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On the Equivalence of Interleavers for Turbo Codes

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Abstract—Three of the most common interleavers for Turbo Codes (TCs) are Dithered Relative Prime (DRP) interleavers, Quadratic Permutation Polynomial (QPP) interleavers and Almost Regular Permutation (ARP) interleavers. In this paper it is shown that DRP and QPP interleavers can be expressed in the ARP interleaver function form. Furthermore, QPP interleavers can be seen as a particular case of ARP interleavers, in which the values of the periodic shifts follow the quadratic term of the QPP interleaver function. Some application examples of the equivalent expressions are provided. Particularly, in the QPP interleaver case, the different instances in the Long Term Evolution (LTE) standard are considered. Obtained results are useful when investigating a suitable and general permutation model for TCs.

Index Terms—Turbo codes, ARP interleaver, DRP interleaver, QPP interleaver, equivalence.

I. INTRODUCTION

THE interleaver is a key component of Turbo Codes (TCs). Its role is twofold. First, it has an important impact on the achievable minimum Hamming distance of the TC [1]. Second, due to its scattering properties, it also acts on the correlation of exchanged extrinsic information during the iterative decoding process [2]. In practical turbo coded systems, algebraic permutations are preferred to random-based permutations. In this case, permuted addresses can be computed via the application of a mathematical expression avoiding the use of storage elements or a look-up table. Therefore, they are easier to specify and implement. Three of the most popular interleavers with the above mentioned properties are Dithered Relative Prime (DRP) interleavers [3], Quadratic Permutation Polynomial (QPP) interleavers [4] adopted in LTE [5], and Almost Regular Permutation (ARP) interleavers [6] adopted in the DVB-RCS/RCS2 [7], [8] and WiMAX [9] standards. Until now, no generic permutation model for TCs has been established. In this paper, we show that a relation exists among these three families of interleavers. Indeed, we demonstrate that any DRP or QPP interleaver can be expressed as an ARP interleaver.

II. ALGEBRAIC INTERLEAVER MODELS FOR TCs

This section gives an overview of the algebraic interleavers analyzed in this letter.

A. The ARP Interleaver

The ARP interleaver was proposed by Berrou *et al.* [6]. It is based on a regular permutation of period P and a vector of

shifts S . The interleaving function is defined as:

$$\Pi_{\text{ARP}}(i) = (P \cdot i + S_{(i \bmod Q)}) \bmod K \quad (1)$$

where $i = 0, \dots, K-1$ denotes the address of the data symbol after interleaving and $\Pi_{\text{ARP}}(i)$ represents its corresponding address before interleaving. P is a positive integer relatively prime to K , K being the interleaver size. The disorder cycle or disorder degree in the permutation is denoted by Q , which corresponds to the number of shifts in S . K must be a multiple of Q .

B. The DRP Interleaver

As introduced by Crozier and Guinand [3], the DRP interleaver is composed of three interleaving stages:

$$\Pi_a(i) = R \lfloor i/R \rfloor + r_{(i \bmod R)} \quad (2)$$

$$\Pi_b(i) = (s + P \cdot i) \bmod K \quad (3)$$

$$\Pi_c(i) = W \lfloor i/W \rfloor + w_{(i \bmod W)} \quad (4)$$

where $\lfloor x \rfloor$ denotes the closest lower integer value with respect to x ; r and w are the read and write dither vectors with lengths R and W , respectively. The interleaver length K must be a multiple of the length of both dither vectors. P denotes the regular interleaver period, relatively prime to K , and s represents a constant shift. Then, the complete interleaver function is defined as:

$$\Pi_{\text{DRP}}(i) = \Pi_a(\Pi_b(\Pi_c(i))) \quad (5)$$

where $i = 0, \dots, K-1$ is the address of the data symbol after interleaving and $\Pi_{\text{DRP}}(i)$ represents its corresponding address before interleaving.

