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► **To cite this version:**

| Stéphane Crépey, Armenti Yannick. Central Clearing Valuation Adjustment. 2015. hal-01169169v1

**HAL Id: hal-01169169**

**<https://hal.science/hal-01169169v1>**

Preprint submitted on 29 Jun 2015 (v1), last revised 3 Feb 2017 (v3)

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# Central Clearing Valuation Adjustment

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June 29, 2015

## Abstract

We develop an XVA analysis of centrally cleared trading, parallel to the one that has been developed in the last years for bilateral transactions. A dynamic framework incorporates the sequence of the cash-flows involved in the waterfall of resources of the CCP. The total cost of the clearance framework for a member of the clearinghouse, called CCVA for central clearing valuation adjustment, is decomposed into a nonstandard CVA corresponding to the cost of the losses on the default fund in case of defaults of other members, an FVA corresponding to the cost of funding its position (including all the margins) and a KVA corresponding to the cost of regulatory capital (and for completeness we also incorporate a DVA term). This framework can be used by a clearinghouse to assess the right balance between initial margins and default fund in order to minimize the CCVA, hence optimize its costs for a given level of resilience. A clearinghouse can also use it to analyze the benefit for a dealer to trade centrally as a member, rather than on a bilateral basis, or to help clearing members risk manage their CCVA. The potential netting benefit of central clearing and the impact of the credit risk of the members are illustrated numerically.

**Keywords:** Counterparty risk, central counterparty (CCP), margins, default fund, cost of funding, cost of capital, netting.

## 1 Introduction

To cope with counterparty risk, the current trend of the regulation is to push dealers to clear their trades via CCPs, i.e. central counterparties (or clearinghouses). Progressively, central clearing is even becoming mandatory for flow products. Centrally cleared trading mitigates counterparty risk through an extensive netting of all transactions. Moreover, on top of the

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\*The research of Stéphane Crépey benefited from the support of the Chair Markets in Transition under the aegis of Louis Bachelier laboratory, a joint initiative of École polytechnique, Université d'Évry Val d'Essonne and Fédération Bancaire Française.

variation and initial margins that are also used in the context of bilateral transactions, a clearinghouse deals with extreme and systemic risk on a mutualization basis, through an additional layer of protection, called the default or guarantee fund, which is pooled between the clearing members. In this paper we study the resulting cost of the clearance framework for a member of the clearinghouse.

## 1.1 Review of the Literature

Most of the CCP literature deals with the related systemic and liquidity issues. First, there is the danger of creating “too big to fail” clearinghouses (see Duffie (2010) and Cont, Santos, and Moussa (2013)). A second problem is fragmentation. In practice, clearing is typically organized by asset classes, so that “default” on one asset class (called service closure) does not harm the activity of the clearinghouse on other markets—and also because otherwise, in case of a default, holders of more liquid assets (e.g. interest rate swaps) would be advantaged with respect to holders of less liquid assets (e.g. CDS). But this implies a large number of clearinghouses (or clearinghouse services), whereas Duffie and Zhu (2011) have argued that a large number of clearinghouses fragments the market. A contrario Cont and Kokholm (2014) claim that this conclusion only holds under unrealistic homogeneity assumptions on the financial network. Third, the extensive collateralization prompted by the expansion of centrally cleared trading (and also bilateral trading under the current “standard CSA” procedures) requires a huge amount of cash or liquid assets, which puts a high pressure on liquidity (see Singh and Aitken (2009), Singh (2010), Levels and Capel (2012) and Duffie, Scheicher, and Vuillemeys (2014)). Liquidity in the sense of the optimal liquidation of the portfolio of a defaulted member by the clearinghouse is a fourth important issue that is considered in Avellaneda and Cont (2013).

