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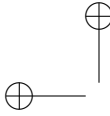
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# Design of a PI Control using Operator Theory for the nonlinear de Saint-Venant equations: Some numerical extension

Valérie Dos Santos Martins, Mickael Rodrigues

**Abstract**— This paper considers the control design of a nonlinear distributed parameter system in infinite dimension, described by the hyperbolic Partial Differential Equations (PDEs) of de Saint-Venant. The nonlinear system dynamic is formulated by a Multi-Models approach over a wide operating range, where each local model is defined around a set of operating regimes. A Proportional Integral (PI) feedback was designed and performed through Bilinear Operator Inequality (BOI) and Linear Operator Inequality (LOI) techniques for infinite dimensional systems. The authors propose in this paper to improve the numerical part by introducing weight  $\mu_i$  not only equal to  $\{0,1\}$ , but  $\mu_i \in [0, 1]$ .

## I. INTRODUCTION

Regulation of irrigation channels has received an increasing interest over the last three decades. Water losses in open channels are very large due to inefficient management and control. A large number of instruments of automation equipment (e.g., automatized gates, level sensors, etc...) are required in open channel networks in order to avoid overflows and to satisfy the water demand [25]. In order to deliver water, it is important to ensure that the water level and the flow rate in open channels remain at certain values. The difficulty of this regulation problem is that only the gates positions can meet performance specifications. Such problems can be solved by designing boundary control laws in order to satisfy the control objectives: to maintain water level or flow rate at given values.

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The open surface channels couple transport phenomena and delay phenomena, so they have complex nonlinear dynamics. The dynamics of such distributed parameter systems can be represented by hyperbolic Partial Differential Equations (PDE): the equations of de Saint-Venant, which depend on time and space [24], [28]. Some studies take into account the uncertainties and apply robust control approaches [23], [22]. Studying the nonlinear dynamics directly is also possible as in [10], [15], [22], [37]. The Riemann approach has also been used to prove stability results for systems of two conservation laws [19], and for systems of larger dimensions in [21]. Recently, it has been also coupled with LMI [2]. The Lyapunov techniques have been used in [5], [9], [10].

In practice, industrial processes such as mining, chemical or water treatment processes are complex systems characterized by multiple operating regimes. Multi-Models methods split the operating range of a system into separate regions where local models describe each region [27] for control and Fault Diagnosis purposes [3], [17], [29]. Each local model is defined as a Linear Time Invariant (LTI) model for each operating point. The Multi-Models philosophy is based on weighting functions which ensure the transition between the different local models. Some authors have studied gain scheduling strategy for example in [20] or Linear Parameter Varying (LPV) controllers [31].

The use of Multi-Models representation for the study of the stability of a system described by nonlinear PDE has been examined in [2], [12], [13]. The nonlinear PDE stability is studied by extending the common approaches based on finite dimension

to infinite dimension. The theoretical proof has been given for the closed loop stability under a Proportional and a Proportional Integral (PI) controller with identical gains or with a general PI, using Multi-Models and the Internal Model Boundary Control (IMBC) structure [14]. A variable elimination technique, as for finite-dimensional systems [4], [30], [33], has been used in order to solve a BOI (well known as Bilinear Matrix Inequality (BMI) in finite dimension) problem by the resolution of two LOI (well known as Linear Matrix Inequalities (LMI) in finite dimension). This paper proposes a brief extension of [14] from a simulation part.

The first parts of the paper briefly recall the theoretical part developed in [14]: the equations of de Saint-Venant, the control, the linearized models, the LOI formalism, the design of the feedback gains by LOI & BOI techniques which ensures the stability of the system. The last section IV is the major contribution of this paper. Comparisons are done between the simulations with constant piecewise weighting functions with values  $\in \{0,1\}$  done previously and non-piecewise weighting functions with values  $\in [0,1]$ , and show the significant improvement brought by the second case.

## II. PROBLEM STATEMENT ABOUT CHANNEL REGULATION

The control problem concerns the stabilization of the water flow rate and/or the water height around an equilibrium for a reach denoted by  $(Z_e(x), Q_e(x))$ .

