

Speculative Bubble Burst

Hye-jin CHO

*University of Paris1 - Panthéon Sorbonne
Hyejin.Cho@malix.univ-paris1.fr

Thu, 16/07/2015

Undefined Financial Object (UFO) in in financial crisis

- A **fundamental dichotomy** – a partition of a whole into two parts, nothing can belong simultaneously to both parts
 - Financial crises directly result in **a loss of paper wealth** (=wealth as measured by monetary value, as reflected in the price of assets - how much money one's assets could be sold for) but do not necessarily result in changes in the real economy: **Banking crisis, Currency crisis, Sovereign defaults, Wider economic crisis (recession: negative GDP growth lasting two or more years) and Speculative bubbles and crashes**

A speculative bubble exists in the event of large, sustained **overpricing of stock and other assets** which is not equivalent to fundamentals.

Definition of bubbles

- Economic bubble, a speculative bubble, a market bubble, a price bubble, a financial bubble, a speculative mania (or a balloon)
- Asset price in **trading**, strongly deviated from the intrinsic value (fundamental value)
- crash, a bubble burst: sudden drops in prices
- **British South Sea Company** rose by over 700 percent during the last half of 1720 and the price fell back to about fifty percent above its value at the start of the year (allen-gorton, 1993)

Consequence of bubbles and theories

- What is (market) fundamental?: Leroy and Porter (1981), Shiller (1981)
- Positive feedback mechanism: boom and burst
- Negative feedback mechanism: prediction from supply and demand alone without externality is impossible at equilibrium price
 - Bubbles can appear without uncertainty, speculation, bounded rationality?
 - Forecasting probability with a Markov switching model
- Mainstream economics: Bubbles cannot be identified in advance. After burst, price coordination is progressed via monetary policy and fiscal policy
- Austrian economics: Mis-allocation of resources in Austrian business cycle theory
- Shrinking credit cycle

What is (market) fundamental?

Market fundamentals

Market fundamentals rank the price level x_t in a discrete-time framework according to

$$x_t = f_t + aE(x_{t+1} | \Omega_t), a \in (0, 1), t \in \mathbb{N}$$

where x_t is the **price level**, f_t is fundamentals and Ω_t is the information set.

Three examples suggested by O. Blanchard (1979):

- 1 Reduced form of a money market equilibrium: x_t is price level and f_t is **nominal money** (Flood and Garber, 1980)
- 2 Arbitrage equation: x_t is price of share and f_t is **dividend** (Scheinkman and Xiong, 2003)
- 3 Equilibrium of a material market (for example the gold market): x_t is price and f_t is **existing stock** (Harrison and Kreps, 1978)

Why price vector setting with fundamental is difficult?

- Bubbles are **inflation or not?**
- Asset price inflation is inflation or not?

CASE 1

There is a fad to use a computer named Apple, demand of Apple increase, agents can give up to buy

CASE 2

50 cents become 1 euro, the price of everything immediately goes up, raised (comparative) price of Apple, agents cannot escape from this Inflation.

- Even though asset price inflation may be related to the monetary policy, in a partial sense not to feel as total about pricing, it's hard to say inflation.
ef. housing, dotcome company and so on.

Aumann's agreement theorem (1976)

- No bubble, no betting pareto optimal, [I know it's bubble](#)
- Risk neutral, Aumann's agreement theorem (1976), "You cannot actually agree to disagree." Two "perfectly rational" agents with the same prior estimate of an event's probability and "common knowledge" of one another's posterior estimates cannot come to different posteriors. [Bayesian Nash Equilibrium](#)

money has positive value

- Samuelson (1958) money has positive value in spite of uselessness of intrinsic value (ef. Market [fundamental](#) equals [zero](#))

and Technical problems of OLG

- No exchange **from old to young**, assets should be sold to young, a stationary equilibrium to diminish automatically
- Price vectors as many components as contingent goods. **Inter**-temporal budget. constraints and **inter**-states, **multi**-maturity of several assets
- Complete information in the financial contract, sequential transactions. At the stationary asset of **zero fundamental value** exchanged at a positive price.