Margin schemes are studied under various respects in Hurlin and Perignon (2012), Cruz-Lopez, Harris, Hurlin, and Perignon (2013) and Menkveld (2014). The first paper proposes a validation framework. The other two papers (see also Armenti, Crépey, Drapeau, and Papapantoleon (2015)) propose methodologies to assess central clearing margin requirements. Closer to the pricing perspective of this paper, Cont, Mondescu, and Yu (2011) and Pallavicini and Brigo (2013) analyze the pricing implications of the differences between the margining procedures involved in bilateral and centrally cleared transactions. Brigo and Pallavicini (2014) and Crépey and Song (2015) adapt their respective bilateral counterparty risk analyses to the case of centrally cleared trades viewed from the perspective of a client (as opposed to a member) of a clearinghouse. Hence, they can ignore the default fund and credit risk dependence issues that are inherent to the position of a clearing member.

Until recently, the cost analysis of CCPs, our focus in this paper, was only considered in an old business finance literature reviewed in Knott and Mills (2002), notably Fenn and Kupiec (1993). In the last years, new papers have appeared in this direction. Under stylized assumptions, Arnsdorf (2012) derives an explicit approximation to a CCVA (using the terminology of the present paper), including effects such as wrong way risk (meant as procyclicality of the margins), credit dependence between members and left tailed distributions of the P&Ls. Ghamami (2014) proposes a static one-period model where a CCVA can be priced by Monte-Carlo. We also include in our setup margining funding costs, for which Green and Kenyon (2014) present a computationally efficient method of calculation, and capital costs (KVA), following the lines introduced in the bilateral case by Green, Kenyon, and Dennis (2014).

## 1.2 Contribution and Outline

This paper develops an XVA (costs) analysis of centrally cleared trading, parallel to the one that has been developed in the last years for bilateral transactions (see e.g. Brigo, Morini, and Pallavicini (2013) or Crépey, Bielecki, and Brigo (2014, Parts II and III)). A dynamic framework incorporates the sequence of the cash-flows involved in the waterfall of resources of the clearinghouse. Moreover, as opposed to Arnsdorf (2012) and Ghamami (2014), Our CCVA accounts for the central clearing analog of the CVA, which is the cost for a member of the losses on the default fund due to realized breaches, for the FVA cost of funding its position (including all the margins) and for the KVA cost of the regulatory capital that is required for being a member of the clearinghouse (and for completeness we also incorporate a DVA term). Beside the theoretical interest, the framework of this paper can be used by a clearinghouse to find the right balance between initial margins and default fund in order to minimize the CCVA, hence optimize its costs for a given level of resilience. A clearinghouse can also use it to analyze the benefit for a dealer to trade centrally as a member, rather than on a bilateral basis, or to help its members risk manage their CCVA. To demonstrate the practicality of our approach, a numerical simulation assesses the netting benefit of central clearing and the impact of the credit risk of the members.

The paper is organized as follows. Sect. 2 presents our clearinghouse setup. The margin waterfall is described in Sect. 3. A BSDE based pricing and CCVA analysis is conducted in Sect. 4. Sect. 5 specializes the CCVA BSDE to a common shock model that is used for modeling the default times of the members of the clearinghouse. Sect. 6 provides an executive summary of the centrally cleared analysis of this paper and of a bilateral CSA methodology adapted, for comparison purposes, from Crépey and Song (2015). Sect. 7 designs an experimental framework used in the numerics of Sect. 8. Sect. 9 concludes.

## 1.3 Basic Notation and Terminology

$\int_a^b = \int_{(a,b]}$  with, in particular,  $\int_a^b = 0$  whenever  $a \geq b$ ;  $x^+ = \max(x, 0)$ ,  $x^- = \max(-x, 0) = (-x)^+$ ;  $\delta_a$  represents a Dirac measure at the point  $a$ ;  $\lambda$  denotes the Lebesgue measure on the nonnegative half-line  $\mathbb{R}_+$ . A time dependence is denoted in functional form by  $\cdot(t)$ , when deterministic, and as a subscript, by  $\cdot_t$ , for a stochastic process. Unless otherwise stated, a “deterministic function” of real arguments is measurable with respect to the corresponding Borel  $\sigma$  field (and a function involving discrete arguments is always considered continuous with respect to these, in reference to the discrete topology); a filtration satisfies the usual conditions; a price process is a special semimartingale in a càdlàg version; all inequalities between random quantities are to be understood almost surely or almost everywhere, as suitable; all the cash flows are assumed to be integrable whenever required; by “martingale” we mean local martingale, but true martingality is assumed whenever necessary. The last point means that we only derive local martingality properties. Usually in applications one needs true martingales, but this is not a real issue in our case, where even square integrability typically follows separately from additional assumptions classically postulated when dealing with backward stochastic differential equations (BSDEs for short), which are our main pricing tool in this paper.

