Basins of Attraction Plasticity of a Strange Attractor with a Swirling Scroll

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The topology of the attraction basins provides the first characterization of the global behavior of any dynamical system. Therefore, the paper explores the morphological plasticity of the attraction basins of an asymmetric strange attractor with swirling scroll. Indeed, the bifurcation diagrams reports such basins of attractor transformation. The attractor is confined in a restricted domain of the phase space, and then the whole space is unified only for a single value of the control parameter, exhibiting a periodic cycle. Beyond this narrow field, the attractor reappears embodied in a basin utterly different from the first one. Eventually, the 3D chaotic attractor wings and scrolls shrink and/or expand monitor the contours of the basins.

Key words: Chaos; 3D nonlinear system; Strange attractor; Diagram of bifurcation; Largest Lyapunov exponent;

1. Introduction

The detection of chaotic systems and the derived 3D strange attractors have been widely explored from more than three decades [1-3] following the late (re)discovery of the Lorenzian strange attractor in 1963 [4]. Recently, chaotic system studies established that the strange attractor discoveries pursued its vigorous trend [5-10]. Purposefully creating chaos in the 3D phase space with a few number of nonlinearity can be a nontrivial task to explore such unpredictable patterns. This paper introduces a new chaotic system not developed from previous 3d models and embedding five nonlinear terms. The scope of this intentionally constructed system is the exploration of the domains of attraction plasticity. It denotes the remarkable (or not) amplitude of its turbulence scale.

Section 2 investigates the basic characteristics of the introduced autonomous three-dimensional system of first order differential equations.

Section 3 points chiefly to the morphological plasticity of the basins of attraction.

The final remarks report the extension/contraction of the wings and scrolls of the chaotic attractor.

2. The new 3D chaotic system

The new system is governed by the following three-dimensional nonlinear differential equations:

$$\begin{cases} \mathbf{dx/dt} = \alpha\, \mathbf{x}\,(\mathbf{y} - \mathbf{1}) + \beta\, \mathbf{y\,z} \\ \mathbf{dy/dt} = \varphi\,(\mathbf{1} - \mathbf{x}^2)\,\mathbf{y} + \mu\, \mathbf{x\,z} \\ \mathbf{dz/dt} = \eta\, \mathbf{x\,y} + s\, \mathbf{z} \end{cases}$$

where x, y, and z, the state variables of the model, and α, β, φ, μ, η, and s positive parameters.

Equations embed height terms on the right-hand side, five of which are nonlinear, four cross terms, i.e. two *xy*, xz, and yz, and a cubic cross-term x^2y.

The withdrawal of the z-equation leads to a 2D system constituted by a Lotka-Volterra-like system [11-12] applied also in previous 3D and 4D chaotic systems [13-14]. Moreover, the determination of the equilibrium points can be easily found by solving the steady-state condition:

$$\mathbf{dx/dt = dy/dt = dz/dt = 0}$$

For the set of parameters C_0 (α, β, φ, μ, η, s) = (2, - 3, 0.8, 1, -2, 0.3), the unique equilibrium of the system in R^3 is the origin S_0 (0, 0, 0). The four other solutions emerge in C^3:

