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***Abstract*— Simulated radar observations of the sea surface dynamics as used in the MODENA project, are based on an original methodology for sea states : the "groupy" wave model (GWM). Random wave fields can have very different modulations but nearly identical spectra. Nevertheless, the response of a floating object to waves depends strongly on the likelihood of large wave encounters. Sea surface fluxes also depend on wave breaking and air-flow separation, both being consequences of large-amplitude events. So wave group structure is one key description to simulate radar clutter under various environmental and instrumental configurations. The GWM builds on random distributions of wave groups and conditionally distributed breaking waves over these groups. Each wave group travels across the simulated area, and breaking waves appear dynamically on the wave crest at the rear of a group, propagating and disappearing at the front of this group. The generation of sea states follows a prescribed sea wave directional spectrum, and any breaking wave statistical distribution such as $\Lambda(\vec{c})d\vec{c}$ describing the total length of breaker per unit area and time with phase speed between $\vec{c}$ and $\vec{c}+d\vec{c}$. Accordingly, the group density per surface unit can lead to very different sea state structures, and the results will be discussed.**

I. Introduction

Description of sea states rests on statistical distributions for waves and breakings. Wind waves for example are described by a spectrum associating to each sea wave vector an average energy. These descriptions are associated to observations of sea-states during almost 20 minutes[Longuet 84]. This observation period averages the modulations of the wave field.

Random wave fields can have very different modulations but nearly identical spectra. Furthermore, the response of a floating object to waves depends strongly on the likelihood of large wave encounters. Sea surface fluxes also depend on wave breaking and air-flow separation, both being consequences of large-amplitude events.

If one takes the sinusoidal map as a primitive for the simulation of waves, the energy will be homogeneously distributed over the simulation surface and we'll have no explicit way to control the different modulations occuring in a wave field. Furthermore, to coherently distribute multiscale breaking events over such a wave description may not be so easy.

So wave group structure is one key description to simulate radar clutter under various environmental and instrumental configurations. Following previous work on group-based sea state simulations [Parenthoen 04], [LeGal 07], we propose an alternative to sinusoidal maps named "wave group maps", which does explicit the localisation of multiscale phenomena observed in sea-states : wave groups and breakers.

The Groupy Wave Model (GWM) builds on random distributions of wave groups and conditionally distributed breaking waves over these groups. Each wave group travels across the simulated area, and breaking waves appear dynamically on the wave crest at the rear of a group, propagating and disappearing at the front of this group.

The generation of sea states by the GWM can follow a prescribed sea wave directional spectrum, and any breaking wave statistical distribution such as $\Lambda(\vec{c})d\vec{c}$ describing the total length of breaker per unit area and time with phase speed between $\vec{c}$ and $\vec{c}+d\vec{c}$. Accordingly, the group density per surface unit can lead to very different sea state structures, and the results will be discussed.

II. The groupy wave model (GWM)

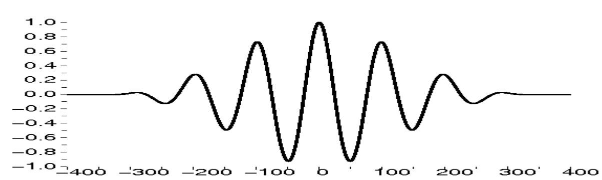
We present here the groupy wave model (GWM) for sea state simulation, based on a simple model for independant dynamical wave groups, which are distributed into wave group maps according to a prescribed directional spectrum.

A. Wave group model

Our wave group model is a reified Morlet wavelet, with its mean wave vector $\vec{k}$, its maximum amplitude a at its center $\vec{x}_0$ at $t=0$, its phase θ_0 at this center at $t=0$, its pulsation ω and its enveloppe extension ϱ. The pulsation ω also follows the open sea dispersion relationship :

$$\omega^2 = gk\left(1+(k/k_m)^2\right) \tag{1}$$

where $k_m = \sqrt{\rho_w g/T} \approx 370 rad.m^{-1}$.



