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Abstract. A well known strategy for belief revision is the use of an operator which takes as input a belief base and formula and outputs a new consistent revised belief base. Many operators require additional information such as epistemic entrenchment relations, system of spheres, faithful orderings, subformulae relation, etc. However, in many applications this extra information does not exist and all beliefs have to be equally considered. Other operators that can do without background information are dependent on the syntax. Among the few operators that possess both kinds of independence: of extra information and of the syntax, Dalal's operator is the most outstanding. Dalal's revision moves from the models of the base to the models of the input formula which are closest in terms of Hamming distance. A drawback of Dalal's approach is that it fails when faced with inconsistent belief bases. This paper proposes a new method for computing Dalal's revision that avoids the computation of belief bases models. We propose a new distance between formulae based on distances between terms of formulae in DNF and a revision operator based on these distances. The proposed operator produces Dalal's equivalent results when the belief base and new input are both consistent. Moreover, this new operator is able to handle inconsistent belief bases. We also analyze several properties of the new operator. While the input belief base and formula need a compilation to DNF, the operator meets desirable properties making the approach suitable for implementation.

1 Introduction

Belief revision is a framework that characterizes the process of belief change in which an agent revises its beliefs when newly received evidence contradicts them. Logic-based belief revision has been studied extensively [1–3]. Usually an agent's beliefs are represented as a theory or base K and a new input is in the form of a propositional formula μ which must be preserved after the revision. Many belief revision operators $\circ$ have been proposed to tackle the problem, they take the base and the formula as input and reach a new consistent revised belief base $K \circ \mu$ as output. Diverse operators existing in the literature need additional information such as epistemic entrenchment relations [4], system of spheres [5], faithful orderings [1], subformulae relations[6], etc. However, in most of the cases we do not have this extra information. There exist formula-based belief operators which do not need extra information; however, they are sensitive to the syntax, i.e., two equivalent inputs may produce different outputs. So they lose the desirable property of independence of syntax that is met by most of the operator mentioned previously. Dalal's operator is the most outstanding revision technique that meets both: independence of syntax and independence of extra information. The revision is based on the Hamming distance between interpretations once it is extended to distances between interpretations and bases. Dalal's operator takes the interpretations which are models of the input formula and which are closest to the belief base. In practice this framework entails a costly computation of models. For example, suppose that $K = \{a \to b\}$ and $\mu = a \wedge \neg c \wedge d \wedge e \wedge f \wedge g$. The approach needs to consider 96 models for the base and 2 models for the input formula, so the approach calculates 192 distances between interpretations in order to select the models of μ closest to K.

Another drawback of Dalal approach is its inability to revise inconsistent bases. For example suppose an agent who holds the following information: it is raining, if it is raining it is cloudy, it is not cloudy, and the sky is blue, represented by the following base $\{a, a \to b, \neg b, c\}$, now suppose the agent receives the new information: it is cloudy. In this case Dalal revision needs a preprocessing to transform an inconsistent base into a consistent one and then revise by b. A possible solution may be to consider each formula as a base and then merge the formulae of the belief base, however, the process became more expensive and the merging phase does not take into account the new information who may be the key to recover consistency.

On the other hand, efforts have been made to reduce the computational costs associated with models by compiling the initial belief base and new input into prime implicant and prime implicate forms [7, 8]. Belief compilation has been proposed recently for dealing with the computational intractability of general propositional reasoning [9]. A propositional theory is compiled off-line into a target language, which is used on-line to answer a large number of queries in polynomial time. The idea behind belief compilation is to push as much of the computational overhead into the off-line phase as possible, as it is then amortized over all on-line queries. Target compilation languages and their associated algorithms allow us to develop on-line reasoning systems requiring fewer resources. Thus, we propose reducing computation by compiling the belief base and formula representing new evidence to their Disjunctive Normal Form (DNF), and then avoiding computation of distances from a interpretation to another by computing a new distance directly between terms of formulae in DNF. The idea behind this new distance is that instead of measuring how different the models of the belief base are from a model of the new evidence, we compute how different the terms of the belief base are from the terms of the formula representing new evidence. This notion of *distance between terms* avoids reaching the level of models and measures distances between sets of models represented by subformulae (terms) instead. In the case of the previous example, the computed distances are only

2 instead of 192, see Section 3 for more details. While the operator based on this new distance meets the desirable properties of independence of syntax and of extra information, a compilation of the belief base and formula to a DNF is required.

Classical belief revision always trust new information and thus revises the current beliefs to accommodate new evidence to reach a consistent belief base. Most studies on belief revision are based on the AGM (Alchourron, Gardenfors & Makinson) postulates [6] which captures this notion of priority and describe minimal properties a revision process should have. The AGM postulates formulated in the propositional setting in [1], denoted as R_1-R_6, characterize the requirements with which a revision operator should comply. For example, postulate R_1, also called the success postulate, captures the priority of new evidence over the belief base, it requires that the revision result of a belief base K by a proposition μ (new information) should always maintain μ being believed. R_3 is the previously mentioned principle of independence of syntax. In this paper we analyze the satisfaction of R_1-R_6 by the new operator.