S_1 (i 0.294, -0.227, - i 0.667),

S_2 (-i 0.294, -0.227, i 0.667),

S_3 (i 0.294, 0.293, i 0.861), and

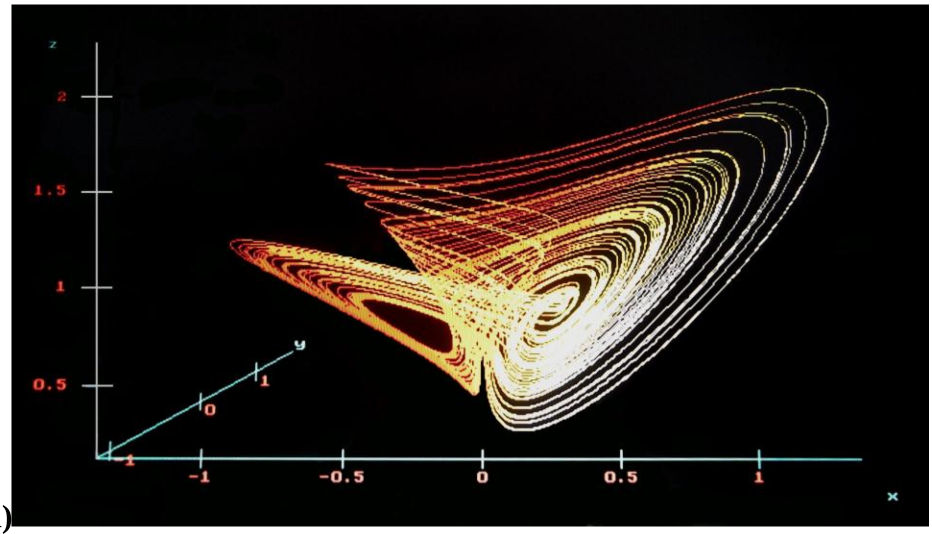
S_4 (-i 0.294, 0.293, - i 0.861).

The elementary attributes of the real equilibria are given by the corresponding eigenvalues λ_i and found by solving the characteristic equation $|J - \lambda I| = 0$, where ***I***, the unit matrix and ***J***, the Jacobian of the model:

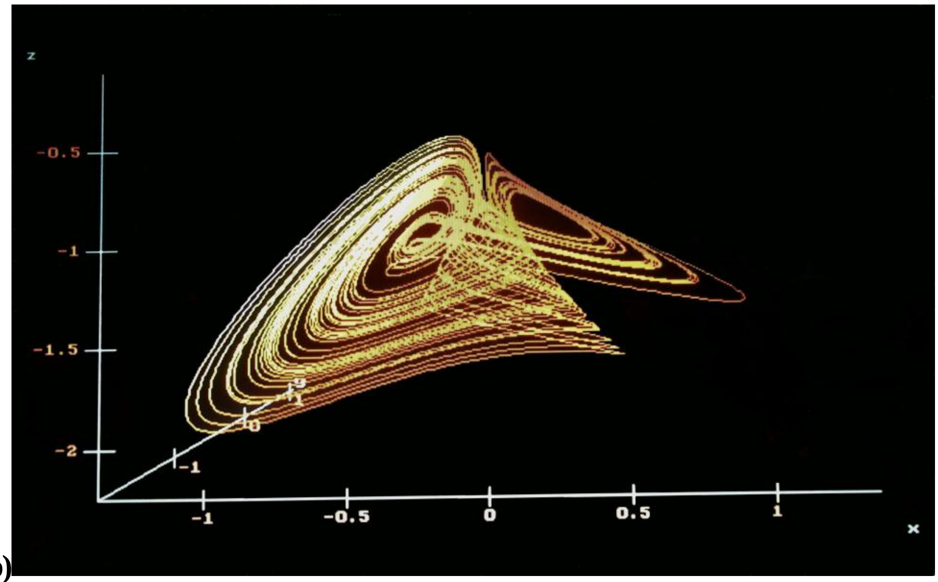
$$J = \begin{vmatrix} \alpha\,(y - 1) & \alpha\, x + \beta\, z & \beta\, y \\ -\, 2\, \varphi\, xy + \mu\, z & \varphi\,(1 - x^2) & \mu\, x \\ \eta\, y & \eta\, x & s \end{vmatrix}$$

The solution S_0 reports a highly aspect of instability since all the three eigenvalues of its characteristic equation have positive real parts (Re (λ) > 0).