Surface elevation $[m]$ in the $(\vec{k},\vec{z})$ plane function of position $[m]$ for a group whith parameters $k = 0.01 * 2\pi\ rad\,m^{-1}$, $a = 1\ m, \varrho = 5*\pi/k$ and θ such that the center $\vec{x}_c$ is a crest position.

Fig. 1. Our primitive for the groupy wave model

Such a single group acts on sea surface elevation $\eta(\vec{x}, t)$ as following :

$$\eta(\vec{x},t) = a \cdot \exp\left(\frac{-(\vec{x}-\vec{x}_c(t))^2}{2\varrho^2}\right) \cdot \sin\left(\vec{k}.(\vec{x}-\vec{x}_0)+\theta_0-\omega t\right) \quad (2)$$

with its center $\vec{x}_c$ moving at group speed in open sea from $\vec{x}_0$ at $t = 0$:

$$\vec{x}_c = \vec{x}_0 + t \cdot \frac{\vec{c}}{2} \quad (3)$$

where $\vec{c} = \omega \vec{k}/k^2$, is its phase speed. Waves are travelling at phase speed from the backward to the forward of the group.

Finally, the enveloppe extension ϱ of the group is chosen greater than the wavelength $2\pi/k$ and is proportional to the number of waves $n_{waves} \geq 2$ actually expressed by the group :

$$\varrho = n_{waves} \frac{\pi}{k} \quad (4)$$

Figure 1 illustrates the shape of such a group on the surface.

We make the assumption that sea surface can be described by the linear superposition of such independant wave groups : when several groups act, the resulting elevation is the sum of elevations due to each group.

B. Wave group maps

We organize groups with the same wave vector $\vec{k}$ into what we call a wave group map. Our map for a given wave vector $\vec{k}$ at a given time t, is the sum of a Morlet wavelet family, each wavelet has its own phase θ_n uniformly distributed in $[0, 2\pi[$, its own amplitude a_n following a Rayleigh law with parameter σ, and its own position center $\vec{x}_n$ uniformly distributed on a surface S. The number N of wavelets on a given surface S follows a Poisson law with parameter λ. The surface elevation η due to this map at this fixed time is then :

$$\eta(\vec{x}) = \sum_{n=1}^{N} h_n(\vec{x}, a_n, \vec{x}_n, \theta_n) \quad (5)$$

where h_n is the contribution of the n^{th} group :

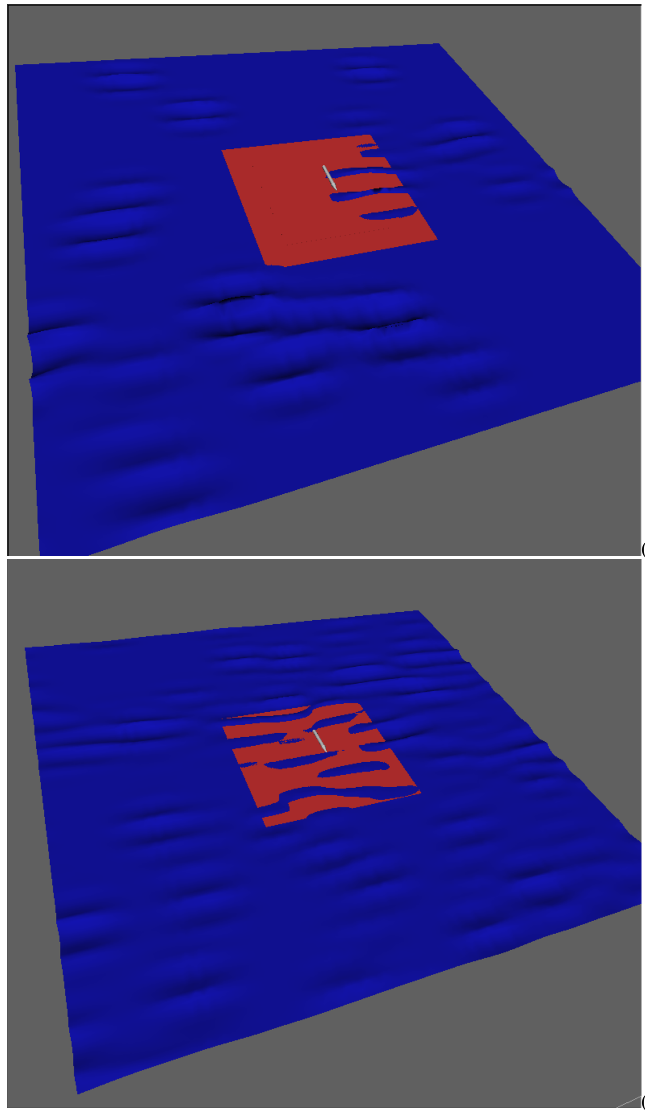
$$h_n(\vec{x}, a_n, \vec{x}_n, \theta_n, \varrho_n) = a_n \cdot \exp\left(\frac{-(\vec{x}-\vec{x}_n)^2}{2\varrho_n^2}\right) sin\left(\vec{k}.\vec{x}+\theta_n\right) \quad (6)$$