To summarize, the major contribution of this paper is to propose a new method that reproduces Dalal's results and is able to handle two drawbacks of Dalal's revision: the need to compute all the models of formulae and the inability to handle inconsistent belief bases. The new method satisfies postulates R_1-R_6 when both inputs: the belief base and new evidence are consistent, and it satisfies some of postulates when the inputs are inconsistent. The complexity of the new method, once the formula is in DNF, is polynomial. The rest of the paper is organized as follows. After providing some technical preliminaries and reviewing the characterization of revision process, in Section 3 we introduce the new distance and its respective operator. Then we analyze the satisfaction of the postulates and the complexity issues. Finally, we conclude with some future work.

2 Preliminaries

We consider a language $\mathcal{L}$ of propositional logic using a finite ordered set of symbols or atoms $P := \{p_1, p_2, ..., p_n\}$. A *belief base/theory* K is a finite set of propositional formulae of $\mathcal{L}$ representing the beliefs from a source (we identify K with the conjunction of its elements). A literal l is an atom or the negation of an atom. A term D is a conjunction of literals: $D = l_{r_1} \wedge ... \wedge l_{r_m}$, where, $r_i \in \{1, ..., n\}$ and l_{r_i} concerns atom p_{r_i}. A minterm is term in which each atoms of language appears exactly once. A Disjunctive Normal Form of a formula ϕ is a disjunction of terms $\mathrm{DNF}_\phi = D_1 \vee ... \vee D_k$ which is equivalent to ϕ. If a literal l appears in a term D, it is denoted by $l \in D$ and if D appears in DNF_ϕ, it is denoted by $D \in \mathrm{DNF}_\phi$. If D is a term, $index(D)$ denotes the set of indexes of the literals appearing in D. For example, if $D = p_4 \wedge \neg p_2 \wedge p_8$, then $index(D) = \{2, 4, 8\}$.

A set of possible interpretations from P of language $\mathcal{L}$ is denoted as $\mathcal{W}$. $w \in \mathcal{W}$ is denoted as vectors of the form $(w(p_1), ..., w(p_n))$, where $w(p_i) = 1$ or $w(p_i) = 0$

for $i = 1, ..., n$. A interpretation w is a model of $\phi \in \mathcal{L}$ if and only if ϕ is true under w in the classical truth-functional manner. The set of models of a formula ϕ is denoted by $mod(\phi)$. K is consistent iff there exists model of K.

$|X|$ denotes the cardinality of X if X is a set or $|X|$ denotes the number of literals occurring in X if X is a term, finally, it denotes the absolute value of X if X is a number. $|l|_b$ denotes 1 (respectively 0) if l is an atom (respectively the negation of an atom). Let $\leq_\psi$ be a relation over a set of possible interpretations; $x =_\psi y$ is a notation for $x \leq_\psi y$ and $y \leq_\psi x$, and $x <_\psi y$ is a notation for $x \leq_\psi y$ and $y \not\leq_\psi x$.

In [6] eight postulates have been proposed to characterize the process of belief revision, which are known as the AGM Postulates. Assuming a proposition setting, in [10, 1] this characterization is rephrased producing the following R_1-R_6 postulates, where K, K_1 and K_2 are belief bases to be revised and μ, μ_1 and μ_2 are new evidence:

R_1. $K \circ \mu$ implies μ.
R_2. If $K \wedge \mu$ is satisfiable, then $K \circ \mu \equiv K \wedge \mu$.
R_3. If μ is satisfiable, then $K \circ \mu$ is also satisfiable.
R_4. If $K_1 \equiv K_2$ and $\mu_1 \equiv \mu_2$, then $K_1 \circ \mu_1 \equiv K_2 \circ \mu_2$.
R_5. $(K \circ \mu_1) \wedge \mu_2$ implies $K \circ (\mu_1 \wedge \mu_2)$.
R_6. If $(K \circ \mu_1) \wedge \mu_2$ is satisfiable, then $K \circ (\mu_1 \wedge \mu_2)$ implies $(K \circ \mu_1) \wedge \mu_2$.