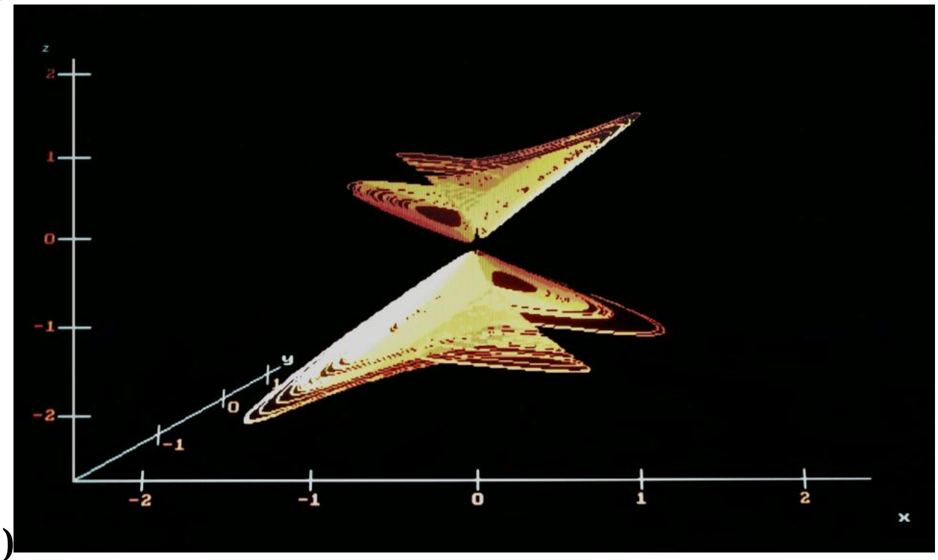
In addition, the system exhibits an explicit chaotic nature since its *Largest Lyapunov Exponent* reaches the value: ***LLE* ≈ 0.85**. The dynamics become unpredictable. For the set of parameters C_0, and the initial conditions IC_0 (0.1, 0.1, 0.1), the simulations display an asymmetric strange attractor with a swirling scroll, connecting the trajectory to the second roll (Fig.1.a). It is confined in the upper domain of the phase space (x, z, y).



(a)



(b)



(c)

Fig.1. Phase portraits of the attractor for C_0 (α, β, φ, μ, η, s) = (2, - 3, 0.8, 1, -2, 0.3)

(a) The attractor appears in the upper basin of the space (x, z, y) for the Initial Conditions IC_0 (0.1, 0.1, 0.1), (b) the attractor in the lower basin for IC_1 (-0.1, -0.1, -0.1), and (c) the disposition of the two chaotic trajectories gathered in the same representation

Besides, for the same parameters C_0, and the negative initial conditions IC_1 (-0.1, -0.1, -0.1), the attractor reappears reversed and located in the lower part of the phase space (Fig.1.b).

The position of the two attractors, independently sited in each basin of attraction, is exhibited in the figure 1.c.

3. Basins of attraction Structure

The computation of the diagrams of bifurcation can describe the array of the trajectory path of the chaotic dynamics.

We retain s as the parameter of control of the bifurcations, varying in the interval [0.1, 6[.

