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► **To cite this version:**

Haifa Fares Jridi, Charlotte Langlais. Finite-SNR Diversity-Multiplexing-Delay Tradeoff in Half-Duplex Hybrid ARQ Relay Channels. IET Communications, 2015, 9 (16), pp.872 - 879. 10.1049/iet-com.2014.0516 . hal-01174420

**HAL Id: hal-01174420**

**<https://hal.science/hal-01174420>**

Submitted on 9 Jul 2015

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# Finite-SNR Diversity-Multiplexing-Delay Tradeoff in Half-Duplex Hybrid ARQ Relay Channels

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## Abstract

In this paper, we consider a delay-limited hybrid automatic repeat request (HARQ) protocol that makes use of incremental redundancy over the three-node decode-and-forward (DF) relay fading channel where one source cooperates with a relay to transmit information to the destination. We provide an estimate of the diversity-multiplexing tradeoff (DMT) at finite signal to noise ratio (SNR) based on tight bounds on outage probabilities for two channel models. The results for the long term quasi-static channel highlight the distributed diversity, ie. the cooperative space diversity, and the HARQ coding gain, achieved by soft combining the successive transmitted punctured codewords via incremental redundancy. On the other hand, the results for the short term quasi-static channel illustrate the diversity gains obtained thanks to cooperative space diversity and time diversity, along with the HARQ coding gain. Using the DMT formulation, we show that equal power partitioning between the source and the relay nodes provides close to optimal performance. Furthermore, thanks to the extension of the finite-SNR DMT to the finite-SNR diversity-multiplexing-delay tradeoff, we show that, unlike the asymptotic SNR analysis, the ARQ delay, defined as the number of retransmissions rounds, impacts the performance of the HARQ relay protocol for high effective multiplexing gain.

## I. INTRODUCTION

Cooperative communication has emerged as an efficient concept that draws some of the benefits of multi-input multi-output (MIMO) systems, such as spatial diversity gain. One of the simplest cooperative scheme is the decode-and-forward (DF) relay channel, introduced in [1]: a relay assists a source by decoding its original message and forwarding extra information to the destination.

Hybrid automatic repeat request (HARQ) technique, and particularly type-II hybrid ARQ that makes use of incremental redundancy [2], was proven to be particularly suited for wireless communication systems with appropriate feedback link [3]. Recently, HARQ protocol has been successfully incorporated into cooperative communications where retransmissions can be performed by the source or/and the relay [4–9]. In this case, the HARQ relay protocol based on the DF relay channel exploits two kinds of diversity: space diversity available via the relay node, and ARQ diversity. The latter is obtained from: i) the available time diversity and the retransmissions for short term quasi-static channels, ii) only the retransmissions and the combination of coded blocks at the destination thanks to incremental redundancy for long term quasi-static channels.

In [5], Tabet *et al.* used the diversity-multiplexing tradeoff (DMT), originally defined for MIMO systems in [10], as an analytical tool to point out the gain achieved by the HARQ relay protocol under asymptotic signal to noise ratio (SNR) condition. They assumed a delay-limited network whose delay constraint is defined by the maximum number of HARQ rounds  $L$ . So they further extended the DMT to the three-dimensional diversity-multiplexing-delay tradeoff (DMDT), originally defined in [11], in order to quantify the diversity gain as a function of the effective multiplexing gain, taking into account the ARQ delay. However, the asymptotic DMT and DMDT do not illustrate the system behavior at low to moderate SNRs, where communication systems operate conventionally. Thus, recent works have extended the concept of finite-SNR DMT, proposed in [12] in the MIMO context, to cooperative relaying techniques, e.g. [13–15].

In this paper, we analyze the performance of the HARQ relay protocol proposed in [5] thanks to the finite-SNR DMT. First, based on the methodology provided in [13], we derive a closed-form expression of the outage

probability of this protocol, based on tight bounds, for both short term and long term quasi-static channels. Then the finite-SNR DMT is derived thanks to the expression of the outage probability. We show that the asymptotic DMT analysis is inaccurate at finite-SNR since it overestimates the DMT. Furthermore, the results highlight the impact of the power partitioning between the source and the relay on the system performance. The third major contribution is the derivation of the diversity-multiplexing-delay tradeoff (DMDT) of the HARQ relay protocol. Unlike the asymptotic SNR case where the DMDT is asymptotically equal to the DMT, the DMDT is non trivially derived at finite SNRs, since the effective multiplexing gain depends on the SNR value. Finally, we compare the finite-SNR DMT with the finite-SNR DMDT to evaluate the impact of the number of HARQ rounds on the performance of the HARQ relay protocol. Furthermore, this powerful theoretical performance metric allows us to fairly compare the HARQ relay protocol with the non-ARQ relay channel and the non-cooperative HARQ scheme.

Unlike [8], during the cooperation phase, we allow the superposition of the signals, coming from the source and the relay, at the destination side. Hence, further spatial diversity and better throughput efficiency are achieved. In addition, as the end-to-end performance in [8] is determined for *uncoded* transmissions, the term of incremental relaying is used with some abuse of language. Finally, in the DF scheme, as uncoded transmissions are assumed, the relay does not check its decoding state and errors propagation can occur if the relay decoding is erroneous. In our work, we consider that the source and the relay are able to transmit a punctured version of the codeword simultaneously, different at each HARQ round, so that the most powerful HARQ protocol of type II making use of the proper incremental redundancy process is carried out.

Compared with [7], we focus on the same HARQ relay protocol. Unlike them, to cope with both long term and short term quasi-static channels, we choose to derive the closed-form expression of the outage probability based on tight bounds, obtained via numerical optimization. In [7], the authors observe the outage probability and the average throughput independently. Thanks to the applied methodology, we are able to derive an accurate estimation of the finite-SNR DMT for both channels. Finally, the derived finite-SNR DMDT precisely describes the interplay between diversity, i.e. reliability, and effective multiplexing gain, i.e. average throughput, taking into account the number of HARQ rounds. Thus, conclusions different from those in [7] can be drawn.

The remainder of the paper is as follows: Section II describes the system model, including the HARQ relay protocol. Section III is dedicated to the derivation of the finite-SNR DMT of the considered protocol, based on the computation of the outage probability for both short term and long term quasi-static channels. In Section IV, we extend the DMT performance metric to the DMDT, in order to take into account the ARQ delay. The numerical results are presented in Section V to confirm the validity of the analytical expressions. Conclusions are presented in Section VI.

## II. SYSTEM MODEL AND SETTINGS

We consider the wireless relay channel, consisting in one source  $s$  assisted by one relay  $r$  transmitting to a single destination  $d$ . All nodes are equipped with single antenna. Moreover, half-duplex transmission is assumed. Each channel is modeled as Rayleigh flat fading with additive white Gaussian noise. We consider that  $h_{sd} \sim \mathcal{CN}(0, \sigma_{sd}^2)$ ,  $h_{sr} \sim \mathcal{CN}(0, \sigma_{sr}^2)$  and  $h_{rd} \sim \mathcal{CN}(0, \sigma_{rd}^2)$ , where  $\mathcal{CN}(a, b)$  denotes a complex Gaussian distribution with mean  $a$  and variance  $b$ . The differences in the variances account for path loss and shadowing. In the numerical examples,  $\sigma_{sd}^2 = 1$  and  $\sigma_{sr}^2 = \sigma_{rd}^2 = 2$ . The additive noises at the relay and the destination are complex Gaussian with zero mean and variance  $\sigma_n^2$ . The instantaneous receive SNR over the source-destination link (respectively over source-relay and relay-destination links) is then denoted by  $\gamma_{sd} = \alpha\rho|h_{sd}|^2$  (respectively,  $\gamma_{sr} = \alpha\rho|h_{sr}|^2$  and  $\gamma_{rd} = \beta\rho|h_{rd}|^2$ ).  $\bar{\gamma}_{sd} = \alpha\rho\sigma_{sd}^2$ ,  $\bar{\gamma}_{sr} = \alpha\rho\sigma_{sr}^2$  and  $\bar{\gamma}_{rd} = \beta\rho\sigma_{rd}^2$  denote their corresponding average SNR. We define by  $\rho_s = \alpha\rho$  and  $\rho_r = \beta\rho$  the transmit SNRs for the source-destination and relay-destination links, respectively, with  $\rho = E_s/\sigma_n^2$  the network SNR and  $E_s$  the average total network transmit power. Thus, the parameters  $\alpha$  and  $\beta$  illustrate the static power partitioning between the source and the relay nodes, subject to  $\alpha + \beta = 1$ .