A representational theorem has been provided which shows equivalence between the six postulates and a revision strategy based on total pre-orders. The theorem is based on the notion of *faithful assignment*. The formal definitions are as follows [10]:

Definition 1. *Let $\mathcal{W}$ be the set of all interpretations of a propositional language $\mathcal{L}$. A function that maps each sentence ψ in $\mathcal{L}$ to a total pre-order $\leq_\psi$ on interpretations W is called a faithful assignment if and only if:*

1. *$w_1, w_2 \models \psi$ only if $w_1 =_\psi w_2$;*
2. *$w_1 \models \psi$ and $w_2 \not\models \psi$ only if $w_1 <_\psi w_2$; and*
3. *$\psi \equiv \phi$ only if $\leq_\psi = \leq_\phi$.*

Theorem 1 (Representation Theorem). *A revision operator $\circ$ satisfies Postulates R_1-R_6, iff there exists a faithful assignment that maps each sentence ψ into a total pre-order $\leq_\psi$ such that: $mod(\psi \circ \mu) = min(mod(\mu), \leq_\psi)$.*

3 Distance between Terms

Without loss of generality we consider only compiled languages so that each belief base is taken as a DNF, and each formula representing new evidence is taken as a DNF too. For example, for the belief base $\{a, a \to b, \neg b, c\}$, we consider the compiled belief base $(a \wedge \neg a \wedge \neg b \wedge c) \vee (a \wedge b \wedge \neg b \wedge c)$. Moreover, we consider only terms with non repeated literals, then terms such as $a \wedge a \wedge a \wedge b$ will be considered simply as $a \wedge b$.

Classically in Dalal's revision the process uses two type of distances: Hamming distance which is a distance from a interpretation to another one defined as follows: $d(w_1, w_2) = \sum_{p \in P} |w_1(p) - w_2(p)|$ and a distance from a interpretation to a belief base defined as follows: $d(w, K) = min_{w' \in mod(K)} d(w, w')$. The latter distance allows the definition of a pre-order over the models of the input information, $w_1 \leq_K w_2 \quad iff \quad d(w_1, K) \leq d(w_2, K)$. The closest interpretations to the belief base are the models of the revision process $mod(K \circ_D \mu) = min(mod(\mu), \leq_K)$. Our proposal of belief revision is quite similar; the process defines a distance between terms as follows:

Definition 2 (Distance between terms). *Let $D = l_{r_1} \wedge ... \wedge l_{r_m}$ and $D' = l'_{s_1} \wedge ... \wedge l'_{s_k}$ be two terms, the distance between D and D', denoted $d(D, D')$, is defined as: $d(D, D') = \sum_{i \in \{r_1, ..., r_m\}} \left(||l_i|_b - |l'_i|_b| \quad s.t. \quad i \in \{s_1, ..., s_k\} \right)$.*

Or equivalently $d(D, D') = \sum_{i \in index(D)} \left(||l_i|_b - |l'_i|_b| \quad s.t. \quad i \in index(D') \right)$, when both D and D' are consistent, $d(D, D') = \sum_{i \in index(D) \cap index(D')} ||l_i|_b - |l'_i|_b|$ can be used instead. If both D and D' are minterms, we can consider them as possible interpretations and then recover Hamming distance. Moreover, the following desirable properties are satisfied: $d(D, D') = d(D', D)$ and $d(D, D') = 0$ if $D = D'$.

This term-based distance allows us to define a succinct process to reproduce minimal Hamming distances. Consider the example in the Introduction, where $K = \{\neg a \vee b\}$ and $\mu = a \wedge \neg c \wedge d \wedge e \wedge f \wedge g$. The 96 models of K: $\{(0,0,0,0,0,0,0), (0,0,0,0,0,0,1), ..., (0,1,1,1,1,1,1), (1,1,0,0,0,0,0), (1,1,0,0,0,0,1), ..., (1,1,1,1,1,1,1)\}$ can be represented in a succinct form as $(0, x_2, x_3, x_4, x_5, x_6, x_7)$ or $(x'_1, 1, x'_3, x'_4, x'_5, x'_6, x'_7)$ where every x_i and x'_i can take the value 0 or 1. The models of μ can be represented in a succinct form as $(1, y_2, 0, 1, 1, 1, 1)$ where y_2 can take the value 0 or 1. Now, it is easy to verify that a Hamming distance between two interpretations is the number of positions for which the corresponding valuation of symbols is different. In other words, it measures the minimum number of substitutions required to change one interpretation into the other. We want to transform the models of K into models of μ with minimal change. The notion of minimal change is expressed by substitutions as follows: in order to change a model of K expressed as $(0, x_2, x_3, x_4, x_5, x_6, x_7)$ into a model of μ expressed as $(1, y_2, 0, 1, 1, 1, 1)$ we need to substitute 0 by 1 in the first position and assign the following values to the variables: $x_2 = y_2$, $x_3 = 0$, $x_4 = 1$, $x_5 = 1$, $x_6 = 1$, and $x_7 = 1$, i.e., the minimal change, the minimal Hamming distance for these two patterns is 1. Considering now the second succinct form of expressing models of K: $(x'_1, 1, x'_3, x'_4, x'_5, x'_6, x'_7)$, in order to transform it into the succinct form of models of μ $(1, y_2, 0, 1, 1, 1, 1)$, no substitution is required, solely an assignment of Boolean variables as follows: $x'_1 = 1$, $x'_3 = 0$, $x'_4 = 1$, $x'_5 = 1$, $x'_6 = 1$, $x'_7 = 1$ and $y_2 = 1$. This means the minimal Hamming distance for these two patterns is 0. Thus, we found the model of μ (1,1,0,1,1,1,1) that represents the revision of K by μ in terms of minimal Hamming distance, i.e. the minimal change found is 0: none substitution is required.