with a constant enveloppe extension $\varrho_n = \varrho(\vec{k}) = \frac{\pi n_{waves}(k)}{k}$, that specifies the number of waves per group for this map.

We suppose that random variables h_n follow the same law and are independant as function of independant variables :

- amplitude : $a_n \sim Rayleigh(\sigma)$
- position : $\vec{x}_n \sim Uniform_S$
- phase : $\theta_n \sim Uniform_{[0,2\pi[}$

Under these assumptions, to build a sea spectrum as a superposition of such maps rests on the simple summation of the spectra due to each map involved.



(1)

(2)

Two single wave group maps with the same wave number $k = 0.2\,rad\,m^{-1}$ and the same enveloppe extention $\varrho = 3 * \pi/k$. The surface S of the red squares (at level $z = 0$) is such that $\lambda_1/S = 1$ (up) while $\lambda_2/S = 4$ (down). The average steepnesses at the center of the groups of these maps are $\sigma_1 k = \pi.0.06$ and $\sigma_2 k = \pi.0.03$. Since $\lambda_1\sigma_1^2 = \lambda_2\sigma_2^2$, both blue sea-surfaces ($500\,m \times 500\,m$) present the same map spectrum.

Fig. 2. Maps with different wave structures but identical spectra

C. Wave group map spectrum

A group map on a surface S is fully determined by its wave vector $\vec{k}_0$, its Poisson parameter λ, its Rayleigh parameter σ and its enveloppe extention ϱ.

Lets start the computation of such a map spectrum by the two point caracteristical function :

$$\Phi(u, v) = \langle \exp\left((iu\eta(\vec{r}) - iv\eta(\vec{r}+\vec{x})\right)\rangle \quad (7)$$

where $\langle . \rangle$ is the mean over the whole processus. For our map, this can be rewriten as :

$$\Phi(u, v) = \exp\left(\lambda \left\langle \exp(iu\eta(\vec{r}) - iv\eta(\vec{r}+\vec{x})\right\rangle - 1\right) \quad (8)$$

and the mean is over $\vec{r}, a_n, \vec{x}_n, \theta_n$.

The autocorrelation function ρ_0 obeys :

$$\rho_0(\vec{x}) = \frac{\partial^2 \Phi(u,v)}{\partial_u \partial_v}|_{u,v=0} \tag{9}$$

Then we obtain the energy spectral density Ψ for this group map as the Fourier Transform (FT) of the autocorrelation function :

$$\Psi(\vec{k}) = \int \rho_0(\vec{x}) e^{-i\vec{k}.\vec{x}}\, d\vec{x}$$
$$= \frac{2\pi^2\lambda\sigma^2\varrho^4}{S}\left(e^{-2\pi^2\varrho^2\|\vec{k}-\vec{k_0}\|_2^2} - e^{-2\pi^2\varrho^2\|\vec{k}+\vec{k_0}\|_2^2}\right)^2 \tag{10}$$

where $\vec{k}_0$ is the wave vector of the map, λ/S is the mean number of groups per surface unit (or the group density), σ is the mean amplitude at the center of these groups and ϱ is their enveloppe extention.