C. The QPP Interleaver

QPP interleavers, proposed by Sun and Takeshita [4], are based on permutation polynomials over the integer ring \mathbb{Z}_K where K corresponds to the interleaver length. Such an interleaver is completely defined by the algebraic expression:

$$\Pi_{\text{QPP}}(i) = (f_1 i + f_2 i^2) \bmod K \quad (6)$$

where $i = 0, \dots, K-1$ denotes the address of the data symbol after interleaving and $\Pi_{\text{QPP}}(i)$ represents its corresponding address before interleaving. For even data sequence lengths, the necessary and sufficient condition for the polynomial in (6) to define a valid permutation (i.e., one to one mapping) can have two different expressions [10], [11]:

- 1) 2^n divides K for $n > 1$: then, f_1 is relatively prime to K and all prime factors of K are also factors of f_2 .
- 2) 2^n divides K for $n = 1$, but not for $n > 1$: then, $f_1 + f_2$ is odd, f_1 is relatively prime to $\frac{K}{2}$ and all prime factors of K , excluding 2, are also factors of f_2 .

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According to these conditions, K and f_2 can be factorized in their prime factors as:

$$K = 2^{\alpha_{K,1}} \prod_{i=2}^{\omega(K)} p_i^{\alpha_{K,i}} \quad (7)$$

$$f_2 = 2^{\alpha_{f,1}} \prod_{i=2}^{\omega(K)} p_i^{\alpha_{f,i}} \prod_{i=\omega(K)+1}^{\omega(f)} p_i^{\alpha_{f,i}} \quad (8)$$

where $\omega(K)$ and $\omega(f)$ represent the number of different prime factors of K and f_2 , respectively. $\alpha_{f,1} = 0$ for $\alpha_{K,1} = 1$ and $\alpha_{f,1} > 1$ for $\alpha_{K,1} > 1$.

III. A UNIFIED DESCRIPTION FORM OF ALGEBRAIC INTERLEAVERS BASED ON THE ARP MODEL

In this section it is shown that DRP and QPP interleavers can be expressed as ARP interleavers.

A. DRP Interleavers Expressed as ARP Interleavers

Noting that for natural integers n and k , $n \lfloor k/n \rfloor$ is equal to $k - k \bmod n$, expressions (2) and (4) can be rewritten as:

$$\Pi_a(i) = (i - i \bmod R + r_{(i \bmod R)}) \bmod K \quad (9)$$

$$\Pi_c(i) = (i - i \bmod W + w_{(i \bmod W)}) \bmod K \quad (10)$$

Then, the term $\Pi_b(\Pi_c(i))$ in (5) can be expressed as:

$$\begin{aligned} \Pi_b(\Pi_c(i)) &= (s + P(i - i \bmod W + w_{(i \bmod W)})) \bmod K \\ &= (P \cdot i + s + P(w_{(i \bmod W)} - i \bmod W)) \bmod K \end{aligned} \quad (11)$$

Therefore, a vector with W periodic shifts, \mathbf{S}_W , can be defined as:

$$S_{W(i \bmod W)} = (s + P(w_{(i \bmod W)} - i \bmod W)) \bmod K \quad (12)$$

and (11) can be expressed as:

$$\Pi_b(\Pi_c(i)) = (P \cdot i + S_{W(i \bmod W)}) \bmod K \quad (13)$$

Substituting (13) and (9) into (5) gives the overall expression of the DRP interleaver:

$$\begin{aligned} \Pi_{\text{DRP}}(i) &= (P \cdot i + S_{W(i \bmod W)} - \\ &\quad (P \cdot i + S_{W(i \bmod W)}) \bmod R + \\ &\quad r_{((Pi + S_{W(i \bmod W)}) \bmod R)}) \bmod K \end{aligned} \quad (14)$$