The succinct forms of models can be represented by terms of a DNF, hence, $\neg a$ represents $(0, x_2, x_3, x_4, x_5, x_6, x_7)$, b represents $(x'_1, 1, x'_3, x'_4, x'_5, x'_6, x'_7)$ and $a \wedge \neg c \wedge d \wedge e \wedge f \wedge g$ represents $(1, y_2, 0, 1, 1, 1, 1)$. Actually, in this case the models of μ leave free solely the second position, which means that the models of K can fix a Boolean value only in the second position if it is required to hold a minimal change, in this case the second position is fixed with 1 by the second pattern of K. To capture this notion of fixing a model of μ with the help of literal belonging to the models of K, we introduce the notion of extension of terms as follows:

Definition 3 (Extension of terms). *The extension of term D_1 by a term D_2, denoted $ext(D_1, D_2)$, is defined as: $ext(D_1, D_2) = D_1 \wedge \bigwedge_{l_i \in D_2 | i \notin index(D_1)} l_i$.*

I.e. the result of extending a term with a second term is a term that includes all the literals of the former and the literals of the second term that do not consider atoms appearing in the former. Notice that in the running example, $ext(\neg a, a \wedge \neg c \wedge d \wedge e \wedge f \wedge g) = \neg a \wedge \neg c \wedge d \wedge e \wedge f \wedge g$ and $ext(a \wedge \neg c \wedge d \wedge e \wedge f \wedge g, \neg a) = a \wedge \neg c \wedge d \wedge e \wedge f \wedge g$, then the extension of terms is not commutative. This notion of extension can be extended to formulae. Thus, we will be able to extend the terms of μ by terms of belief base K preserving consistency.

Definition 4 (Extension of formulae). *We define the extension of formula ϕ_1 by a formula ϕ_2, denoted $ext(\phi_1, \phi_2)$, as the following multiset:*

$$ext(\phi_1, \phi_2) = \{ext(D_1, D_2) | D_1 \in \phi_1 \quad and \quad D_2 \in \phi_2\}.$$

This definition can help us to find the potential extended terms that will form part of the revision result. In the running example considering the term of μ and the two terms of K, we have $ext(\mu, K) = \{a \wedge \neg c \wedge d \wedge e \wedge f \wedge g, a \wedge b \wedge \neg c \wedge d \wedge e \wedge f \wedge g\}$. If we see a term as a subformula, we can find the models of a term. Then $mod(ext(D_1, D_2)) \subseteq mod(D_1)$ and the union of the models of every term appearing in a formula equals the models of the formula: $\cup_{D \in \phi} mod(D) = mod(\phi)$. Thus $\cup_{D \in ext(\mu,K)} mod(D) \subseteq mod(\mu)$, i.e., the models of the extension of μ by K are a refinement of the models of μ such that the refinement models are the closest to the models of base K, and then the extended terms belonging to such extension are the potential candidates to forming part of the revision result.

Once we compute the potential terms that may be part of the revision result, the question arises of how to select from all the extended terms the ones that will constitute the revision result? A solution is to deploy the notion of minimal change, i.e., minimal substitutions for transforming a model of K into a model of μ. Definition 2 measures the change required for such transformation. Note that the distance between terms is a succinct form of computing Hamming distances where the sum considers solely the atoms appearing in both terms and, as in Hamming distance, the sum increases only when the related literals are opposite. Definition 2 allows us to define a pre-order over the extended terms as follows: $ext(D_1, D_2) \leq ext(D_3, D_4) \quad iff \quad d(D_1, D_2) \leq d(D_3, D_4)$.

Which means that the extension of D_1 by D_2 is preferred to the extension of D_3 by D_4. So, finally, the terms forming part of the operator's outcome are the extended terms of μ by terms of K that required minimal change to transform a model of K into a model of μ, i.e.

Definition 5 (Dalal's Revision without Hamming distance). *Let K be a belief base and μ a formula representing new evidence. The revision of K by μ, $K \circ \mu$, is defined as follows: $K \circ \mu = \bigvee min(ext(\mu, K), \leq)$.*

It should be noted that our process of revision inputs formulae in DNF and outputs formulae in DNF, i.e. we propose a syntactical framework which is desirable for a framework of iterated belief revision: from the second iteration the compilation of formulae to DNF is no longer required. Classical Dalal's revision inputs formulae and outputs models.