Equation (10) shows that for a given wave number $\vec{k}_0$ and group extention ϱ, any product $\lambda \cdot \sigma^2 = C^{te}$ for the map parameters gives the same spectrum for those maps. If the number of wave group per surface unit is low ($\lambda/S < 1$), the group amplitude is high and the energy distribution could be very heterogeneous, while if the group density is high ($\lambda/S > 4$), the group amplitude is moderated and the energy is more homogeneously distributed over the surface as it is illustrated by figure 2.

D. Directional spectrum simulation by the GWM

We aim at simulating a prescribed directional spectrum Ψ_{target} by our wave group model with wave numbers range from k_{min} to k_{max}, and directions range from $-\pi/2$ to $\pi/2$, as for example a Donelan and Pierson spectrum, an Elfouhaily spectrum or any experimental spectrum. Naturally, we use our wave group maps. To do so, we not only need to discretize wave numbers and directions, but we also need to choose for each wave vector a density of groups per surface unit $\lambda(\vec{k})/S$ and a enveloppe extention $\varrho(\vec{k})$ depending on hypothesis about the heterogeneity of energy distribution in function of scale.

For example, the enveloppe extention could be :

$$\varrho(\vec{k}) = \frac{\pi n_{waves}}{k} \tag{11}$$

with a constant $n_{waves} > 3$; the corresponding assumption is an invariant of scale for group shapes. And the Poisson law parameters could be :

$$\lambda(\vec{k})/S = \frac{\varrho^2(\vec{k}) d_{groups}}{S} \tag{12}$$

with a constant $d_{groups} > 0$; corresponding to an invariant of scale for group density per surface unit.

The given spectrum can approximativly be covered by wave group maps so that :

$$\forall \vec{k},\ \Psi_{target}(\vec{k}) \approx \sum_j \Psi(\vec{k})_{\vec{k}_j, \lambda(\vec{k}_j), \sigma_j(\vec{k}_j), \varrho(\vec{k}_j)} \tag{13}$$

where $\Psi(\vec{k})_{\vec{k}_j, \lambda(\vec{k}_j), \sigma_j(\vec{k}_j), \varrho(\vec{k}_j)}$ given by the equation (10) is the spectral contribution of the j^{th} wave group map with its parameters $\vec{k}_j, \lambda(\vec{k}_j), \sigma_j(\vec{k}_j), \varrho(\vec{k}_j)$.

To numerically solve the previous equation with a given number n_{maps} of maps, we use the following algorithm :

`Initialisation` :

No wave group map exists yet : $j = 0$

We initialize the 0^{th} residual spectrum :

$$\Psi_0 = \Psi_{target}$$

`While` $j < n_{maps}$:

1) To find a $\vec{k}_j$ such that : $\Psi_j(\vec{k}_j) = \max(\Psi_j)$
2) To compute $\sigma_j = \sigma_j(\vec{k}_j)$ such that $\Psi(\vec{k}_j) = \max(\Psi_j)$ where $\Psi(\vec{k}_j)$ is given by equation (10, used with $\vec{k}_j$ instead of both $\vec{k}_0$ and $\vec{k}$), in function of group density λ_j/S_j and enveloppe extention ϱ_j parameters :
$$\sigma_j = \sqrt{\frac{S_j \max(\Psi_j)}{2\lambda_j \pi^2 \varrho_j^4 (1 - e^{-8\pi^2\varrho_j^2 k_j^2})}} \tag{14}$$
3) To update the j^{th} residual spectrum :
$$\Psi_{j+1} = \Psi_j - \Psi$$
where Ψ is the spectrum due to the j^{th} map given by replacing $\vec{k}_0, \frac{\lambda}{S}, \sigma, \varrho$ by $\vec{k}_j, \frac{\lambda_j}{S_j}, \sigma_j, \varrho_j$ in equation (10).
4) To increment j

`End`

where Ψ_j is the j^{th} residual spectrum, *e.g :* the difference between the target spectrum and the spectrum due to the j first maps.