We investigate two channel models: i) long term quasi-static channel, where all the ARQ rounds occur during the same channel realization, and ii) short term quasi-static channel where the fading is constant over each transmission round and changes independently from one round to another one, such that time diversity is then available. The study for the long term quasi-static channel will demonstrate if HARQ and incremental redundancy are still beneficial even if time diversity is not available. Note that, for short term quasi-static channel model, we distinguish different instantaneous SNRs for each HARQ rounds, i.e.  $\gamma_{sd}^i$ ,  $\gamma_{sr}^i$  and  $\gamma_{rd}^i$  for  $i = 1, \dots, L$  over source-destination, source-relay and relay-destination links.

### A. Protocol description

The HARQ DF relay protocol considered in this paper is a type II HARQ protocol that makes use of incremental redundancy [2][3]. Thus, we assume that the relay and destination nodes keep the past received signals in memory and mutual information is accumulated at both nodes thanks to Chase combining, a.k.a. code combining [2]. Each node is equipped with a cyclic redundancy check (CRC) encoder and/or decoder to detect unsuccessful decoding. We assume that the feedback channels are error free and that the system is delay-limited such that the maximum number of retransmission rounds is finite, limited to  $L$ . First, the source encodes the information message linearly thanks to the linear code  $\mathcal{C}$ . If the destination is able to decode this first coded block, it forwards an ACK (positive acknowledgment) message to the source via the feedback channel and the source goes to the next information message to encode. If the destination is not able to decode the message and the maximum number of retransmission rounds ( $L$ ) is not attained, the destination forwards a NACK (negative acknowledgment) message and the source retransmits a new version, e.g. a punctured version, of the codeword according to the principle of incremental redundancy. The destination performs Chase combining over the successive transmitted coded blocks. Similarly, the relay tries to recover the message sent by the source at each ARQ round thanks to Chase combining. After successful decoding, the relay starts to cooperate by forwarding extra versions of the codeword. To improve the spectral efficiency of the half-duplex relay channel, the relay and the source transmit the same coded block *simultaneously* during the cooperation phase. In [13], in a non-HARQ context, this non orthogonal transmission was shown to provide the best outage probability performance, compared to protocols that allow only the relay to transmit. To cope with the interference generated by this non-orthogonal transmission, the most famous technique is probably the Alamouti space-time block coding in its distributed DF version [16]. We also proposed a receiver based on multi-user detection, and more particularly interleaved division multiple access (IDMA), that can deal with the non orthogonal transmission in a user cooperation context [17]. Distributed MIMO precoding based on singular value decomposition is another possible technical solution that makes use of channel state information at the transmitter (even partial or quantized) [18].

### B. System Model

The system model is as follows. The source information message  $\mathbf{u}$  of  $K$  symbols is encoded by the encoder  $\mathcal{C}$ , located at the source and the relay.  $\mathbf{x}$  denotes the generated codeword of maximal length  $LN$  where  $N$  is the number of channel uses to transmit one round and  $L$  is the number of allowed ARQ rounds. Then, we define  $R = K/N$  as the first-round transmission rate. We denote by  $\mathbf{x}^i$ ,  $i = 1, \dots, L$ , the punctured codewords of length  $N$  generated by the source and the relay nodes during the  $i$ -th ARQ round. The observations of  $\mathbf{x}^i$  received at the destination and the relay are, respectively, denoted by  $\mathbf{y}_{sd}^i$  and  $\mathbf{y}_{sr}^i$  for the  $i$ -th ARQ round. During the cooperation phase, the source and the relay transmit simultaneously to compensate for the loss of spectral efficiency due to the half-duplex transmission. To cope with the interference generated by the non orthogonal transmission, the codewords generated at the source and at the relay are first block encoded, in a distributed fashion, thanks to the Alamouti code. Thus, the destination performs a maximum ratio combining (MRC) and space diversity is achieved. The received observations at the destination are denoted by  $\mathbf{y}_{srd}^i$ .

Based on the system model, the transmission strategy can be summarized as follows. According to the acknowledgment messages from both the source and the relay, four different cases are possible:

- 1) The destination is able to decode at the first transmission round. No outage is declared. The source goes to the next information message to encode.
- 2) If the relay and the destination successfully decode the source message at the  $k$ -th round and at the  $l$ -th round, respectively, we distinguish between two subcases:
  - if  $k < l$ : During the first  $k$  ARQ rounds, the source transmits the codewords  $\mathbf{x}^i$ ,  $i = 1, \dots, k$ . At each transmission round, the destination tries to decode by applying a Chase combining on the last received observation  $\mathbf{y}_{sd}^i$  and the previous ones  $\mathbf{y}_{sd}^l$ ,  $l = 1, \dots, i - 1$ . During the following  $(l - k)$  ARQ rounds, the relay becomes able to cooperate. Thus, at the destination, the maximal ratio combining is followed by a Chase combining on  $\mathbf{y}_{srd}^i$  and the previous received observations, until successful decoding at the destination at  $i = l$ .
  - if  $k \geq l$ : since the source-relay channel is always in outage, the relay is not able to cooperate and only  $\mathbf{y}_{sd}^i$ ,  $i = 1, \dots, l$  are exploited to perform Chase combining at the destination.

3) If decoding at the destination fails after  $L$  ARQ rounds, an outage is declared.

Many possible extensions of this described transmission strategy to the multiple relay case could be found in the literature [19][20][7]. The most trivial one is that given in [19]: in each transmission round, among  $M$  available relays, only relays able to decode correctly the original source message retransmit the same codeword, different from that transmitted in the previous rounds. Thus, at the destination, the maximal ratio combining is performed among the source contribution and the relay contributions, before performing the Chase combining operation with previously received observations from previous transmission rounds. Despite the additional cost on complexity and delay related to additional feedbacks, the extension to multiple relay case offers further distributed space diversity, which is beneficial especially for the long-term static channels, where temporal diversity is not available.

However, unlike the single relay scheme, from this protocol description, we see that many transmission cases arise for the multiple relay scenario. The number of cases increases with the number of cooperating relays involved in each transmission round (those able to correctly recover the source message), i.e., from no relay contribution to full cooperative relaying case. Therefore, the analysis given in this paper is not trivially extensible to the case of multiple relay scheme, and major changes are needed to derive the finite-SNR diversity-multiplexing-delay tradeoff. Nevertheless, this work can be considered as a starting point and a useful tool for performance prediction of the more general scheme.

### C. Settings

In this sub-section, we recall main definitions, given originally in [12], to derive the finite-SNR DMT. The multiplexing gain  $r$  as a function of the network SNR  $\rho$  is defined as the ratio of the first-round transmission rate  $R$  to the capacity of an additive white Gaussian noise channel, written as

$$r = \frac{R}{\log(1 + \rho)} \quad (1)$$

The multiplexing gain can also be seen as the normalized transmission rate of the first round. The HARQ relay protocol has a variable rate by nature due to the varying number of HARQ rounds depending on the channel realizations. Thus a very useful performance metric for ARQ protocols is the average throughput efficiency  $\eta$ , a.k.a the long term average throughput in [7][11], in transmitted bits per channel use, given by

$$\eta = \frac{R}{1 + \sum_{l=1}^{L-1} P_{\text{out}}(r, \rho, l)} \quad (2)$$

where  $P_{\text{out}}(r, \rho, l)$  denotes the probability that the received codeword at the  $l$ -th transmission round is not correctly decoded. The denominator can be seen as the average number of HARQ rounds. The throughput efficiency  $\eta$  corresponds to the average transmission rate of the system, not just the first round. From (1) and (2), we define the effective multiplexing gain, as a function of the finite network SNR  $\rho$ , as follows:

$$\begin{aligned} r_e &= \frac{\eta}{\log(1 + \rho)} \\ &= \frac{r}{1 + \sum_{l=1}^{L-1} P_{\text{out}}(r, \rho, l)} \end{aligned} \quad (3)$$

For a given multiplexing gain  $r$ , the effective multiplexing gain can vary according to the average number of HARQ rounds, which, itself, depends on the SNR.

