Delay and state observer for SISO LTI systems

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*Index Terms***— Input delay, time delay estimation, robust observer.**

*Abstract***— This paper deals with the problem of state and delay estimation for SISO LTI systems, with unknown time-varying delay in the input. Thanks to an adequate approximation of the delayed input by the Taylor's theorem, an original approach based on observer design is proposed in order to estimate both state and delay. This new technique allows the estimation of time-varying delay. The convergence of the observer is formally proved. The efficiency of the method is widely illustrated by simulations.**

I. Introduction

Input delay systems are a subcategory of time-delay systems (TDS). They especially include all remote controlled devices. The source of delay is multiple: the network configuration (see the extensive literature on networked control system [14], [24]), computational delays or physical transport delays. When the delay is small or the system is open loop stable, delay free controllers can often achieve stabilization. However, predictive techniques are often required as soon as the delay becomes larger and cannot be neglected anymore [18], [23]. To use such methods, the exact value of the delay is needed. However, in real applications, it is quite difficult to measure the delay with precision so it has to be estimated. For an exhaustive review of time delay estimation (TDE) techniques, the reader can refer to the report [20] by O'Dwyer. To build the prediction not only the delay is needed but also all the state. However, standard observation techniques cannot be applied when the delay is unknown. In this paper, both problem are addressed : delay estimation and state observation.

A. Delay identification

Time delay identification has often been based on a signal processing approach and particularly in the acoustic field [7][16]. These methods are not well adapted in the control context because they are usually offline methods and because they require the knowledge of the delayed signal. A survey of TDE techniques with a signal processing focus is given in [4]. On the contrary, some works use control oriented tools. In these approaches, the delay is considered as a parameter of the system and its identification is often combined to the identification of other parameters. Some authors use the frequency domain where the delay appears as a parameter in the exponential e^{-hs}. In [1], the term e^{-hs} is approximated by a rational transfer function of the Padé form; then a standard discrete least-square algorithm is used to minimize an objective function. Tuch *et al.* [25] also based their approach on the frequency domain and proposed a continuous recursive least square algorithm. However, this method does not work if the initial conditions of the system are not perfectly known. In [19], a PDE approximation is used to extract the delay. In [8], a similar techniques as in [25] is applied but the value of $u(t-h)$ is required. In [9], observers have been used to identify the delay. However, all the state and its time-derivatives are needed; so the method is very sensitive to noise measurement. In [2], a convolution approach is discussed for transfer function systems.

Note that, in all previously mentioned articles the delay is constant.

B. State observation of TDS with unknown delay

In previous methods, transfer function models are often considered and the problem of state observation is not adressed. On the contrary, some papers deal with the problem of state observation with an unknown delay but do not estimate the delay [11], [21], [22]. The references on that topic are much more scarce[1]. As far as the authors knowledge, the only paper that deals with both delay identification and state reconstruction is [10]. The design of their state observer is largely based on a particular sampling/holding technique.

C. Contribution

The main contribution of this paper is to offer an online identification method, based on the theory of robust observation, for both state and delay. The method works for time-varying delays and only requires the knowledge of the input value and the output at time t. The observer convergence is formally proven even for time-varying delays.

D. Paper's structure

The paper is organized as follows. The problem presentation and an observability study are provided in Section II. Section III is dedicated to the observer construction and the convergence analysis. The results are illustrated by simulations in Section IV and a conclusion and some future developments are given in Section V.

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[1]However, there are a lot of works on state observation of TDS with known delay (see [11] and references therein).

II. Problem statement and observability analysis

A. Problem statement

The considered systems are SISO LTI systems with a time varying delay $h(t)$ acting on the control input u:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t-h(t)) \\ y(t) = Cx(t) \\ x(0) = x_0 \end{cases} \tag{1}$$

with $x \in \mathbb{R}^n$, $u \in \mathbb{R}$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times 1}$, $C \in \mathbb{R}^{1\times n}$. This work is focused on the estimation of a single time-varying delay in the input of LTI SISO systems; no parameter identification is considered here.

Assumption 1: The matrices A, B and C are constant and known. The pair (A, C) is observable.

Assumption 2: The relative degree of system (1) is n.