With such a method, if n_{maps} is large enough, the last update $\Psi_{n_{maps}}$ of the residual spectrum should be nearly null. That is to say that the spectrum Ψ of the superposition of such n_{maps} maps equals approximativly Ψ_{target}.

Thus, we can obtain various sea-state multiscale structures with the same spectrum, as each map involved in the simulation can offer various wave structures at a given scale (the wave length of that map) with identical spectra.

III. Breaking wave distribution over wave groups

The sea surface is peopled by independant wave groups localising energy nearby their center. We'll see now how breaking events are added over these groups, respecting breaking statistics and offering some spatial coherency between breaking front and high wave crest positions.

A. Models for breaking statistics

Incremental breaking statistics $\Lambda(\vec{c})$ $[m^{-2}s]$ is the average length of breaking fronts per unit surface per unit speed interval [Phillips 85]. When a single breaking event starts, a turbulent foam patch is generally initiated at some point on the wave crest, and during the active breaking period, the pach speads laterally along the direction of travel of the wave. At any instant, its lateral dimention can be represented by a main axis having the shape of an arc segment. The length of the breaking front Λ at that particular time is a mesure of the length of this arc segment. As the wind blows over the water surface, at any instant, the fronts of the breaking waves therefore define a distribution of isolated lines or arc segments. The distribution $\Lambda(\vec{c})$ is such that $\Lambda(\vec{c})d\vec{c}$ represents the average total length per unit surface of breaking fronts that have velocities in the range $\vec{c}$ to $\vec{c}+d\vec{c}$.

The omni-directional distribution of breaking front length $\Lambda(c)$ is :

$$\Lambda(c) = \int_{-\pi/2}^{\pi/2} c\Lambda(\vec{c})d\theta \qquad (15)$$

Different models for incremental breaking statistics $\Lambda(\vec{c})$ or the omni-directional breaking statistics $\Lambda(c)$ are reviewed for example in [Reul 03] :

eq : a model for fully developped sea states [Phillips 85] :

$$\Lambda_{eq}(\vec{c}) = -4b'^{-1}mg^5c^{-14}u_*^2\Psi(\vec{k})cos(\theta) \qquad (16)$$

where $b' \approx 9.10^{-3}$ is the average value for unsteady breaker modified similarity factor, $m \approx 0.04 \pm 0.02$ is a parameter for wind induce wave growth rate, g is the acceleration due to gravity, u_* is the wind friction velocity, Ψ is the directional spectrum, and θ is the angle between $\vec{c}$ (or $\vec{k}$) and the wind vector.

emp : a model for the omnidirectional distribution of breaking front length based on experimental measure from an aircraft [Melville 02] :

$$\Lambda_{emp}(c) = (U_{10}/10)^3 \times 3.3 \times 10^{-4}e^{-0.64c} \qquad (17)$$

where U_{10} is the wind speed at 10 meters above the sea surface. The validity of this formula is for U_{10} ranges 7 to 14 ms^{-1}, and well developped sea states (wind wave fetch ranges 100 to 150 km).

dom : a statistical model for a narrow band spectrum (wave number k ranges $k_{peak}/2$ to $2k_{peak}$) :

$$c\Lambda_{dom}(c) = \frac{k}{2\pi}\exp\left(-\frac{\varepsilon_T^2}{\varepsilon_s^2}\right) \qquad (18)$$

where $\varepsilon_T \approx 0.24$ is a tuning constant [Makin 02], $\varepsilon_s = 2k_m M_{00}^{1/2}$ is the dominant wave steepness, $k_m = \sqrt{M_{20}/M_{00}}$ defines the mean wavenumber and $M_{mn} = \int k_x^m k_y^n \Psi(\vec{k})d\vec{k}$ are the spectral moments of order mn.

Thus, one can choose any appropriate model for incremental breaking statistics that we'll distribute over wave groups.