Example 1. The following example was presented in [8]: $K = (\neg p_2 \wedge \neg p_3) \vee (\neg p_1 \wedge \neg p_3 \wedge p_4) \vee (\neg p_2 \wedge p_4)$ and $\mu = (p_3 \wedge \neg p_4) \vee (p_1 \wedge p_2)$. Dalal's revision must find the models of the result on the models of μ. As we can see in Table 1 the models of μ that are in the revision result using Dalal $K \circ_D \mu$ are (0,0,1,0), (1,0,1,0), (1,1,0,0), (1,1,0,1) and (1,1,1,1) which minimal Hamming distance is 1. An equivalent result is produced with our operator $K \circ \mu = (\neg p_2 \wedge p_3 \wedge \neg p_4) \vee (\neg p_2 \wedge p_3 \wedge \neg p_4) \vee (p_1 \wedge p_2 \wedge \neg p_3) \vee (p_1 \wedge p_2 \wedge \neg p_3 \wedge p_4) \vee (p_1 \wedge p_2 \wedge p_4)$ by computing solely 6 distances instead of 49 (7 models of μ by 7 models of K), see Table 2.

Table 1. Distances between interpretations required for $K \circ_D \mu$

	$w' \in mod(K)$						
	(0,0,0,0)	(0,0,0,1)	(0,0,1,1)	(0,1,0,1)	(1,0,0,0)	(1,0,0,1)	(1,0,1,1)
$w \in mod(\mu)$	$d(w, w')$	$d(w, w')$	$d(w, w')$	$d(w, w')$	$d(w, w')$	$d(w, w')$	$d(w, w')$
(0,0,1,0)	1	2	1	3	2	3	2
(0,1,1,0)	2	3	2	2	3	4	3
(1,0,1,0)	2	3	2	4	1	2	1
(1,1,0,0)	2	3	4	2	1	2	3
(1,1,0,1)	3	2	3	1	2	1	2
(1,1,1,0)	3	4	3	3	2	3	2
(1,1,1,1)	4	3	2	2	3	2	1

Table 2. Distances between terms required for $K \circ \mu$

	$D_1 = p_3 \wedge \neg p_4 \in \mu$		$D_1 = p_1 \wedge p_2 \in \mu$	
$D_2 \in K$	$ext(D_1, D_2)$	$d(D_1, D_2)$	$ext(D_1, D_2)$	$d(D_1, D_2)$
$\neg p_2 \wedge \neg p_3$	$\neg p_2 \wedge p_3 \wedge \neg p_4$	1	$p_1 \wedge p_2 \wedge \neg p_3$	1
$\neg p_1 \wedge \neg p_3 \wedge p_4$	$\neg p_1 \wedge p_3 \wedge \neg p_4$	2	$p_1 \wedge p_2 \wedge \neg p_3 \wedge p_4$	1
$\neg p_2 \wedge p_4$	$\neg p_2 \wedge p_3 \wedge \neg p_4$	1	$p_1 \wedge p_2 \wedge p_4$	1

Example 2. In [7] the following example is presented: $K = a \vee (a \wedge b) \vee (a \wedge c) \vee (b \wedge c)$ and $\mu = \neg a \wedge \neg b$. From now we suppose that atoms are ordered alphabetically. The models of μ are $(0,0,1)$ and $(0,0,0)$. As we can see in Table 3 the models of $K \circ_D \mu$ are $(0,0,0)$ and $(0,0,1)$ too. Given 5 models of K, the number of Hamming distances computed is 10. An equivalent result is found with our operator $K \circ \mu = (\neg a \wedge \neg b) \vee (\neg a \wedge \neg b \wedge c) \vee (\neg a \wedge \neg b \wedge c)$, which computes only 4 distances between terms.

Table 3. $K \circ_D \mu$

$w \in mod(\mu)$	$d(w, K)$
(0,0,0)	1
(0,0,1)	1

Table 4. $K \circ \mu$

$ext(D_1, D_2) \in ext(\mu, K)$	$d(D_1, D_2)$
$\neg a \wedge \neg b$	1
$\neg a \wedge \neg b$	2
$\neg a \wedge \neg b \wedge c$	1
$\neg a \wedge \neg b \wedge c$	1

As we can see in Table 4 the extension of terms of μ by terms of K can hold repeated elements as a result of Definition 4 where a multiset is considered instead of a set. However, the repeated elements do not necessarily hold the same distance; we may compute different distances for repeated elements of the multiset given that the repeated elements come from different extensions to different terms. In this case the extended terms $ext(\neg a \wedge \neg b, a)$ and $ext(\neg a \wedge \neg b, a \wedge b)$ hold the same result $\neg a \wedge \neg b$, even when the second operand is not the same in both cases. Indeed, this difference is the cause of producing a different distance between the corresponding terms: $d(\neg a \wedge \neg b, a) = 1$ and $d(\neg a \wedge \neg b, a \wedge b) = 2$. This means that any model represented[1] by a needs 1 substitution for transforming it to a model represented by $\neg a \wedge \neg b$ while any model represented by $a \wedge b$ needs 2 substitutions for transforming it to a model represented by $\neg a \wedge \neg b$, see 2nd and 3rd rows in Table 4. Actually, it is simpler considering the terms as subformulae, then we can say the models of a need at least 1 substitution for being transformed to models of $\neg a \wedge \neg b$, while the models of $a \wedge b$ need at least 2 substitutions for being transformed to models of $\neg a \wedge \neg b$.

