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Quantifying the Impact of Unpredictable Generation on Market Coupling*

Hélène Le Cadre[†] Mathilde Didier[‡]

Abstract

Modeling Market Coupling using an agent-based approach, we compare two organizations: centralized versus decentralized. To perform this comparison we analytically study the impact of wind farm concentration and the uncertainty resulting from the increasing penetration of renewables on the total cost of procurement, market welfare and the ratio of renewable generation to conventional supplies. We prove that the existence and uniqueness of equilibrium depend on the number of interacting demand markets. In a decentralized organization, forecast errors heavily impact the behavior of the electrical system. Simulations show that suppliers have incentives to certify the forecast uncertainty of other markets. We analytically derive the uncertainty price that might be charged by a risk certifier depending on the required confidence level.

Keywords: Uncertainty ; Optimization ; Energy Markets ; Intermittent Sources

1 Introduction

Market Coupling was developed jointly by power exchanges and transport operators following the liberalization of energy markets and aims at improving the use of available cross-border capacity and promises a greater harmonization of prices between countries [22]. It creates a unique platform for daily electricity transactions. It implicitly allocates interconnection capacity in the day-ahead and real timescales until a uniform market clearing price is achieved or the available capacity is fully utilized. In this latter case i.e., when capacity becomes limiting, a congestion rent is paid to the grid operator to encourage it to invest in capacity upgrading. The increasing penetration of renewable energy in the energy mix, required by governments, complexifies the coupling mechanisms, due to their non-controllability¹ and their intermittency. Their

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¹It is still possible to adapt the wind turbine speed.

intermittency results from their strong dependence on meteorological conditions: if the weather is favorable (i.e., wind and sun²) they will produce a lot ; very little otherwise.

Intermittent energy sources are decentralized. Non stationarity has been observed in the histories of generation and extreme values are frequently reached [16]. This implies that statistical techniques based on regression, times series, etc., can no longer provide accurate forecasts of the generation of such sources. In the following parts of the Introduction, we describe the major steps of energy market liberalization that has challenged the design of new multi-tiered markets, the main questions raised by increasing renewable penetration and, finally, give a quick overview of existing coupling mechanisms.

Energy market liberalization and the emergence of multi-tiered market designs: Energy markets, and especially electricity markets, were traditionally considered as natural monopolies due to the huge investments required and relatively low marginal costs, and hence they were managed by national firms that owned both production plants and the distribution network. Some interconnections were created between countries, but only for adjustment purpose [10], [21]. However, at the end of the 1990s the European Union (EU) decided to progressively liberalize energy markets, i.e., gas and electricity, and to create a global competitive European market. In France, one consequence of electricity market liberalization is that different activities that were handled by a sole national company, such as production, transport, distribution and electricity sales are now divided up several firms. This measure clearly aims at mitigating vertical market power [26]. However, the mitigation of horizontal market power remains challenging. A successful attempt in the United States deregularized power market lies in the design of multi-tiered markets [12], [13], [20], [26], [27]. Kamat and Oren consider a two-tiered approach where day-ahead contracts and real-time transactions are settled at different prices for a two- and three-node networks with potential congestion [13]. They show that welfare impacts of two sided systems are highly sensitive to the probability that a network contingency reduces the transmission capacity of the lines. Yao et al. generalize the previous work to more realistic multi-node and multi-zone networks considering flow constraints, system contingencies and demand uncertainty in the real-time market [26], [27]. The tractability of their models to real size power systems is guaranteed through the use of mathematical programs with equilibrium constraints algorithms based on quadratic programming solving and on parametric linear complementarity problem pivoting. However their models do not incorporate renewable generation and focus on competition modeling at the generator level, though an attempt to model market interactions through multi-agent simulation is provided by Veit et al. [25], as well as by Li and Shi [15].

The increasing penetration of renewables: The energy world is being totally restructured, both at the level of the final consumer, whose usages are more finely analyzed to enable the launching of smart solutions such as demand response [12], but

²Other, more predictable sources of renewable energy such as those coming from marine currents, are still under study. Déporte et al. studied a way to collect energy using a wavering membrane [6]. Although this approach seems promising to overcome the uncertainties associated with renewable energy generation, much remains to be done.

also at the level of the electricity market, the design of which has stimulated lively debates [18]. These debates originate from the increasing penetration of intermittent energy sources. The European Commission has set of a 20% share of renewables in the energy mix by 2020 ; measures encouraged by the launching of feed-in tariffs. We have already mentioned the high uncertainty associated with generating of intermittent sources. It is therefore essential for the agents involved in these markets to elaborate accurate forecasts. Indeed, forecast errors generate penalties at the supplier level [16], [17] and, at the global level, create disruptions in electricity market operations with considerable consequences, such as negative prices [7], [8], crashes of price-based coupling mechanisms and congestion which could result in the interruption of end user supply, etc.

Some articles tackle the difficult problem of renewable energy integration in electricity markets. Modeling the generation of intermittent sources as random individual sequences, which require no underlying stochastic assumptions on the generating process, Le Cadre and Bedo study the impact of distributed learning strategies in the real-time balancing market [16], [17]. Focusing on the day ahead and assuming perfect coordination of the power exchanges and of the Transmission System Operators (TSOs), Oggioni et al. compare the effect of two wind policies ("priority dispatch" under which the TSO must accomodate all wind energy produced and "no priority dispatch" under which the TSO can decide not to inject all potential wind power into the grid in order to limit congestion problems) in a context of Market Coupling organization [22]. Morales et al. consider a two-tiered electricity market made of a day-ahead and a real-time balancing market, including a number of stochastic generators [19]. They conclude that generation scheduling should be driven by the cost of its uncertainty i.e., its economic impact due to system balancing. However their model does not tackle market interactions that might result from the current restructuring of the electricity market. In a multi-tiered market context, Nair et al. explicitly characterize the impact of growing renewable penetration on the procurement policy by considering a scaling regime that models the aggregation of unpredictable renewable sources [20]. They introduce a scaling regime for wind penetration, which models the effect of aggregating the output of several wind generators. A key feature of their model is that it takes into account the relative concentration (or, inversely, scattering) of the intermittent energy sources being aggregated. Based on this scaling model, they measure the optimal reserves, the amount of conventional generation produced, as well as the cost of procurement, in line with increasing wind penetration. Finally, using a market design close to that of Nair et al., Jiang and Low propose a real-time demand response algorithm while studying the effects of renewable generation on social welfare.

Market coupling: Competitiveness, sustainability and energy supply security are essential issues in the pursuit of European energy market integration and the creation of a single energy area. Energy markets were initially liberalized autonomously at national level, with domestic scope, but there has been a growing need for an optimal management of cross-border transmissions and congestion. However, optimal network governance of a centralized authority depends on the balance between different interests accross countries [8]. Market functioning in terms of competition among generators can be obstructed by limited transmission capacity at the borders of the interconnected

markets. Therefore two mechanisms have been put forward to solve the allocation of such scarce-border capacity: the first is implicit auctioning, and the second is coordinated explicit auctioning which has not been implemented yet. The latter system will allow countries to keep their power exchanges running, but it proves less efficient than the former system when uncoordinated [8]. The implicit auction mechanism adopted in Europe is designed to include cross-border trades in the day-ahead auction mechanism on individual power exchanges, to avoid inefficiency. Different implementations of the implicit coupling mechanism exist, such as no coupling, volume coupling, one way price coupling, etc. The implementation of price coupling, which is the focus of this article, optimizes cross-border flows to reflect energy-only price differences between coupled markets. It implicitly allocates the interconnection capacity in the day-ahead and real-timescales until a uniform market clearing price is achieved or the available capacity is fully utilized.

From the end user’s point of view, Market Coupling should impact favorably on his energy bill [23]: by maximizing the use of cross-border interconnection capacity, Market Coupling increases the level of market integration and facilitates access to low-cost generation for consumers located in high-cost generation countries, such as Italy. However, the associated congestion management costs may increase significantly in the future due to a higher share of renewables and potential divergences in the developments of transmission and generation infrastructure [14].

Article organization: The literature dealing with energy market economic models can be roughly divided into two areas: Market Coupling mechanisms [7], [8], [10], [11], [21], [22], [23], and models for multi-tiered markets with uncertain supply [12], [13], [19], [20], [25], [27], [26]. In this article, we try to reconcile both approaches.

In Section 2, we detail the involved agents (suppliers, generators), wind generation modeling, derive coupling prices and optimal conventional supplies. In Section 3, we compare first theoretically two organizations of Market Coupling: centralized versus decentralized. Simulations are then run in Subsection 3.3. Finally in Section 4, we simulate the suppliers’ behavior under asymmetric and partial knowledge of the forecast uncertainty in a decentralized Market Coupling organization. We derive analytically the price of uncertainty.

2 The model

Market Coupling is progressively moving from a decentralized to a more centralized organization. This distinction may become more relevant with the increasing wind penetration, which is a result of both European and national policies that complicate this comparison. In a decentralized Market Coupling organization, each market selfishly maximizes its objective function. A good illustration of this type of organization is Germany, where each market participants are responsible for planning their unit commitment [14]. On the contrary, in a centralized Market Coupling organization, a central agent rules the markets.

Market Coupling is organized as a two-tiered system with a day-ahead market, occurring at t_f , and a real-time system, occurring at t_0 . In the EU, the real-time systems

introduced in this article can either be assimilated to intra-day markets, whose design and timing are still debated, or to an EU balancing mechanism in which the imbalance price settlement mechanism is designed so that no compensation is provided in case of over-provisioning [18]. This choice of modeling can be justified by the fact that our model aims at determining how suppliers share the risk of under-provisioning between day-ahead and real-time markets. Furthermore we assume price coupling in both day-ahead and real-time markets.

We consider a certain number of geographic demand markets, each characterized by price-insensitive demand. This assumption is justified by the fact that nowadays in France, demand response deals mainly with (relatively) low consumption reports/reductions. This is because end-user pricing is still restricted to flat rates and on/off peak hour tariffs. Wind generation in each geographic demand market is price insensitive and random. It is unknown on the day ahead and revealed in real time. Each geographic demand market is composed of:

(i) A *conventional generation* system. Marginal costs are linear [9], [11], [15] and higher in real time than on the day ahead. The ordering of marginal costs can be justified by the fact that any conventional energy that is demanded closer to real time is provided by generators that require several hours to start up [20]. Conventional generators being price takers, they do not exercise market power i.e., they cannot charge margins on top of marginal costs. This implies that suppliers buy electricity at marginal costs.