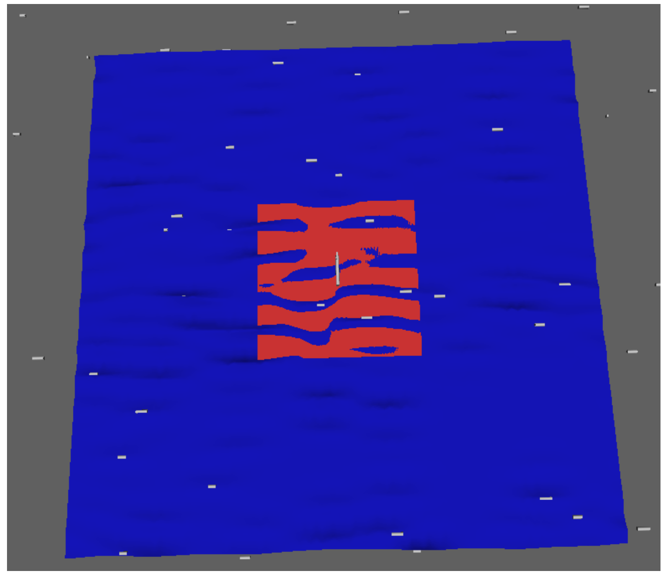
B. Conditional distribution of a breaking front length over a group map

We equip the wave groups of a wave group map with breaking fronts attached to wave crests. For that, we define a breaking area as a circle centred at the center $\vec{x}_c$ of the group with a radius $r_{breaking}$. Any crest belonging to this area is considered as a breaking wave (inside the breaking area) and its velocity $\vec{c}$ is then the phase speed of that group.

Lets call $l_{breaking}$ the mean breaking front length carried by such a group. Because for a given crest, its temporal mean length inside the breaking area during the crossing period is $\pi r_{breaking}^2/(2r_{breaking}) = \pi r_{breaking}/2$, and because the mean number of breaking crests is $2r_{breaking} \times k/2\pi$, we have :

$$r_{breaking} = \sqrt{\frac{2l_{breaking}}{k}} \qquad (19)$$

where k is the wave number of the group.



A wave group map expresses breaking fronts (white patches) on wave group crests with a breaking front length per surface unit $L = 10^{-3}\,m^{-1}$. Wave group map parameters are $k = 0.2\,rad\,m^{-1}$, $\varrho = 5 * 2\pi/k$. The surface S of the red square (at level $z = 0$) is such that $\lambda/S = 4$. The group map steepnesses is $\sigma k = \pi.0.03$. The cumulated breaking length over the blue sea-surface ($25000\,m^2$) is worth on average $25\,m$. Here, every groups are equiped with the same breaking area. This is a screenshot of a coherent dynamical simulation.

Fig. 3. Breaking fronts over the groups of a map

If one wants now to distribute a breaking front length L_j per unit area over the crests of a wave group map $\vec{k}_j, d_j, \sigma_j, \varrho_j$ where $\vec{k}_j$ is the wave vector, $d_j = \lambda_j/S_j$ the group density per unit surface (λ_j is the Poisson law parameter, *e.g :* the average number of groups covering the surface S_j), σ_j the average amplitude at the group center which follows a Rayleigh law and ϱ_j their enveloppe extention, the appropriate radius $\bar{r}_{breaking_j}$ so that this map j expresses the given breaking length is :

$$\bar{r}_{breaking_j} = \sqrt{\frac{2L_j}{k_j d_j}} = \sqrt{\frac{2L_j S_j}{k_j \lambda_j}} \qquad (20)$$

We also specify the breaker geometry using similarity laws [Bortkovskii 87]. Accordingly, the average whitecap thickness $\bar{\delta}_j$ for the breakers of the j^{th} map shall apply the following similarity law :

$$\bar{\delta}_j = b \cdot \frac{2\pi}{k_j} \qquad (21)$$

where b is an empirical constant estimated to be $b \approx 0.03$ for quasi-steady breakers [Ducan 81].

Figure 3 illustrates the spatial coherency that can be easily obtained by our conditional distribution of breaking fronts over the wave crests of the group map.

