Supplemental Appendix to Optimal Food Price Stabilization in a Small Open Developing Country

Christophe Gouel* Sébastien Jean†

April 22, 2013

S1 First-order conditions of the optimal policy problem

To solve the optimal policy problem presented in section IV, we need to reformulate the complementarity equations (5)–(7) because they cannot be included directly as constraints in a maximization problem. We restate these equations as a combination of inequalities and equations. For equation (5), we introduce a positive slack variable, ϕ, with its associated complementarity slackness conditions

$$\phi_t = P_t + k - \beta \, \mathrm{E}_t \left(P_{t+1}\right) - \zeta_t, \tag{S1}$$

$$S_t \phi_t = 0. \tag{S2}$$

To limit the number of equations and variables, the two trade policy instruments are merged into one with $\nu_t = \nu_t^X - \nu_t^M$, which is equivalent since each instrument is redundant when the other is active. The equations governing trade, (6)–(7), can be restated as

$$X_t \left[P_t^{\mathrm{T}} - \left(P_t^w - \tau\right)\right] = 0, \tag{S3}$$

$$M_t \left[\left(P_t^w + \tau\right) - P_t^{\mathrm{T}}\right] = 0, \tag{S4}$$

$$P_t^{\mathrm{T}} = P_t + \nu_t, \tag{S5}$$

where $X_t \geq 0$, $M_t \geq 0$, and $P_t^w - \tau \leq P_t^{\mathrm{T}} \leq P_t^w + \tau$.

The Markov-perfect equilibria of optimal policies under discretion are known to be difficult to characterize since they involve functional equations, often called generalized Euler equations (Klein et al., 2008, Ambler and Pelgrin, 2010). A model setting with occasionally binding constraints is even more complex because first-order conditions cannot be derived (see appendix S3). For optimal policies with both instruments and with storage subsidy alone, this difficulty can be sidestepped by noting that the policy maker acts identically under commitment and under discretion. This peculiarity makes it possible to use the solution to the problem under commitment, more easily computed, as a solution to the problem under discretion.

*INRA, Économie Publique, and CEPII (christophe.gouel@grignon.inra.fr)
†CEPII and INRA, Économie Publique (sebastien.jean@cepii.fr)

The equivalence between commitment and discretion for these two policies arises because the storage subsidy allows full control of the only intertemporal trade-off, namely the storage decision. Formally, following Marcet and Marimon (2011), the optimal policy problem under commitment can be expressed as a saddle-point functional equation problem:

$$
\begin{aligned}
J\left(A_t, P_t^w, \lambda_{t-1}\right) = \min_{\Phi_t} \max_{\Omega_t} \{ & v\left(P_t, Y\right) + w\left[P_t A_t - \left(P_t + k\right) S_t + \nu_t \left(X_t - M_t\right)\right] \\
& + \chi_t \left[A_t + M_t - D\left(P_t\right) - S_t - X_t\right] \\
& + \lambda_t \left(\phi_t + \zeta_t - P_t - k\right) \\
& + \lambda_{t-1} P_t \\
& + \delta_t^S S_t \phi_t \\
& + \delta_t^M M_t \left(P_t^w + \tau - P_t^{\mathrm{T}}\right) \\
& + \delta_t^X X_t \left(P_t^{\mathrm{T}} - P_t^w + \tau\right) \\
& + \kappa_t \left(P_t^{\mathrm{T}} - P_t - \nu_t\right) \\
& + \beta \,\mathrm{E}_t \left[J\left(S_t + \varepsilon_{t+1}^H, f\left(P_t^w, \varepsilon_{t+1}^w\right), \lambda_t\right)\right] \},
\end{aligned}
\tag{S6}
$$

where $\Omega_t = \{S_t \geq 0, P_t, M_t \geq 0, X_t \geq 0, P_t^w - \tau \leq P_t^{\mathrm{T}} \leq P_t^w + \tau, \zeta_t, \phi_t \geq 0, \nu_t\}$ and $\Phi_t = \{\chi_t, \lambda_t, \delta_t^S, \delta_t^M, \delta_t^X, \kappa_t\}$. From the first-order conditions and the envelope theorem, and following some manipulation, the recursive equilibrium under an optimal stabilization policy can be characterized by the following system of complementarity conditions:[1]