The diversity gain  $d(r, \rho)$  of a system is defined by the negative slope of the log-log curve of the outage probability versus SNR, leading to:

$$d(r, \rho) = -\frac{\partial \log P_{\text{out}}(r, \rho)}{\partial \log \rho} = -\frac{\rho}{P_{\text{out}}(r, \rho)} \frac{\partial P_{\text{out}}(r, \rho)}{\partial \rho} \quad (4)$$

For the HARQ relay protocol,  $d(r, \rho)$  is given for  $P_{\text{out}}(r, \rho) = P_{\text{out}}(r, \rho, L)$ , for a given  $L$ . We will show in the next section that there is a direct interplay between  $P_{\text{out}}$ ,  $r$ , or  $r_e$ , and  $\rho$ . This interplay can be conveniently illustrated thanks to the diversity-multiplexing tradeoff.

### III. FINITE-SNR DIVERSITY-MULTIPLEXING TRADEOFF

Thanks to HARQ protocol, the receiver accumulates mutual information on successive rounds and decoding is consequently performed after soft combining, i.e. Chase combining, all received messages. The outage event at a particular decoder (either at the relay or the destination), after the  $l^{\text{th}}$  retransmission, for a given multiplexing gain  $r$  and at SNR  $\rho$  is defined as  $\mathcal{O}_l = \left\{ \sum_{i=1}^l I_i < r \log(1 + \rho) \right\}$ . Its complement,  $\bar{\mathcal{O}}_l$ , refers then to successful decoding after  $l$  rounds.  $I_i$  is the average mutual information of the  $i$ -th ARQ round. Depending on which links are involved in the transmission, we distinguish between three different mutual informations

$$\begin{aligned} I_{s,d}^i &= I(\mathbf{y}_{sd}^i | h_{sd}^i) = \log\left(1 + \alpha\rho |h_{sd}^i|^2\right) \\ I_{s,r}^i &= I(\mathbf{y}_{sr}^i | h_{sr}^i) = \log\left(1 + \alpha\rho |h_{sr}^i|^2\right) \\ I_{s,r,d}^i &= I(\mathbf{y}_{srd}^i | h_{sd}^i, h_{rd}^i) = \log\left(1 + \alpha\rho |h_{sd}^i|^2 + \beta\rho |h_{rd}^i|^2\right) \end{aligned} \quad (5)$$

The last equation illustrates the cooperation phase with an MRC operation on the received observations  $\mathbf{y}_{srd}^i$ . The outage probability on the link s-d at time instant  $l$  is defined as  $P_{sd}(l) = \Pr\left\{\sum_{i=1}^l I_{s,d}^i < r \log(1 + \rho)\right\}$ . Similar expressions are obtained for the links s-r and r-d. Like in [5],  $\tau_r$  denotes the number of ARQ rounds to ensure successful decoding at the relay. The probability that  $k$  ARQ rounds are needed to correctly recover the information message  $\mathbf{u}$  at the relay is then defined as

$$\begin{aligned} p_k &= \Pr\{\tau_r = k\} = \Pr\{\mathcal{O}_{r,1}, \dots, \mathcal{O}_{r,k-1}, \bar{\mathcal{O}}_{r,k}\} \\ &= \Pr\{\mathcal{O}_{r,k-1}\} - \Pr\{\mathcal{O}_{r,k}\} = P_{sr}(k-1) - P_{sr}(k) \end{aligned} \quad (6)$$

with  $P_{sr}(0) = \Pr\{\mathcal{O}_{r,0}\} = 1$ .

As in Section II, we assume that the relay successfully decodes the source message at the  $k$ -th round. The outage probability for the HARQ relay protocol at the  $l$ -th round for all possible  $k$  is given by

$$\begin{aligned} P_{\text{out}}(r, \rho, l) &= \sum_{k=1}^{l-1} \Pr\left\{\sum_{i=1}^k I_{s,d}^i + \sum_{i=k+1}^l I_{s,r,d}^i < r \log(1 + \rho)\right\} p_k \\ &\quad + \sum_{k=l}^L \Pr\left\{\sum_{i=1}^l I_{s,d}^i < r \log(1 + \rho)\right\} p_k \\ &= \sum_{k=1}^{l-1} P_{\text{coop}}(k, l) p_k + P_{sd}(l) [P_{sr}(l-1) - P_{sr}(L)] \end{aligned} \quad (7)$$

with  $P_{\text{coop}}(k, l)$  denotes the outage probability of the HARQ relay protocol at the  $l$ -th round when the relay is able to cooperate from the  $k$ -th round. The variable  $\rho$  is omitted to shorten the notations. The first term corresponds to the case when  $k < l$  whereas the second term corresponds to the case when  $k \geq l$ . Additionally, the derivative of the outage probability with respect to  $\rho$  is

$$\begin{aligned} \frac{\partial P_{\text{out}}(r, \rho, l)}{\partial \rho} &= \sum_{k=1}^{l-1} \frac{\partial P_{\text{coop}}(k, l)}{\partial \rho} p_k + P_{\text{coop}}(k, l) \frac{\partial p_k}{\partial \rho} + \frac{\partial P_{sd}(l)}{\partial \rho} \times \\ &\quad [P_{sr}(l-1) - P_{sr}(L)] + P_{sd}(l) \left[ \frac{\partial P_{sr}(l-1)}{\partial \rho} - \frac{\partial P_{sr}(L)}{\partial \rho} \right] \end{aligned} \quad (8)$$

Using (7) and (8),  $d(r, \rho)$  is given based on (4). In order to compute  $P_{\text{out}}(r, \rho, l)$  and  $\frac{\partial P_{\text{out}}(r, \rho, l)}{\partial \rho}$ , we need  $P_{sr}(m)$ ,  $P_{sd}(l)$ ,  $P_{\text{coop}}(k, l)$  and their derivatives, whose expressions depend on the statistics of the channel.

### A. Long Term Quasi-Static Channel

As the fading remains constant over all ARQ rounds, the mutual informations at each round do not vary until the transmission of the current message is stopped. From (5) and for  $m = l - 1, \dots, L$ , we obtain

$$P_{\text{sr}}(m) = \Pr\{mI_{\text{s,r}} < r\log(1+\rho)\} = 1 - \exp\left(-\frac{(1+\rho)^{\frac{r}{m}} - 1}{\alpha\rho\sigma_{\text{sr}}^2}\right) \quad (9)$$

The derivative of  $P_{\text{sr}}(m)$  with respect to  $\rho$  is given by

$$\frac{\partial P_{\text{sr}}(m)}{\partial \rho} = -\exp\left(-\frac{(1+\rho)^{\frac{r}{m}} - 1}{\alpha\rho\sigma_{\text{sr}}^2}\right) \left[ \frac{-r(1+\rho)^{\frac{r}{m}-1}}{m\alpha\rho\sigma_{\text{sr}}^2} + \frac{(1+\rho)^{\frac{r}{m}} - 1}{\alpha\rho^2\sigma_{\text{sr}}^2} \right] \quad (10)$$

Similar expressions are obtained for  $P_{\text{sd}}(l)$  and  $\frac{\partial P_{\text{sd}}(l)}{\partial \rho}$  by replacing  $m$  by  $l$  and  $\sigma_{\text{sr}}^2$  by  $\sigma_{\text{sd}}^2$ .