### A. A model of a reach

The channel is supposed to have a sufficient length  $L$  (from  $x = 0$  to  $x = L$ ) such that one can consider that the lateral movement is uniform. The water flow rate  $Q(x, t)$  and the height of the water  $Z(x, t)$  are the state variables. The nonlinear PDE of de Saint-Venant, which describes the flow on the channel, are [8], [18]:

$$\begin{cases} \partial_t Z = -\partial_x \frac{Q}{b}, \\ \partial_t Q = -\partial_x \left( \frac{Q^2}{bZ} + \frac{1}{2}gbZ^2 \right) + gbZ(I - J), \end{cases}^{(1)}$$

$$y(t) = C[Z(x, t) \ Q(x, t)]^T \quad (2)$$

$$Z_0(x) = Z(x, 0), Q_0(x) = Q(x, 0) \quad (3)$$

$\forall x \in \Omega = (0, L), t > 0, C : (L^2(0, L))^2 \rightarrow \mathbb{R}$ .  $I$  is the slope,  $b$  is the channel width,  $g$  is the gravity constant.  $J$  is the friction slope from the formula of Manning-Strickler and  $R$  is the hydraulic radius. The considered boundary conditions  $\forall x \in \Gamma = \partial\Omega$  are two underflow gates. The controlled variable is defined as follows:

$$Q(x, t) = U(t)\Psi(Z(x, t)) \quad (4)$$

with  $\Psi(Z) = \kappa\sqrt{2g(Z_{up} - Z_{do})}$ .  $Z_{up}$  is the water height upstream of the gate,  $Z_{do}$  is the water height downstream of the gate,  $\kappa$  is the product of the channel width and the water flow rate coefficient of the gate. The gate opening  $U(t)$  is the control at upstream ( $U_{up}$ ) and at downstream ( $U_{do}$ ). The output variable is the downstream water level i.e.  $Z(L)$ .

### B. A regulation model

The fluvial case, i.e. the subcritical case [24], is considered. Let  $\xi(t) = (z(t) \ q(t))^T$  be the linearized state variable, then the model around the equilibrium state  $(Z_e(x) \ Q_e(x))^T$  is:

$$\begin{aligned} \partial_t \xi(x, t) &= \mathcal{A}\xi(x, t) \\ &= A_1(x)\partial_x \xi(x, t) + A_2(x)\xi(x, t) \end{aligned} \quad (5)$$

$$F_b \xi(t) = B_b u(t) \text{ and } \xi(0, t) = \xi_0(t), \quad (6)$$

where  $A_1$  and  $A_2$  are matrices of the space variable  $x$ . The linearized boundary conditions (6) are equivalent to:

$$\begin{aligned} q(0, t) &= U_{up,e} \partial_z \Psi(Z_e(0, t)) z(0, t) \\ &\quad + u_{up}(t) \Psi(Z_e(0, t)), \end{aligned} \quad (7)$$

$$\begin{aligned} q(L, t) &= U_{do,e} \partial_z \Psi(Z_e(L, t)) z(L, t) \\ &\quad + u_{do}(t) \Psi(Z_e(L, t)), \end{aligned} \quad (8)$$

where  $U_{up,e}$  and  $U_{do,e}$  are the upstream and downstream gate openings respectively at the equilibrium and  $u_{up}(t)$ ,  $u_{do}(t)$  are the variations of these gate openings to be controlled.

The initial control problem is to find the variations of  $u_{up}(t)$  at extremity  $x = 0$  and  $u_{do}(t)$  at extremity  $x = L$  of the reach such that the downstream water level,  $Z(L, t)$  (measured variable) tracks a reference signal  $r(t)$ . [14]

The control scheme is based on the Internal Model Boundary Control (IMBC) [11].

### III. MULTI-MODELS APPROACH

#### A. A Multi-Models representation of de Saint-Venant's Equation

The Multi-Models representation [30], [13] of de Saint-Venant's PDE around  $N$  operating points is defined by the following equations:

$$\begin{aligned} \partial_t \xi(x, t) &= \sum_{i=1}^N \mu_i(\zeta(t)) \mathcal{A}_i(x) \xi(x, t) \\ \text{with } \mathcal{A}_i(x) &= A_{1,i}(x) \partial_x + A_{2,i}(x), \quad (9) \\ \xi_0(x) &= \xi(x, 0), \end{aligned}$$

where  $\mathcal{A}_i(x)$  is the operator which corresponds to the  $i^{th}$  equilibrium state. The function  $\zeta(t)$  depends on some decision variables directly linked with the measurable state variables and eventually to the input. The weighting functions  $\mu_i(\zeta(t))$  activate the control law in function of the output of the process  $Z_L$ . They belong to a convex set such that

$$\sum_i \mu_i(\zeta(t)) = 1 \text{ and } \mu_i(\zeta(t)) \geq 0.$$

In previous works, the weight were chosen for the simulations as follows, one was equal to 1 the others equal to 0. We propose to apply our theoretical approach to the simulations for the general case, i.e.  $\mu_i \neq \{0, 1\}$  but functions with values in  $[0, 1]$ .