(ii) *Suppliers* who, contrary to standard assumptions, are not price takers. They are aware that their decision modifies prices and take that knowledge into account to minimize their procurement cost. They purchase energy in the day-ahead market on the basis of forecast of demand and wind generation. Then they procure energy in real time on the basis of wind realization. This procurement process is complex in the sense that suppliers consider their impact on (both day-ahead and real-time) energy market prices. This, in turn, assumes that suppliers can anticipate the equilibrium that will prevail on these markets under different assumptions of demand and wind generation.

2.1 Description of the geographic demand markets

Geographic demand market $i = 1, \dots, N$ is defined by:

- d_i the end users' total demand of energy at time t_0 . It is price insensitive.
- w_i the wind generation produced at time t_0 . It is price insensitive and random i.e., there exists a random variable ϵ_i representing the forecast error such that: $w_i = \hat{w}_i - \epsilon_i$ where \hat{w}_i is the forecast made at t_f of the quantity of renewable energy that market i generator will produce at t_0 . ϵ_i is distributed according to a density function f_{ϵ_i} with support $[L_i; B_i]$ where $L_i \in \{-\infty\} \cup \mathbb{R}$ and $B_i \in \mathbb{R} \cup \{+\infty\}$ and F_{ϵ_i} (resp. $\bar{F}_{\epsilon_i} = 1 - F_{\epsilon_i}$) is the associated (resp. complementary) cumulative distribution function. In the rest of the article, the forecast generating density function will coincide with a Gaussian distribution function centered in

zero and of standard deviation σ_i . More general distribution functions can, of course, be considered. Furthermore there is no correlation between any two geographic demand market forecast errors.

- q_i^f (resp. q_i^0) market i demand of conventional energy in day-ahead (resp. real-time) markets.
- s_i^f (resp. s_i^0) market i supply of conventional energy in day-ahead (resp. real-time) markets.
- $c_i^f(s_i^f) = a_i^f + b_i^f s_i^f$ (resp. $c_i^0(s_i^0) = a_i^0 + b_i^0 s_i^0$) the marginal cost function of conventional energy produced by market i and purchased at t_f (resp. at t_0). For convenience, they are supposed to be linear in the supply [9], [11], [15]. We assume that $a_i^0 > a_i^f > 0$ and that $b_i^0 > b_i^f > 0$ guaranteeing that the marginal cost in the real-time market remains higher than in the day-ahead market.

At EU level, the global market is characterized by the equilibrium between supply and demand: $q_{\text{tot}}^f(N) = \sum_{i=1, \dots, N} q_i^f = \sum_{i=1, \dots, N} s_i^f$ (resp. $q_{\text{tot}}^0(N) = \sum_{i=1, \dots, N} q_i^0 = \sum_{i=1, \dots, N} s_i^0$) which is the global quantity of conventional energy exchanged in day-ahead (resp. real-time) markets.

The amounts of energy purchased by market i at t_f and at t_0 are defined as follows: $q_i^f = (d_i - \hat{w}_i + r_i)_+$ and $q_i^0 = (d_i - w_i - q_i^f)_+$ where r_i is a reserve of energy purchased in the day-ahead (lower cost) market because of the uncertainty of supply at t_0 . Following Kamat and Oren's financial terminology, it can be interpreted as the day-ahead position of demand market i supplier [13]. The hypothesis that $q_i^f > 0$ holds as long as the demand exceeds the average wind capacity. In the rest of the article, we will assume that: $q_i^f \triangleq d_i - \hat{w}_i + r_i$.

As mentioned above, since generators do not exercise market power, we make the assumption that the prices p_i^f and p_i^0 paid by market i suppliers for the energy purchased at t_f and t_0 respectively equal the marginal costs: $p_i^f \triangleq c_i^f(s_i^f)$ and $p_i^0 \triangleq c_i^0(s_i^0)$. As stated in the Introduction, the fundamental idea behind Market Coupling is to create an integrated energy market with uniform energy prices among the countries involved. Therefore, we assume that a clearing price is reached at t_f i.e., $p_i^f = p_j^f \triangleq p^f, \forall i, j = 1, \dots, N, i \neq j$. Because the transfers are limited by the available transmission capacities, it will be harder to align the market prices at t_0 : if there is equilibrium then $p_i^0 = p_j^0 \triangleq p^0, \forall i, j = 1, \dots, N, i \neq j$; otherwise there exists at least one market $i \in \{1, \dots, N\}$ in which the supplier pays $p_i^0 \neq p_j^0$ for $j \in \{1, \dots, N\}$ and $j \neq i$ ³. In the rest of the article, we will make the hypothesis that, at t_f , the markets are myopic and do not anticipate the potential congestion of the lines. In practical terms, this means that network capacities are sufficient to guarantee that demand market prices

³In case where $p_i^0 \neq p_j^0$, a congestion rent $CR = (p_i^0 - p_j^0)t_{j \rightarrow i}^0$ is paid to the transmission operator; $t_{j \rightarrow i}^0$ represents the traded flow of energy from market j to market i in a real-time market. CR is: positive if the lower price market is exporting energy to the higher price market; null if the interconnection line, binding market i to market j , is not congested and $p_i^0 = p_j^0 = p^0$; negative if the lower price market is importing energy from the higher price market.

will be aligned on the day ahead and in real time⁴. The case where markets anticipate congestion in real time is detailed in [18].

2.2 Suppliers' total costs, generators' profits and social welfare

We define U_i , as the expectation of the total cost, TC_i , that the supplier has to pay for its end-user energy consumption⁵:

$$U_i = \mathbb{E}[TC_i] = q_i^f p^f + \mathbb{E}[q_i^0 p_i^0] \quad (1)$$

We take Π_i to be the expected profit of market i energy generator. It is defined as the difference between the price conventional energy is sold each time to the demand markets and the cost of the conventional energy. We assume that all the supply is sold at each time. Then:

$$\Pi_i = s_i^f p^f + \mathbb{E}[s_i^0 p_i^0] - \int_0^{s_i^f} c_i^f(s) ds - \mathbb{E}\left[\int_0^{s_i^0} c_i^0(s) ds\right] \quad (2)$$

Finally, we define W_i , the welfare of market i , as the difference between the generator's expected profit and the supplier's expected total cost:

$$W_i = \Pi_i - U_i$$

As usual in Game Theory [5], social welfare is defined as the sum of the welfares of all the agents involved:

$$W = \sum_{i=1, \dots, N} W_i$$

A simultaneous game is played, involving on the one side, the generators, and on the other side, the suppliers (resp. the market operators). It is described as follows:

- Generators selfishly and simultaneously determine their conventional supplies on the day ahead and in real time so as to maximize their profits
- Suppliers (resp. market operators) selfishly and simultaneously determine their reserve on the day ahead so as to minimize their total costs (resp. their welfare)

The last step can be decentralized provided the suppliers (resp. the market operators) operate independently or it can be centralized provided a supervisor (like a regulator at the EU level) takes the decisions for all the suppliers (resp. the market operators).

2.3 Wind generation modeling

For each geographic demand market, its wind generation is a function of the number of wind farms and their concentration, which is characterized by their spatial distribution. To determine the renewable procurement for market i , we use Nair et al. model [20]. For geographic demand market i , we introduce:

⁴Under this assumption, the congestion rent is expected to be null.

⁵Note that the expectation of TC_i is taken with respect to $e_i, (e_j)_{j \neq i}$.

- α_i the average generation of a single wind farm
- γ_i the number of wind farms
- $\theta_i \in [\frac{1}{2}; 1]$ (resp. $1-\theta_i \in [0; \frac{1}{2}]$) a constant capturing the concentration (resp. the scattering) of the wind farm locations over market i geographic coverage area. The more (resp. the less) concentration, the more (resp. the less) correlation there is between the wind farm generations

We suppose that, at t_f , α_i is the best forecast of wind energy procurement of a wind farm [20]. Then: $\hat{w}_i(\gamma_i) = \alpha_i \gamma_i$. The forecast error will also depend on the wind penetration, and we choose the coefficient θ_i so that $\epsilon_i(\gamma_i) = \gamma_i^{\theta_i} \tilde{\epsilon}_i$ where $\tilde{\epsilon}_i$ represents the forecast error for the generation of a single wind farm and $\tilde{\sigma}_i$ its standard deviation. If the wind farms are co-located they will all generate the same quantity of energy at the same time i.e., their generations are strongly correlated. This is the case when $\theta_i = 1$. This implies in turn that: $\epsilon_i = \gamma_i \tilde{\epsilon}_i$ and that: $\hat{w}_i = w_i + \gamma_i \tilde{\epsilon}_i$. If their generations are independent from one another i.e., uncorrelated, and under the assumption that the forecast errors are distributed according to Gaussian distribution functions, the Central Limit Theorem tells us that: $\sigma_i = \sqrt{\gamma_i} \tilde{\sigma}_i$ [20]. Therefore, the wind farm generations are independent from one another if, and only if, $\theta_i = \frac{1}{2}$. Note that in case of more general forecast error distribution functions, it can be interpreted as a sufficiently broad approximation for γ_i large enough.

With these notations, we obtain $w_i(\gamma_i) = \hat{w}_i(\gamma_i) - \epsilon_i(\gamma_i) = \gamma_i \alpha_i - \gamma_i^{\theta_i} \tilde{\epsilon}_i$ and $\sigma_i(\gamma_i) = \gamma_i^{\theta_i} \tilde{\sigma}_i$.

2.4 Coupling prices

In this subsection, we determine the analytical expressions of the coupling prices in day-ahead and real-time markets.

We set: $A^f \triangleq \sum_{i=1, \dots, N} \frac{a_i^f}{b_i^f}$ and $B^f \triangleq \sum_{i=1, \dots, N} \frac{1}{b_i^f}$. Furthermore, we make the assumption that the marginal cost parameters at t_f are chosen so that $B^f \neq 0$.