$$S_t : S_t \geq 0 \quad \perp \quad -w\left[P_t + k - \beta \,\mathrm{E}_t\left(P_{t+1}\right)\right] - \chi_t + \beta \,\mathrm{E}_t\left(\chi_{t+1}\right) + \delta_t^S \phi_t \leq 0, \tag{S7}$$

$$P_t : v_P\left(t\right) + wD\left(P_t\right) - \chi_t D'\left(P_t\right) - \lambda_t + \lambda_{t-1} = 0, \tag{S8}$$

$$M_t : M_t \geq 0 \quad \perp \quad -w\nu_t + \chi_t + \delta_t^M\left(P_t^w + \tau - P_t^{\mathrm{T}}\right) \leq 0, \tag{S9}$$

$$X_t : X_t \geq 0 \quad \perp \quad w\nu_t - \chi_t + \delta_t^X\left(P_t^{\mathrm{T}} - P_t^w + \tau\right) \leq 0, \tag{S10}$$

$$P_t^{\mathrm{T}} : P_t^w - \tau \leq P_t^{\mathrm{T}} \leq P_t^w + \tau \quad \perp \quad -\delta_t^M M_t + \delta_t^X X_t \tag{S11}$$

$$\zeta_t : \lambda_t = 0, \tag{S12}$$

$$\phi_t : \phi_t \geq 0 \quad \perp \quad \lambda_t + \delta_t^S S_t \leq 0, \tag{S13}$$

$$\nu_t : P_t^{\mathrm{T}} - P_t - \nu_t = 0, \tag{S14}$$

$$\chi_t : A_t + M_t = D\left(P_t\right) + S_t + X_t, \tag{S15}$$

$$\lambda_t : \phi_t + \zeta_t - P_t - k + \beta \,\mathrm{E}_t\left(P_{t+1}\right) = 0, \tag{S16}$$

$$\delta_t^S : S_t \phi_t = 0, \tag{S17}$$

$$\delta_t^M : M_t\left(P_t^w + \tau - P_t^{\mathrm{T}}\right) = 0, \tag{S18}$$

$$\delta_t^X : X_t\left(P_t^{\mathrm{T}} - P_t^w + \tau\right) = 0. \tag{S19}$$

The transition from one period to the next is still governed by equations (9)–(10). Combining equations (S9)–(S10) and (S18)–(S19) gives $\nu_t = \chi_t / w$ for positive trade. It means that the optimal trade policy is to base trade decisions on the total marginal value of the commodity (the sum of private marginal value, domestic price, and social marginal value, χ_t / w) instead of the price.

[1] Here the "perp" notation ($\perp$) is extended to situations with two complementarity constraints. The expression $a \leq X \leq b \perp F(X)$ is a compact formulation for $X = a \Rightarrow F(X) \geq 0, X \in (a,b) \Rightarrow F(X) = 0, X = b \Rightarrow F(X) \leq 0$.

Usually, the time-inconsistency of policies under commitment shows up in the lagged Lagrange multipliers. Here, the multiplier λ is always null (equation (S12)), which demonstrates that there is no commitment problem. The equilibrium, therefore, is recursive in its natural state variables and is time-consistent. It is identical under commitment and under discretion.

S2 First-order conditions of the optimal storage subsidy

The first-order conditions of the optimal storage subsidy are obtained similarly, except that variables ν and P^{T} have to be dropped from (S6). From the first-order conditions with both instruments, equations (S7), (S12), (S13), and (S15)–(S17) remain valid. The following first-order conditions must be considered in addition:

$$P_t : P_t^w - \tau \leq P_t \leq P_t^w + \tau \quad \perp \quad v_P(t) + w(A_t - S_t) - \chi_t D'(P_t) - \delta_t^M M_t + \delta_t^X X_t, \tag{S20}$$

$$M_t : M_t \geq 0 \quad \perp \quad \chi_t + \delta_t^M (P_t^w + \tau - P_t) \leq 0, \tag{S21}$$

$$X_t : X_t \geq 0 \quad \perp \quad -\chi_t + \delta_t^X (P_t - P_t^w + \tau) \leq 0, \tag{S22}$$

$$\delta_t^M : M_t (P_t^w + \tau - P_t) = 0, \tag{S23}$$

$$\delta_t^X : X_t (P_t - P_t^w + \tau) = 0. \tag{S24}$$

S3 Characterization of the optimal trade policy

For the optimal trade policy, the solutions under commitment and under discretion being different, we have to characterize the discretionary solution directly. Since we focus on a Markovian equilibrium, following Klein et al. (2008), we can characterize price by a function of the state variables: $P_t = \mathscr{P}(A_t, P_t^w)$. Substituting this function $\mathscr{P}$ for price expectations in equation (S1), the value function is defined by the following Bellman equation