#### B. Closed-loop structure for a proportional integral feedback

In this part, the closed loop structure is studied under a proportional integral feedback [14].

Let  $K_i$  and  $K_p$  be the integral and proportional gains respectively. It follows that [11]:

$$u(t) = K_i \int_0^L [r(\tau) - y(\tau)] d\tau + K_p [r(t) - y(t)]$$

where  $r(t)$  is the physical water level wanted. With  $\tilde{K}_{int} = D K_i$ ,  $\tilde{K}_{pr} = D K_p$ , the expression of

the closed-loop system can be expressed as follows:

$$\begin{aligned} \partial_t \varphi(x, t) &= \sum_{i=1}^N [(\mathcal{A}_i(x) + \tilde{K}_{int} C + \tilde{K}_{pr} C \mathcal{A}_i(x)) \varphi(x, t) \\ &\quad + \tilde{K}_{int} (C D u(t) + C E q(x, t) - r(t))] \\ &\quad \times \mu_i(\zeta(t)) = \sum_{i=1}^N \mathcal{M}_i(x, t). \end{aligned} \quad (10)$$

The stability conditions are ensured by using a quadratic Lyapunov function [31] guaranteeing the convergence of the water height to the reference  $r(t)$  over the widest operating range.

#### C. Stability

Let us consider:

$$V(\varphi(x, t), t) = \langle \varphi(x, t), P \varphi(x, t) \rangle, \quad (11)$$

where  $\langle \cdot, \cdot \rangle$  is the considered inner product. The Multi-Models representation of the linearized de Saint-Venant equations defined by equation (10) is asymptotically stable if there exists an operator  $P > 0$ , such that [14]:

$$\langle \dot{\varphi}, P \varphi \rangle + \langle \varphi, P \dot{\varphi} \rangle < -\langle \varphi, \varphi \rangle. \quad (12)$$

Then, by taking into account (10)-(12), one has to prove the following inequality:

$$\langle \mathcal{M}_i, P \varphi \rangle + \langle \varphi, P \mathcal{M}_i \rangle < 0, \quad (13)$$

where  $\mathcal{M}_i$  is defined in (10).

Both propositions developed in [14] are applied to prove the stability. The coefficients  $\beta$  and  $\sigma$  of the proposition 1 in [14] are negative in both cases simulated, so the condition  $\|X\| \leq -2\sigma$  is always satisfied whatever  $X = \beta Id$ .

The contribution of this extended paper is in the following part: the weight functions are taken as functions, and the improvements are significant.

### IV. SIMULATION RESULTS

Two benchmarks are used for the simulations: the micro-channel (benchmark) of Valence (France) and the channel of Gignac (France). The simulations are based on a Chang and Cooper scheme [10], [13].

For both applications, the weighting function  $\mu_i(\zeta(t))$  is no more equal to 1 or 0. It is a function

such that  $\mu_i(\zeta(t)) \in [0, 1]$  and  $\sum_i \mu_i = 1$ . The parameter  $\zeta(t)$  is a function of the output of the system which is the decision variable.

#### A. The micro-channel of Valence

The following set of parameters of this channel is considered:  $L = 8 \text{ m}$  is the length of the channel,  $b = 0.1 \text{ m}$  is the width of the channel,  $N = 40$  is the number of the discretized points,  $Z_L$  is the water height to regulate such that  $0.06 \text{ m} < Z < 0.19 \text{ m}$ .

The equilibrium profiles have been chosen such that the calculated control law from the local models can be efficient over all the operating range of the water height [11][14].

#### B. The channel of Gignac

We also study a channel which is located in Gignac (France). The following set of parameters of this channel is considered:  $L = 2272 \text{ m}$  is the length of the channel,  $b = 3 \text{ m}$  is the width of the channel,  $N = 40$  is the number of the discretized points,  $Z_L$  is the water height to regulate such that  $1.7 \text{ m} < Z < 2.5 \text{ m}$ .

#### C. Simulation results

The improvements here are achieved by the introduction of weighting functions  $\mu_i$  taken different from  $\{0, 1\}$ .