Assumption 3: The delay is a unknown and time-varying. It is modeled as a continuous and differentiable function that satisfies $h(t) \in [0, \bar{h}]$. Its dynamics $\dot{h}(t) = \eta(t)$ is unknown and bounded[2]: for all $t > 0$, $|\eta(t)| \leq H$. The bounds $\bar{h}$ and H can be unknown as well.

Assumption 4: The input u is at least twice time differentiable and the derivatives are bounded for all $t > -\bar{h}$. In particular, there exists $M > 0$ such that

$$|\ddot{u}(t)| \leq M$$

for all $t \geq -\bar{h}$.

The objective is to design an observer that reconstructs the state $x(t)$ and the delay $h(t)$ from the only knowledge of the output $y(t)$ and the input $u(t)$ and its time-derivatives.

The Taylor's theorem is used to kick the delay out of the control. The input u is differentiable for all $t > -\bar{h}$. Then, there exists a function $\gamma : \ ]-\bar{h}; +\infty[\to \mathbb{R}$ such that for all $t' > -\bar{h}$,

$$u(t') = u(t) + (t - t')\dot{u}(t) + \gamma(t') \tag{2}$$

where γ is called the remainder. In particular for $t' = t - h(t) > -\bar{h}$, it leads to

$$u(t - h(t)) = u(t) - h(t)\dot{u}(t) + \gamma(t - h(t)) \tag{3}$$

for all $t > -\bar{h}$. Besides, since u is twice differentiable from Assumption 4, the remainder γ is such that

$$|\gamma(t - h(t))| \leq \frac{h^2(t)}{2} M. \tag{4}$$

From expression (3), the first order approximation of $u(t - h(t))$ is

$$u(t - h(t)) \approx u(t) - h(t)\dot{u}(t). \tag{5}$$

Note that it could be possible to extend the approximation to higher order to make it more accurate. However, from a practical point of view, it has been arbitrarily decided to stop at order one as a tradeoff between problems induced by numerical differentiation and approximation precision.

[2]No restriction is made on the delay rate; H can be larger than one (fast-varying delays)

By substituting $u(t - h(t))$ by (3) in (1), an extended system with perturbation is obtained:

$$\begin{cases} \begin{bmatrix} \dot{x} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} A & -B\dot{u} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ h \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} B\gamma(t-h(t)) \\ \eta(t) \end{bmatrix} \\ y = Cx \end{cases}$$

Denoting the extended vector $X = [x^T \ h]^T$, $X \in \mathbb{R}^{n+1}$, the system can be rewritten in the general form:

$$\begin{cases} \dot{X} = \bar{A}(\dot{u})X + \bar{B}u(t) + \Gamma(t, t-h(t)) \\ y = Y(X) = \bar{C}X \end{cases} \tag{6}$$

where
$\bar{A}(\dot{u}) = \begin{bmatrix} A & -B\dot{u} \\ 0 & 0 \end{bmatrix}$, $\bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$, $\bar{C} = [C \ 0]$ and

$$\Gamma(t, t-h(t)) = \begin{bmatrix} B\gamma(t-h(t)) \\ \eta(t) \end{bmatrix}.$$

It is important to note that the extended system (6) is delay-free, with respect to the input, thanks to the expression (3). However, systems (1) and (6) are equivalent in the sense that they have the same state trajectories. The transformation is only a convenient way to rewrite the system in order to apply existing results from observer literature. The error of approximation γ is going to be considered as a perturbation in the design of the observer as well as the dynamics of the delay η. This is a key point of the method : considering the delay dynamics as perturbation and designing a robust observer that is able to reconstruct the state and the delay in spite of the uncertainty Γ. In the next part, the observability of the extended system (6) is investigated.

B. Delay observability

First the following assumption is made:

Assumption 5: The perturbation Γ does not modify the observability of (6).

Then, the observability condition for extended system (6) is given in the next theorem. See [13] for observability definitions.

Theorem 1: Extended system (6) is observable if and only if

$$\dot{u}(t) \neq 0 \quad \forall t > 0. \tag{7}$$

Proof: First note that from Assumption 2, one has

- $CA^iB = 0$ for all i such that $0 \leq i \leq n-2$
- $CA^{n-1}B \neq 0$.