Since  $P_{\text{coop}}(k, l)$  denotes the outage probability at the  $l$ -th round when the relay is able to cooperate from the  $k$ -th round, it is given by

$$\begin{aligned} P_{\text{coop}}(k, l) &= \Pr\{kI_{\text{s,d}} + (l-k)I_{\text{s,r,d}} < r\log(1+\rho)\} \\ &= \iint_A \frac{1}{\tilde{\gamma}_{\text{sd}}} \frac{1}{\tilde{\gamma}_{\text{rd}}} \exp\left(\frac{-\gamma_{\text{sd}}}{\tilde{\gamma}_{\text{sd}}}\right) \exp\left(\frac{-\gamma_{\text{rd}}}{\tilde{\gamma}_{\text{rd}}}\right) d\gamma_{\text{sd}} d\gamma_{\text{rd}} \end{aligned} \quad (11)$$

where  $A \equiv \{(1 + \gamma_{\text{sd}})^k (1 + \gamma_{\text{sd}} + \gamma_{\text{rd}})^{l-k} < (1 + \rho)^r\}$ . Rewriting the constraint  $A$  as  $\gamma_{\text{rd}} \leq \frac{(1+\rho)^{\frac{r}{l-k}}}{(1+\gamma_{\text{sd}})^{\frac{k}{l-k}}} - 1 - \gamma_{\text{sd}}$ , and assuming that  $\gamma_{\text{rd}}$  is always greater than zero, we give a precise constraint for  $\gamma_{\text{sd}}$ , such that  $\gamma_{\text{sd}} < (1 + \rho)^{\frac{r}{l-k}} - 1$ . Consequently,  $P_{\text{coop}}(k, l)$  can be evaluated numerically from

$$\begin{aligned} P_{\text{coop}}(k, l) &= \int_{\gamma_{\text{sd}}=0}^{\gamma_{\text{sd}}=(1+\rho)^{\frac{r}{l-k}}-1} \frac{1}{\tilde{\gamma}_{\text{sd}}} \exp\left(\frac{-\gamma_{\text{sd}}}{\tilde{\gamma}_{\text{sd}}}\right) \times \\ &\quad \int_{\gamma_{\text{rd}}=0}^{\gamma_{\text{rd}}=\frac{(1+\rho)^{\frac{r}{l-k}}}{(1+\gamma_{\text{sd}})^{\frac{k}{l-k}}}-1-\gamma_{\text{sd}}} \frac{1}{\tilde{\gamma}_{\text{rd}}} \exp\left(\frac{-\gamma_{\text{rd}}}{\tilde{\gamma}_{\text{rd}}}\right) d\gamma_{\text{rd}} d\gamma_{\text{sd}} \end{aligned} \quad (12)$$

However, to obtain a closed-form expression of the finite-SNR DMT, we prefer to estimate  $P_{\text{coop}}(k, l)$ , based on the bounding technique developed in [13]. In Section V, we show the tightness of the resulting bounds on the outage probabilities compared to the numerical exact evaluations using (12).  $P_{\text{coop}}(k, l)$  is bounded as follows:

$$\begin{aligned} P_{\text{coop}}(k, l) &\geq \Pr\left\{\left(1 + \alpha\rho|h_{\text{sd}}|^2\right)^k < (1 + \rho)^{a_1}\right\} \times \\ &\quad \Pr\left\{\left(1 + \alpha\rho|h_{\text{sd}}|^2 + \beta\rho|h_{\text{rd}}|^2\right)^{l-k} < (1 + \rho)^{a_2}\right\} \end{aligned} \quad (13)$$

where  $(1 + \rho)^r = \prod_{i=1}^2 (1 + \rho)^{a_i}$ .

Let  $z = \alpha\rho|h_{\text{sd}}|^2 + \beta\rho|h_{\text{rd}}|^2$ . We distinguish between two cases

1) If  $\alpha\sigma_{\text{sd}}^2 \neq \beta\sigma_{\text{rd}}^2$ , the probability density function (pdf) of  $z$  is given in [13]

$$p_z(z) = \frac{1}{\beta\sigma_{\text{rd}}^2 - \alpha\sigma_{\text{sd}}^2} \left[ \exp\left(\frac{-z}{\beta\rho\sigma_{\text{rd}}^2}\right) - \exp\left(\frac{-z}{\alpha\rho\sigma_{\text{sd}}^2}\right) \right] \quad (14)$$

Consequently, this yields

$$\begin{aligned} P_{\text{coop}}(k, l) &\geq \left[ 1 - \exp\left(-\frac{(1+\rho)^{\frac{a_1}{k}} - 1}{\alpha\rho\sigma_{\text{sd}}^2}\right) \right] \left[ 1 - \frac{\beta\sigma_{\text{rd}}^2}{\beta\sigma_{\text{rd}}^2 - \alpha\sigma_{\text{sd}}^2} \times \right. \\ &\quad \left. \exp\left(-\frac{(1+\rho)^{\frac{a_2}{l-k}} - 1}{\beta\rho\sigma_{\text{rd}}^2}\right) + \frac{\alpha\sigma_{\text{sd}}^2}{\beta\sigma_{\text{rd}}^2 - \alpha\sigma_{\text{sd}}^2} \exp\left(-\frac{(1+\rho)^{\frac{a_2}{l-k}} - 1}{\beta\rho\sigma_{\text{rd}}^2}\right) \right] \\ &= \bar{P}_{\text{coop}}(k, l) \end{aligned} \quad (15)$$

2) If  $\alpha\sigma_{\text{sd}}^2 = \beta\sigma_{\text{rd}}^2 = \sigma^2$ , the probability density function (pdf) of  $z$  is then given by

$$p_z(z) = \frac{z}{(\sigma^2)^2} \exp\left(\frac{-z}{\rho\sigma^2}\right) \quad (16)$$

Consequently, this yields

$$\begin{aligned} P_{\text{coop}}(k,l) &\geq \left[1 - \exp\left(-\frac{(1+\rho)^{\frac{a_1}{k}} - 1}{\rho\sigma^2}\right)\right] \times \\ &\quad \left[1 - \left(1 + \frac{(1+\rho)^{\frac{a_2}{l-k}} - 1}{\rho\sigma^2}\right) \exp\left(-\frac{(1+\rho)^{\frac{a_2}{l-k}} - 1}{\rho\sigma^2}\right)\right] \\ &= \bar{P}_{\text{coop}}(k,l) \end{aligned} \quad (17)$$

An accurate estimate of the outage probability is obtained by maximizing this lower bound numerically, for each  $\rho$ , over the exponents  $a_1$  and  $a_2$ , subject to the three constraints,  $0 \leq a_1 \leq r$ ,  $0 \leq a_2 \leq r$  and  $a_1 + a_2 = r$ .

In order to compute the diversity order, we need to compute the derivative of  $P_{\text{coop}}(k,l)$ . Following the setup in [13], the derivative of  $P_{\text{coop}}(k,l)$  is approximated by the derivative of its lower bound  $\bar{P}_{\text{coop}}(k,l)$  and is then used in (8).

$$\frac{\partial P_{\text{coop}}(k,l)}{\partial \rho} \approx \frac{\partial \bar{P}_{\text{coop}}(k,l)}{\partial \rho} \quad (18)$$

### B. Short Term Quasi-Static Channel

Unlike the long term quasi-static channel, the instantaneous mutual informations vary from one transmission round to another. For  $m = l - 1, \dots, L$ , we obtain

$$\begin{aligned} P_{\text{sr}}(m) &= \Pr\left\{\sum_{i=1}^m I_{\text{sr},r}^i < r \log(1+\rho)\right\} \\ &= \int_{\gamma_{\text{sr}}^1=0}^{\gamma_{\text{sr}}^1=(1+\rho)^r-1} \int_{\gamma_{\text{sr}}^2=0}^{\gamma_{\text{sr}}^2=\frac{(1+\rho)^r}{1+\gamma_{\text{sr}}^1}-1} \dots \\ &\quad \int_{\gamma_{\text{sr}}^m=0}^{\gamma_{\text{sr}}^m=\frac{(1+\rho)^r}{\prod_{i=1}^{m-1}(1+\gamma_{\text{sr}}^i)}-1} \prod_{i=1}^m \frac{1}{\tilde{\gamma}_{\text{sr}}^i} \exp\left(\frac{-\gamma_{\text{sr}}^i}{\tilde{\gamma}_{\text{sr}}^i}\right) d\gamma_{\text{sr}}^i \end{aligned} \quad (19)$$