Let M be the Least Common Multiple (LCM) of R and W . Then, a vector with M periodic shifts, \mathbf{S}_M , can be defined as:

$$\begin{aligned} S_{M(i \bmod M)} &= (S_{W(i \bmod W)} - (P \cdot i + S_{W(i \bmod W)}) \bmod R \\ &\quad + r_{((Pi + S_{W(i \bmod W)}) \bmod R)}) \bmod K \end{aligned} \quad (15)$$

Finally, (14) can be expressed in the form:

$$\Pi_{\text{DRP}}(i) = (P \cdot i + S_{M(i \bmod M)}) \bmod K \quad (16)$$

Given that K is a multiple of both M and Q , we can take $Q = M$ and the expression in (16) corresponds to the ARP interleaver function in (1). Therefore, it is shown that any DRP interleaver can be expressed in the ARP interleaver function form.

B. QPP Interleavers as Special Cases of ARP Interleavers

It can be shown that the QPP interleaver is a particular case of the ARP interleaver. Let us first recall the addition property of congruence [12]: if $a \equiv b \bmod n$ and $c \equiv d \bmod n$ then $ax + cy \equiv bx + dy \bmod n$ for all integers x and y .

Therefore, a sufficient condition for the existence of an ARP-equivalent form of a valid QPP interleaver is that the following equations hold:

$$\begin{cases} (P \cdot i) \bmod K = (f_1 i) \bmod K \\ S_{(i \bmod Q)} \bmod K = (f_2 i^2) \bmod K \end{cases} \quad (17)$$

(17) holds for:

$$P = f_1 \quad (19)$$

(18) holds for the trivial case in which $Q = K$. However, it is not a suitable value for implementation purposes, since K shift values have to be stored. It can be shown that a valid value for Q , smaller than the trivial case $Q = K$, can be found. Given that \mathbf{S} is a vector of Q periodic shifts, equation (18) is satisfied for:

$$(f_2 i^2) \bmod K = (f_2 (i + Q)^2) \bmod K \quad (20)$$

$$(f_2 i^2) \bmod K = (f_2 i^2 + 2f_2 Qi + f_2 Q^2) \bmod K$$

Then, it must be verified that:

$$(2f_2 Qi + f_2 Q^2) \bmod K = 0 \quad (21)$$

which holds for:

$$\begin{cases} (2f_2 Qi) \bmod K = 0 \\ (f_2 Q^2) \bmod K = 0 \end{cases} \quad (22)$$

(22) implies that:

$$2f_2 Q = lK, \quad l \in \mathbb{N}^+ \quad (24)$$

which is satisfied for:

$$Q = (lK)/(2f_2) \quad (25)$$

Let us factorize l in its prime factors as:

$$l = 2^{\alpha_{l,1}} \prod_{i=2}^{\omega(K)} p_i^{\alpha_{l,i}} \prod_{i=\omega(K)+1}^{\omega(f)} p_i^{\alpha_{l,i}} \quad (26)$$

Then, (25) with Q as a divisor of K holds for:

$$\begin{cases} \alpha_{l,1} \in \{0, 1\}, \quad \text{for } \alpha_{K,1} = 1 \end{cases} \quad (27)$$

$$\begin{cases} \alpha_{f,1} - \alpha_{K,1} + 1 \leq \alpha_{l,1} \leq \alpha_{f,1} + 1, \quad \text{for } \alpha_{K,1} > 1 \end{cases} \quad (28)$$

$$\begin{cases} \alpha_{f,i} - \alpha_{K,i} \leq \alpha_{l,i} \leq \alpha_{f,i}, \quad i = 2, \dots, \omega(K) \end{cases} \quad (29)$$

$$\begin{cases} \alpha_{l,i} = \alpha_{f,i}, \quad i = \omega(K) + 1, \dots, \omega(f) \end{cases} \quad (30)$$