Lemma 1. *The coupling price in the day-ahead market is: $p^f = \frac{\sum_{i=1, \dots, N} q_i^f + A^f}{B^f}$.*

Proof of Lemma 1. Using the assumption of the supply and demand equilibrium guaranteed by the market rules, we have:

$$\begin{aligned}
 q_{\text{tot}}^f(N) &= \sum_{i=1, \dots, N} q_i^f = \sum_{i=1, \dots, N} s_i^f \\
 &= \sum_{i=1, \dots, N} \frac{p_i^f - a_i^f}{b_i^f} \text{ under the assumption that } p_i^f = c_i^f \\
 &= \sum_{i=1, \dots, N} \frac{p^f - a_i^f}{b_i^f} \text{ since the } N \text{ markets are coupled at } t_f
 \end{aligned}$$

$$= p^f \left(\sum_{i=1, \dots, N} \frac{1}{b_i^f} \right) - \sum_{i=1, \dots, N} \frac{a_i^f}{b_i^f}$$

We infer from the following equations the day-ahead price for the coupled demand

$$\text{markets: } p^f = \frac{\sum_{i=1, \dots, N} q_i^f + A^f}{B^f}. \quad \square$$

We also set: $A^0 \triangleq \sum_{i=1, \dots, N} \frac{a_i^0}{b_i^0}$ and $B^0 \triangleq \sum_{i=1, \dots, N} \frac{1}{b_i^0}$. Furthermore, we make the

assumption that the marginal cost parameters at t_0 are chosen so that $B^0 \neq 0$. As in Lemma 1 proof, we infer the real-time price for the coupled demand markets:

Lemma 2. *The N demand markets being coupled at time t_0 , the coupling price in the real-time market is:*
$$p^0 = \frac{\sum_{i=1, \dots, N} q_i^0 + A^0}{B^0}.$$

2.5 Conventional supplies

Substituting the coupling prices derived in Lemmas 1 and 2 in demand market i generator's profit Π_i defined in Equation (2), we obtain:

$$\begin{aligned} \Pi_i = & s_i^f \frac{\sum_{j=1, \dots, N} (d_j - \hat{w}_j + r_j) + A^f}{B^f} + s_i^0 \mathbb{E} \left[\frac{\sum_{j=1, \dots, N} (\epsilon_j - r_j)_+ + A^0}{B^0} \right] \\ & - a_i^f s_i^f - b_i^f \frac{(s_i^f)^2}{2} - a_i^0 s_i^0 - b_i^0 \frac{(s_i^0)^2}{2} \end{aligned} \quad (3)$$

The conventional energy procurement can be optimized before the market takes place i.e., at t_f and t_0 . It should be optimized so as to maximize the generator's profit at t_f and at t_0 . It is straightforward to observe, judging by Equation (3), that market i generator's profit at t_f (resp. t_0) is concave in s_i^f (resp. s_i^0) since it is a second order polynomial equation in s_i^f (resp. s_i^0) with a negative highest order coefficient: $-\frac{b_i^f}{2}$ (resp. $-\frac{b_i^0}{2}$).

Proposition 3. *In demand market i , the supply of conventional energy at time t_f maximizing the generator's profit is:*

$$s_i^{f*} = \frac{1}{b_i^f} \left[\frac{\sum_{j=1, \dots, N} (d_j - \hat{w}_j + r_j) + A^f}{B^f} - a_i^f \right] \text{ and,}$$

$$\text{at time } t_0: s_i^{0*} = \frac{1}{b_i^0} \left[\frac{\sum_{j=1, \dots, N} \mathbb{E}[(\epsilon_j - r_j) | \epsilon_j \geq r_j] + A^0}{B^0} - a_i^0 \right].$$

Proof of Proposition 3. Before the market takes place at t_f , the generator optimizes its conventional energy procurement so as to maximize its profit: Π_i . But, as already mentioned, Π_i is a second order polynomial equation in s_i^f with a negative highest order coefficient: $-\frac{b_i^f}{2}$. Therefore Π_i admits a unique maximum in s_i^f . It is obtained as a solution of $\frac{\partial \Pi_i}{\partial s_i^f} = 0$. Similarly the generator determines s_i^0 just before the market occurs at t_0 . \square

3 How should Market Coupling be organized?

In this section, we determine the optimal quantities of energy to be purchased by demand markets in the day-ahead market, $(q_i^d)_i$, or equivalently, their optimal reserves, $(r_i)_i$ since the demands $(d_i)_i$ and the wind generation forecasts $(\hat{w}_i)_i$ are common knowledge in the day-ahead market. The optimization programs can either be centralized by a supervisor (for instance, a regulator at EU level) who determines the optimal reserves for all demand markets (cf. Subsection 3.2) or, they can be decentralized provided each demand market optimizes its reserve selfishly (cf. Subsection 3.1). Furthermore, depending on which agent takes the decision (i.e., either the suppliers or the demand markets ruled by their own operators) and on the timing of the game, it might be relevant to minimize the supplier's total cost or maximize the market welfare. Indeed, if we consider the short term effect, the costs of the suppliers remain fixed and, in perfect competition, they will bid at their marginal cost, so that the energy suppliers, who buy energy from the generators, have most of the economic power and will try to minimize their own total costs. However, if we consider the effects in the longer term, the generators can choose to change their costs, by investing in new technologies or by scaling their plants for example, in order to optimize their own profits, so that the economic power is shared between the generators and the suppliers. In this latter case, it is more appropriate to maximize the market welfare.

3.1 Decentralized organization of Market Coupling

In this subsection, the suppliers (resp. the market operators) selfishly and simultaneously optimize their reserves in Subsection 3.1.1 (resp. Subsection 3.1.2).

3.1.1 Suppliers' total cost minimization

In each demand market i , the suppliers independently and simultaneously determine their reserve r_i so as to minimize their procurement total cost:

$$\min_{r_i \geq 0} U_i = \mathbf{E} \left[\text{TC}_i \right] \quad (4)$$

In all the optimization programs described in this article we make the assumption that $r_i \geq 0$ since otherwise a supplier could fall short in the day-ahead market, which may not seem realistic given that conventional plants are more expensive in real time.

Market i supplier determines the best response, $r_i^{\text{BR}}(\mathbf{r}_{-i})$, where \mathbf{r}_{-i} is a $N - 1$ dimensional vector containing the reserves of all the suppliers except market i supplier, which minimizes its total cost. The decentralized program output is a Nash equilibrium, $(r_i^{\text{NE}})_{i=1, \dots, N}$, defined by: $r_i^{\text{NE}} = r_i^{\text{BR}}(\mathbf{r}_{-i}), \forall i = 1, \dots, N$.

Proposition 4. *There exists a Nash equilibrium solution of Program 4. It is unique if the number of interacting demand markets, N , is such that: $\frac{N-3}{B\bar{r}} < C_i^0 f_{\epsilon_i}(r_i) + \frac{1}{B^0} \bar{F}_{\epsilon_i}(r_i) \left[2 - \sum_{j=1, \dots, N, j \neq i} \bar{F}_{\epsilon_j}(r_j) \right], \forall i = 1, \dots, N$.*

Proof of Proposition 4. We let: $C_i^0 \triangleq \frac{\sum_{j=1, \dots, N, j \neq i} \mathbb{E}[(\epsilon_j - r_j) | \epsilon_j \geq r_j] + A^0}{B^0}$
and $C_i^f \triangleq \frac{\sum_{j=1, \dots, N, j \neq i} (d_j - \hat{w}_j + r_j) + A^f}{B^f}$. The Lagrangian function associated with Program 4 is: $\mathcal{L}_i^U(r_i, \mu) = U_i - \mu r_i$ where $\mu \in \mathbb{R}_+$ is a Lagrange multiplier. According to the Karush-Kuhn-Tucker (KKT) conditions, at the optimum in r_i : $\frac{\partial \mathcal{L}_i^U(r_i, \mu)}{\partial r_i} = 0$, $r_i = 0$ or $\mu = 0$.

Going back to the definition of the supplier's total cost, as defined in Equation (1), it can be rewritten by substituting the coupling price expressions derived in Lemmas 1 and 2:

$$\begin{aligned} U_i &= (d_i - \hat{w}_i + r_i) \frac{\sum_{j=1, \dots, N} (d_j - \hat{w}_j + r_j) + A^f}{B^f} \\ &+ \mathbb{E} \left[(\epsilon_i - r_i)_+ \frac{\sum_{j=1, \dots, N} (\epsilon_j - r_j)_+ + A^0}{B^0} \right] \end{aligned} \quad (5)$$

Differentiating U_i with respect to r_i , we obtain:

$$\begin{aligned} \frac{\partial U_i}{\partial r_i} &= \frac{\sum_{j=1, \dots, N} (d_j - \hat{w}_j + r_j) + A^f}{B^f} + \frac{1}{B^f} (d_i - \hat{w}_i + r_i) \\ &+ C_i^0 \frac{\partial}{\partial r_i} \mathbb{E}[(\epsilon_i - r_i) | \epsilon_i \geq r_i] + \frac{1}{B^0} \frac{\partial}{\partial r_i} \mathbb{E}[(\epsilon_i - r_i)^2 | \epsilon_i \geq r_i] \end{aligned}$$

But: $\frac{\partial}{\partial r_i} \mathbb{E}[(\epsilon_i - r_i) | \epsilon_i \geq r_i] = -\bar{F}_{\epsilon_i}(r_i)$ and

$$\begin{aligned} \frac{\partial}{\partial r_i} \mathbb{E}[(\epsilon_i - r_i)^2 | \epsilon_i \geq r_i] &= \frac{\partial}{\partial r_i} \int_{r_i}^{+\infty} (x_i - r_i)^2 f_{\epsilon_i}(x_i) dx_i \\ &= 2r_i \int_{r_i}^{+\infty} f_{\epsilon_i}(x_i) dx_i - 2 \int_{r_i}^{+\infty} x_i f_{\epsilon_i}(x_i) dx_i \\ &= 2r_i \bar{F}_{\epsilon_i}(r_i) - 2\mathbb{E}[\epsilon_i | \epsilon_i \geq r_i] \end{aligned}$$

By substitution in $\frac{\partial U_i}{\partial r_i}$ expression, we obtain the simplified expression: $\frac{\partial U_i}{\partial r_i} = \frac{2}{B^f} (d_i - \hat{w}_i + r_i) + C_i^f + \left(\frac{2r_i}{B^0} - C_i^0 \right) \bar{F}_{\epsilon_i}(r_i) - \frac{2}{B^0} \mathbb{E}[\epsilon_i | \epsilon_i \geq r_i]$. Differentiating twice U_i with respect to r_i we obtain: $\frac{\partial^2 U_i}{\partial r_i^2} = \frac{2}{B^f} + \left(C_i^0 - \frac{2r_i}{B^0} \right) f_{\epsilon_i}(r_i) + \frac{2}{B^0} \bar{F}_{\epsilon_i}(r_i) + \frac{2}{B^0} r_i f_{\epsilon_i}(r_i) = \frac{2}{B^f} + C_i^0 f_{\epsilon_i}(r_i) + \frac{2}{B^0} \bar{F}_{\epsilon_i}(r_i) > 0$. This proves that r_{-i} being fixed, function U_i is convex with respect to r_i .