Although the notion of multiset helps to define the process, this notion leads into duplicate terms in the final result. Then, an elimination phase of repeated terms will be desirable. A simple transformation from a multiset to a set will be enough for erasing the repeated elements. However, there are non-desirable elements as $\neg a \wedge \neg b \wedge c$ that is model inclusion subsumed by $\neg a \wedge \neg b$, i.e. $mod(\neg a \wedge \neg b \wedge c) \subseteq mod(\neg a \wedge \neg b)$. Thus, we propose cleaning the result as follows: first create a set with terms that are not model inclusion subsumed by other terms, $\mathsf{NonSubsum}(K \circ \mu) = \{D \in K \circ \mu | \forall_{D' \in K \circ \mu} mod(D') \subseteq mod(D)\}$, then take the disjunction of such set $\mathsf{Clean}(K \circ \mu) = \vee_{D \in \mathsf{NonSubsum}(K \circ \mu)} D$. We can argue about the necessity of computing models but actually this set can be defined through indexes sets as follows: $\mathsf{NonSubsum}(\phi) = \{D \in \phi | \forall_{D' \in \phi | index(D) \subseteq index(D')} |D| \leq$

[1] Recall, a representing a model means a represents the model pattern $(1, x_2, x_3)$ where x_2 and x_3 can take value of 1 or 0.

$|D'|$ *and* $\forall_{l \in D} l \in D'\}$. I.e., if two or more terms share the same literals the set will keep only the term that has the minimal number of literals. So, $K \circ \mu = (\neg a \wedge \neg b) \vee (\neg a \wedge \neg b \wedge c) \vee (\neg a \wedge \neg b \wedge c)$ can be written in an equivalent form as $\mathsf{Clean}(K \circ \mu) = (\neg a \wedge \neg b)$ which makes more apparent the equivalence with Dalal's result: $mod(K \circ_D \mu) = \{(0,0,0),(0,0,1)\}$.

Example 3. Let us now consider the inconsistent base presented at the beginning of this section $K = (a \wedge \neg a \wedge \neg b \wedge c) \vee (a \wedge b \wedge \neg b \wedge c)$ and suppose that the new evidence b is received, then as we can see in Table 5 the models of μ can be computed, however, there are no models of K, which disqualify Dalal's revision: the Hamming distances cannot be computed. The result of our operator is $\mathsf{Clean}(K \circ \mu) = a \wedge b \wedge c$ which means the agent gives up its belief concerning $\neg b$ but keeps the rest. The process transforms the inconsistent base K into a consistent base $K \circ \mu$ with a minimal change. Notice that the extended term $a \wedge \neg a \wedge b \wedge c$ is model inclusion subsumed by $a \wedge b \wedge c$, due to an inconsistent term being subsumed by a consistent one.

Table 5. $K \circ_D \mu$

$w \in mod(\mu)$	$d(w, K)$
(0,1,0)	?
(0,1,1)	?
(1,1,0)	?
(1,1,1)	?

Table 6. $K \circ \mu$

$ext(D_1, D_2) \in ext(\mu, K)$	$d(D_1, D_2)$
$a \wedge \neg a \wedge b \wedge c$	1
$a \wedge b \wedge c$	1

Our operator can deal with inconsistent beliefs bases, which do not have any models; in contrast Dalal's operator does not operate without belief base models. Some authors such as [11] consider $K \circ_D \mu = \mu$ when K is inconsistent, however, the revised result loses too much consistent information which can be retained. Let $K = a \wedge \neg a \wedge b \wedge c \wedge \neg c$ and new evidence $\mu = c$, the revised result is $K \circ_D \mu = c$, which is consistent but the revision itself violates the minimal change principle. The agent actually gives up all the previous information keeping on the new information. Iteratively, an agent would forget everything every time it has inconsistent information and retains only the newest information. Notice that between the inconsistencies there is consistent information about b which is lost. The result of our approach conserves as much as possible the information of K, i.e. $K \circ \mu = a \wedge \neg a \wedge b \wedge c$, even when the result is inconsistent, the agent keeps the information concerning b, gives up the contradiction about c and retains the contradiction about a. Actually, the method can be easily extended for recovering consistency when the result is not consistent: merely erasing the contradictory information; then for this example the result would be $b \wedge c$.