Using the same bounding technique developed in [13],  $P_{\text{sr}}(m)$  can be approximated by its lower bound given by

$$\begin{aligned} P_{\text{sr}}(m) &\geq \prod_{i=1}^m \Pr\left\{(1+\alpha\rho|h_{\text{sr}}^i|^2) < (1+\rho)^{a_i}\right\} \\ &\geq \prod_{i=1}^m \left[1 - \exp\left(-\frac{(1+\rho)^{a_i} - 1}{\alpha\rho\sigma_{\text{sr}}^2}\right)\right] = \bar{P}_{\text{sr}}(m) \end{aligned} \quad (20)$$

with  $0 \leq a_i \leq r$ ,  $\forall i = 1, \dots, m$  and  $\sum_{i=1}^m a_i = r$ . The values  $a_i$  are obtained by maximizing this lower bound numerically. The derivative of  $P_{\text{sr}}(m)$  with respect to  $\rho$  is approximated by

$$\begin{aligned} \frac{\partial P_{\text{sr}}(m)}{\partial \rho} &\approx \frac{\partial \bar{P}_{\text{sr}}(m)}{\partial \rho} \\ &= \sum_{i=1}^m -\exp\left(-\frac{(1+\rho)^{a_i} - 1}{\alpha\rho\sigma_{\text{sr}}^2}\right) \left[\frac{-a_i(1+\rho)^{a_i-1}}{\alpha\rho\sigma_{\text{sr}}^2} + \frac{(1+\rho)^{a_i} - 1}{\alpha\rho^2\sigma_{\text{sr}}^2}\right] \prod_{j=1, j \neq i}^m \left[1 - \exp\left(-\frac{(1+\rho)^{a_j} - 1}{\alpha\rho\sigma_{\text{sr}}^2}\right)\right] \end{aligned} \quad (21)$$

Similar expressions are obtained for  $P_{\text{sd}}(l)$  and  $\frac{\partial P_{\text{sd}}(l)}{\partial \rho}$  by replacing  $m$  by  $l$  and  $\sigma_{\text{sr}}^2$  by  $\sigma_{\text{sd}}^2$ .



$$\begin{aligned}
 P_{\text{coop}}(k, l) &= \Pr \left\{ \sum_{i=1}^k I_{s,d}^i + \sum_{i=k+1}^l I_{s,r,d}^i < r \log(1+\rho) \right\} \\
 &= \int \int \dots \int_{(\gamma_{sd}^1, \dots, \gamma_{sd}^l, \gamma_{rd}^{k+1}, \dots, \gamma_{rd}^l) \in \Gamma} \prod_{i=1}^l \frac{1}{\bar{\gamma}_{sd}^i} \exp\left(\frac{-\gamma_{sd}^i}{\bar{\gamma}_{sd}^i}\right) d\gamma_{sd}^i \prod_{j=k+1}^l \frac{1}{\bar{\gamma}_{sr}^j} \exp\left(\frac{-\gamma_{rd}^j}{\bar{\gamma}_{sr}^j}\right) d\gamma_{rd}^j \\
 &= \int_{\gamma_{sd}^1=0}^{\gamma_{sd}^1=(1+\rho)^r-1} \dots \int_{\gamma_{sd}^k=0}^{\gamma_{sd}^k=\frac{(1+\rho)^r}{\prod_{i=1}^{k-1}(1+\gamma_{sd}^i)}-1} \int_{\gamma_{sd}^{k+1}=0}^{\gamma_{sd}^{k+1}=\frac{(1+\rho)^r}{\prod_{i=1}^k(1+\gamma_{sd}^i)}-1} \int_{\gamma_{rd}^{k+1}=0}^{\gamma_{rd}^{k+1}=\frac{(1+\rho)^r}{\prod_{i=1}^k(1+\gamma_{sd}^i)}-1-\gamma_{sd}^{k+1}} \dots \\
 &\quad \int_{\gamma_{sd}^l=0}^{\gamma_{sd}^l=\frac{(1+\rho)^r}{\prod_{i=1}^k(1+\gamma_{sd}^i) \prod_{j=k+1}^{l-1}(1+\gamma_{sd}^j + \gamma_{rd}^j)}-1} \int_{\gamma_{rd}^l=0}^{\gamma_{rd}^l=\frac{(1+\rho)^r}{\prod_{i=1}^k(1+\gamma_{sd}^i) \prod_{j=k+1}^{l-1}(1+\gamma_{sd}^j + \gamma_{rd}^j)}-1-\gamma_{sd}^l} \\
 &\quad \prod_{i=1}^l \frac{1}{\bar{\gamma}_{sd}^i} \exp\left(\frac{-\gamma_{sd}^i}{\bar{\gamma}_{sd}^i}\right) d\gamma_{sd}^i \prod_{j=k+1}^l \frac{1}{\bar{\gamma}_{sr}^j} \exp\left(\frac{-\gamma_{rd}^j}{\bar{\gamma}_{sr}^j}\right) d\gamma_{rd}^j
 \end{aligned} \tag{23}$$

$$\text{with } \Gamma = \left\{ \prod_{i=1}^k (1 + \gamma_{sd}^i) \prod_{j=k+1}^l (1 + \gamma_{sd}^j + \gamma_{rd}^j) < (1 + \rho)^r \right\}.$$

Furthermore,  $P_{\text{coop}}(k, l)$  can be exactly evaluated using (23). Note that a numerical exact evaluation of  $P_{\text{coop}}(k, l)$ , using nested integrations, becomes laborious when  $l > 3$ . Therefore, in Section V, we restrict the numerical results of exact evaluation of the outage probabilities over the short term quasi-static channels to the cases  $L = 2$  ( $l = 1, 2$ ) and  $L = 3$  ( $l = 1, 2, 3$ ).

However,  $P_{\text{coop}}(k, l)$  can be approximated by its lower bound given by

$$\begin{aligned}
 P_{\text{coop}}(k, l) &\geq \prod_{i=1}^k \Pr \left\{ (1 + \alpha\rho |h_{sd}^i|^2) < (1 + \rho)^{a_i} \right\} \times \\
 &\quad \prod_{i=k+1}^l \Pr \left\{ (1 + \alpha\rho |h_{sd}^i|^2 + \beta\rho |h_{rd}^i|^2) < (1 + \rho)^{a_i} \right\} \\
 &= \prod_{i=1}^k \left[ 1 - \exp\left(-\frac{(1+\rho)^{a_i} - 1}{\alpha\rho\sigma_{sd}^2}\right) \right] \prod_{i=k+1}^l \left[ 1 - \frac{\beta\sigma_{rd}^2}{\beta\sigma_{rd}^2 - \alpha\sigma_{sd}^2} \times \right. \\
 &\quad \left. \exp\left(-\frac{(1+\rho)^{a_i} - 1}{\beta\rho\sigma_{rd}^2}\right) + \frac{\alpha\sigma_{sd}^2}{\beta\sigma_{rd}^2 - \alpha\sigma_{sd}^2} \exp\left(-\frac{(1+\rho)^{a_i} - 1}{\beta\rho\sigma_{rd}^2}\right) \right] \\
 &= \bar{P}_{\text{coop}}(k, l)
 \end{aligned} \tag{22}$$

The values  $a_i$  are determined by numerically maximizing the corresponding lower bounds for each  $\rho$  with  $0 \leq a_i \leq r$ ,  $\forall i = 1, \dots, l$  and  $\sum_{i=1}^l a_i = r$ . The derivative of  $P_{\text{coop}}(k, l)$  is approximated by the derivative of its lower bound  $\bar{P}_{\text{coop}}(k, l)$  and is then used in (8).