$$\begin{aligned} J(A_t, P_t^w) = \min_{\Phi_t} \max_{\Omega_t} \Big(& v(P_t, Y) + w\left[P_t A_t - (P_t + k) S_t + \nu_t (X_t - M_t)\right] \\ & + \chi_t \left[A_t + M_t - D(P_t) - S_t - X_t\right] \\ & + \lambda_t \left\{\beta \,\mathrm{E}_t\left[\mathscr{P}\left(S_t + \varepsilon_{t+1}^H, f\left(P_t^w, \varepsilon_{t+1}^w\right)\right)\right] + \phi_t - P_t - k\right\} \\ & + \delta_t^S S_t \phi_t \\ & + \delta_t^M M_t \left(P_t^w + \tau - P_t^{\mathrm{T}}\right) \\ & + \delta_t^X X_t \left(P_t^{\mathrm{T}} - P_t^w + \tau\right) \\ & + \kappa_t \left(P_t^{\mathrm{T}} - P_t - \nu_t\right) \\ & + \beta \,\mathrm{E}_t\left[J\left(S_t + \varepsilon_{t+1}^H, f\left(P_t^w, \varepsilon_{t+1}^w\right)\right)\right] \Big). \end{aligned} \tag{S25}$$

In this setting with occasionally binding constraints, we cannot assume $\mathscr{P}$ to be differentiable everywhere. Thus, in theory, we cannot derive the first-order conditions of this problem since they would imply derivatives of $\mathscr{P}$. But since, in practice, $\mathscr{P}$ is approximated by a spline, which is differentiable everywhere, we can solve the dynamic programming problem numerically by solving the first-order conditions of the approximated problem:

$$S_t : S_t \geq 0 \perp -w\left(P_t+k\right)-\chi_t+\beta\lambda_t \operatorname{E}_t \mathscr{P}_A\left(A_{t+1},P^w_{t+1}\right)+\delta^S_t\phi_t+\beta \operatorname{E}_t\left(wP_{t+1}+\chi_{t+1}\right)\leq 0, \tag{S26}$$

$$P_t : v_P\left(t\right)+wD\left(P_t\right)-\chi_t D'\left(P_t\right)-\lambda_t=0, \tag{S27}$$

$$M_t : M_t \geq 0 \quad \perp \quad -w\nu_t+\chi_t+\delta^M_t\left(P^w_t+\tau-P^{\mathrm{T}}_t\right)\leq 0, \tag{S28}$$

$$X_t : X_t \geq 0 \quad \perp \quad w\nu_t-\chi_t+\delta^X_t\left(P^{\mathrm{T}}_t-P^w_t+\tau\right)\leq 0, \tag{S29}$$

$$P^{\mathrm{T}}_t : P^w_t-\tau \leq P^{\mathrm{T}}_t \leq P^w_t+\tau \quad \perp \quad -\delta^M_t M_t+\delta^X_t X_t, \tag{S30}$$

$$\phi_t : \phi_t \geq 0 \quad \perp \quad \lambda_t+\delta^S_t S_t \leq 0, \tag{S31}$$

$$\chi_t : A_t+M_t = D\left(P_t\right)+S_t+X_t, \tag{S32}$$

$$\lambda_t : \beta \operatorname{E}_t\left(P_{t+1}\right)+\phi_t-P_t-k=0, \tag{S33}$$

$$\delta^S_t : S_t\phi_t=0, \tag{S34}$$

$$\delta^M_t : M_t\left(P^w_t+\tau-P^{\mathrm{T}}_t\right)=0, \tag{S35}$$

$$\delta^X_t : X_t\left(P^{\mathrm{T}}_t-P^w_t+\tau\right)=0, \tag{S36}$$

$$\nu_t : P^{\mathrm{T}}_t-P_t-\nu_t=0. \tag{S37}$$

S4 Optimal policy when the country is not self-sufficient on average

In the paper, we have considered a situation where the country trades with the rest of the world because yield shocks are distinct in the two regions. Now we consider in addition that the mean price in autarky differs from the mean world price, so that trade will take place even in the absence of stochastic shocks. For convenience, the change is implemented through μ, which parameterizes the world yield distribution. Its default value is 1 and we consider two alternatives, 0.9 and 1.1—a 10% increase or decrease in the world mean production—where the country is respectively a structural exporter or a structural importer.