4 Postulates and Complexity

We show now that the newly proposed operator satisfies postulates R_1-R_6. In [10] the representation theorem is used for proving Dalal's operator satisfies the postulates, therefore, we show our proposal and Dalal's revision provide equivalent results.

Proposition 1. *If both K and μ are consistent formulae in DNF, then $K \circ_D \mu \equiv K \circ \mu$ where $\circ_D$ denotes Dalal's revision operator and $\circ$ denotes terms distance-based revision operator.*

Proof. First, if a interpretation w belongs to the set of models of formula ϕ: $w \in mod(\phi)$, there exists at least one term $D \in \phi$ such that $w \in mod(D)$.
($\Rightarrow$) Let $w \in mod(K \circ_D \mu)$ iff $w \in mod(\mu)$ and $\forall_{w' \in mod(\mu)}\ w \leq_K w'$, then $\forall_{w' \in mod(\mu)}\ min_{x \in mod(K)} d(w, x) \leq min_{x \in mod(K)} d(w', x)$. Let $x' \in mod(K)$ such that $d(w, x') = min_{x \in mod(K)} d(w, x)$ and call m the Hamming distance between w and x', i.e. $d(w, x') = m$; note that m is the minimal Hamming distance between w and base K, in other interpretations the minimal change for transforming a model of K into a model of μ is m. Now, we use the notation introduced above: $p_i = 1$ and $\neg p_i = 0$, where p_i is an atom and $i = 1, ..., n$, then w can be seen as $(l_1, ..., l_n)$ and x' can be seen as $(l'_1, ..., l'_n)$ where $l_i = p_i$ if $w(p_i) = 1$ and $l_i = \neg p_i$ if $w(p_i) = 0$ and similarly for the $l'_i s$. Notice that there are m opposite literals between w and x', i.e. Hamming distance between w and x' can be calculated by $\sum_{i=1}^{n} ||l_i|_b - |l'_i|_b| = m$.

Also, it is worth to note that for every term $D_1 \in \mu$ such that $w \in mod(D_1)$ it must be satisfied that if $l \in D_1$, $l \in \{l_1, ..., l_n\}$, similarly, for every term $D_2 \in K$ such that $x' \in mod(D_2)$ it must be satisfied that if $l \in D_2$, $l \in \{l'_1, ..., l'_n\}$; thus $\sum_{i \in index(D_1) \cap index(D_2)} ||l_i|_b - |l'_i|_b| \leq \sum_{i=1}^{n} ||l_i|_b - |l'_i|_b| = m$, i.e. $d(D_1, D_2) \leq m$. Now, suppose that $d(D_1, D_2) < m$, then there exists a model of K that can be transformed to a model of μ with strictly less substitutions than m, contradicting the fact that m is the minimal Hamming distance. Therefore, $d(D_1, D_2) = m$, which means that all the opposite literals appear in both terms, thus $ext(D_1, D_2)$ extends D_1 with literals of D_2 that do not oppose the literals of D_1, therefore if $l \in D_2$ and $l \in ext(D_1, D_2)$ then $l \in \{l_1, ..., l_n\}$, which means that $w \in ext(D_1, D_2)$. Now, suppose that there is a $ext(D_3, D_4) \in ext(\mu, K)$ such that $ext(D_3, D_4) < ext(D_1, D_2)$, then $d(D_3, D_4) < d(D_1, D_2)$, i.e., we can find a model of D_4 (model of K) that can be transformed into a model of D_3 (model of μ) with strictly less substitutions than m, which is not possible. Therefore, $\forall_{ext(D_3, D_4) \in ext(\mu, K)}\ ext(D_1, D_2) \leq ext(D_3, D_4)$ and given that $w \in ext(D_1, D_2)$, we can conclude that $w \in mod(K \circ \mu)$.
($\Leftarrow$) The proof in the other direction is straightforward. Let $w \in mod(K \circ \mu)$, then there exist k terms $D'_1, ..., D'_k$ in μ such that $w \in mod(D'_i)$, $i = 1, ...k$. Let's take $D_1 \in \{D'_1, ..., D'_k\}$, $D_2 \in K$ such that $w \in ext(D_1, D_2)$ and $\forall_{ext(D_3, D_4) \in ext(\mu, K)}$ $ext(D_1, D_2) \leq ext(D_3, D_4)$, notice that we assure the existence of such D_1 and D_2, given that $w \in mod(K \circ \mu)$. Thus $\forall_{ext(D_3, D_4) \in ext(\mu, K)}\ d(D_1, D_2) \leq d(D_3, D_4)$, which means that the minimal change for transforming a model of K into a model of μ is the same distance required for transforming a model of D_2 into w, which in terms of Hamming distance is expressed as $\forall_{w' \in mod(\mu)}$ $min_{x \in mod(D_2)} d(w, x) \leq min_{x \in mod(K)} d(w', x)$. Given that $mod(D_2) \subseteq mod(K)$, $\forall_{w' \in mod(\mu)}\ min_{x \in mod(K)} d(w, x) \leq min_{x \in mod(K)} d(w', x)$ holds, which means that

$\forall_{w' \in mod(\mu)}\ w \leq_K w'$, clearly $w \in mod(\mu)$ and finally, we can conclude that $w \in mod(K \circ_D \mu)$.