Considering $\Gamma = 0$ (Assumption (5)), the observation space is defined by

$$\mathcal{O}(Y) = \text{span}\{Cx, CAx, \ldots, CA^{n-1}x, CA^n x - CA^{n-1}B\dot{u}h\}.$$

Since the initial system is observable, one has

$$\dim \text{span}\{Cdx, CAdx, \ldots, CA^{n-1}dx\} = n$$

so

$$d\mathcal{O}(Y) = \text{span}\{Cdx, CAdx, \ldots, CA^{n-1}dx, -CA^{n-1}B\dot{u}dh\}$$

and $\dim(d\mathcal{O}(Y)) = n+1$ if and only if $CA^{n-1}B\dot{u} \neq 0$. Finally, from [13], system (6) is observable if and only if condition (7) is verified because $CA^{n-1}B \neq 0$. ■

This is a logical condition because if the input is constant, the delay as no influence on the system so it cannot be observed. This condition is very restrictive because it means that the input has to be strictly monotonic. However, this condition can be relaxed using the notion of persistence defined in [3]. The definition is recalled below.

Definition 1: A measurable bounded signal $\dot{u}$ is said to be regularly persistent for system (6) if there exists $T > 0$, $\alpha > 0$ and $t_0 > 0$ such that $\min_i(\lambda_i(W(t,T,\dot{u})) > \alpha$ for all $t > t_0$ where $W(t,T,\dot{u})$ is the Observability Gramian and $\lambda_i(M)$ denotes the i^{th} eigenvalue of the matrix M.
Roughly speaking, it allows $\dot{u}$ to cancel at some isolated time instants without deteriorating the estimation.

Assumption 6: The signal $\dot{u}$ is regularly persistent.

III. A NEW SCHEME OF DELAY-STATE OBSERVER

A. Kalman like observer design [12]

Kalman-like observer is easy to tune because it only has one parameter to adjust and it is well adapted for linear systems with matrix A depending on an external signal. That is why, it has been chosen in this work. From [12], a Kalman-like observer for (6) reads as

$$\dot{\hat{X}} = \bar{A}(\dot{u})\hat{X} + \bar{B}u - S^{-1}R\bar{C}^T\bar{C}(\hat{X} - X) \tag{8}$$

where the matrix S is the solution of

$$\begin{cases} \dot{S} = -\rho S - \bar{A}(\dot{u})^T S - S\bar{A}(\dot{u}) + \bar{C}^T\bar{C} \\ S(0) > 0 \end{cases} \tag{9}$$

with ρ a positive constant and R a diagonal matrix acting as a filter. In the noise-free case, $R = I_n$ (identity matrix of order n).

B. Practical stability of the observer

The time-varying perturbations $\gamma(t - h(t))$ and $\eta(t)$ prevent the asymptotic convergence of the observer error. Consequently, only a practical convergence to a ball of radius r around the origin is achievable. The size of r is tightly related to the size of the perturbation and the observer gain. The following lemma, given in [3], will be useful to formulate the main result.

Lemma 1: Consider that S is defined by (9) and that Assumption 6 holds. Then there exists a real ρ_0 such that for any symmetric positive definite matrix $S(0)$, for all $\rho \geq \rho_0$, there exists $\bar{\alpha} > 0,\ \bar{\beta} > 0,\ t_0 > 0$ such that for all $t > t_0$

$$\bar{\alpha} I_{n+1} \leq S(t) \leq \bar{\beta} I_{n+1}$$

where I_{n+1} is the identity matrix of order $n+1$.
Now, the main result can be stated.

Theorem 2: Consider system (6) and any input $u(t)$ and delay $h(t)$ such that Assumptions 1-6 are fulfilled. Then, there exist positive scalars t_0, k, r, θ such that for all $t \geq t_0$ the following inequality holds:

$$\left\|\hat{X}(t) - X(t)\right\| \leq k||e(t_0)|| \exp(-\theta(t - t_0)) + r. \tag{10}$$

Proof:

Define the Lyapunov candidate function as

$$V(e) = e^T S e \tag{11}$$

with S given by (9) and $e = \hat{X} - X$, the error dynamics of the observer. The objective is to show that (11) complies with the assumptions of Lemma 9.4 in [15].