## 2 Clearinghouse Setup

We model a service of a clearinghouse dedicated to proprietary trading (typically on a given market) between its members, labeled by  $i \in N = \{0, \dots, n\}$ . By comparison with proprietary trading, customer (external) trades are heavily overmarginated, as treated on a trade-by-trade basis, without any offsetting benefit. As a consequence, proprietary trading between members is the most important risk management issue for the clearinghouse, whence our focus in the paper.

### 2.1 From Bilateral to Centrally Cleared Trading

In a centrally cleared setup, the clearinghouse interposes itself in all transactions, becoming, quoting Knott and Mills (2002), “the buyer to every seller and the seller to every seller”. All the transactions between the clearinghouse and any given counterparty are then netted together. See Figure 1 for an example, where the circled numbers 50, 70 and 80 in the left diagram (resp. the numbers in the right diagram) show the gross positions of  $n = 3$  counterparties in a bilateral CSA setup (resp. their net positions after interfacing of all transactions by the CCP added in the middle). In addition, the clearinghouse asks for several layers of margins to be posted by the members as a guarantee against counterparty risk. Margins include a default fund that is pooled between the members. The expected benefits of centrally cleared trading are reduction of contagion risk and netting benefit. The expected drawbacks are an increase of systemic risk, where too big to fail hubs (CCPs) are created, and liquidity risk, due to the liquidity requirements for the margins.

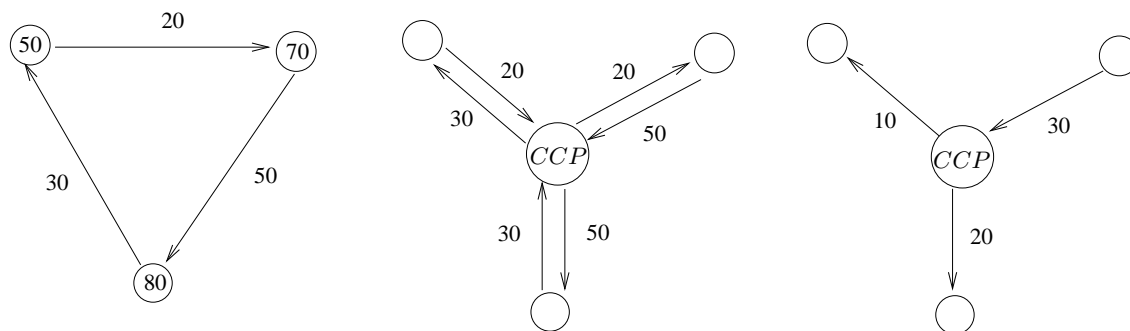


Figure 1: From bilateral to centrally cleared trading.

### 2.2 Liquidation Procedure

In practice, transactions with defaulted members are typically reallocated through a gradual liquidation of assets in the market (see Avellaneda and Cont (2013)) and/or auctions among the surviving members for the residual assets at the end of the liquidation period, a time interval of length  $\delta$  usually estimated to a few days. For ease of analysis in this paper, we simply assume the existence of a risk-free “buffer” that is used by the clearinghouse for replacing defaulted members in their transactions with others after a period of length  $\delta$  (the defaulted transactions already involving the buffer as one counterparty are simply terminated). The buffer can be viewed as an additional, risk-free member, which therefore doesn’t post any margins. But, in practice, the buffer need not correspond to an actual

member. It can be implemented by the clearinghouse through back to back hedges of the defaulted transactions with (virtually) risk-free counterparties. In addition, consistent with the practice of clearinghouses to protect themselves against speculation on the portfolio of a defaulted member, we assume that during the liquidation period (time interval of length  $\delta$  between default and buffer substitution), the promised contractual cashflows and the hedge of a defaulted member are taken over by the clearinghouse. Hence, the clearinghouse bears all the risks of the position of the defaulted member during the liquidation period.