Program 4 is solved simultaneously by all the demand market suppliers. The Nash equilibria are obtained at the intersections of the best responses: $r_i^{BR}(\mathbf{r}_{-i}), \forall i =$

1, ..., N. To show uniqueness of the resulting Nash equilibrium, we apply the contraction mapping approach. Due to Bertsekas [2], it is sufficient to show that the Hessians of the suppliers' total costs fulfill the diagonal dominance condition i.e.,

$$\sum_{j=1, \dots, N, j \neq i} \left| \frac{\partial^2 U_i}{\partial r_i \partial r_j} \right| < \left| \frac{\partial^2 U_i}{\partial r_i^2} \right|, \forall i = 1, \dots, N. \text{ Since } \frac{\partial^2 U_i}{\partial r_i \partial r_j} = \frac{1}{B^f} + \frac{1}{B^0} \bar{F}_{\epsilon_i}(r_i) \bar{F}_{\epsilon_j}(r_j),$$

the diagonal dominance condition becomes: $\frac{N-3}{B^f} < \underbrace{C_i^0 f_{\epsilon_i}(r_i)}_{>0} + \underbrace{\frac{1}{B^0} \bar{F}_{\epsilon_i}(r_i)}_{>0} \left[2 - \sum_{j=1, \dots, N, j \neq i} \bar{F}_{\epsilon_j}(r_j) \right].$

$r_i = 0$ checks the KKT conditions. Provided the diagonal dominance condition is checked, U_i has a unique minimum in \mathbb{R}^+ . Therefore either it is reached in $r_i^{NE} = 0$ or in $r_i^{NE} > 0$ such that $\frac{\partial U_i}{\partial r_i} |_{r_i=r_i^{NE}} = 0$. \square

We note that if there are no more than three interacting demand markets, the sufficient condition of Proposition 4 is checked, guaranteeing the uniqueness of the Nash equilibrium at the intersection of the suppliers' best responses.

3.1.2 Maximization of welfare

Each market i independently and simultaneously determines its reserve, r_i , so as to maximize its welfare:

$$\max_{r_i \geq 0} W_i \quad (6)$$

Proposition 5. *There exists a Nash equilibrium solution of Program 6 if the conventional supply in the real-time is strictly lower than $B^0 C_i^0$. It is unique if the number of interacting demand markets, N , is such that: $\frac{N-3}{B^f} < \left[C_i^0 - \frac{s_i^0}{B^0} \right] f_{\epsilon_i}(r_i) + \frac{1}{B^0} \bar{F}_{\epsilon_i}(r_i) \left[2 - \sum_{j=1, \dots, N, j \neq i} \bar{F}_{\epsilon_j}(r_j) \right].$*

Proof of Proposition 5. To solve Program 6, we need to determine the zero(s) of the differentiate of W_i with respect to r_i : $\frac{\partial W_i}{\partial r_i} = -\frac{\partial U_i}{\partial r_i} + \frac{s_i^f}{B^f} - \frac{s_i^0}{B^0} \bar{F}_{\epsilon_i}(r_i)$. Differentiating twice W_i with respect to r_i , we obtain: $\frac{\partial^2 W_i}{\partial r_i^2} = -\frac{2}{B^f} + \left(\frac{s_i^0}{B^0} - C_i^0 \right) f_{\epsilon_i}(r_i) - \frac{2}{B^0} \bar{F}_{\epsilon_i}(r_i)$. If $s_i^0 < B^0 C_i^0$ then $\frac{\partial^2 W_i}{\partial r_i^2} < 0$. This proves that r_{-i} being fixed, function W_i is concave with respect to r_i .

As in Proposition 4 proof, the resulting Nash equilibria are obtained at the intersections of the best responses. Since $\frac{\partial^2 W_i}{\partial r_i \partial r_j} = -\frac{1}{B^f} - \frac{1}{B^0} \bar{F}_{\epsilon_i}(r_i) \bar{F}_{\epsilon_j}(r_j)$ and under the assumption that $s_i^0 < B^0 C_i^0$, the diagonal dominance condition introduced in Proposition 4 proof becomes: $\frac{N-3}{B^f} < \left[C_i^0 - \frac{s_i^0}{B^0} \right] f_{\epsilon_i}(r_i) + \frac{1}{B^0} \bar{F}_{\epsilon_i}(r_i) \left[2 - \sum_{j \neq i} \bar{F}_{\epsilon_j}(r_j) \right].$

$r_i = 0$ is a solution to Program 6. Under the diagonal dominance condition, W_i has a unique maximum in \mathbb{R}^+ . Therefore either it is reached in $r_i^{NE} = 0$ or in $r_i^{NE} > 0$ such that $\frac{\partial W_i}{\partial r_i} |_{r_i=r_i^{NE}} = 0$. \square

As in Proposition 4, we note that if there are no more than three interacting demand markets, the sufficient condition of Proposition 5 is trivial to check, provided $s_i^0 < B^0 C_i^0$.

3.2 Centralized organization of Market Coupling

In this subsection, a supervisor is introduced. He optimizes the suppliers' (resp. the market operators') reserves in Subsection 3.2.1 (resp. 3.2.2).

3.2.1 Minimization of the sum of the suppliers' total costs

A supervisor determines the N market reserves $(r_i)_{i=1,\dots,N}$ minimizing the sum of the suppliers' total costs over the N demand markets:

$$\begin{aligned} \min_{(r_i)_{i=1,\dots,N}} \quad & \mathcal{U} = \mathbf{E} \left[\sum_{i=1,\dots,N} \text{TC}_i \right] \\ \text{s.t.} \quad & r_i \geq 0, \forall i = 1, \dots, N \end{aligned} \quad (7)$$

Proposition 6. *There exists a unique global optimum solution of Program 7 if the number of interacting demand markets, N , is such that:*

$$\begin{aligned} \frac{2(N-2)}{B^f} < & \left[C_i^0 + \sum_{j=1,\dots,N,j \neq i} \frac{\mathbb{E}[(\epsilon_j - r_j | \epsilon_j > r_j)]}{B^0} \right] f_{\epsilon_i}(r_i) \\ & + \frac{2}{B^0} \bar{F}_{\epsilon_i}(r_i) \left[1 - \sum_{j=1,\dots,N,j \neq i} \bar{F}_{\epsilon_j}(r_j) \right] \end{aligned}$$

Proof of Proposition 6. The Lagrangian function associated with Program 7 is: $\mathcal{L}^{\mathcal{U}}((r_i)_i, \tilde{\mu}) = \mathcal{U} - \sum_{i=1,\dots,N} \tilde{\mu}_i r_i$ where $\tilde{\mu} = (\tilde{\mu}_i)_{i=1,\dots,N} \in \mathbb{R}_+^N$ are Lagrange multipliers. According to KKT conditions, at the optimum in $(r_i)_i$: $\frac{\partial \mathcal{L}^{\mathcal{U}}((r_i)_i, \tilde{\mu})}{\partial r_j} = 0, \forall j = 1, \dots, N, r_i = 0$ or $\tilde{\mu}_i = 0, \forall i = 1, \dots, N$.

Going back to the definition of \mathcal{U} : $\mathcal{U} = \mathbb{E} \left[\sum_{i=1,\dots,N} \text{TC}_i \right] = \sum_{i=1,\dots,N} \mathbb{E} [\text{TC}_i] = \sum_{i=1,\dots,N} \mathcal{U}_i$. The differentiation of \mathcal{U} with respect to r_i gives:

$$\begin{aligned} \frac{\partial \mathcal{U}}{\partial r_i} &= \frac{1}{B^f} (d_i - \hat{w}_i + r_i) + \sum_{j=1,\dots,N} \frac{d_j - \hat{w}_j + r_j}{B^f} + C_i^f \\ &- \bar{F}_{\epsilon_i}(r_i) \sum_{j=1,\dots,N,j \neq i} \mathbb{E} \left[\frac{(\epsilon_j - r_j)}{B^0} | \epsilon_j \geq r_j \right] + \left(\frac{2r_i}{B^0} - C_i^0 \right) \bar{F}_{\epsilon_i}(r_i) \\ &- \frac{2}{B^0} \mathbb{E} [\epsilon_i | \epsilon_i \geq r_i] \end{aligned}$$

Differentiating twice U with respect to r_i we obtain:

$$\frac{\partial^2 U}{\partial r_i^2} = \frac{2}{B^f} + \left[C_i^0 + \sum_{j=1, \dots, N, j \neq i} \frac{\mathbb{E}[(\epsilon_j - r_j) | \epsilon_j \geq r_j]}{B^0} \right] f_{\epsilon_i}(r_i) + \frac{2}{B^0} \bar{F}_{\epsilon_i}(r_i) > 0$$

and for any $j = 1, \dots, N, j \neq i$: $\frac{\partial^2 U}{\partial r_i \partial r_j} = \frac{2}{B^f} + \frac{2}{B^0} \bar{F}_{\epsilon_i}(r_i) \bar{F}_{\epsilon_j}(r_j) > 0$. Hence, the Hessian matrix associated with U is non-negative. This implies that function U is convex with respect to each of its components.

The diagonal dominance condition, introduced by Bertsekas in [2], becomes:

$$\begin{aligned} & \sum_{j \neq i} \left| \frac{\partial^2 U}{\partial r_i \partial r_j} \right| < \left| \frac{\partial^2 U}{\partial r_i^2} \right|, \forall i = 1, \dots, N \\ \Leftrightarrow & \frac{2(N-2)}{B^f} < \underbrace{\left[C_i^0 + \sum_{j=1, \dots, N, j \neq i} \frac{\mathbb{E}[(\epsilon_j - r_j) | \epsilon_j > r_j]}{B^0} \right]}_{>0} \underbrace{f_{\epsilon_i}(r_i)}_{>0} \\ & + \underbrace{\frac{2}{B^0} \bar{F}_{\epsilon_i}(r_i)}_{>0} \left[1 - \sum_{j=1, \dots, N, j \neq i} \bar{F}_{\epsilon_j}(r_j) \right], \forall i = 1, \dots, N \end{aligned} \quad (8)$$

Assuming Equation (8) is checked, the Hessian matrix associated with U is a Hermitian (since real-valued and symmetric) strictly diagonally dominant matrix with real positive diagonal entries. As a result it is positive definite. This implies in turn that U is strictly convex [3]. To sum up: if the diagonal dominance condition defined by Equation (8) is checked, the minimization of U over \mathbb{R}_+^N admits a unique global optimum.