From Lemma 1, there exists ρ_0 and t_0 such that

$$\bar{\alpha}||e||^2 \leq V(e) \leq \bar{\beta}||e||^2 \tag{12}$$

for all $\rho \geq \rho_0$ and $t \geq t_0$. Furthermore, the dynamics of the observer of the **undisturbed** system is

$$\dot{e} = [\bar{A}(\dot{u}) - S^{-1}\bar{C}^T\bar{C}]e. \tag{13}$$

Then, by using (9) and (13), the time derivative of (11) is

$$\dot{V}(e) = -\rho e^T S e - e^T\bar{C}^T\bar{C}e.$$

Since $e^T\bar{C}^T\bar{C}e \geq 0$, one has the inequality

$$\dot{V}(e) \leq -\rho\bar{\alpha}||e||^2. \tag{14}$$

In addition, V satisfies the relation

$$\left\|\frac{\partial V}{\partial e}\right\| \leq 2\bar{\beta}||e||. \tag{15}$$

Equations (12), (14) and (15) holds globally so Lemma 9.4 from [15] ensures that

$$||e(t)|| \leq k||e(t_0)|| \exp\left(-\theta(t - t_0)\right) + r$$

with $k = \sqrt{\frac{\bar{\beta}}{\bar{\alpha}}}$, $\theta = \frac{\rho\bar{\alpha}}{2\bar{\beta}}$ and

$$r = \frac{2\bar{\beta}^2}{\rho\bar{\alpha}^2} \sup_{t > t_0} ||\Gamma(t)||. \tag{16}$$

■

In a particular case, it is possible to evaluate the value of r.

Corollary 1: For constant delays and ramp inputs, the observation error converges exponentially to zero and one has $r = 0$.

Proof: Assumptions 3 and 4 gives

$$\sup_{t > t_0} ||\Gamma(t)|| \leq c_1\bar{h}^2 M + c_2 H$$

with c_1 and c_2 stricly positive scalars. Furthermore, if the delay is constant then its dynamics is equal to zero so $H = 0$; if the input is a ramp, its second time-derivative is 0 so $M = 0$. As a consequence, the upper bound of Γ is 0 and the radius r given in (16) cancels which ends the proof. ■

Note that if the delay is slowly-varying then the approximation will be more accurate because H will be smaller. Similarly, the smaller M and $\bar{h}$, the finer the approximation (5) and the smaller the convergence radius r. Theoretically, it is possible to add higher order terms in the approximation (5) to reduce the uncertain term γ; however, it would require to compute high order time-derivatives of u. Observer (8) does not guarantee the boundedness of $\hat{h}$ to $[0, \bar{h}]$ so a projection of $\hat{h}$ on $[0, \bar{h}]$ can be made if the bounds are known [5][6][10]. Simulation results are provided in the next section to illustrate the efficiency and the limits of this new method.

IV. SIMULATIONS

A. Model presentation and observer design

A second order system has been chosen to illustrate previous result. Its input-output representation reads as

$$\ddot{y} + \beta_1 \dot{y} + \beta_0 y = u(t - h(t)), \tag{17}$$

and its state space representation is

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -\beta_0 & -\beta_1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t - h(t)) \\ y(t) = Cx(t) = x_1(t). \end{cases}$$

The system is observable and the relative degree of y equals two; Assumptions 1 and 2 hold. The parameters chosen for all the simulations are $x(0) = [1.5, 1]^T$, $\beta_1 = 2$, $\beta_2 = 3$ and $h(t) \in [0, 1]$. The extended system is defined by matrices:

$$\bar{A}(\dot{u}) = \begin{bmatrix} 0 & 1 & 0 \\ -\beta_0 & -\beta_1 & 0 \\ 0 & 0 & -\dot{u} \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ and } \bar{C} = [1\ 0\ 0].$$

The parameter ρ is chosen equal to 5, $S(0) = I_3$ (the identity matrix of dimension 3). The value of ρ is a compromise between fast time-response and noise amplification (in the real case). The initial conditions of the observer are $\hat{x}(0) = [0, 0]^T$ and $\hat{h}(0) = 0.4$.

B. Noise-free simulations

Two kinds of delay are used in the sequel:

- $h_1(t) = \begin{cases} 0.15 \text{ for } 0 \leq t \leq 15 \\ 0.6 \text{ for } 15 < t \leq 30 \\ 0.3 \text{ otherwise} \end{cases}$
- $h_2(t) = 0.4 + 0.2\sin(0.4t)$

The delay h_1 is a piecewise function whose each sub function complies with Assumption 3. The delay h_2 complies with Assumption 3. Two cases of input signals are tested:

- a ramp : $u_1(t) = 0.2t$
- a sinusoidal input: $u_2(t) = \sin(0.1t)$

For the input u_1, one has $\dot{u}_1(t) = 0.2$ and $\ddot{u}_1(t) = 0$ so Assumption 4 and 6 are true. The input u_2 is regularly persistent because the condition (7) holds almost everywhere so Assumption 6 is true. Figure 1 displays inputs u_1, u_2 and their derivatives.