### 2.3 Pricing Framework

Let  $(\Omega, \mathcal{G}, \mathbb{G}, \mathbb{Q})$ , with  $\mathbb{G} = (\mathcal{G}_t)_{t \in \mathbb{R}_+}$ , represent a stochastic pricing basis, such that all our processes are  $\mathbb{G}$  adapted and all the random times of interest are  $\mathbb{G}$  stopping times. The meaning of a pricing (or risk-neutral) measure in our setup, with different funding rates in particular, will be specified in Sect. 4.2-4.3 by a martingale condition regarding prices on the hedging market, along with a pricing BSDE regarding the valuation of the position of a member. We denote by  $r_t$  a progressive OIS rate process (overnight indexed swap rate, the best market proxy for a risk-free rate as well as the reference rate for the remuneration of the collateral) and by  $\beta_t = e^{-\int_0^t r_s ds}$  the corresponding discount factor. For each member  $i$ , we represent by  $D_t^i$  a finite variation cumulative promised dividend process of its portfolio (contractual cash-flows ignoring counterparty risk) and by  $P_t^i$  the corresponding mark-to-market (risk-neutral conditional expectation of future promised cash flows discounted at the OIS rate  $r_t$ ), both considered from the point of view of the clearinghouse, e.g.  $P_t^i = 1$  means that the members  $i$  is short of a mark-to-market value equal to one (disregarding margins etc.) toward the clearinghouse at time  $t$ . Writing  $\mathbb{E}_t$  for the  $(\mathcal{G}_t, \mathbb{Q})$  conditional expectation, the mark-to-market  $P_t^i$  corresponds to the classical no arbitrage risk-neutral valuation formula

$$\beta_t P_t^i = \mathbb{E}_t \left( \int_t^{\bar{T}} \beta_s dD_s^i \right), \quad t \in [0, \bar{T}], \quad (2.1)$$

where  $\bar{T}$  is a time horizon relevant for the clearinghouse (if there is some residual value in the portfolio at that time, it is treated as a terminal dividend  $(D_{\bar{T}}^i - D_{\bar{T}-}^i)$ ). Since all trades are between the members, we have  $\sum_{i \in N} P^i = 0$ . The portfolio of any member is assumed fixed over  $[0, \bar{T}]$  (unless the member defaults). In practice, portfolios may evolve in time (even if, in the case of derivative portfolios, it's not uncommon that positions are kept relatively constant), but one can only develop the analysis by making some assumption in this regard and the constant portfolio assumption is also the one that is prevailing in the classical bilateral XVA analysis.

## 3 Margin Waterfall Analysis

Ignored by the mark-to-market pricing formula (2.1), any member  $i$  is defaultable, with default time  $\tau_i$  and survival indicator process  $J^i = \mathbb{1}_{[0, \tau_i)}$ . As a first counterparty risk mitigation tool, the members are required to post/withdraw variation margins that track the mark-to-market of their portfolios. A clearinghouse can call for variation margins several times per day (versus at most daily in practice in the case of bilateral transactions). But various frictions and delays, notably the liquidation period  $\delta$ , imply gap risk, i.e. the risk of a gap between the variation margin and the debt of a defaulted member at the time