(23) implies that:

$$f_2 Q^2 = mK, \quad m \in \mathbb{N}^+ \quad (31)$$

With Q as defined in (25), (31) evaluates to:

$$(l/2)^2 (K/f_2) K = mK \quad (32)$$

According to (32), $(l/2)^2 (K/f_2) \in \mathbb{N}^+$, which holds for:

$$\begin{cases} \alpha_{l,1} = 1, \quad \text{for } \alpha_{K,1} = 1 \end{cases} \quad (33)$$

$$\begin{cases} \frac{\alpha_{f,1} - \alpha_{K,1}}{2} + 1 \leq \alpha_{l,1}, \quad \text{for } \alpha_{K,1} > 1 \end{cases} \quad (34)$$

$$\begin{cases} \frac{\alpha_{f,i} - \alpha_{K,i}}{2} \leq \alpha_{l,i}, \quad i = 2, \dots, \omega(K) \end{cases} \quad (35)$$

Afterwards, we have to find a range for the exponents of the prime factors of l , $\alpha_{l,i}$, validating the conditions that verify equations (25) and (32).

First, $\alpha_{l,1} = 1$ validates both conditions (33) and (27) for $\alpha_{K,1} = 1$. Then, for $\alpha_{K,1} > 1$, two cases have to be considered:

$$\begin{cases} \alpha_{K,1} < \alpha_{f,1} : \text{the valid range for } \alpha_{l,1} \text{ is given by (28)} \\ \alpha_{K,1} \geq \alpha_{f,1} : \frac{\alpha_{f,1} - \alpha_{K,1}}{2} + 1 \leq \alpha_{l,1} \leq \alpha_{f,1} + 1 \end{cases} \quad (36)$$

The same applies for $\alpha_{l,i}$, $i = 2, \dots, \omega(K)$:

$$\begin{cases} \alpha_{K,i} < \alpha_{f,i} : \text{the valid range for } \alpha_{l,i} \text{ is defined by (29)} \\ \alpha_{K,i} \geq \alpha_{f,i} : \frac{\alpha_{f,i} - \alpha_{K,i}}{2} \leq \alpha_{l,i} \leq \alpha_{f,i} \end{cases} \quad (37)$$

Finally, for $\alpha_{l,i}$, $i = \omega(K) + 1, \dots, \omega(f)$, the exponent values are obtained from (30). It is noteworthy that negative lower bounds for $\alpha_{f,i}$, in (36) or (37) must be replaced with zero since $\alpha_{f,i} \in \mathbb{N}^0$.

Therefore, an equivalent ARP function for a valid QPP interleaver is obtained by choosing P equal to f_1 with a vector of shifts \mathbf{S} of length Q . The possible values for Q are determined by (25), for l defined in (26). Let Q_s be the smallest possible value for Q . It corresponds to the minimum disorder degree for which an equivalent ARP can be found. Other possible values for Q are then multiples of Q_s provided that K remains a multiple of Q . Then, the Q periodic shifts of \mathbf{S} are obtained from (18) for $i = 0, \dots, Q-1$. Note that the shift values of \mathbf{S} for multiples of Q_s are just periodic repetitions of those obtained for Q_s . Then, only the shift values for Q_s have to be found. Thus, the QPP interleaver is a particular case of the ARP interleaver in which the values of the periodic shifts follow the relation (18).

IV. ADVANTAGES OF THE PROPOSED UNIFIED REPRESENTATION OF INTERLEAVERS FOR TCS

As shown in [13], differences seem to exist between achievable Hamming distances of QPP and DRP-based interleavers (mostly in favor of DRP ones). Since any DRP or QPP interleaver can be represented by an ARP interleaver (the other way around is not necessarily true), the Hamming distances achievable by these former structures are attainable by the ARP interleaver. Thus, one can argue that the corresponding values set a lower bound on the achievable distances by ARP-based interleavers. Furthermore, since TCs have found their way to several standards with different interleaving structure in each (e.g., QPP for LTE and ARP for WiMAX and DVB-RCS), unifying the interleaving structure have an advantage for implementation purposes. Indeed, for a hardware implementation, one can design an ARP interleaver that can be used for ARP interleavers and that can support QPP ones. Therefore, the overall complexity can be reduced when both interleaving structure have to be supported in the same chip.