On Figure 2, the simulation is carried out with the delay h_1 and the input u_1. It can be noted that the observation errors tend to zero asymptotically (exponentially) because

- the delay dynamics $\eta(t)$ is 0 since the delay is piecewise constant;
- the Taylor approximation (5) is exact $u_1(t - h(t)) = u_1(t) - h(t)\dot{u}_1(t)$.

Consequently, the pertubation term Γ in (6) is equal to 0 and the convergence radius is reduced to 0. This result illustrates Corollary 1.

Figure 3 shows the result for the time-varying delay h_2 and the ramp input u_1. The Taylor approximation is still exact so γ in (6) is equal to 0. However, the observation error $x - \hat{x}$ does not tend to zero exponentially. Indeed, the observation error, $h - \hat{h}$, is introduced in the state observation.

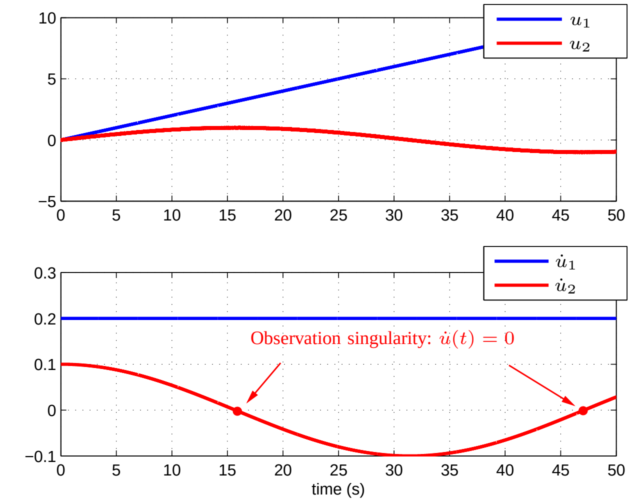

Fig. 1. Inputs and observation singularity

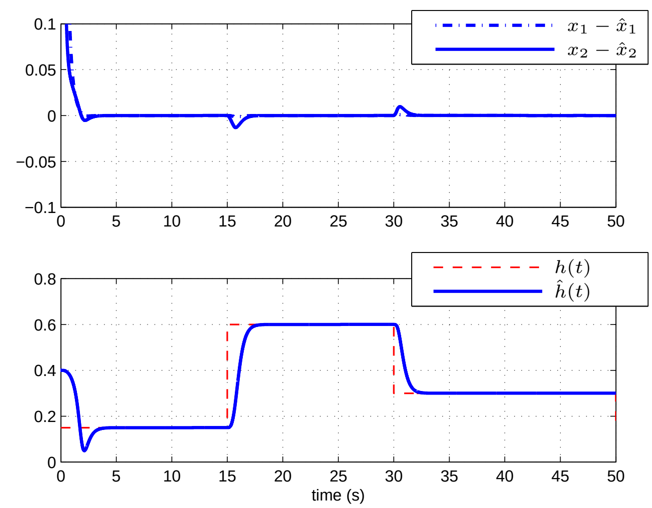

Fig. 2. Simulation 1 with $h(t) = h_1(t)$ and $u(t) = u_1(t)$ (ramp)

It is clear that $e_x = \hat{x} - x$ and $e_h = \hat{h} - h$ converge to a neighborhood around the origin and the size of this ball can be adjusted thanks to the gain ρ. Figure 4 shows this feature, with $\rho = 15$, the convergence radius has decreased.

On Figure 5, the piecewise constant delay h_1 is associated to the sinusoidal input u_2. One can notice that the convergence radius for e_h is smaller when h is small. This is is mainly due to the accuracy of the approximation (5) that is better for small delays. The peaks are caused by the singularity in the observer gain (S^{-1}) and the poor accuracy of the approximation (5) for large delays.