of liquidation of its portfolio. This is a special concern for certain classes of assets, such as credit derivatives, that may have quite unpredictable cashflows. This is why another layer of collateralization, called initial margins (as opposed to the variation margin that only accounts for market risk), is maintained in centrally cleared transactions as well as under bilateral transactions under a sCSA (standard CSA). Initial margins are updated too, up to a daily basis, based on risk measures of the variation-margined P&L of each member computed over a time horizon reflecting the length of the so called margin period of risk (liquidation period  $\delta$  plus maximal time  $h$  elapsed since the last margin call before the default). However, variation and initial margins still leave room for residual risk. Gap risk is magnified by wrong-way risk, i.e. adverse dependence between the portfolios and the credit risk of the members. One also needs to account for credit contagion effects between members (again, these are of special concern regarding credit derivatives). Clearinghouses deal with such extreme and systemic risks through an additional layer of margins, namely a default fund contributed by the members. The default fund contribution of each member is primarily intended to reimburse the losses that occur in case it defaults, but, if rendered necessary by exhaustion of the previous layers of the waterfall, it can also be used for reimbursing the losses due to the defaults of other members.

### 3.1 Margins

Let  $lh$  (respectively  $lT$ ),  $l \geq 0$ , with  $T$  a multiple of  $h$ , e.g.  $h =$  one day and  $T =$  one month, represent the times of the variation and initial (respectively default) margin calls. Consistent with our sign convention that  $P^i$  is the mark-to-market of the portfolio of the member  $i$  from the perspective of the clearinghouse, we count a margin positively when it is posted by the member and we define its variation margin  $VM^i$ , initial margin  $IM^i$  and default fund contribution  $DF^i$  as piecewise constant processes reset at their respective margin call grid times (whilst  $i$  is alive) following, respectively:

$$VM_{lh}^i = P_{lh-}^i, \quad IM_{lh}^i = \rho_{lh}^i, \quad DF_{lT}^i = \varrho_{lT}^i, \quad (3.1)$$

based on suitable risk measures as explained below. Note that (3.1) defines the level of reset of the cumulative margins. Starting from  $VM_0^i = P_{0-}^i, IM_0^i = \rho_0^i, DF_0^i = \varrho_0^i$ , the corresponding margin calls at the subsequent margin grid times (as long as the member  $i$  is alive) are  $(P_{lh-}^i - P_{(lh-h)-}^i), (\rho_{lh}^i - \rho_{lh-h}^i)$  and  $(\varrho_{lT}^i - \varrho_{lT-T}^i)$ .

**Remark 3.1** In practice, the variation margin only tracks the mark-to-market of the portfolio up to some thresholds (or free credit lines of the members) and minimal transfer amounts (to avoid useless transfers). These frictions, which can be important in the case of bilateral transactions (depending on the CSA), are omitted here as negligible in the case of cleared transactions.

The risk measure that is used for fixing the initial margins is a marginal risk measure, at the level of each member individually, which we formalize by

$$\rho_{lh}^i = \rho \left( P_{lh+(\delta+h)}^i + \int_{[lh, lh+(\delta+h)]} e^{\int_s^{lh+(\delta+h)} r_u du} dD_s^i - VM_{lh}^i \right), \quad (3.2)$$

where  $\rho$  can be value at risk, expected shortfall, etc.. Here the dependence between the portfolios of the members is only represented by the structural constraint that  $\sum_{i \in N} P^i = 0$ .

**Remark 3.2** Traditionally, for fixing the initial margins, CCPs have been mostly using the SPAN methodology introduced by CME in the 80s, which is based, for each member, on the most defavorable among sixteen reference scenarios (see Kupiec and White (1996)).

Given the specification of the variation margin by the first identity in (3.1), the quantity

$$\begin{aligned} P_{lh+(\delta+h)}^i + \int_{[lh, lh+(\delta+h)]} e^{\int_s^{lh+(\delta+h)} r_u du} dD_s^i - VM_{lh}^i \\ = P_{lh+(\delta+h)}^i + \int_{[lh, lh+(\delta+h)]} e^{\int_s^{lh+(\delta+h)} r_u du} dD_s^i - P_{lh-}^i =: P\&L_{lh, lh+(\delta+h)}^i \end{aligned}$$

corresponds to the variation-margined P&L of the member  $i$  at the time horizon  $(\delta + h)$  of the margin period of risk. Note that  $P\&L_{lh, lh+(\delta+h)}^i$  is a cumulative P&L also accounting for the promised dividends capitalized at the risk-free rate on the margin period of risk  $[lh, lh + (\delta + h)]$ . For the determination of the default fund contributions, in principle, stressed and multivariate risk measures would make sense (but are not really implemented in practice), i.e.