For $\mu = 0.9$ the deterministic steady-state price in the world market is equal to 1.3, whereas in autarky the steady-state price is still 1 in the domestic market. Thus, when open to trade, the country exports and has a mean price higher than its mean autarkic price (see table S1). Storage behavior is consistent with what was previously described for the trade/storage relationship: the mean stock level is very low when the country is an importer, and is much higher if it is an exporter. At the aggregate level, the parameter μ does not seem to affect the optimal policy pattern, which consistently increases stock levels and produces very limited changes in average trade. As mentioned in the paper, the optimal policy's trade effect consists of distorting the distribution of trade, not in isolating the economy from the world market, which would be too costly. In the benchmark case, the distortion of excess supply curves is achieved mainly through the use of import subsidies and export taxes. With $\mu = 0.9$, export taxes and subsidies become the most important trade policy instruments. For prices higher than the mean price, export restrictions are used to avoid importing high prices from the world market. For prices below the mean price, exports are subsidized so as to move domestic price closer to its mean. For $\mu = 1.1$, the behavior is similar but applied to imports. Imports are subsidized when the domestic price is above its mean and taxed when it is below it. So the optimal behavior is to exploit the world market so as to stabilize the domestic price around its mean.

The welfare analysis shows that policy intervention is more beneficial from a social point of view for structural exporters, while the gains to structural importers are sharply reduced in comparison to the benchmark. If

Table S1: Sensitivity to the assumption of self-sufficiency

	Structural exporter (μ =0.9)		Self-sufficient (μ =1)[a]		Structural importer (μ =1.1)	
	No Policy	Optimal Policy	No Policy	Optimal Policy	No Policy	Optimal Policy
Descriptive statistics						
Mean price	1.230	1.208	1.045	1.034	0.915	0.913
CV of price	0.216	0.134	0.173	0.121	0.156	0.109
Mean stocks	0.090	0.120	0.033	0.047	0.013	0.020
Mean imports	0.003	0.004	0.018	0.018	0.050	0.048
Mean exports	0.074	0.074	0.028	0.028	0.008	0.008
Welfare						
Consumers gains		1.34		1.05		0.25
Producers gains		−1.38		−0.92		−0.10
Government		0.27		−0.03		−0.09
Total gains		0.23		0.10		0.07

Notes: Columns No Policy and Optimal Policy display respectively results for the model without public intervention and for the optimal policy using the two instruments.
[a] Benchmark

anything, this only adds to the policy concerns surrounding the use of export restrictions, since structural exporters seem to have potentially serious motivations to persist in using them.

S5 Numerical algorithm

The numerical algorithm used here is inspired by Fackler (2005) and Miranda and Glauber (1995). It is a projection method with a collocation approach. Since several models are solved, we present here a general method that can be applied to all of them. Following Fackler (2005), rational expectations problems can be expressed using three groups of equations. State variables s are updated through a transition equation:

$$\dot{s} = g(s, x, \dot{e}), \tag{S38}$$

where x are response variables, e are stochastic shocks, and next-period variables are indicated with a dot on top of the character. Response variables are defined by solving a system of complementarity equilibrium equations:

$$\underline{x}(s) \leq x \leq \overline{x}(s) \quad \perp \quad f(s, x, z). \tag{S39}$$

Response variables can have lower and upper bounds, $\underline{x}$ and $\overline{x}$, which can themselves be functions of state variables.[2] For generality, equilibrium equations have been expressed as complementarity problems. In cases where response variables have no lower and upper bounds, equation (S39) simplifies to a traditional equation:

$$f(s, x, z) = 0. \tag{S40}$$

z is a variable representing the expectations about next period and is defined by

$$z = \mathrm{E}\left[h(s, x, \dot{e}, \dot{s}, \dot{x})\right]. \tag{S41}$$

[2] It is the case in equation (S11) where lower and upper bounds on P_t^{T} are functions of P_t^w.

One way to solve this problem is to find a function that approximates well the behavior of response variables. We consider a spline approximation of response variables as a function of state variables,

$$x \approx \mathscr{X}(s,\theta), \tag{S42}$$

where θ are the parameters defining the spline approximation. To calculate this spline, we discretize the state space (using, for the storage-trade model, 41 points for availability and 23 for world price), and the spline has to hold exactly for all points of the grid.