From a theoretical point of view, the results were developed for any functions  $\mu_i$  with the regularity needed, but numerically it was implemented only with constant functions.

First simulations are done with the Valence channel data. It can be seen in figures (1)-(2) that around time instants  $t = 250, 1000 \text{ s}$  and  $2180 \text{ s}$ , the new weighting functions  $\mu_i \in [0, 1]$  (depicted on Fig (3)) outperform previous results obtained with functions  $\mu_i \in \{0, 1\}$ , [14]. Indeed, the downstream water level tracks the reference better and avoids some overshoot. It can be noticed that the gate openings, Figure (4), get very similar dynamics in spite of different weighting functions  $\mu_i$ : however, just around time instant  $t = 900 \text{ s}$  and  $t = 2000 \text{ s}$ , the gate openings are little bit different which corresponds to the transition between the weighting

functions. In [14] the weighting functions were considered only as a switch between models and the system model id not remain correct during this switching. New weighting functions  $\mu_i \in [0, 1]$  provide a real interpolation between the local models, to improve the dynamics of the nonlinear system at every time instant.

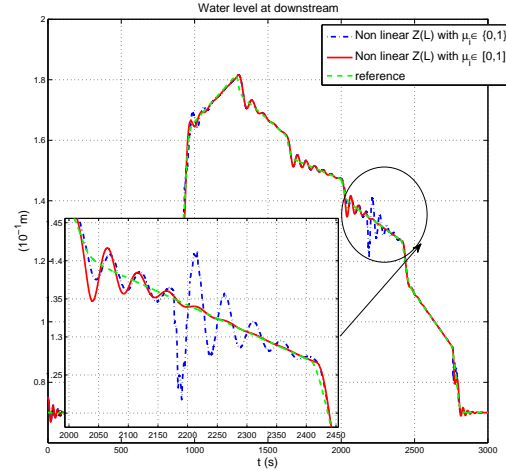


Fig. 1. Valence channel simulation; Comparison of the downstream water level

The channel of Gignac is a real channel located in the south of France. In Fig (5), it can be seen that the use of weighting functions  $\mu_i \in [0, 1]$  (Fig (7)) allow to avoid some overtaking at time instant  $t = 1200 \text{ s}$  and  $t = 2800 \text{ s}$  that correspond also to the transition of models. Again, the old weighting functions were only a switch that have been replaced in this paper by a real interpolation between the local models to better track the reference water level. Moreover, in Fig. (6), it can be seen that the use of new weighting functions allow to avoid major damping for the upstream water flow around the same time instant  $t = 1200 \text{ s}$  and  $t = 2800 \text{ s}$ . The downstream water flow is also improved although the improvement is less important. The gates opening in Figure (8) at the time instant  $t = 1200 \text{ s}$  and  $t = 2800 \text{ s}$ , are more

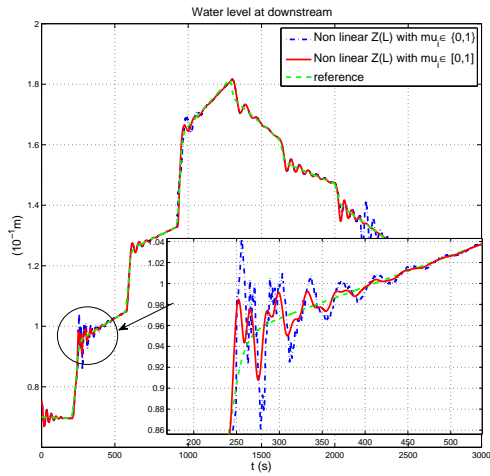
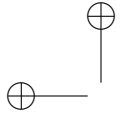