□

Thus, we can be sure that the distance-based operator $\circ$ based on terms satisfies postulates R_1-R_6 when both the belief base and new evidence are consistent. When the belief base or the new evidence are inconsistent, then only some of the properties are satisfied. For instance, it is evident R_2 is satisfied, however, R_1 and R_3 are not satisfied, let's take $K = \neg a \wedge a$ and $\mu = b$ then $K \circ \mu = \neg a \wedge a \wedge b$, which intuitive interpretation is if the agent holds inconsistent beliefs concerning a and he receives information concerning b he keeps holding its inconsistency concerning a because the new information does not help him to give up the inconsistency. R_4 is not satisfied, we can find inconsistent belief bases or formulae for which results are not equivalent. Finally, our operator satisfy R_5 and R_6, for the sake of space we omit the proofs.

Complexity: An important issue is the computational complexity of the operators, even when the revision methods are intractable in the general case, it is not clear under which restrictions the methods would became tractable. The most widely investigated computational task in the literature is deciding the following relation: $K \circ \mu \models \phi$ where K, μ and ϕ are inputs. I.e., Given a knowledge base K, a new formula μ and a formula query ϕ, decide whether ϕ is a logical consequence of the revised belief base $K \circ \mu$. The complexity of Dalal's revision operator belongs to, in the general case, $P^{NP[O(log\ n)]}$-complete (the class of problems solvable in polynomial time using a logarithmic number of calls to an NP oracle, where an NP oracle is a subroutine solving an NP-complete problem) [11]. Another problem studied is the complexity of model checking for belief revision: given a knowledge base K, a new formula μ and a interpretation w, decide if $w \in mod(K \circ \mu)$. The complexity of Dalal's revision in this case is in $P^{NP[O(log\ n)]}$-complete too [12]. The authors in both cases have proved that the complexity remains the same whether inputs are restricted to those in Horn format (conjunctions of Horn clauses) or not. However, as far as we know there is no formal analysis when inputs are restricted to those in the dual format (disjunctions of terms) even when it is evident that the problem of determining the satisfiability of a Boolean formula in DNF is polynomial time.

Once the inputs are in DNF the proposed method can be implemented in polynomial time. The extension of terms can be computed in $n_1 * n_2 * n_3$, where n_1 is the number of terms in K, n_2 is the number of terms in μ and n_3 is the maximum number of literals appearing in a term, if both K and μ are consistent n_3 is the number of atoms of the language. Thus, for realistic implementations we propose maintaining an algorithm in class polynomial using a method that transforms a formula to its DNF, we know that the worst case, when the input is a formula in Conjunctive Normal Form (CNF), has exponential complexity. However, given the quantity of research about SAT problems, we can find many efficient examples in the literature transforming a formula to CNF, which can be adapted distributing conjunction over disjunctions rather than disjunctions over conjunctions in the final step of the conversion and then obtain an algorithm for dealing with realistic scenarios, in particular we are interested in adapting the algorithm used in [13].

5 Conclusion

One of the most established methods of revising belief bases without extra information is Dalal's operator, which takes as input a belief base K and a formula μ and gives as result a revised consistent belief base. Suitable implementations of Dalal's operator must deal with the calculation of belief base models. In this paper, we have proposed a new method for computing Dalal's revision which does not need to compute Hamming distances and calculates distance between terms instead. Given that the classical revision framework gives priority to new evidence, the proposed method uses definitions considering this principle, thus the extension of formulae gives priority to the new formula μ keeping all the literals of μ and complementing the term with literals of K. However, there are some attempts that consider μ should not have the priority, and our approach is flexible enough that Definition 3 can be easily adapted in order to take extensions of K by μ instead of μ by K or we can consider a weighted formulae to compute the extension. The operator meets the desirable properties of R_1-R_6 when both inputs are consistent. When the belief base or new information is inconsistent some properties are satisfied such as R_2, R_4, R_5, independence of extra information and the first property of iterated belief revision framework [2]. Properties R_1, R_3 and R_4 cannot be accomplished for inconsistent inputs, however, the results seem intuitive.

Our method has another advantage over Dalal's result: its representational succinctness at once erasing both repeated and subsumed terms. As future work, a deep analysis of the definitions will be carried out in order to combine this approach with an algorithm transforming formulae to its DNF and solve realistic cases. Moreover, an analysis and extension of the proposal will be considered in order to satisfy the four properties of iterated belief revision framework.

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