Overlapping generation, Tirole (1985):

Existence of unique eq with bubbleless, maximum feasible bubble with initial bubble, existence of no bubbleless eq

Aggregate bubble per capita $b_{t+1} = (1 + r_{t+1}/1 + n) \times b_t$

Bubbles, Scheinkman and Xiong (2003)

$$q(-k^*) = h(-k^*)/(r + \lambda) [h'(k^*) + h'(-k^*)]$$

Fundamentals from the cumulative dividend process

Let's induce the speculative bubble model from the dividend process.

Cumulative dividend process

$$dD_t = Af_t dt + \sigma_D dZ_t^D$$

where

- 1 f is the fundamental random variable not depending on dividend noise of the standard Brownian motion Z^D of a constant volatility parameter σ_D ,
- 2 taking the value $df_t = -\lambda(f_t - \bar{f})dt + \sigma_f dZ_t^f$,
- 3 given mean reversion (moving to average) $\lambda \geq 0$, \bar{f} denotes long-run mean of f and
- 4 A is the infinitesimal generator in $Af = \lim_{t \rightarrow 0} \frac{T_t f - f}{t}$ of a parameter T_t

How long? finite maturity?

- Temporary bubbles (it cannot be) or a durable bubble

The bubble:

- finite or infinite horizon,
- a deterministic price path Vs. Stochastic price path,
- existence of a non-risky asset with interest rate r ,
- no bubble possibility in the finite horizon,
- possibility of “equilibrium price=0” and “common knowledge=0”,
- “backward induction = forecasting prediction” with historic choices or non-historic choices,
- topological space about possible exit from the fundamental value path

How long? finite maturity?

- Deterministic durable bubble of infinite life by rational agents
- Stochastic durable bubble with positive probability by agents involved in trading
 - How to measure bubbles in advance? Probability theory of fundamentals, Heterogeneous belief: Overconfidence

Def Overconfidence parameter

Overconfidence parameter ϕ increases as a larger ϕ increases, agents attribute to their own forecast of the current level of fundamentals where $0 < \phi < 1$.

Lemma Stationary variance

Stationary variance γ decreases with ϕ .

Nature of Bubbles

- Existence of an aggregate bubble, Store of value
 - Difference between the market price and the market fundamental
- A finite number of infinitely lived trader in a discrete time finite horizon setting stock prices, tirole (1982)
 - Scheinkman (1980) If the long-run interest rate is positive, the asset bubble-which must grow at the interest rate-eventually becomes so big that young generations cannot buy the asset

Nature of Bubbles

- Asymptotically bubbly path which is not small relative to the economy
 - Bubbles with default of debt mixed with the problem of solvency and liquidity
- Exogeneous borrowing constraint, Scheinkman and Weiss (1986), Santos-Woodford (1997)
 - Endogeneous borrowing constraints, Kehoe-Levine (1993)
 - Hellwig (2009) no default, endogenous debt, choice of a profile of consumption and net amount of asset holdings

Memoryless property:

Future states only depends on the present states

Brownian motion is the feller processes. Every feller process satisfies the strong Markov property (memoryless). A stochastic process has the Markov property if the conditional probability distribution of future states of the process depends only upon the present states defined in terms of a random variable known as a stopping time.

Def: Speculative Bubble Crush Index A

The Agents attribute to their own forecast of the current level of fundamentals as a larger overconfidence ϕ increases (stationary of Rogers and Williams (1987), Lipster and Shiryaev (1977))

$$\gamma \equiv \frac{\sqrt{(A\lambda + \phi \frac{\sigma_f}{\sigma_s})^2 + (1 - \phi^2)(2 \frac{\sigma_f^2}{\sigma_s^2} + \frac{\sigma_f^2}{\sigma_D^2}) - (A\lambda + \phi \frac{\sigma_f}{\sigma_s})}{\frac{1}{\sigma_D^2} + \frac{2}{\sigma_s^2}}$$

Lemma: Stability and mean reversion

If $A (= \theta'(\lambda))$ is infinitesimal and $B (= \phi\sigma_f/\sigma_s)$ is significantly larger than A , the stability Variance γ does not increase as mean reversion λ increases and:

$$\frac{d\gamma}{d\lambda} = \frac{A}{(A\lambda + B)} - A < 0.$$

Hence, $B (= \phi\sigma_f/\sigma_s) > A\lambda$.