#### IV. FINITE-SNR DIVERSITY-MULTIPLEXING-DELAY TRADEOFF

To quantify the impact of the HARQ delay, i.e. the number of HARQ rounds, on the performance, we extend the previous formulation of the finite-SNR DMT to the finite-SNR DMDT  $d(r_e, \rho)$ , by introducing the effective multiplexing gain defined in (3). In the case of finite-SNR DMT, first, we fix the multiplexing gain and second, we compute  $P_{\text{out}}(r, \rho, l)$  for a given  $\rho$  and  $l$ . However, at the light of (3), it is clear that the effective multiplexing gain can not be fixed independently from the network SNR. Note that the high SNR analysis conducted in [5] leads to an effective multiplexing gain that tends to the first-round rate, i.e.  $r_e \doteq r$ . This has been explained by the fact that most information messages are decoded successfully in the first round. On the contrary, at finite SNR, the DMDT is not trivially derived, and no more equivalent to the DMT, since the effective multiplexing gain depends on the SNR.

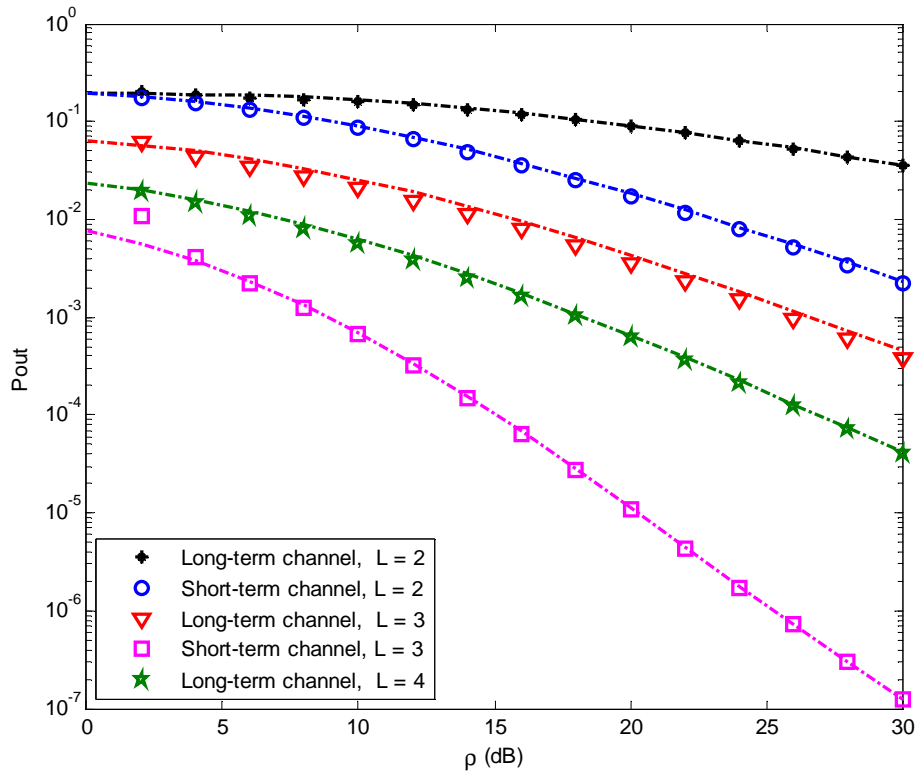


Fig. 1. Outage probabilities  $P_{out}(L)$ : Numerical exact evaluations (markers) versus approximations based on lower bounds (dashed lines).

Hence, we need to compute the function  $f$  defined by  $r_e = f(r, \rho)$ . As no exact form is available, we resort to numerical approximations. We use a set of discretized values of  $\rho$  and  $r$  on the range of interest. For each couple  $(\rho^i, r^i)$ , we construct a look-up table (LUT) representing  $f$  according to (3). Reversely, we get  $r = g_{\rho^i}(r_e)$ , over the corresponding set of  $r$  and  $r_e$ , where  $g_{\rho^i}(r_e) = f_{\rho^i}^{-1}(r_e)$  and  $f_{\rho^i}(r_e) = f(r_e, \rho^i)$  for a given  $\rho^i$ . Using the previously constructed LUT and performing a cubic interpolation, we obtain an analytical approximation of  $r$  as a function of  $r_e$ . Consequently, at a given SNR value  $\rho$ , using the appropriate value of  $r$ , depending on  $r_e$ , into the finite-SNR DMT given in Section III, we obtain the DMDT of the HARQ relay protocol, computing the diversity as a function of  $r_e$ .

### V. NUMERICAL RESULTS

In this section, first, we illustrate the tightness of the bounds used to derive the outage probability of the HARQ relay protocol. Second, we give numerical results of the finite-SNR DMT, derived in Section III-A and Section III-B for two different channel models: long term and short term quasi-static channels. For comparison purpose, we also give the numerical results of the asymptotic DMT of the HARQ relay protocol [5], the finite-SNR DMT of the non-ARQ relay channel [13] and the finite-SNR DMT of the non-cooperative HARQ scheme. Finally, we provide the finite-SNR DMDT results of the HARQ relay protocol derived in Section IV.

#### A. Outage probability results

Unlike [7], we propose to approximate the outage probability, given in (7), based on lower bounds on  $P_{coop}(k, l)$ , for both long term and short term static channel models (in (15), (17) and (22)), and  $P_{sr}(m)$  for short term static channels, in (20). In this section, we illustrate the tightness of the approximations, by comparing them with the exact numerical evaluation, given in (12) and (23). Fig. 1 confirms that the analytical bounds adopted to derive an estimate of the outage probability, and consequently on the finite-SNR DMT and DMDT, provide accurate approximations of the numerical exact evaluation results. The comparisons are illustrated for both channel models (long term and short term) and for different maximum ARQ rounds  $L$ .

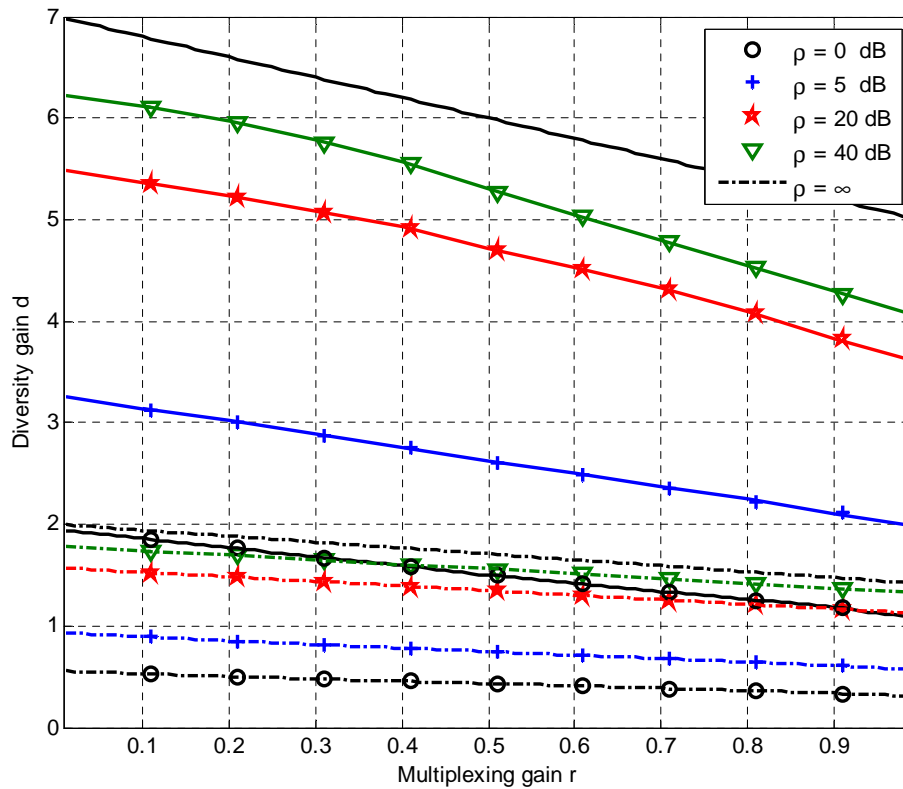


Fig. 2. Finite-SNR DMT of the HARQ relay protocol for  $L = 4$  with  $\alpha = \beta = 0.5$ . Both long term static channel (dashed lines) and short term static channel (solid lines) are considered.