$$\varrho_{IT}^i = \varrho_i \left( \left( P\&L_{IT, IT+(\delta+h)}^j - IM_{IT}^j \right)_{j; J_{IT}^j=1} \right) \quad (3.3)$$

(the horizon can be stressed too, i.e. using a “greater  $\delta$ ” here than for the initial margins in  $\rho^i$ ). The formula (3.3) suggests a top down definition of the  $\varrho_{IT}^i$  (as opposed to a bottom up approach as of (3.2)), e.g. by Euler allocation of a global risk measure at the clearinghouse service level, in line with the mutualization rationale for the default fund.

**Remark 3.3** For instance, the recent “cover two” EMIR rule prescribes that the clearinghouse should set default fund contributions at a level ensuring its resilience to the joint default of the two riskiest clearing members, where riskiest is meant in the sense of the largest breaches, or exposures at default, i.e. losses (beyond the margins) in case defaults would happen (see Sect. 3.2).

**Remark 3.4** Margining schemes as above, even if possibly based on multivariate risk measures (cf. (3.3)), only account for dependence from the point of view of asset dependence between the portfolios of the members, ignoring credit contagion effects between members. This is in line with the mandate of a clearinghouse to mitigate (i.e. put a cap on) the breaches by means of margins, in case defaults would happen, whereby defaults are viewed as totally unpredictable events. On top of the margins, add-ons are sometimes required from members to account for their credit risk and concentration risk, among others.

Regarding the distributions that are used for the  $P\&L^i$  in all these risk measure computations, since the crisis, the focus has shifted from the cores of the distributions, dominated by volatility effects, to their queues, dominated by scenarios of crisis and default events. Hence, already for the determination of the initial margins, Gaussian (especially in combination with VaR) models are generally banned since the crisis and CCPs typically focus on either Pareto laws or on historical VaR (sometimes bootstrapped to make it a bit richer). Stressed scenarios and parameters are used for the determination of the default fund contributions. Good margining schemes should guarantee the required level of resilience to the clearinghouse at a bearable cost for the members. Two points of concern are procyclicality,



in particular with haircuts that increase with the distress of the posting party, and liquidity, given the generalization of central clearing and collateralization. We refer to Ghamami (2014), Cruz-Lopez et al. (2013), Menkveld (2014) and Armenti et al. (2015) for alternative margin schemes proposals.

**Remark 3.5** Posted collateral does not imply any transfer of ownership (unless the default happens and the member is liquidated) and can be seen in this sense as a loan by the posting member. However, CCPs actually charge a spread above OIS (e.g. 20 bp) on initial margins and (though not always) default fund contributions.

## 3.2 Breaches

The default time of the member  $i$  is modeled as a stopping time  $\tau_i$  with an intensity process  $\gamma^i$ . In particular, any event  $\{\tau_i = t\}$ , for a fixed time  $t$ , has zero probability and can be ignored in the analysis. For every time  $t \geq 0$ , let

$$\bar{t} = t \wedge \bar{T}, \quad t^\delta = t + \delta, \quad \bar{t}^\delta = \mathbf{1}_{t < \bar{T}} t^\delta + \mathbf{1}_{t \geq \bar{T}} \bar{T}$$

and let  $\hat{t}$  denote the greatest  $lh \leq t$ . We denote by

$$\mathcal{C}^i = VM^i + IM^i + DF^i \tag{3.4}$$

the overall collateral process of the member  $i$ . In view of the above, we have  $\mathcal{C}_t^i = \mathcal{C}_{\hat{t}}^i$ ,  $t \leq \tau_i$ , and the process  $\mathcal{C}$  is stopped at time  $\hat{\tau}_i$ . For each member  $i$ , we write