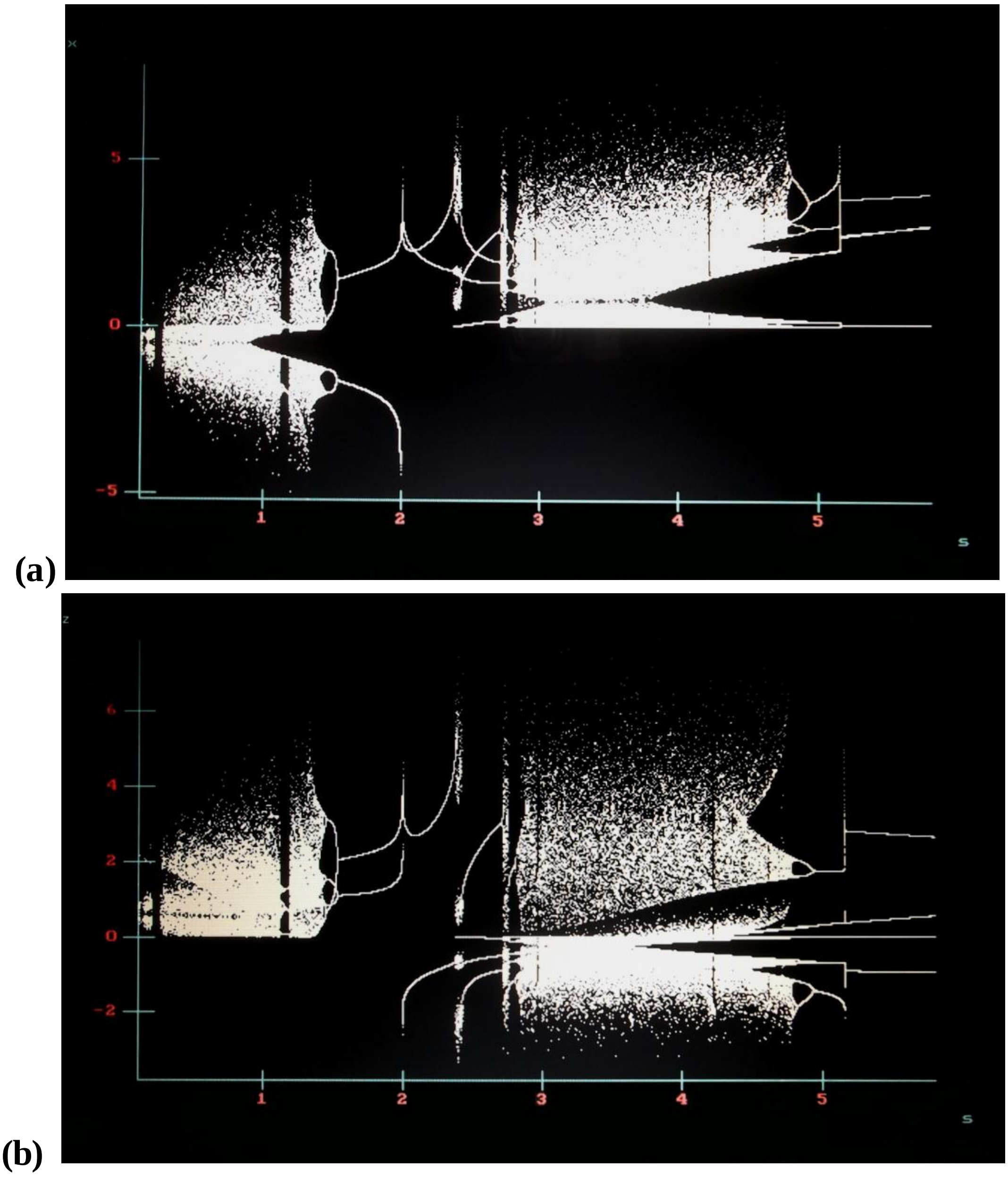


Fig.2. Diagrams of Bifurcation for ŷ = 0.5, and s ∈ [0.1, 6[, with s step-size = 10^{-5}. Sequences of chaotic bubbles and stability windows are displayed in (a) for the diagram of bifurcation of x, and (b) the diagram of bifurcation of z

The dots report the orbit crossing the level $\hat{y} = 0.5$. We notice the shrink of the dynamical amplitude of the variable x (Fig.2.a) beyond the value s = 2. At the contrary, the dynamical amplitude of the variable z (Fig.2.b) expands beyond the same threshold. It indicates the extension/contraction of the basin of attraction in two different directions. Besides, one can observe the wide range of dynamical patterns, within chaotic bubbles and several windows of stability.

Focusing the distinctive value s = 2, the phase space is unified in a same domain of attraction for all variables, displaying a periodic orbit with peculiar shape (Fig.3).