Aggregation of Belief

Assume that the heterogenous beliefs offer the joint dynamics of the D, f, s^A, s^B . All agents observe a vector of signals s^A and s^B that satisfy:

$$\text{remind: } dD_t = Af_t dt + \sigma_D dZ_t^D$$



$$ds_t^A = A^A f_t dt + \sigma_s dZ_t^A$$

$$ds_t^B = A^B f_t dt + \sigma_s dZ_t^B$$

Agents of group A(B) believe that innovations dZ^A (dZ^B) in the signal s^A (s^B) are correlated with the innovation dZ^f in the fundamental process, with ϕ ($0 < \phi < 1$) as the correlation parameter.

Question) Infinitesimal Generator A is the summation of $A^A + A^B$?

Theorem (Origin of Speculative Bubble Crush)

- 1 Mean reversion λ doesn't work to maintain the stability variance γ of random process,
- 2 the infinitesimal generator A is positive oscillation of fundamental value A to maintain the Brownian motion affects mean reversion λ ,
- 3 then threshold of fundamental oscillation $A\lambda$ is captured by the noise movement between $B(= \phi\sigma_f/\sigma_s)$ and $-(\sigma_f/\sigma_s)$,

$$\text{hence, } -\sigma_f/\sigma_s < A\lambda < B(= \phi\sigma_f/\sigma_s).$$

Equilibrium setting: Accumulation factor

Accumulation factor a_h

The price system is the l -tuple $p = (p_1, \dots, p_h, \dots, p_l)$. The value of an action a (arrow, 1959) relative to the price system p is $\sum_{h=1}^l p_h a_h$ where a_h is accumulation factor.

An equilibrium in trading driven by inertia

Inertia drives an equilibrium in trading only at which is the status quo for all traders so T is the set of agents who are involved in trading in this equilibrium,

$$T = \{i : x^i \neq w^i\}.$$

An equilibrium with inertia (Rigotti and Shannon, 2005)

- A feasible allocation (x^1, \dots, x^I) induced from $\sum_i x^i \leq \sum_i w^i$ where i are finitely many agents indexed by $i = 1, \dots, I$ and a non-zero price vector $p \in \mathbb{R}_+^S$ where S are possible states of nature indexed by $s = 1, \dots, S$ if for all i ,
 $x \succ^i x^i \implies p \cdot x > p \cdot w^i$
and endowment $w^i \in \mathbb{R}_+^S$ is started at date 1
assumed that consumption doesn't occur at date 0 and
consumption balance left after trading Arrow securities and

- 1 for all i , $p \cdot x^i = p \cdot w^i$,
- 2 and for each i , either $x^i = w^i$, or $E_{\pi^i} [u^i(x^i)] \geq E_{\pi^i} [u^i(w^i)]$.
where E_{π^i} denotes the expected value with respect to the probability distribution π^i , each $\pi^i \in \Pi^i$, \exists a closed, convex set Π^i and $u(x)$ denotes the vector $u(x_1), \dots, u(x_S)$.

Equilibrium setting:

Trading point with an accumulation factor

accumulation factor

- The number l of commodities is a given positive integer.
- An action a of an agent is a point of R^l , the commodity space. A price system p is a point of R^l .
- The value of an action a relative to a price system p is the inner product $p \cdot a$. The equilibrium allocation (w^1, \dots, w^l) is supported by p as below, $p \cdot a \cdot x^i = p \cdot a \cdot w^i$, where a is accumulation factor.

Trading

- For $x < k^*$ where k^* is a trading point, $p \cdot \psi(-k, x) \cdot x^i = p \cdot \psi(-k, x) \cdot w^i$.
- For $x \geq k^*$, $p \cdot \psi(r, \lambda, -k, x) \cdot x^i = p \cdot \psi(r, \lambda, -k, x) \cdot w^i$.
Bubble crushes at $B(= \phi\sigma_f/\sigma_s)/A > \lambda > -(\sigma_f/\sigma_s)/A$.

Theorem

We assumed that u^i is strictly concave and a closed, convex set of the probability distribution $\Pi^i \subset \text{rint}\Delta$.

- If (w^1, \dots, w^I) is an equilibrium allocation, then there is the unique equilibrium allocation with inertia at the speculative bubble crush when $x < k^*$.