$$\begin{aligned} \Delta_t^i &= \int_{[\tau_i, t]} e^{\int_s^t r_u du} dD_s^i, \quad Q_t^i = P_t^i + \Delta_t^i, \quad \chi_i = (Q_{\tau_i^\delta}^i - \mathcal{C}_{\hat{\tau}_i}^i)^+, \\ \mathfrak{R}_i &= -\mathbf{1}_{\chi_i > 0} (\mathcal{C}_{\hat{\tau}_i}^i + R_i \chi_i) - \mathbf{1}_{\chi_i = 0} Q_{\tau_i^\delta}^i, \\ \xi_i &= Q_{\tau_i^\delta}^i + \mathfrak{R}_i = \mathbf{1}_{\chi_i > 0} (Q_{\tau_i^\delta}^i - \mathcal{C}_{\hat{\tau}_i}^i - R_i \chi_i) = (1 - R_i) \chi_i, \end{aligned} \tag{3.5}$$

where  $R_i$  denotes a related recovery rate. Note that in the context of centrally cleared trading, by liquidation, we simply mean liquidation of the portfolio of a defaulted member by the clearinghouse, as opposed to the legal liquidation, by a mandatory liquidator, that can take years to complete (the New York branch of Lehman was legally liquidated in December 2013, more than five years after Lehman's default). Hence, unless a defaulting member is also involved with the clearinghouse bilaterally, as it can happen through reinvestment of the margins by the clearinghouse, there is no recovery to expect on a defaulted member, i.e.  $R_i = 0$ . We only introduce recovery coefficients for the discussion regarding DVA and FVA / DVA2 in Sect. 4 and for comparison with the bilateral counterparty risk framework in Sect. 6 and 7.

Note that we don't exclude joint defaults in our setup. In fact, joint defaults, which can be viewed as a form of "instantaneous contagion", is the way we will introduce credit dependence between members in Sect. 5. For  $Z \subseteq N$ , let  $\tau_Z \in \mathbb{R}_+ \cup \{\infty\}$  denote the time of joint default of names in  $Z$  and only in  $Z$ .

**Lemma 3.1** *At each liquidation time  $\tau_Z^\delta = \tau_Z + \delta$  such that  $\tau_Z < \bar{T}$ , the realized breach for the clearinghouse (residual cost after the margins of the defaulted members have been used) is given by*

$$B_{\tau_Z^\delta} = \sum_{i \in Z} \xi_i. \tag{3.6}$$

**Proof.** At that stage we consider all the costs or benefits from the perspective of the clearinghouse and the community of the surviving members altogether. The allocation of these costs between the clearinghouse and the surviving members will be considered in Sect. 3.3. Consistent with our stylized model of the liquidation procedure in Sect. 2.2, during the liquidation period  $[\tau_Z, \tau_Z^\delta]$ , where  $\tau_Z = \tau_i$  if and only if  $i \in Z$ , the clearinghouse substitutes itself to the defaulting members, taking care of all their dividend cash flows, which represent a cumulative cost of  $\sum_{i \in Z} \Delta_{\tau_i^\delta}^i$  (including a cost of funding at the risk-free rate). Subsequently, at the liquidation time  $\tau_Z^\delta$ , the clearinghouse substitutes the buffer to itself as counterparties in all the involved contracts (or simply puts an end to the contracts that were already with the buffer), at a cost of  $\sum_{i \in Z} P_{\tau_i^\delta}^i$ . In addition, for any  $i \in Z$ :