### B. Finite-SNR DMT results

First, we consider an equal power partitioning between the source and the relay, i.e.,  $\alpha = \beta = 0.5$ . In Fig. 2, we plot the finite-SNR DMT of the HARQ relay protocol for various SNRs with the maximum number of HARQ rounds  $L$  equal to 4. Results are given for the long term quasi-static channel (dashed lines) and the short term quasi-static channel (solid lines). As expected, the diversity gains at finite-SNR are significantly lower than the asymptotic ones. For the same SNR, the short term quasi-static channel gives additional diversity, compared with the long term one, since time diversity is available. However, the more meaningful result given by Fig. 2 is that, at low SNR, the gap between finite-SNR DMT of the two channel models is significantly lower than the one at high SNR, or asymptotic SNR.

In Fig. 3, we compare the finite-SNR DMT of the HARQ relay protocol with  $L = 3$ , at a fixed SNR value ( $\rho = 10$  dB), for various power partitioning setups ( $\alpha$ ). For reference, we also give the finite-SNR DMT for the non-cooperative HARQ protocol, which corresponds to  $\alpha = 1$  (all the network power is allocated to the source). We notice that, for  $\alpha = 0.1$ , the DMT curves are significantly degraded for both channel models compared to equal power partitioning. However, for  $\alpha = 0.25, 0.75, 0.9$ , their corresponding DMT curves are very similar to the case of  $\alpha = 0.5$ . These results indicate that, regardless the channel model, the performance of the HARQ relay protocol at finite SNR is relatively insensitive to the power partitioning for  $0.1 < \alpha \leq 0.9$ . This observation corroborates the conclusion given in [13], where authors state that minor variations in the diversity performance of the relay channel (where no ARQ is performed) are noted for  $0.1 < \alpha \leq 0.9$ . Similar results are obtained for other SNRs and are omitted for the sake of brevity. On the other hand, we notice sizable improvements with respect to the non-cooperative HARQ protocol, except for  $\alpha = 0.1$  and high-rate regime. Indeed, this power allocation is not favorable to the relay protocol because too little energy is allocated to the source. Thus, with equal power partitioning, cooperation with a relay will always bring some extra diversity gains to the HARQ protocol, whatever the statistics of the channel model. With respect to the non-cooperative HARQ protocol, the diversity gain increase is more visible for low multiplexing gains.

Fig. 4 shows the finite-SNR DMT of the HARQ relay protocol with  $L = 4$  over the short term quasi-static

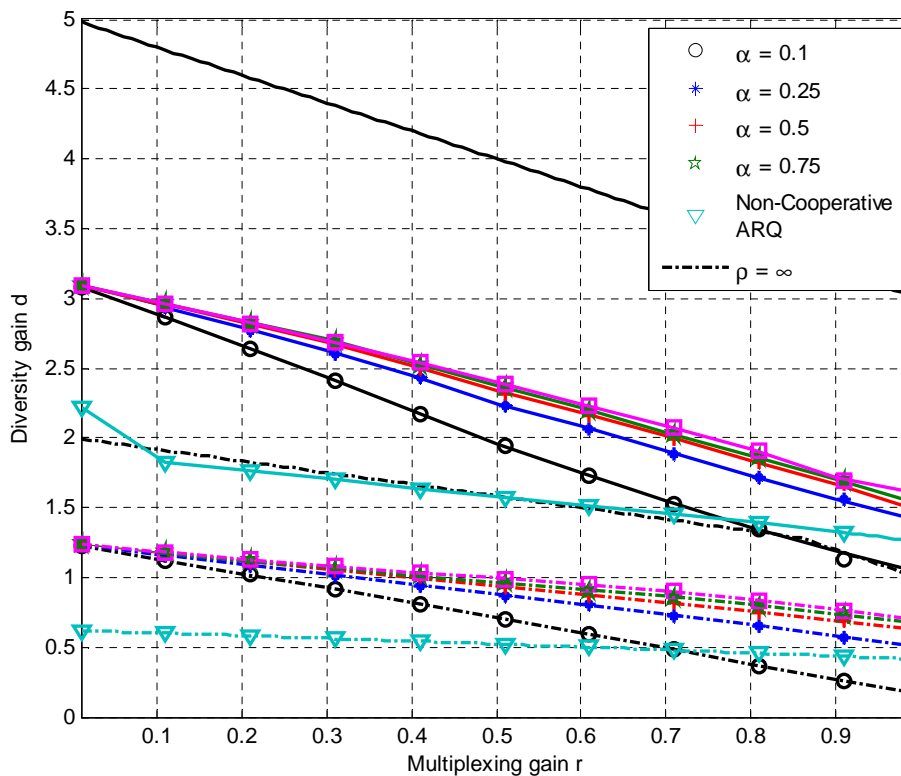


Fig. 3. Power partitioning effect on the finite-SNR DMT of the ARQ relay protocol for  $\rho = 10$  dB,  $L = 3$  with  $\beta = 1 - \alpha$ . Both short term (solid lines) and long term (dashed lines) channels are considered. The non-cooperative HARQ protocol performance is drawn for comparison purposes.

channel compared to the finite-SNR DMT of the non-ARQ relay channel, as presented in [13], for various SNRs. Notice that, for the non-ARQ relay channel, it does not make sense to distinguish between long term and short term quasi-static channels, since there is no retransmission attempt. The channel realization is unique for one transmission phase and changes independently for the next transmission. Outstanding gains are observed using the HARQ relay protocol with respect to the non-ARQ relay channel. Comparing the results of the non-ARQ relay channel, from Fig. 4, with those of the HARQ relay protocol over the long term quasi-static channel, given in Fig. 2, shows that performing retransmissions with incremental redundancy in the relay network is still beneficial even over long term quasi-static channel, especially for medium to high multiplexing rates.

However, the comparison thanks to the DMT metric is not completely fair since the HARQ rounds imply a loss in spectral efficiency, and so in multiplexing gain, that is not taken into account with this metric. That's why we present the results in terms of DMDT in the next section.

### C. Finite-SNR DMDT results

Fig. 5 depicts the finite-SNR DMDT of the HARQ relay protocol for both long term and short term quasi-static channels and for various SNRs with  $L = 4$ . Unlike the asymptotic analysis [5], at finite SNR, we note from Fig. 2 and Fig. 5 that the DMDT curves can be lower than the DMT curves, depending on the targeted network SNR. Indeed, unlike the multiplexing gain  $r$ , the effective multiplexing gain  $r_e$  does not only depend on the first-round rate  $R$  but also on the number of HARQ rounds, which varies along with the network SNR. This degradation is more visible from a certain threshold rate  $r_e^{\text{th}}$  that depends on  $\rho$ . We recall that the multiplexing gain is defined as the normalized first-round rate. Consequently, the threshold rate  $r_e^{\text{th}}$  corresponds to the case when  $r$  becomes greater than 1, which impacts the outage probability  $P_{\text{out}}(r, \rho, l)$  dramatically. In the case of long term statistics, for low SNR values ( $\rho = 0$  or 5 dB), the diversity gain even vanishes completely for  $r_e \geq 0.7$ .