The last configuration with the sinusoidal input u_2 and the time varying-delay h_2 is presented on Figure 6. The analysis is the same as the one of simulations 2 and 3:

- some peaks appear on the graph of $\hat{h}$ due to the observation singularity;
- the convergence radiuses for e_x and e_h are larger than

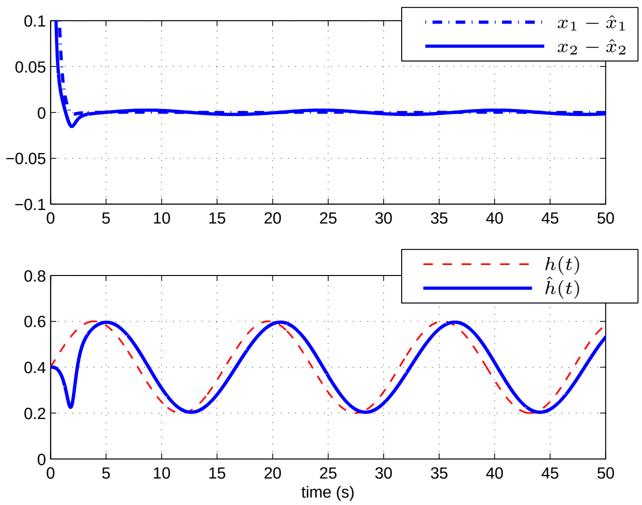

Fig. 3. Simulation 2 with $h(t) = h_2(t)$ and $u(t) = u_1(t)$ (ramp) and $\rho = 5$

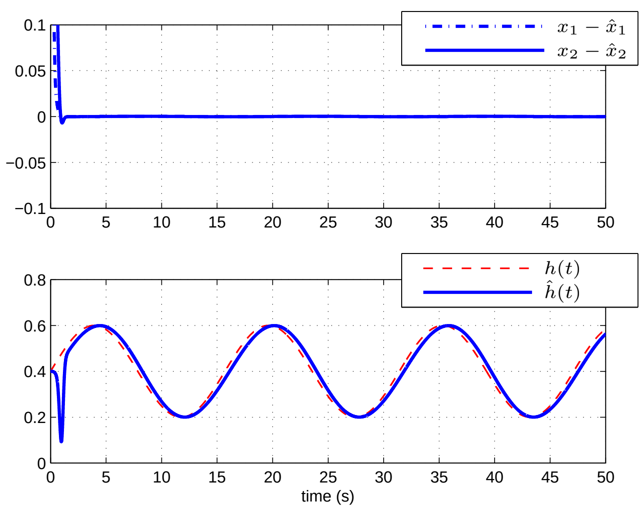

Fig. 4. Simulation 2 with $h(t) = h_2(t)$ and $u(t) = u_1(t)$ (ramp) and $\rho = 15$

for constant delay because of the term $\eta(t) \neq 0$.

Previous simulations have confirmed theoretical results. They illustrate the efficiency of the proposed robust observer technique

- to reconstruct the state of a system with an unknown and possibly time-varying delay in the input;
- to estimate the delay value.

The choice of the input is crucial. Indeed, the quality of the delay estimation highly depends on the input dynamics. As shown before, the ramp input is the best choice because it does not introduce any observation singularity and because the Taylor approximation is exact in this case. However, this is not always possible to apply it in practice. Consequently, a basic idea is to design inputs that are similar to a ramp and turned off the observer when it gets closer to the singularity

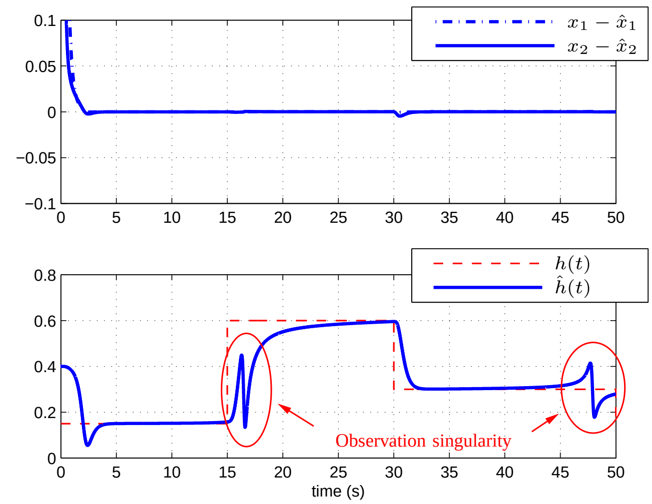

Fig. 5. Simulation 3 with $h(t)=h_1(t)$ and $u(t)=u_2(t)$ (sine)