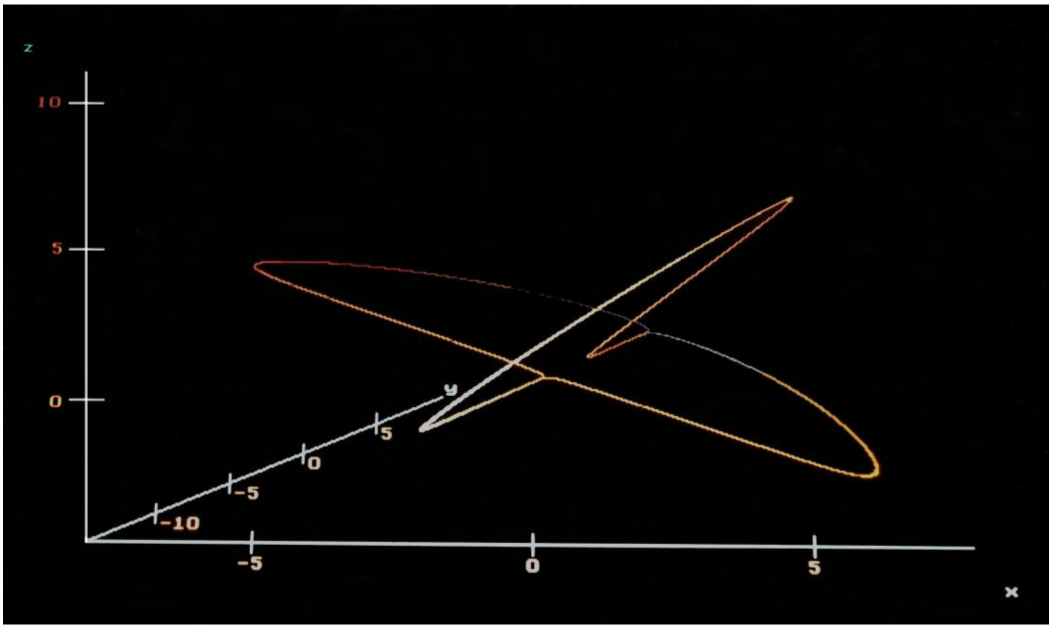


Fig.3. Periodic orbit for the parameters C_1 (2, - 3, 0.8, 1, -2, 2)

Beyond this value, the phase space shrinks in the direction of x and expands in the other direction z. On the other hand, for s = 3, the system, leads to another attractor with dissimilar topology when the initial conditions are IC_0 (Fig.4a.) Besides, with the IC_1, the posture of the attractor is inversed, reporting also a shrink but in the direction of the negative values of the x-axis (Fig.4.b).

The figure 4.c displays the two simulations, each attractor in its basin of attraction. We notice the intricate arrangement of the rolls, so close to appear associated in a single contour. The remarkable morphological versatility of the attractors in figures 1 and 4 seems determined by the cubic nonlinear term in Eq.2.