The expectations operator in equation (S41) is approximated through a Gaussian quadrature (with 5 points on each dimension), which defines a set of pairs $\{e_l, w_l\}$ in which e_l represents a possible realization of shocks and w_l the associated probability. Using this discretization, and equations (S38) and (S41)–(S42), we can express the equilibrium equation (S39) as

$$\underline{x}(s) \leq x \leq \overline{x}(s) \quad \perp \quad f\left(s,x,\sum_l w_l h\left(s,x,e_l,g\left(s,x,e_l\right),\mathscr{X}\left(g\left(s,x,e_l\right),\theta\right)\right)\right). \tag{S43}$$

For a given spline approximation, θ, and a given s, equation (S43) is a function of x and can be solved using a mixed complementarity solver.

Once all the above elements are defined, we can proceed to the algorithm, which runs as follows:

1. Initialize the spline approximation, θ_0, based on a first-guess, x^0.
2. For each point of the grid of state variables, s_i, solve for x_i equation (S43) using the solver `PATH` (Dirkse and Ferris, 1995):

$$\underline{x}(s_i) \leq x_i \leq \overline{x}(s_i) \quad \perp \quad f\left(s_i,x_i,\sum_l w_l h\left(s_i,x_i,e_l,g\left(s_i,x_i,e_l\right),\mathscr{X}\left(g\left(s_i,x_i,e_l\right),\theta_n\right)\right)\right). \tag{S44}$$

3. Update the spline approximation using the new values of response variables, $x = \mathscr{X}(s,\theta_{n+1})$.
4. If $\|\theta_{n+1}-\theta_n\|_2 \geq 10^{-8}$ then increment n to $n+1$ and go to step 2.

Once the rational expectations equilibrium is identified, the spline approximation of the decision rules is used to simulate the model.

The storage model representing the world market is solved using this algorithm and a 50-node spline approximation. This spline approximation defines the continuous Markov chain (10) representing the world price dynamics. It is subsequently used in the storage-trade model as a transition equation, which means that it is a part of equation (S38).

For the optimal trade policy, the method requires a few modifications, since equation (S26) includes a derivative of $\mathscr{P}$, $\mathscr{P}$ being an approximation of the behavior of price with respect to state variables. The calculation of this approximation is actually carried out in the above method in step 3 (with $\mathscr{P}$ being an element of $\mathscr{X}$). Since derivatives of $\mathscr{P}$ appears only in expectations terms, to apply the method we have to make sure that the definition of the function h includes these terms, which leads to a new equation (S41):

$$z = \mathrm{E}\left[h(s,x,\dot{e},\dot{s},\dot{x},\mathscr{X}_s(\dot{s},\theta))\right], \tag{S45}$$

where $\mathscr{X}_s(\dot{s},\theta)$ is the derivative of $\mathscr{X}$ with respect to the next-period state variables.

This is only a sketch of the solution method. In fact, several methods are used in this paper to solve the models, depending on which one is the most efficient. For example, instead of using the simple updating rule in step 3, a Newton, or inexact Newton, updating is used when feasible. For more precisions, see the programs that generate all the results, and the RECS solver (Gouel, 2013), with which the models are solved.

References

Ambler, S. and Pelgrin, F. (2010). Time-consistent control in non-linear models. *Journal of Economic Dynamics and Control*, 34(10), 2215–2228.

Dirkse, S. P. and Ferris, M. C. (1995). The PATH solver: A non-monotone stabilization scheme for mixed complementarity problems. *Optimization Methods and Software*, 5, 123–156.

Fackler, P. L. (2005). A MATLAB solver for nonlinear rational expectations models. *Computational Economics*, 26(2), 173–181.

Gouel, C. (2013). RECS: MATLAB solver for rational expectations models with complementarity equations. Available at http://www.recs-solver.org.

Klein, P., Krusell, P. and Ríos-Rull, J.-V. (2008). Time-consistent public policy. *Review of Economic Studies*, 75(3), 789–808.

Marcet, A. and Marimon, R. (2011). *Recursive Contracts*. Discussion Paper 1055, CEP.

Miranda, M. J. and Glauber, J. W. (1995). Solving stochastic models of competitive storage and trade by Chebychev collocation methods. *Agricultural and Resource Economics Review*, 24(1), 70–77.