Fig. 2. Valence channel simulation; Comparison of the downstream water level

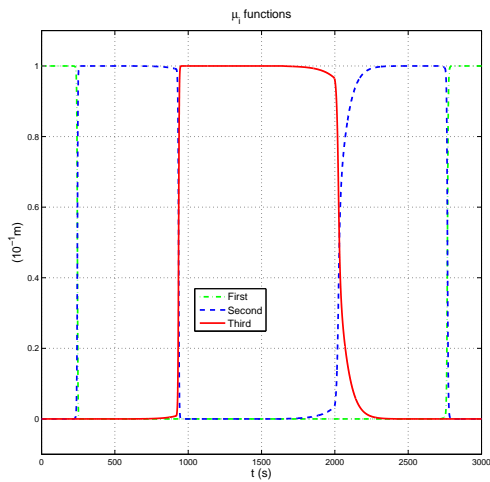


Fig. 3. Valence channel simulation;  $\mu_i$  functions

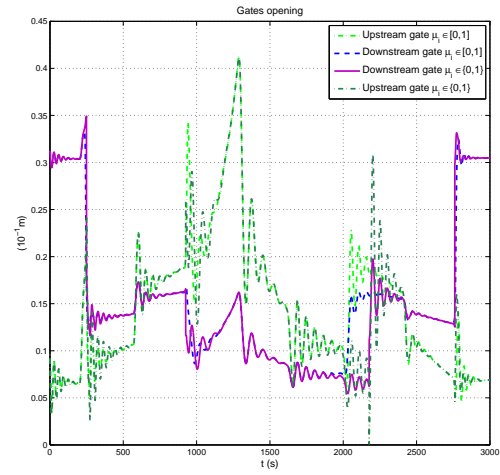


Fig. 4. Valence channel simulation; gates opening

linear without big variations.

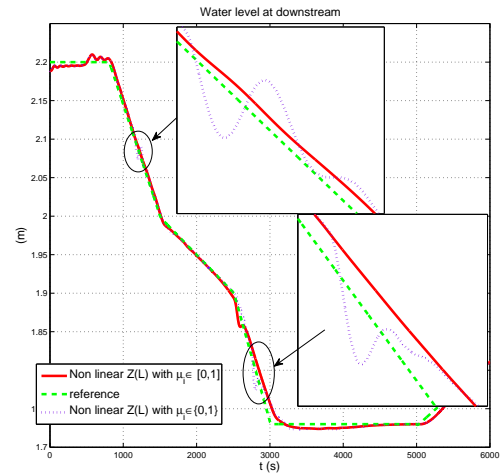


Fig. 5. Gignac channel simulation; Comparison of the downstream water level

The last figure (9) presents another type of reference tracking for the downstream water level. It can be seen that by the use of the new weighting functions, all the overtaking have