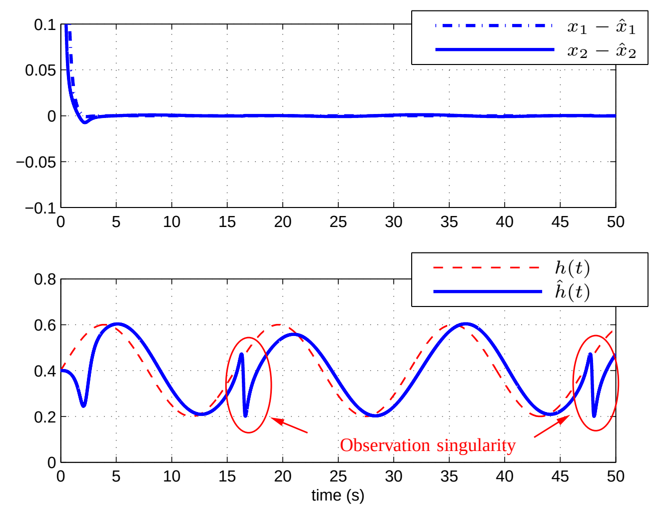

Fig. 6. Simulation 4 with $h(t)=h_2(t)$ and $u(t)=u_2(t)$ (sine)

($\dot{u} = 0$). This method will be tested in the next subsection.

C. Simulations with noisy measurement and noisy input

In practice, the measurement and the input can be affected by noise. In the next simulation, a 5% white noise has been added to the output (measurement) and the input. A diagonal matrix R has been implemented in the observer (see (8)) to filter the noise and a Levant differentiator [17] has been used to compute the input derivative. Furthermore, to overcome the observability singularity, the observer is turned off as soon as $|\dot{u}(t)| \leq \epsilon$. More precisely, only the delay-observer part of (8) is turned off, the state-observer part still runs. The parameter ϵ has to be tuned according to the input dynamics, in the next simulation $\epsilon = 0.03$.

Figure 7 shows that the estimation accuracy is degraded but the observer still converges. Note also that the convergence is slower because of the filter. When the observer is turned off, the gain of the last equation in (8) is set to zero so $\dot{\hat{h}} = 0$ that is why $\hat{h}$ is constant. Because of this observation, the technique is efficient when the delay varies slowly; the tuning of ϵ has to be a tradeoff between avoiding the singularity peaks and keeping an accurate estimation of h.

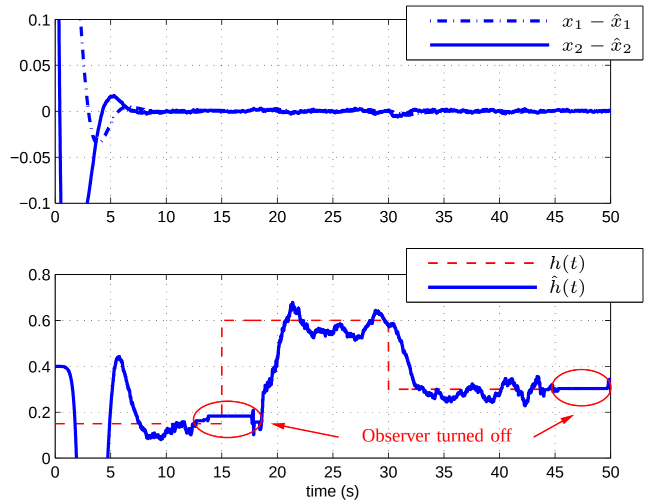

Fig. 7. Simulation 5 with $h(t) = h_1(t)$ and $u(t) = u_2(t)$ (sine) with measurement noise

V. Conclusion

This paper presents a new and original approach for observer design of input delay systems. This observation solution allows to estimate both state and delay (time-varying). The Taylor approximation is exploited to take out the delay from the retarded input. Then an extended system is created with the delay as a part of the augmented state. An observability condition is derived from the analysis of this extended system. Finally, a Kalman-like observer is design and practical stability is obtained. It is shown that asymptotic convergence can be achieved for constant delay with particular inputs. All the results are illustrated by numerous simulations.

The extension to nonlinear or MIMO systems as well as the observation in closed-loop are considered for future developments.

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