$r_i = 0$ is a solution to Program 7, regarding the i -th direction. The strict convexity of U under the conditions mentioned above implies that it admits a unique minimum in \mathbb{R}_+^N . Therefore in the i -th direction, either it is reached in $r_i^* = 0$ or in $r_i^* > 0$ such that $\frac{\partial U}{\partial r_i} |_{r_i=r_i^*} = 0$. \square

In a centralized Market Coupling organization, we note that if there are no more than two interacting demand markets, the sufficient condition of Proposition 6 is checked. This is less than in the decentralized organization where the diagonal dominance condition was checked for less than three interacting demand markets.

3.2.2 Maximization of the social welfare

A supervisor determines the N market reserves $(r_i)_i$ maximizing the sum of the market welfares i.e., the social welfare:

$$\begin{aligned} \max_{(r_i)_i} \quad & W = \sum_{i=1, \dots, N} W_i \\ \text{s.t.} \quad & r_i \geq 0, \forall i = 1, \dots, N \end{aligned} \quad (9)$$

Proposition 7. *There exists a global optimum solution of Program 9 if the total conventional supply in the real-time is strictly smaller than $B^0 \left(C_i^0 + \sum_{j \neq i} \mathbb{E}[(\epsilon_j - r_j) | \epsilon_j \geq r_j] \right)$*

$r_j]$). It is unique if the number of interacting demand markets, N , is such that:

$$\begin{aligned} \frac{2N}{B^f} < \left[C_i^0 + \frac{1}{B^0} \sum_{j \neq i} \mathbb{E}[(\epsilon_j - r_j) | \epsilon_j \geq r_j] - \frac{\sum_{j=1, \dots, N} s_j^0}{B^0} \right] f_{\epsilon_i}(r_i) \\ + \frac{2}{B^0} \bar{F}_{\epsilon_i}(r_i) \left[1 - \sum_{j \neq i} \bar{F}_{\epsilon_j}(r_j) \right] \end{aligned}$$

Proof of Proposition 7. If the reserves are positive, to solve Program 9, we need to determine the zeros of the differentiate of W with respect to $r_i, \forall i = 1, \dots, N$:

$$\frac{\partial W}{\partial r_i} = -\frac{\partial U}{\partial r_i} + \sum_{j=1, \dots, N} \frac{s_j^f}{B^f} - \sum_{j=1, \dots, N} \frac{s_j^0}{B^0} \bar{F}_{\epsilon_i}(r_i), \forall i = 1, \dots, N \quad (10)$$

Differentiating twice W with respect to r_i we obtain:

$$\begin{aligned} \frac{\partial^2 W}{\partial r_i^2} &= -\frac{\partial^2 U}{\partial r_i^2} + \sum_{j=1, \dots, N} \frac{s_j^0}{B^0} f_{\epsilon_i}(r_i) \\ &= -\frac{2}{B^f} - \left[C_i^0 + \sum_{j=1, \dots, N, j \neq i} \frac{\mathbb{E}[(\epsilon_j - r_j) | \epsilon_j \geq r_j]}{B^0} \right. \\ &\quad \left. - \frac{\sum_{j=1, \dots, N} s_j^0}{B^0} \right] f_{\epsilon_i}(r_i) - \frac{2}{B^0} \bar{F}_{\epsilon_i}(r_i) \end{aligned}$$

and for any $j = 1, \dots, N, j \neq i$: $\frac{\partial^2 W}{\partial r_i \partial r_j} = -\frac{\partial^2 U}{\partial r_i \partial r_j} < 0$. If $\sum_{j=1, \dots, N} s_j^0 < B^0 (C_i^0 +$

$\sum_{j=1, \dots, N, j \neq i} \frac{\mathbb{E}[(\epsilon_j - r_j) | \epsilon_j \geq r_j]}{B^0})$ then $\frac{\partial^2 W}{\partial r_i} < 0$. Hence the Hessian matrix associ-

ated with W is non-positive. This implies that function W is concave with respect to each of its components. Since $\frac{\partial^2 W}{\partial r_i \partial r_j} = -\frac{2}{B^f} - \frac{2}{B^0} \bar{F}_{\epsilon_i}(r_i) \bar{F}_{\epsilon_j}(r_j)$ and under the as-

sumption that $\sum_{j=1, \dots, N} s_j^0 < B^0 (C_i^0 + \sum_{j=1, \dots, N, j \neq i} \frac{\mathbb{E}[(\epsilon_j - r_j) | \epsilon_j \geq r_j]}{B^0})$, the diagonal

dominance condition introduced in Proposition 6 proof becomes: $\frac{2}{B^f} (N-2) < \left[C_i^0 + \frac{1}{B^0} \sum_{j \neq i} \mathbb{E}[(\epsilon_j - r_j) | \epsilon_j \geq r_j] - \frac{\sum_{j=1, \dots, N} s_j^0}{B^0} \right] f_{\epsilon_i}(r_i) + \frac{2}{B^0} \left[1 - \sum_{j \neq i} \bar{F}_{\epsilon_j}(r_j) \bar{F}_{\epsilon_i}(r_i) \right]$.

$r_i = 0$ is a solution to Program 9, regarding the i -th direction. The strict concavity of W under the conditions mentioned above implies that it admits a unique maximum in \mathbb{R}_+^N . Therefore in the i -th direction, either it is reached in $r_i^* = 0$ or in $r_i^* > 0$ such that $\frac{\partial W}{\partial r_i} |_{r_i=r_i^*} = 0$. \square

As in Proposition 6, we note that if there are no more than two interacting demand markets, the sufficient condition of Proposition 7 is trivial to check, provided

$$\sum_{j=1, \dots, N} s_j^0 < B^0 \left(C_i^0 + \sum_{j \neq i} \mathbb{E}[(\epsilon_j - r_j) | \epsilon_j \geq r_j] \right).$$

3.3 Simulations

We consider two interacting demand markets ($N = 2$). This choice is justified by the fact that, according to the results derived in Subsections 3.1 and 3.2, there exists a unique Nash equilibrium and a unique optimum solutions of Programs 4, 6, 7 and 9. The parameters are set so that: $\alpha_1 = \alpha_2 = 1$, $a_i^f = \frac{1}{N}$, $b_i^f = 10^{-2} a_i^f$, $a_i^0 = 10 a_i^f$, $b_i^0 = a_i^f$, $\forall i = 1, 2$ and $d_i = 100$, $\forall i = 1, 2$.

Comparing Market Coupling organizations: We compare the impact of the number of wind farms deployed in each demand market, with the sum of the suppliers' total costs evaluated at the optimum in a centralized organization in Figure 1 (a) and at the Nash equilibrium in a decentralized organization in Figure 2 (a). The centralized optimization is performed using the SciPy L-BFGS-B algorithm for bound constrained optimization [4], [24], [28] while the decentralized optimization is performed using the SciPy multidimensional root finding function [24]. For these simulations, the other parameters are set so that $\theta_1 = \theta_2 = 0.65$, $\sigma_1 = \sigma_2 = 1.1$. In both organizations, the more wind farms deployed in both demand markets, the smaller the sum of the suppliers' total costs. We observe that the sum of the suppliers' total costs is generally smaller in a centralized organization than in a decentralized organization. In Figures 1 (b) and 2 (b), we test the impact of the wind farm concentration on the sum of the suppliers' total costs evaluated at the optimum (resp. at the Nash equilibrium) in a centralized (resp. decentralized) organization. For these simulations, the other parameters are set so that $\gamma_1 = \gamma_2 = 20$ and $\tilde{\sigma}_1 = \tilde{\sigma}_2 = 1.1$. In both organizations, the result is minimal when the wind farms are independent in both demand markets i.e., $\theta_i = \frac{1}{2}$, $\forall i = 1, 2$. In a decentralized organization, the sum of the suppliers' total costs reaches high values when wind farms are co-located in one demand market and almost independent in the other. In Figure 1 (c) and 2 (c), we compare the impact of the forecast uncertainty on the sum of the suppliers' total costs evaluated at the optimum in a centralized organization and at the Nash equilibrium in a decentralized organization. For these simulations, the other parameters are set so that $\gamma_1 = \gamma_2 = 20$ and $\theta_1 = \theta_2 = 0.65$. We observe that the range of values are similar in both organizations and that the sum of the suppliers' total costs increases linearly in the forecast uncertainty.

Replacing the sum of the suppliers' total cost minimization by the social welfare maximization gives rise to the same interpretations when characterizing the impact of the parameters in both market organizations.

For the reserves, we observe that they are larger in a decentralized organization than in a centralized organization. This is due to the fact that in a decentralized organization each supplier selfishly optimizes its reserve so as to minimize its total cost whereas in a centralized organization a supervisor minimizes the joint reserves in all the demand markets. Furthermore, at the optimum, reserves are larger for welfare maximization than for the sum of the suppliers' total cost minimization. This is due to the fact that the generators contribute to market welfare and tend to increase their reserves to avoid costly real-time generation ; the real-time price is presumed to be higher than the day-ahead. Both prices are represented in Figure 2 (d) as functions of the forecast uncertainty. As expected, they increase with the uncertainty ; the slope of the real-time price being higher than the day-ahead price.