Now, to fairly compare the HARQ relay protocol with the non-ARQ relay channel, we use the finite-SNR DMDT of the HARQ relay protocol over short term and long term quasi-static channel from Fig. 5 with the finite-SNR

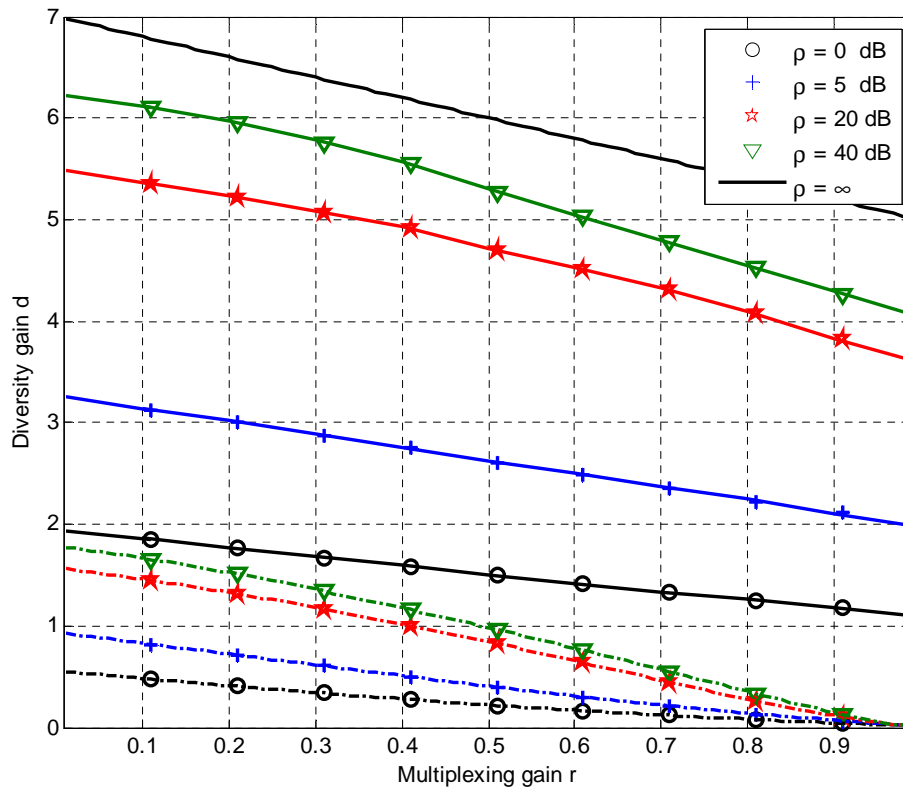


Fig. 4. Finite-SNR DMT comparison between the HARQ relay protocol with  $L = 4$  over the short term quasi-static channel (solid lines) and the non-ARQ relay channel (dashed lines) [13], for  $\alpha = \beta = 0.5$ .

DMT of the non-ARQ relay channel from Fig. 4. First, focusing on the long term quasi-static channel, we observe that the loss in throughput is clearly unfavorable to the HARQ relay protocol. However, over the short term quasi-static channel, the HARQ relay protocol takes advantage of time diversity and we still observe improvements with respect to the finite-SNR DMT performance of the non-ARQ relay channel. Consequently, in the case of short term quasi static channel, we can conclude that performing a retransmission protocol such as HARQ over the relay channel is beneficial, even for low SNR values, where the degradation of the multiplexing gain due to ARQ delay, i.e. the number of retransmissions, is the most visible.

## VI. CONCLUSIONS

In this paper, we are interested in two popular diversity techniques for wireless communications: cooperative communication via the relay channel and retransmission technique for coded schemes via HARQ. In order to compare them not only in terms of reliability, but also throughput, we have focused on a theoretical metric that clearly illustrates the interplay between diversity and multiplexing, namely the diversity-multiplexing tradeoff. We have considered a particular HARQ relay protocol that is delay-limited and known to be particularly efficient in terms of outage probability and throughput, as detailed in the introduction. Since the performance metrics at asymptotic SNR do not illustrate the system behavior at low to moderate SNRs, where communication systems operate conventionally, we have proposed an analysis at finite-SNR.

Thus, we have derived an estimate on the finite-SNR DMT for the HARQ relay protocol, based on a lower bound on the outage probability, for both long term and short term quasi-static channels. The analytical results have shown that, regardless the channel models, the optimization of power partitioning between the source and the relay is not needed and equal power partitioning provides close to optimal finite-SNR DMT curves. Moreover, the comparison between the finite-SNR DMT results of the HARQ relay protocol and the ones of the non cooperative HARQ protocol, as well as the non-ARQ relay channel, has shown that combining both HARQ protocol and cooperative relaying offers sizable performance gains in terms of diversity and multiplexing gains.

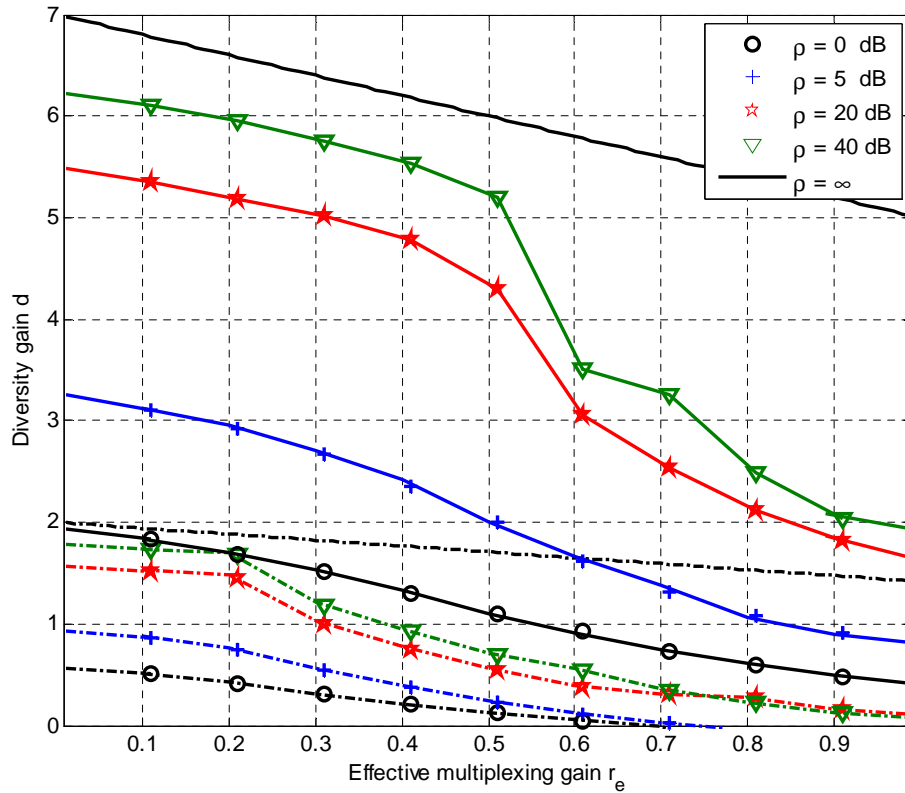


Fig. 5. Finite-SNR DMDT of the ARQ relay protocol for  $L = 4$  with  $\alpha = \beta = 0.5$ . Both long term quasi-static channel (dashed lines) and short term quasi-static channel (solid lines) are considered.

As HARQ introduces a loss in throughput due to the retransmission rounds, we have finally extended the DMT analysis to estimate the DMDT that makes use of the effective multiplexing gain in order to take into account the ARQ delay, i.e. the number of retransmission rounds. Unlike the asymptotic analysis, the finite-SNR DMDT is largely degraded compared with the finite-SNR DMT. This demonstrates the impact of the ARQ delay, i.e. the number of retransmission rounds, on the HARQ relay protocol performance. This degradation is translated in a diversity gain's decrease that is more visible from a certain multiplexing gain threshold, especially for low-SNR regime. Thus the comparison is slightly in favor of the non-ARQ relay channel over the long term quasi-static channel. However, the finite-SNR DMDT performance results over the short-term quasi-static channel have shown that the HARQ protocol combined with the relay channel is largely beneficial in terms of diversity and multiplexing gains compared to the non-ARQ relay channel. In this case, the HARQ protocol takes advantage of time diversity that counteracts the loss in throughput due to the retransmission rounds. This conclusion can not be made using average outage probability analysis and throughput-delay performance separately and only over the long term quasi-static channel, as given in [7].

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