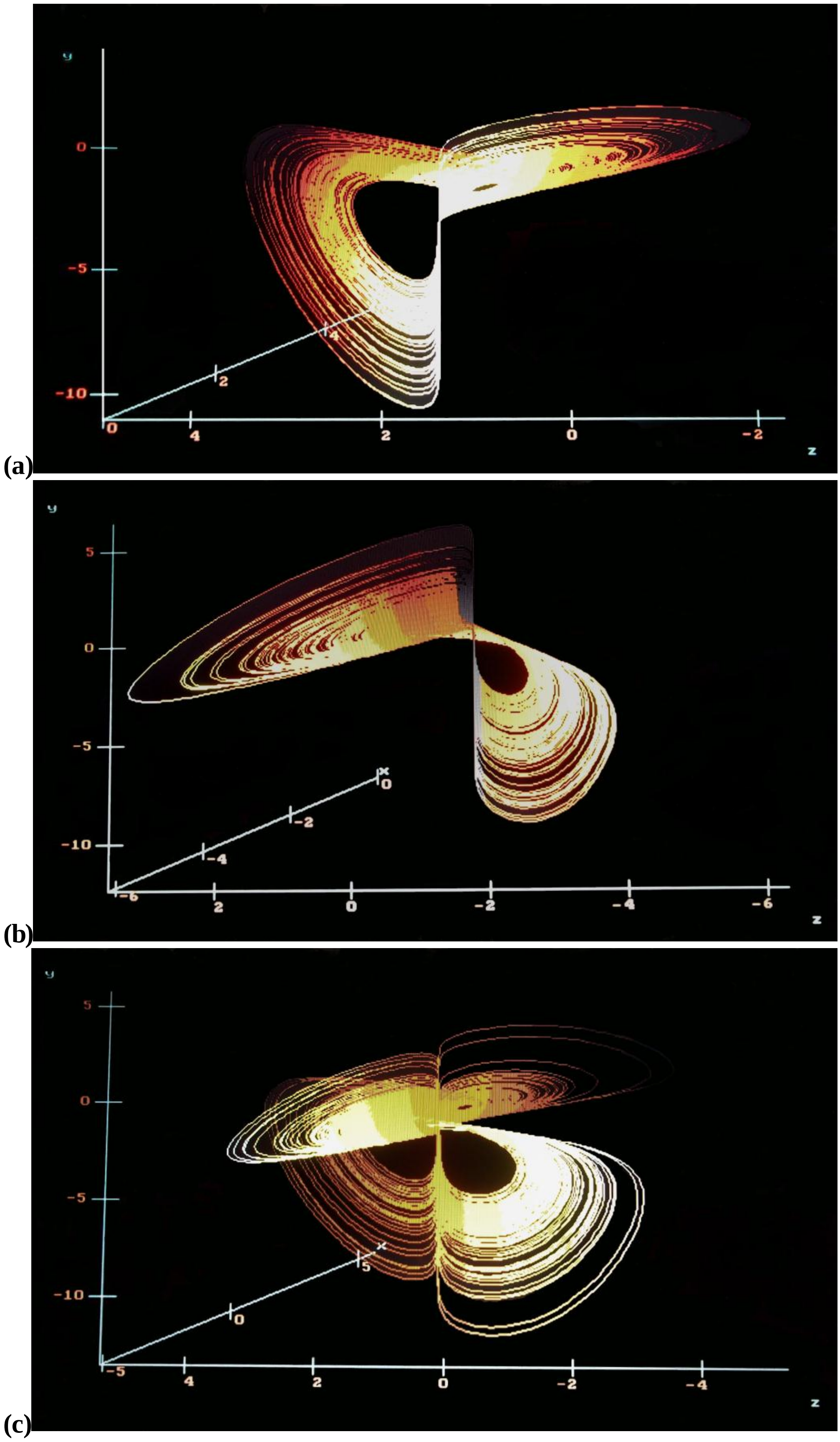


Fig.4. Portraits of the attractor for C_2 (2, - 3, 0.8, 1, -2, 3) in the phase space (z, y, x).
(a) The attractor appears in the positive domain of the x-axis, (b) the attractor located in the negative part of the x-axis, and (c) the disposition of the two chaotic trajectories gathered in the same representation.

4. Final Remarks

At best of our knowledge, the paper introduces an intentionally constructed 3D chaotic attractor distinct from previous attractors. It depicts a shaped complex 2-scroll exhibiting a swirling trajectory in its main loop. Its mutating topology linked to the domain of attraction characterizes a strong sensitive dependency on parameters. This versatility is reported in the bifurcation diagram illustrating the split of the phase space in two domains. Before and beyond an edge of the control parameter, the phase space is restricted in an area with short amplitude of the chaotic frequency. Eventually, the main result of the paper emphasizes the plasticity of the basins of attraction determined by the extension/contraction of the chaotic amplitude.

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