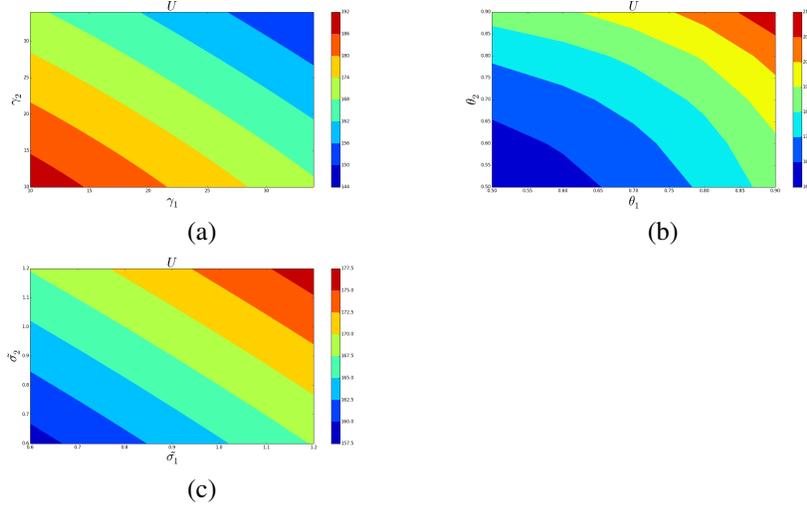


Figure 1: The Market Coupling organization is centralized. In (a) we represent the sum of the suppliers' total costs at the optimum U^* as a function of the number of wind farms in each demand market. In (b) U^* is represented as a function of the concentration of the wind farms in each demand market. In (c) U^* is represented as a function of the forecast uncertainty for each demand market.

From these simulations, run for two interacting demand markets, we conclude that the decentralized organization of Market Coupling gives a very similar performance to that of centralized organization. As seen in Propositions 4 and 6, the uniqueness of the Nash equilibrium and the global optimum relies on the number of interacting demand markets and the choice of the game parameters. The uniqueness property is important because multiple equilibria/optima might cause system instability. Therefore Market Coupling organization should be designed taking into account the number of interacting demand markets and the game parameters.

Quantifying the renewable penetration rate: For these simulations, all the demand markets have the same parameters $\sigma_1 = \sigma_2 = 1.1, \gamma_1 = \gamma_2 = 20, \theta_1 = \theta_2 = 0.65$. We define the renewable penetration rate ρ as the ratio between the wind generation over the total generation ; total generation being defined as the sum of conventional supplies on the day ahead and in real time and renewable generation:

$$\rho \triangleq \frac{\sum_{i=1, \dots, N} w_i}{\sum_{i=1, \dots, N} (q_i^0 + q_i^f + w_i)}. \text{ We let: } \hat{w}_{\text{tot}} \triangleq \sum_i \hat{w}_i \text{ and } d_{\text{tot}} \triangleq \sum_i d_i. \text{ We assume } q_i^0 > 0 \text{ i.e., conventional supply is activated in the real-time for demand market } i. \text{ The}$$

$$\text{expression of } \rho \text{ then becomes: } \rho = \frac{\sum_{i=1, \dots, N} (\hat{w}_i - \epsilon_i)}{\sum_{i=1, \dots, N} d_i}. \text{ Its expectation with respect to}$$

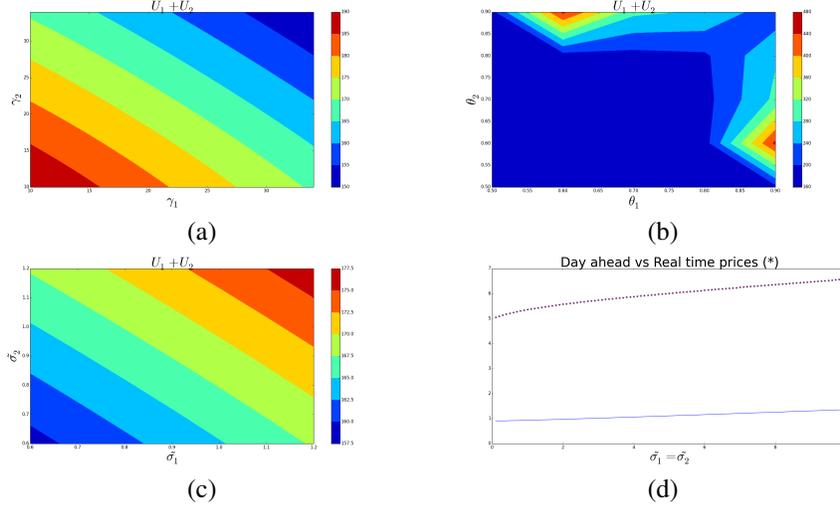


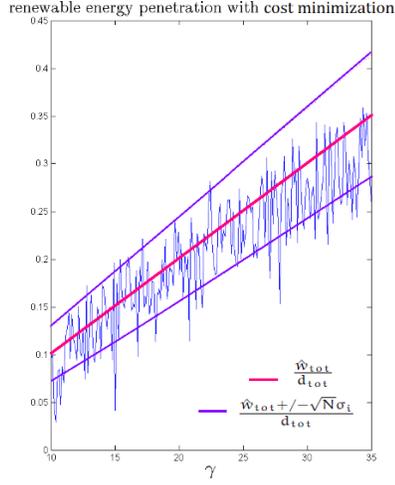
Figure 2: The Market Coupling organization is decentralized. In (a) we represent the sum of the suppliers' total costs evaluated in the Nash equilibrium $U_1(r_1^{NE}, r_2^{NE}) + U_2(r_1^{NE}, r_2^{NE})$ as a function of the number of wind farms in each demand market. In (b) $U_1(r_1^{NE}, r_2^{NE}) + U_2(r_1^{NE}, r_2^{NE})$ is represented as a function of the concentration of the wind farms in each demand market. In (c) $U_1(r_1^{NE}, r_2^{NE}) + U_2(r_1^{NE}, r_2^{NE})$ is represented as a function of the forecast uncertainty for each demand market. In (d) we compare the market clearing prices as functions of the forecast uncertainty.

$$(\epsilon_i)_i \text{ is: } \mathbb{E}[\rho] = \frac{1}{\sum_{i=1, \dots, N} d_i} \mathbb{E} \left[\sum_{i=1, \dots, N} (\hat{w}_i - \epsilon_i) \right] = \frac{\sum_{i=1, \dots, N} \hat{w}_i}{\sum_{i=1, \dots, N} d_i} = \frac{\hat{w}_{tot}}{d_{tot}}$$

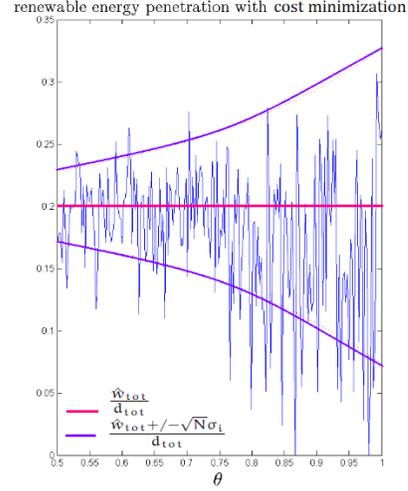
expectation operator is linear and since by assumption $\mathbb{E}[\epsilon_i] = 0, \forall i$. The variance of ρ is:

$$\text{is: } \text{Var}(\rho) = \text{Var} \left(\frac{\sum_{i=1, \dots, N} \hat{w}_i}{\sum_{i=1, \dots, N} d_i} - \frac{\sum_{i=1, \dots, N} \epsilon_i}{\sum_{i=1, \dots, N} d_i} \right) = \frac{\sum_{i=1, \dots, N} \text{Var}(\epsilon_i)}{\left(\sum_{i=1, \dots, N} d_i \right)^2} = \frac{N \sigma_i^2}{d_{tot}^2}$$

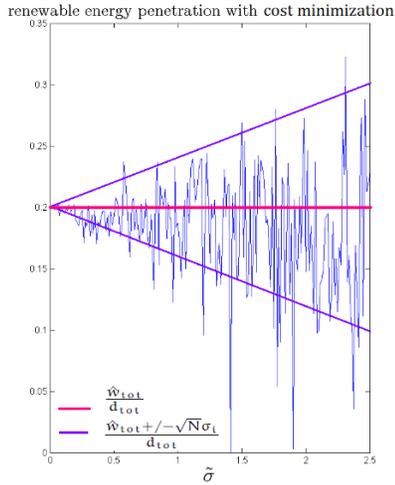
fact that the $(\epsilon_i)_i$ are independent from one another, uncorrelated and that the variance operator is quadratic. We infer that the standard deviation of ρ equals either $-\frac{\sqrt{N} \sigma_i}{d_{tot}}$ or $\frac{\sqrt{N} \sigma_i}{d_{tot}}$. In Figures 3 (a) (resp. (b) and (c)), we represent ρ as a function of γ (resp. of θ and $\tilde{\sigma}$), $(\epsilon_i)_i$ sampled independently. The results are similar for suppliers' total cost minimization and for market welfare maximization. We infer that: increasing the number of wind farms θ makes the expectation of the wind penetration rate increase linearly in θ ; increasing the standard deviation of a single farm forecast error $\tilde{\sigma}$ (resp. the wind farm concentration θ) makes the variance of the wind penetration rate increase quadratically in $\tilde{\sigma}$ (resp. exponentially in θ).



(a)



(b)



(c)

Figure 3: We simulate the renewable penetration rate ρ for different values of γ in (a), θ in (b) and $\tilde{\sigma}$ in (c). The reserves are optimized centrally so as to minimize the suppliers' total costs. ρ is bounded by its standard-deviation values and its expectation is a good approximation of its mean value.

4 Dealing with forecast uncertainty in a decentralized organization

In a decentralized organization, there is no supervisor controlling the forecast errors in each demand market. Each demand market operates selfishly and the forecast er-

rors are generally hidden from the others. However, forecast errors are critical and heavily impact the behavior of the electrical system, as mentioned in the Introduction. Forecast uncertainty generally results from errors made in the decentralized process of wind generation estimation [12], [19], [20]. This uncertainty may be amplified by the decentralized nature of the system. Forecast uncertainty is measured by the standard deviations associated with the wind generation forecast errors: $(\sigma_i)_{i=1,\dots,N}$.