If we want to take attention to the Rayleigh distribution of amplitudes for the wave groups of that map, one can choose to set the breaking radius $r_{breaking_i}$ for a group which amplitude is a_i, with :

$$r_{breaking_i} = \frac{a_i}{\sigma_j} \bar{r}_{breaking_j} \tag{22}$$

this will increase the spatial coherency between breaking fronts and highest crests positions. Futhermore, we can decide that only groups with an amplidude greater than $\alpha_j \sigma_j$ express their breaking fronts ($\alpha_j > 0$) and the precedent formula becomes :

$$r_{breaking_i} = \frac{a_i}{\left(\overline{\{a_{\ (a > \alpha_i \sigma_j)}\}}\right)} \cdot \bar{r}_{breaking_j} \tag{23}$$

where $\overline{\{a_{\ (a > \alpha_j \sigma_j)}\}}$ is the mean value of a random variable a following a σ_j-Rayleigh law restricted by $a > \alpha_j \sigma_j$.

Thus, we know how to distribute a breaking front length L_j per surface unit over a group map by adding to each group i of the map j a breaking area specified by its breaking radius $r_{breaking_i}$ using equations (20) and (22) or (23).

C. Conditonal distribution of incremental breaking statistics $\Lambda(\vec{c})$ over group maps

We distribute now the average total length per unit surface of breaking fronts $\Lambda(\vec{c})d\vec{c}$, that have velocities in the range $\vec{c}$, $\vec{c} + d\vec{c}$ over the groups of a map so that the phase speed of the group crests ranges in the same set. As we don't have a continuum for $\vec{c}$ because we have discretised wave vectors for covering the full spectrum with a finite number of wave group maps, we decide to distribute breaking front lengths so that the total length L_{tot} of breaking fronts per surface unit is preserved :

$$L_{tot} = \int_0^{+\infty} \int_{-\pi/2}^{\pi/2} \Lambda(\vec{c}) d\vec{c} \tag{24}$$

Each map j will then be associated to a breaking front length L_j per surface unit as following :

$$L_j = \frac{\Lambda(\vec{c_j})}{\sum_j \Lambda(\vec{c_j})} \cdot L_{tot} \tag{25}$$

where $\vec{c_j}$ is the phase speed for the j^{th} group map.

If we only have the omni-directional distribution of breaking front length $\Lambda(c)$ cumulating $\Lambda(\vec{c})$ for all directions with the same speed norm c, we use in the precedent formula an angular spreading function (*c.f. :* [Elfouhaily 97]) to distribute $\Lambda(c)$ over different directions.

The precedent section explains how each L_j can be distributed over the groups of the j^{th} wave group map, thanks to equations (20) and (22) or (23).

Thus any incremental breaking statistics can be conditionally distributed over the wave crests of the groups mapping the sea surface. The whole system of waves and breaking fronts evolves dynamically thank to the GWM, while respecing any given directional spectrum and keeping some spatial coherency between phenomena (wave groups and breaking fronts) involved in this sea-state.

IV. Conclusion and discussion

The groupy wave model allows dynamical multiscale simulations of sea states involving simple models for wave groups and breakers. The GWM builds on random distributions of wave groups and conditionally distributed breaking waves over these groups. Each wave group travels across the simulated area, and each breaking wave appears dynamically on a wave crest at the rear of a group, propagating with crests and disappearing at the front of this group. We have shown how the GWM can build a multiscale sea state that follows any prescribed sea wave directional spectrum $\Psi(\vec{k})$ with any breaking wave statistical distribution $\Lambda(\vec{c})$.

In addition to these average statistics, the WGM user needs to choose his own hypothesis about wave field modulations in function of the scale ; namely the mean number of groups per surface unit and the mean number of waves per group. These parameters offer a wide range of different sea state structures, all of them offering the same average statistics. Futur work should study the impact of these parameters on the sea clutter.

Until now, we have used a model for the surface as the sum of random processes specifying vertical displacement. In fact water motion involves orbital movements with horizontal displacements highly correlated to vertical ones. We'll modify our wave group vertical primitive into an orbital primitive and spectral results obtained for the GWM will have to be dressed.

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