V. APPLICATION EXAMPLES

The obtained equivalent expressions of DRP and QPP interleavers in the form of the ARP are applied in some examples.

TABLE I
PARAMETERS OF THE DRP INTERLEAVERS.

DRP	K	s	P	\mathbf{r}	\mathbf{w}
I	784	73	25	(2, 5, 1, 0, 4, 6, 3)	(6, 3, 2, 4, 0, 1, 5)
II	784	13	33	(1, 0, 3, 5, 2, 4, 6)	(3, 2, 6, 4, 5, 1, 0)
III	6144	14	263	(1, 0)	(2, 1, 0)
IV	6144	19	107	(1, 0, 2)	(1, 0)

TABLE II
EQUIVALENT ARP INTERLEAVERS TO THE DRP FROM TABLE I.

ARP	Q	$S(0)$	$S(1)$	$S(2)$	$S(3)$	$S(4)$	$S(5)$	$S(6)$
I	7	220	127	73	99	754	759	47
II	7	113	47	144	45	48	663	599
III	6	541	13	5633	539	15	5631	-
IV	6	127	6055	125	6056	126	6057	-

A. Equivalent ARP interleavers for the DRP Case

In the first application example, a DRP interleaver of length 784 is considered. A length of 7 is selected for the dither vectors. Two different configurations of these vectors are analyzed and listed in Table I. The disorder degree Q of the equivalent ARP interleavers is set to the dither vector size value. The regular permutation period P of each ARP interleaver is the same as the one used in the original DRP version. Then, the 7 shifts of the equivalent ARP interleavers are obtained from (15) and listed in Table II.

In the second example, a DRP of length 6144 is analyzed. Two different dither vector lengths, 2 and 3, as well as two different configurations are considered and listed in Table I. Then, the disorder degree of the equivalent ARP is set to 6, which corresponds to the LCM of 2 and 3. The regular permutation period is again selected as the one used in the DRP interleaver and the 6 shifts of the equivalent ARP interleavers are given by (15). The corresponding shift values are listed in Table II. It was verified that the interleaved addresses obtained with the equivalent ARP interleavers from Table II are the same as those obtained with the respective DRP interleavers from Table I.

B. Equivalent ARP interleavers for the QPP Case

In this case, two instances of the QPP interleaver defined in the LTE standard [5] are detailed. Let us first take the interleaver of length 216 for which $f_2 = 36$. The interleaver length and f_2 are factorized in their prime factors as: $216 = 2^3 3^3$ and $36 = 2^2 3^2$, respectively. Therefore, the values for l , leading to valid values for Q , are factorized as: $l = 2^{\alpha_{l,1}} 3^{\alpha_{l,2}}$, following (26). According to (36), the valid range for $\alpha_{l,1}$ is $1 \leq \alpha_{l,1} \leq 3$. Afterwards, the valid range for $\alpha_{l,2}$, $0 \leq \alpha_{l,2} \leq 2$, is obtained from (37). Then, from these ranges, the group of valid values for l is calculated. Finally, the possible values for Q are obtained from (25). The respective values for Q are listed in Table III. A minimum disorder degree of 6 is found. The shift values of \mathbf{S} for this interleaver and for those of length 432, 720 and 1008, for which $Q_s = 6$, are listed in Table IV.

TABLE III
POSSIBLE VALUES FOR Q IN THE QPP INTERLEAVER WITH $K = 216$,
 $f_1 = 11$ AND $f_2 = 36$.

l	2	4	6	8	12	18	24	36	72
Q	6	12	18	24	36	54	72	108	216

TABLE IV
EQUIVALENT ARP INTERLEAVERS WITH $P = f_1$, $Q_s = 6$ AND $S(0) = 0$.