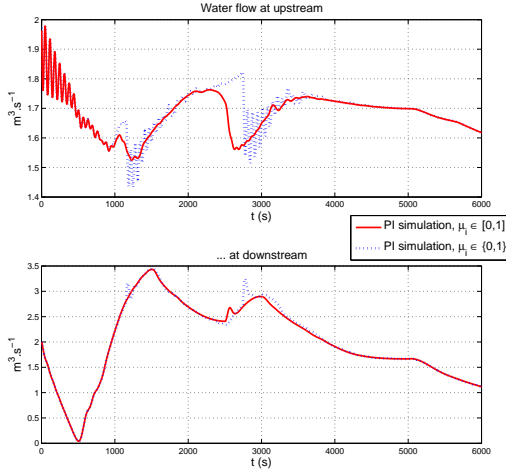


Fig. 6. Gignac channel simulation; Comparison of the water flow

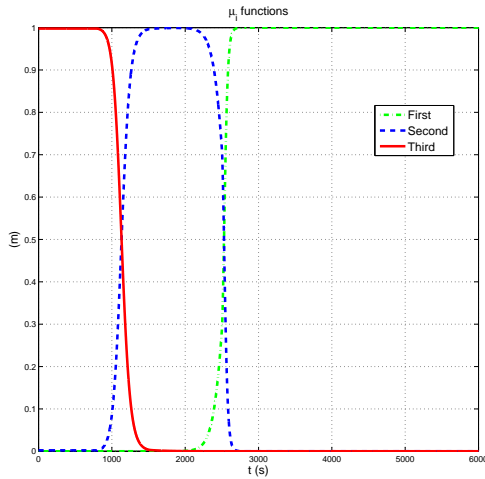


Fig. 7. Gignac channel simulation;  $\mu_i$  functions

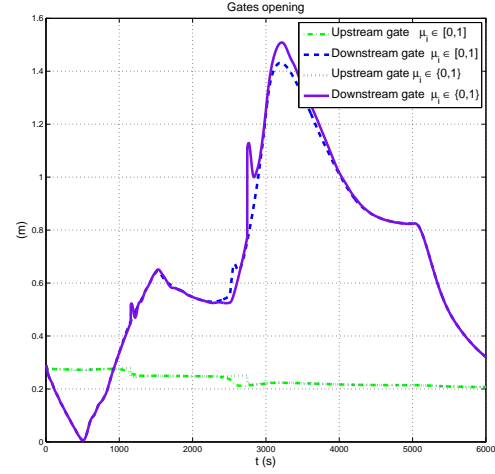


Fig. 8. Gignac channel simulation; gates opening

totally disappeared. Indeed, at time instants  $t = 800s, 4000s, 5800s, 8600s$ , which correspond to the transitions between local models, the new weighting functions contribute to get more smooth water level and better reference tracking. It can be highlighted that the transitions in (10) cover between  $500s$  to  $1500s$  i.e. the Multi-Models strategy takes all its sense by the fact that the use of a single model during these transitions are not judicious. The weighting of local models during these transitions allow to get better performance from a practical point of view by avoiding overshoot and bad behavior of opening gates as presented in Figure (11). We can see that around  $t = 800s$  and  $t = 8600s$ , the improvement is significant.

## V. CONCLUSION

First attempts of a Multi-Models approach on irrigation channels control, through an IMBC structure, have been realized some years ago [11]. Good experimental results were obtained which showed promising results but a theoretical basis was lacking. The first theoretical results in order to design the feedback gain through LMI have been realized in the case of an Integral controller in

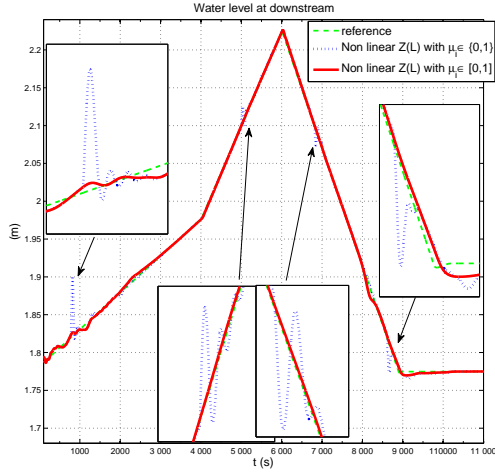


Fig. 9. Gignac channel simulation; Comparison of the downstream water level

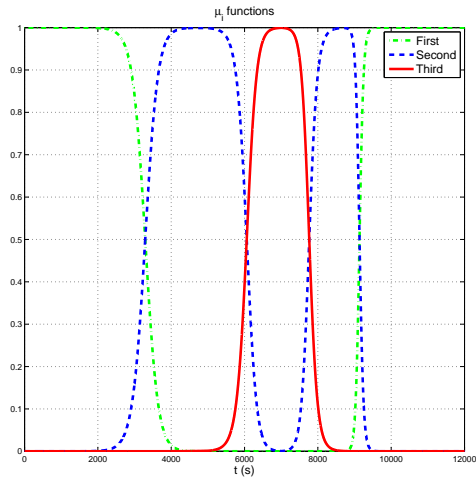


Fig. 10. Gignac channel simulation;  $\mu_i$  functions

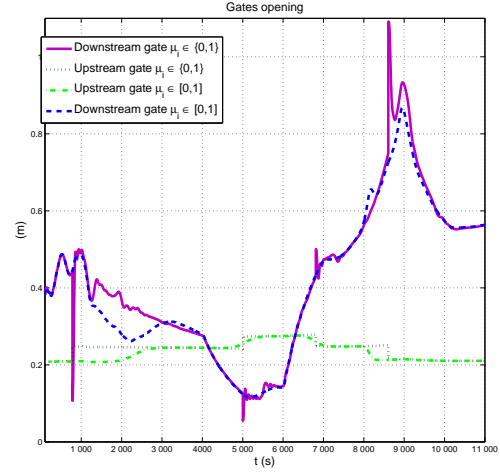


Fig. 11. Gignac channel simulation; gates opening

[13]. Preliminary results of a PI controller in a particular case ( $K_i = K_p$ ) have been published in [12] for infinite dimensional systems and with  $K_i \neq K_p$  in [30] for finite dimensional systems. In the last paper, the authors take into account the more general case of PI controller with  $K_i \neq K_p$  for infinite dimensional systems. They synthesize the new PI controller feedback gains by solving a BOI problem. But the weighting functions were equals to 1 or 0. Here this brief paper improves the previous paper [14] by taking non-piecewise constant ( $\mu_i(\zeta(t)) \in [0, 1]$ ). The last step should be to valid our approach through experimentations.

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