

Measuring enhanced optical correlation induced by transmission open channels in slab geometry (supplementary)

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Consider two random gaussian vectors $\mathbf{E}_1 = \sum_{n=1}^{N_g} c_n$ and $\mathbf{E}'_1 = \sum_{n=1}^{N_g} c'_n$, where c_n and c'_n are random complex gaussian variable with $\langle c_n \rangle = \langle c'_n \rangle = 0$ and $\langle |c_n|^2 \rangle = \langle |c'_n|^2 \rangle = 1$. Let us consider the correlation C_1 :

$$C_1 = \frac{\mathbf{E}_1 \cdot \mathbf{E}'_1}{|\mathbf{E}_1|^2} = \frac{1}{|\mathbf{E}_1|^2} \sum_{n=1}^{N_g} c_n c_n'^* \quad (1)$$

Let us calculate $\langle |C_1|^2 \rangle$ with the Law of large Numbers. We get:

$$\begin{aligned} \langle |C_1|^2 \rangle &= \frac{1}{N_g^2} \left\langle \left| \sum_{n=1}^{N_g} c_n c_n'^* \right|^2 \right\rangle \\ &= \frac{1}{N_g^2} \left\langle \sum_{n=1}^{N_g} \sum_{p=1}^{N_g} c_n c_n'^* c_p'^* c_p \right\rangle \end{aligned} \quad (2)$$

Here, the $n \neq p$ terms do not contribute to $\langle |C_1|^2 \rangle$, because c_n, c'_n, c_p and c'_p are statistically independent. We get thus:

$$\langle |C_1|^2 \rangle = \frac{1}{N_g^2} \sum_{n=1}^{N_g} \langle |c_n|^2 |c_n'|^2 \rangle \quad (3)$$

Since $|c_n|^2$ and $|c_n'|^2$ are also statistically independent, we get:

$$\langle |C_1|^2 \rangle = \frac{1}{N_g^2} \sum_{n=1}^{N_g} \langle |c_n|^2 \rangle \langle |c_n'|^2 \rangle = \frac{1}{N_g} \quad (4)$$

This proves rigorously the result obtained by Monte Carlo for $\langle |C_1|^2 \rangle$.