K	216	432	720	1008
f_1	11	47	79	55
f_2	36	72	120	84
$S(1)$	36	72	120	84
$S(2)$	144	288	480	336
$S(3)$	108	216	360	756
$S(4)$	144	288	480	336
$S(5)$	36	72	120	84

TABLE V
POSSIBLE VALUES FOR Q IN THE QPP INTERLEAVER WITH $K = 1696$,
 $f_1 = 55$ AND $f_2 = 954$.

l	9	18	36	477	954	1908
Q	8	16	32	424	848	1696

TABLE VI
EQUIVALENT ARP INTERLEAVERS WITH $P = f_1$, $Q_s = 8$, $S(0) = S(4) = 0$.

K	1696	768	3264	5376
f_1	55	217	443	251
f_2	954	48	204	336
$S(1)$	954	48	204	336
$S(2)$	424	192	816	1344
$S(3)$	106	432	1836	3024
$S(5)$	106	432	1836	3024
$S(6)$	424	192	816	1344
$S(7)$	954	48	204	336

In another example, the interleaver of length 1696, for which $f_2 = 954$, is considered. K and f_2 are factorized as: $1696 = 2^5 53^1$ and $954 = 2^1 53^1 3^2$, respectively. Thus, according to (26), the valid values for l are factorized as: $l = 2^{\alpha_{l,1}} 53^{\alpha_{l,2}} 3^{\alpha_{l,3}}$. According to (36), the valid range for $\alpha_{l,1}$ is $0 \leq \alpha_{l,1} \leq 2$. Then, the valid range for $\alpha_{l,2}$, $0 \leq \alpha_{l,2} \leq 1$, is obtained from (37). Finally, $\alpha_{l,3} = 2$ as obtained from (30). Table V lists the possible values for Q , obtained from (25) with the valid values of l . A minimum disorder degree of 8 is found. The shift values of S for this interleaver and for those of length 768, 3264 and 5376, for which $Q_s = 8$, are listed in Table VI.

The minimum disorder degree, Q_s , obtained for the 188 different instances of the LTE QPP interleaver, is shown in Fig. 1. It was verified that the interleaved addresses obtained with the corresponding ARP interleavers are the same as those obtained with the respective QPP interleavers.

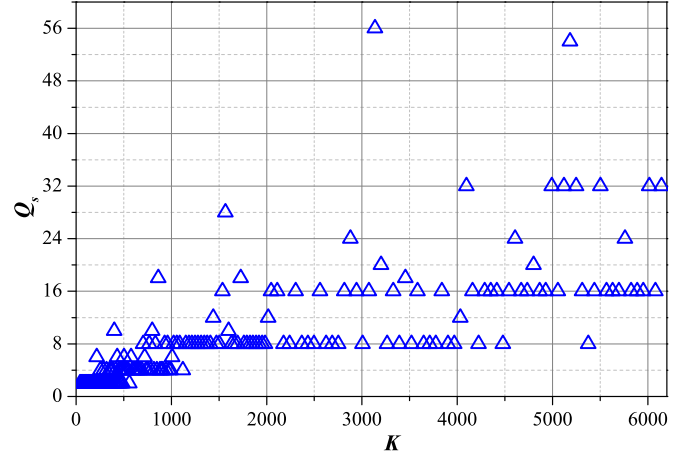


Fig. 1. Minimum disorder degree Q_s of the equivalent ARP interleavers for the different sizes K of the LTE QPP interleaver.

VI. CONCLUSION

In this letter, it was shown that DRP and QPP interleavers can be expressed as ARP interleavers. In addition, QPP interleavers were found to be special cases of ARP interleavers in which the values of the periodic shifts follow the quadratic term of the QPP interleaver function. Thus, the same interleaving properties of DRP or QPP interleavers can be provided by ARP interleavers. Therefore, the ARP interleaver is a sufficient permutation model to design TCs with the achievable asymptotic performance from any of the three interleaver families.

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