The demands $(d_i)_i$ and the renewable generation forecasts $(\hat{w}_i)_i$ are nevertheless presumed to be common knowledge in the day-ahead. In a decentralized Market Coupling organization, to optimize its reserve, market i supplier needs to evaluate $\frac{\partial U_i}{\partial r_i}$ which expression was derived in Subsection 3.1.1. It is a function of r_i, r_j and σ_i, σ_j :

$$\begin{aligned} \frac{\partial U_i}{\partial r_i} = & \frac{2}{B^f} (d_i - \hat{w}_i + r_i) + \frac{(d_j - \hat{w}_j + r_j) + A^f}{B^f} + \left(\frac{2r_i}{B^0} \right. \\ & \left. - \frac{\left(\frac{\sigma_j}{\sqrt{2\pi}} \exp\left(-\frac{r_j^2}{2\sigma_j^2}\right) - r_j \bar{F}_{\epsilon_j}(r_j) \right) + A^0}{B^0} \right) \bar{F}_{\epsilon_i}(r_i) - \frac{2}{B^0} \frac{\sigma_i}{\sqrt{2\pi}} \exp\left(-\frac{r_i^2}{2\sigma_i^2}\right) \end{aligned}$$

At the optimum for market i , r_i^{NE} can be expressed as a function of $r_j^{\text{NE}}, \sigma_i, \sigma_j$; identically, for market j , r_j^{NE} can be expressed as a function of $r_i^{\text{NE}}, \sigma_j, \sigma_i$. Going a step further, market i supplier's optimal reserve depends on its own forecast uncertainty (measured by σ_i) and on the other market supplier's forecast uncertainty (σ_j).

We then consider two cases that depend on which information is known among the agents; for the sake of simplicity, here and in the next subsections, we will consider two interacting demand markets:

(i) *Information asymmetry*: Market 1 knows σ_1 and σ_2 while market 2 knows only σ_2

(ii) *Partial information*: Market 1 knows only σ_1 and market 2 knows only σ_2

We introduce a Principal to whom the suppliers report their standard deviation i.e., the uncertainty associated with their forecast error. We let $\bar{\sigma}_i, i = 1, 2$ be the report made by market $i = 1, 2$ supplier to the Principal. The Principal can be assimilated to a risk certifier.

4.1 Information asymmetry

Firstly, we assume that market 1 supplier has access to its own standard deviation σ_1 but needs to forecast the other market supplier's standard deviation: σ_2 while market 2 has access to the whole information. Of course, both markets' roles can be interchanged. The information is asymmetric since market 2 supplier has access to all the information while market 1 supplier possesses only partial information. We describe the setting of the non-cooperative game with asymmetric information below:

Decentralized Reserve Optimization with Information Asymmetry

Agents: Market 1 and market 2 suppliers

Information:

- σ_1 known by market 1 and market 2
- σ_2 known by market 2 solely
- $\bar{\sigma}_2$ market 2's reported standard deviation on its forecast error

(i) Market 1 supplier determines $r_1(\sigma_1, \bar{\sigma}_2)$ as a solution of Program 4 (resp. 6) in $\sigma_1, \bar{\sigma}_2$ assuming that market 2's optimal reserve is $r_2(\bar{\sigma}_2, \sigma_1)$ obtained as a solution of Program 4 (resp. 6) in $\bar{\sigma}_2, \sigma_1$.

(ii) Simultaneously and independently, market 2 supplier determines $r_2(\sigma_1, \sigma_2)$ as a solution of Program 4 (resp. 6) in σ_1, σ_2 assuming that market 1's optimal reserve is $r_1(\sigma_1, \bar{\sigma}_2)$ obtained as a solution of Program 4 (resp. 6) in $\sigma_1, \bar{\sigma}_2$.

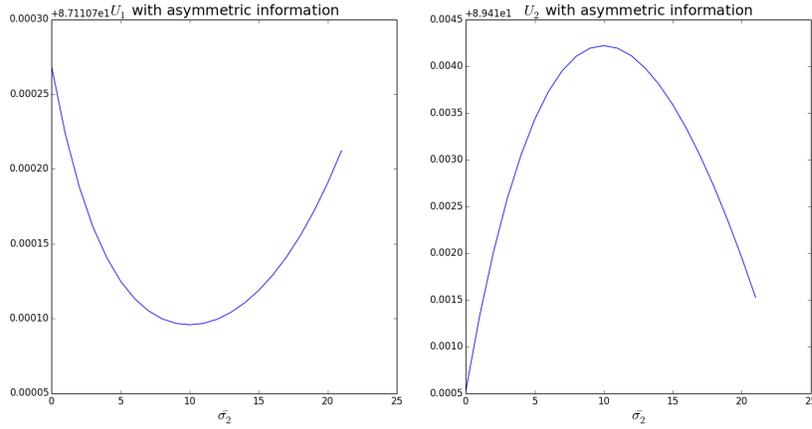


Figure 4: Suppliers' total costs as functions of market 2's reported standard deviation.

In Figure 4, we represent the suppliers' total costs evaluated in the Nash equilibrium as functions of market 2 supplier's reported standard deviation. The (true) standard deviations associated with the forecast errors are: $\sigma_1 = 7.71$ and $\sigma_2 = 9.11$. We observe that market 1 supplier has an incentive to know the true standard deviation of market 2 since its total cost reaches a minimum in $\bar{\sigma}_2 = \sigma_2$. At the same time, market 2 supplier has no incentive to report its true standard deviation since its total cost reaches a maximum in $\bar{\sigma}_2 = \sigma_2$. The simulation results therefore demonstrate that market 2 supplier will bias its reported forecast uncertainty which will cause market 1 supplier's total cost to increase.

4.2 Private information

Secondly, we assume that market 1 and 2 suppliers only know their own standard deviations i.e., market 1 supplier knows σ_1 but ignores σ_2 while market 2 supplier knows

σ_2 but ignores σ_1 . This information is private since each market supplier only has access to its own forecast uncertainty. We describe the setting of the non-cooperative game with private information below:

Decentralized Reserve Optimization with Private Information

Agents: Market 1 and market 2 suppliers

Information:

- σ_1 known by market 1 solely
- σ_2 known by market 2 solely
- $\bar{\sigma}_1$ market 1's reported standard deviation on its forecast error
- $\bar{\sigma}_2$ market 2's reported standard deviation on its forecast error

(i) Market 1 supplier determines $r_1(\sigma_1, \bar{\sigma}_2)$ as a solution of Program 4 (resp. 6) in $\sigma_1, \bar{\sigma}_2$ assuming that market 2's optimal reserve is $r_2(\bar{\sigma}_2, \bar{\sigma}_1)$ obtained as a solution of Program 4 (resp. 6) in $\bar{\sigma}_2, \bar{\sigma}_1$.

(ii) Simultaneously and independently, market 2 supplier determines $r_2(\bar{\sigma}_1, \sigma_2)$ as a solution of Program 4 (resp. 6) in $\bar{\sigma}_1, \sigma_2$ assuming that market 1's optimal reserve is $r_1(\bar{\sigma}_1, \bar{\sigma}_2)$ obtained as a solution of Program 4 (resp. 6) in $\bar{\sigma}_1, \bar{\sigma}_2$.

In Figure 5, we represent the suppliers' total costs evaluated in the Nash equilibrium as functions of the reported standard deviations. The (true) standard deviations associated with the forecast errors are: $\sigma_1 = 7.71$ and $\sigma_2 = 9.11$. To minimize its total cost within $[85.5; 87]$, market 1 supplier should report $\bar{\sigma}_1 \leq -(\bar{\sigma}_2 - 5)(\bar{\sigma}_2 - 10.5) + 2$. Similarly to minimize its total cost within $[88; 90]$, market 2 supplier should report $\bar{\sigma}_2 \leq -(\bar{\sigma}_1 - 6.05)(\bar{\sigma}_1 - 12) + 2$. The true reports i.e., $\bar{\sigma}_1 = \sigma_1, \bar{\sigma}_2 = \sigma_2$, belong to the frontier of both domains. If market i ($i = 1, 2$) supplier truthfully reports its forecast uncertainty i.e., $\bar{\sigma}_i = \sigma_i$, it is highly likely that market j supplier ($j \neq i, j = 1, 2$) will bias its report downwards in the hope of minimizing its total cost.

4.3 The price of uncertainty

We previously observe that the demand markets have incentives to know other demand markets' (true) forecast uncertainty. We now determine the price that the Principal will charge to certify the demand markets' forecast uncertainty.

To obtain an estimate of market $j \neq i$'s forecast uncertainty, the certicator needs to perform a sequence of n observations of market j forecast errors. We introduce the probability that the random variable ϵ distributed according to a Gaussian density function centered in 0 and of standard deviation 1 belongs to the interval $[-\delta; +\delta]$, $\mathcal{C}(\delta) \neq 0$. By definition: $\mathcal{C}(\delta) = F_\epsilon(\delta) - F_\epsilon(-\delta)$ where F_ϵ is the cumulative distribution function associated with the Gaussian distribution function centered in 0 and of standard deviation 1.

We let $\hat{\sigma}_j$ be an estimate of market j forecast error standard deviation, obtained through a second order moment estimation. According to Agard [1], there is a probability, also called confidence level, $\mathcal{C}(\delta)$, that when n is high enough, $\hat{\sigma}_j = \sigma_j$ belongs

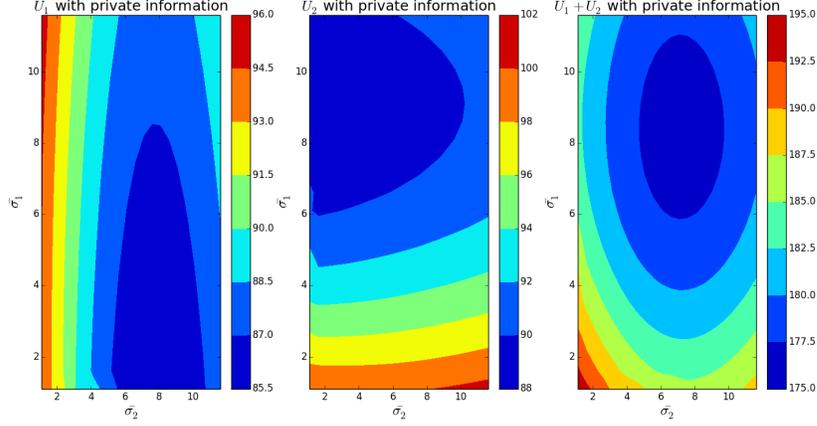


Figure 5: Suppliers' total costs as functions of the markets' reported standard deviations.

to the interval $\left[-\frac{\delta\sigma_j}{\sqrt{n}} \frac{1}{2} \left(\frac{\sqrt{\pi}\Gamma(\frac{5}{2})}{\Gamma^2(\frac{3}{2})} - 1 \right)^{\frac{1}{2}}; \frac{\delta\sigma_j}{\sqrt{n}} \frac{1}{2} \left(\frac{\sqrt{\pi}\Gamma(\frac{5}{2})}{\Gamma^2(\frac{3}{2})} - 1 \right)^{\frac{1}{2}} \right]$ where $\Gamma(a)$ is the Gamma function evaluated in $a \in \mathbb{R}_+$.

