we have $S(\bar{u}(w), w) = 0$. Then, $S(\bar{u}(w), v) = (v^{-\alpha} - w^{-\alpha})\bar{u}(w)^{\alpha-1}\left((1-\alpha)\bar{u}(w) + \alpha t(2 - y_A - y_B)\right)$. Therefore, $\bar{u}(v) > \bar{u}(w)$ if and only if $(1-\alpha)\bar{u}(w) + \alpha t(2 - y_A - y_B) < 0$, i.e., $\bar{u}(w) > \frac{\alpha t(2-y_A-y_B)}{\alpha-1}$. The last inequality is equivalent to $S(\frac{\alpha t(2-y_A-y_B)}{\alpha-1}, w) > 0$, which is equivalent to

$$\frac{(\alpha(2 - y_A - y_B)t/p_0)^{\alpha}}{(\alpha-1)^{\alpha-1}}\left(1 - \frac{1 - y_A}{2 - y_A - y_B}\right) > 1.$$

That last inequality does not depend on w, which implies that $\bar{u}(v)$ is monotonic. □

D Proof of Proposition 5

Proof We prove here that the user welfare (6) at the Nash equilibrium, different from $(0, 0)$, with $t = 0$ is greater than the user welfare at every Nash equilibrium with $t > 0$. Recall that the user welfare is

$$\mathrm{UW} = \log\left(1 + \left(\frac{p_0}{p_A}\right)^{\alpha} + \left(\frac{p_0}{p_B}\right)^{\alpha}\right).$$

The result follows from the fact that, for each ISP, the price at the Nash equilibrium is the lowest when $t = 0$, which we establish below, without loss of generality, for ISP A.

Recall that $\mathrm{BR}_A^t(p_B)$ (resp., $\mathrm{BR}_B^t(p_A)$) is the best-response of ISP A (resp., B) with transit price t. We also denote by $(p_A^{\mathrm{trans,NE}}(t), p_B^{\mathrm{trans,NE}}(t))$ the Nash equilibrium with the smallest price for A. Then, for all $z < p_A^{\mathrm{trans,NE}}(t)$ and every $t > 0$ we have

$$z < \mathrm{BR}_A^t(\mathrm{BR}_B^t(z)). \tag{15}$$

Indeed, the best-response being lower bounded by a strictly positive value, (15) is verified when $z \to 0$. If there exists $z < p_A^{\mathrm{trans,NE}}(t)$ that does not satisfy the inequality, then by continuity of the best-response functions, there is necessarily $\hat{p} < p_A^{\mathrm{trans,NE}}(t)$ for which $\hat{p} = \mathrm{BR}_A^t(\mathrm{BR}_B^t(\hat{p}))$. But this means that $(\hat{p}, \mathrm{BR}_B^t(\hat{p}))$ is a Nash equilibrium, which contradicts the hypothesis of $p_A^{\mathrm{trans,NE}}(t)$ being the smallest price of ISP A at an equilibrium.

Now, recall from Lemma 1 that the best-response is a strictly increasing function of t. Furthermore, while $t^{\alpha}\frac{((2 - y_A - y_B)\alpha/p_0)^{\alpha}}{(\alpha-1)^{\alpha-1}}\left(\frac{1 - y_j}{2 - y_A - y_B}\right) < 1$, the best-response $\mathrm{BR}_i^t(\cdot)$ is a strictly increasing function of the other ISP's price. Let t be small enough so that

$\mathrm{BR}_A^t(p_B)$ is strictly increasing. Let $0 < r < t$, and assume that $p_A^{\mathrm{trans,NE}}(t) < p_A^{\mathrm{trans,NE}}(r)$. Then we have:

$$\begin{aligned} p_A^{\mathrm{trans,NE}}(t) &< \mathrm{BR}_A^r(\mathrm{BR}_B^r(p_A^{\mathrm{trans,NE}}(t))) \\ &< \mathrm{BR}_A^r(\mathrm{BR}_B^t(p_A^{\mathrm{trans,NE}}(t))) \\ &< \mathrm{BR}_A^t(\mathrm{BR}_B^t(p_A^{\mathrm{trans,NE}}(t))) = p_A^{\mathrm{trans,NE}}(t). \end{aligned}$$

The first inequality comes from (15) together with the hypothesis $p_A^{\mathrm{trans,NE}}(t) < p_A^{\mathrm{trans,NE}}(r)$. The second one is due to the fact that $\mathrm{BR}_B^{\cdot}$ increases with the transit price, and that BR_A^r increases with the price set by B. The last one is due to the increasingness of $\mathrm{BR}_A^{\cdot}$ in the transit price, and the last equality stems from $p_A^{\mathrm{trans,NE}}(t)$ being a Nash equilibrium price for ISP A. Finally, this shows a contradiction. Hence $p_A^{\mathrm{trans,NE}}(t)$ increases with t as long as $t^\alpha \dfrac{((2-y_A-y_B)\alpha/p_0)^\alpha}{(\alpha-1)^{\alpha-1}} \left(\dfrac{1-y_j}{2-y_A-y_B}\right) < 1$.

It remains to show that $p_A^{\mathrm{trans,NE}}(t)$ is larger than $p_A^{\mathrm{trans,NE}}(0)$ for large values of t. We have shown that $p_A^{\mathrm{trans,NE}}(t)$ increases with t while the transit price is below $\hat{t}$ that satisfies $t^\alpha \dfrac{((2-y_A-y_B)\alpha/p_0)^\alpha}{(\alpha-1)^{\alpha-1}} \left(\dfrac{1-y_j}{2-y_A-y_B}\right) = 1$. For that value of t, $\mathrm{BR}_A^{\hat{t}}$ is constant, and $\mathrm{BR}_A^{\hat{t}} = p_A^{\mathrm{trans,NE}}(\hat{t}) \geq p_A^{\mathrm{trans,NE}}(0)$. For $t > \hat{t}$, the best-response function is larger than $\mathrm{BR}_A^{\hat{t}}$ according to item (iv) of Lemma 1, and then larger than $p_A^{\mathrm{trans,NE}}(\hat{t})$, independently of p_B. Hence the Nash equilibrium price of ISP A is larger than $p_A^{\mathrm{trans,NE}}(\hat{t})$ and, finally, than $p_A^{\mathrm{trans,NE}}(0)$. □