It is straightforward to observe that the increase of $\mathcal{C}(\delta)$ makes this interval increase but that the accuracy of the forecast decreases, which implies in turn that the risk certification price should decrease. For a confidence level of $\mathcal{C}(\delta)$, we define the uncertainty price as: $\varphi(\delta, \sigma_i, \bar{\sigma}_j, \hat{\sigma}_j) \geq 0$ and $\varphi(\delta, \sigma_i, \bar{\sigma}_j, \hat{\sigma}_j) \rightarrow 0$ when $\delta \rightarrow +\infty$. This is the price that the risk certicator requires from market i to certify the forecast uncertainty of market $j \neq i$. It should be designed so that market i has incentives to make such a certification.

Proposition 8. *Market j having reported $\bar{\sigma}_j$ as the forecast error standard deviation and with the confidence level set to $\mathcal{C}(\delta)$, the uncertainty price is: $\varphi(\delta, \sigma_i, \bar{\sigma}_j, \hat{\sigma}_j) =$*

$\frac{1}{B^0} \left[\frac{\sigma_i}{\sqrt{2\pi}} \exp\left(-\frac{r_i^2}{2\sigma_i^2}\right) - r_i \bar{F}_{e_i}(r_i) \right] \frac{\exp\left(-\frac{r_j^2}{2\sigma_j^2}\right)}{\sqrt{2\pi}} \left(1 + \frac{r_j^2}{\bar{\sigma}_j^2}\right) [\hat{\sigma}_j - \bar{\sigma}_j]$ where $\bar{\sigma}_j$ is defined as follows:

- *It belongs to $\left[\min\{\bar{\sigma}_j; \hat{\sigma}_j\}; \max\{\bar{\sigma}_j; \hat{\sigma}_j\} \right]$*
- *$\frac{1}{B^0} \left[\frac{\sigma_i}{\sqrt{2\pi}} \exp\left(-\frac{r_i^2}{2\sigma_i^2}\right) - r_i \bar{F}_{e_i}(r_i) \right] \frac{\exp\left(-\frac{r_j^2}{2\sigma_j^2}\right)}{\sqrt{2\pi}} \left(1 + \frac{r_j^2}{\bar{\sigma}_j^2}\right) = \frac{U_i(\sigma_i, \hat{\sigma}_j) - U_i(\sigma_i, \bar{\sigma}_j)}{\hat{\sigma}_j - \bar{\sigma}_j}$*

Furthermore the uncertainty price increases in $\bar{\sigma}_j \in \left[\min\{\bar{\sigma}_j; \hat{\sigma}_j\}; \max\{\bar{\sigma}_j; \hat{\sigma}_j\} \right]$.

Proof of Proposition 8. We introduce the opportunity cost for market i to certify the other market's forecast uncertainty. This is defined as the difference between the benefit resulting from the knowledge of the private standard deviation of the other market and

the cost of certification, depending on the required confidence level $\mathcal{C}(\delta)$. Therefore, the opportunity cost for market i to certify the other market's forecast uncertainty is:

$$\underbrace{\mathbb{U}_i(\sigma_i, \bar{\sigma}_j) - \mathbb{U}_i(\sigma_i, \hat{\sigma}_j)}_{\text{Benefit resulting from the knowledge of } \sigma_j} - \varphi(\delta, \sigma_i, \bar{\sigma}_j, \hat{\sigma}_j). \text{ Market } i \text{ will certify the other mar-}$$

ket's forecast uncertainty provided its opportunity cost remains positive or null. But

the risk certicator will propose the highest admissible price so that the market i opportunity cost vanishes: $\varphi(\delta, \sigma_i, \bar{\sigma}_j, \hat{\sigma}_j) = \mathbb{U}_i(\sigma_i, \bar{\sigma}_j) - \mathbb{U}_i(\sigma_i, \hat{\sigma}_j)$.

Using the Mean Value Theorem, we infer that there exists:

$$\bar{\sigma}_j \in \left[\min\{\bar{\sigma}_j; \hat{\sigma}_j\}; \max\{\bar{\sigma}_j; \hat{\sigma}_j\} \right] \text{ such that } \frac{\partial \mathbb{U}_i(\sigma_i)}{\partial \sigma_j} \Big|_{\sigma_j = \bar{\sigma}_j} = \frac{\mathbb{U}_i(\sigma_i, \hat{\sigma}_j) - \mathbb{U}_i(\sigma_i, \bar{\sigma}_j)}{\hat{\sigma}_j - \bar{\sigma}_j}.$$

We go back to $\mathbb{U}_i(\sigma_i, \sigma_j)$ as detailed in Equation (5) and substitute in it the analytical expressions of the conditional expectations: $\mathbb{E}[(\epsilon_i - r_i) | \epsilon_i \geq r_i] = \frac{\sigma_i}{\sqrt{2\pi}} \Gamma(1) P(1, \frac{r_i^2}{2\sigma_i^2}) - r_i \bar{F}_{\epsilon_i}(r_i)$ and $\mathbb{E}[(\epsilon_i - r_i)^2 | \epsilon_i \geq r_i] = \frac{\sigma_i^2}{\sqrt{\pi}} \Gamma(\frac{3}{2}) P(\frac{3}{2}, \frac{r_i^2}{2\sigma_i^2}) - 2r_i \frac{\sigma_i}{\sqrt{2\pi}} \Gamma(1) P(1, \frac{r_i^2}{2\sigma_i^2}) + r_i^2 \bar{F}_{\epsilon_i}(r_i)$ where $P(a, x) = \frac{1}{\Gamma(a)} \int_x^{+\infty} t^{a-1} \exp(-t) dt$ and $x \in \mathbb{R}^+$ is the non-negative lower bound of the integral.

We obtain:

$$\begin{aligned} \mathbb{U}_i(\sigma_i, \sigma_j) &= (d_i - \hat{w}_i + r_i) \frac{\sum_{l=1,2} (d_l - \hat{w}_l + r_l) + A^f}{B^f} \\ &+ \frac{1}{B^0} \mathbb{E}[(\epsilon_i - r_i)^2 | \epsilon_i \geq r_i] + \frac{1}{B^0} \sum_{l \neq i} \mathbb{E}[(\epsilon_i - r_i) | \epsilon_i \geq r_i] \mathbb{E}[(\epsilon_l - r_l) | \epsilon_l \geq r_l] \\ &+ \frac{A^0}{B^0} \mathbb{E}[(\epsilon_i - r_i) | \epsilon_i \geq r_i] \\ &= (d_i - \hat{w}_i + r_i) \frac{\sum_{l=1,2} (d_l - \hat{w}_l + r_l) + A^f}{B^f} + \frac{1}{B^0} \left[\frac{\sigma_j}{\sqrt{2\pi}} \exp\left(-\frac{r_j^2}{2\sigma_j^2}\right) \right. \\ &- r_j \bar{F}_{\epsilon_j}(r_j) \left. \right] \left[\frac{\sigma_i}{\sqrt{2\pi}} \exp\left(-\frac{r_i^2}{2\sigma_i^2}\right) - r_i \bar{F}_{\epsilon_i}(r_i) \right] + \frac{A^0}{B^0} \left[\frac{\sigma_i}{\sqrt{2\pi}} \exp\left(-\frac{r_i^2}{2\sigma_i^2}\right) \right. \\ &- r_i \bar{F}_{\epsilon_i}(r_i) \left. \right] + \frac{1}{B^0} \left[\frac{\sigma_i^2}{\sqrt{\pi}} \Gamma\left(\frac{3}{2}\right) P\left(\frac{3}{2}, \frac{r_i^2}{2\sigma_i^2}\right) - 2r_i \frac{\sigma_i}{\sqrt{2\pi}} \Gamma(1) P\left(1, \frac{r_i^2}{2\sigma_i^2}\right) + r_i^2 \bar{F}_{\epsilon_i}(r_i) \right] \end{aligned}$$

Derivating \mathbb{U}_i with respect to σ_j , we obtain:

$$\frac{\partial \mathbb{U}_i}{\partial \sigma_j} = \frac{1}{B^0} \underbrace{\left[\frac{\sigma_i}{\sqrt{2\pi}} \exp\left(-\frac{r_i^2}{2\sigma_i^2}\right) - r_i \bar{F}_{\epsilon_i}(r_i) \right]}_{=\mathbb{E}[(\epsilon_i - r_i) | \epsilon_i \geq r_i]} \frac{\exp\left(-\frac{r_j^2}{2\sigma_j^2}\right)}{\sqrt{2\pi}} \left(1 + \frac{r_j^2}{\sigma_j^2}\right) > 0$$

This implies that $\frac{\partial \mathbb{U}_i(\sigma_i, \sigma_j)}{\partial \sigma_j} \Big|_{\sigma_j = \bar{\sigma}_j} > 0$ i.e., $\mathbb{U}_i(\sigma_i, \bar{\sigma}_j)$ is increasing in $\bar{\sigma}_j$. \square

5 Conclusion

In this article we model the interactions among a finite number of geographic demand markets as a non-cooperative game, taking into account the forecast uncertainty result-

ing from the increasing penetration of renewables. Price based Market Coupling, based on implicit auction allocation, is impacted by the increasing penetration of renewables. Currently, it can be organized in one of two ways: centralized or decentralized. To compare these two organizations we study how the scaling of the relative concentration of the wind farms being aggregated, their number, and the forecast uncertainty impact the total cost of procurement, the market welfare and the ratio of renewable generation to conventional supplies. The game equilibria are then characterized depending on the number of interacting demand markets and the game parameters. Forecast errors are critical in a decentralized Market Coupling organization. We show, through simulations, that markets have incentives to certify the other markets' forecast uncertainty. The uncertainty price at which this information should be certified is derived analytically as a function of the required confidence level.

In [18], we extend this article by taking into account the network capacity constraints impacting Market Coupling and we determine the optimal wind farm concentration over each geographic demand market. The increasing penetration of intermittent energy sources whose generation may be fairly erratic, the restructuring of the energy market, and the increasing complexity of most energetic systems, make it necessary to couple agent-based techniques with techniques coming from Machine Learning. The latter could enable the automation of renewable generation forecasting. One area of research concerns the scaling effect of these methods, which would require parallelizing online learning algorithms.

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