- if  $\chi_i > 0$ , meaning that the overall margin  $\mathcal{C}^i$  of a member  $i \in Z$  does not cover the totality of its debt to the surviving members, then, at  $\tau_i^\delta$ , the ownership of  $\mathcal{C}^i$  is transferred in totality to the surviving members. If  $R_i > 0$  these also get a recovery  $R_i \chi_i$ ;
- else, i.e. if  $\chi_i = 0$ , meaning  $Q_{\tau_i^\delta}^i \leq \mathcal{C}_{\tau_i^\delta}^i$ , then either  $Q_{\tau_i^\delta}^i \leq 0$  and an amount  $(-Q_{\tau_i^\delta}^i)$  is paid by the surviving members to the member  $i$  (who keeps ownership of all its margin), or  $Q_{\tau_i^\delta}^i \geq 0$  and the ownership of an amount  $Q_{\tau_i^\delta}^i$  of margin is transferred to the surviving members. In both cases, the surviving members get  $Q_{\tau_i^\delta}^i$ .

The total cost (realized exposure not covered by the margins of the defaulting members) is the sum over  $i \in Z$  of the

$$\begin{aligned} P_{\tau_i^\delta}^i + \Delta_{\tau_i^\delta}^i - \mathbb{1}_{\chi_i > 0}(\mathcal{C}_{\tau_i^\delta}^i + R_i \chi_i) - \mathbb{1}_{\chi_i = 0} Q_{\tau_i^\delta}^i &= Q_{\tau_i^\delta}^i - \mathbb{1}_{\chi_i = 0} Q_{\tau_i^\delta}^i - \mathbb{1}_{\chi_i > 0}(\mathcal{C}_{\tau_i^\delta}^i + R_i \chi_i) \\ &= \mathbb{1}_{\chi_i > 0}(Q_{\tau_i^\delta}^i - \mathcal{C}_{\tau_i^\delta}^i - R_i \chi_i) = (1 - R_i) \chi_i = \xi_i. \blacksquare \end{aligned}$$

### 3.3 Equity and Default Fund Replenishment Principle

We proceed with the description of the next layers of the waterfall of resources of the clearinghouse, namely the equity and the default fund.

If the default of a member entails a positive breach, then the first payer, although to a typically quite limited extent, is the clearinghouse itself (before the surviving members), via a proprietary resource of the CCP called the equity process  $E$ . At times  $lY$ ,  $l \geq 0$ , where  $Y$  is a multiple of  $T$  (typically one year whereas  $T$  is one month), the equity  $E$  is reset by the clearinghouse at some target levels  $E_{lY}^*$ , the “skin in the game” of the clearinghouse. In the meantime, the equity is used for covering the realized breaches, i.e., at each  $t = \tau_Z^\delta$  with  $\tau_Z < \bar{T}$ ,

$$\Delta E_t = -(B_t \wedge E_{t-}). \quad (3.7)$$

The part of the realized breach left uncovered by the equity,  $(B_t - E_{t-})^+$ , is covered by the surviving members through the default fund, which they have to refill by the following rule, at each  $t = \tau_Z^\delta$  with  $\tau_Z < \bar{T}$  (see Figure 2):

$$\epsilon_t^i = (B_t - E_{t-})^+ \frac{J_t^i DF_t^i}{\sum_{j \in N} J_t^j DF_t^j}, \quad (3.8)$$

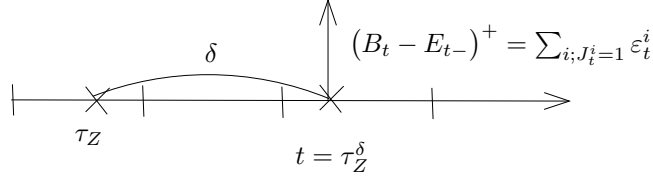


Figure 2: Margin cash flows: resets at margin call grid times and refill of the default fund at liquidation times.

proportional to the default fund contributions (or other keys of repartition such as initial margins or the sizes of the positions).

In sum, the variation, initial and default margins  $VM_{lh}^i$ ,  $IM_{lh}^i$  and  $DF_{lT}^i$  are reset at their respective call grid times by the surviving members according to (3.1); the equity is reset at the times  $lY$  by the clearinghouse and is used for covering the first levels of realized breaches at liquidation times following (3.7); the losses in case of realized breaches above the residual equity are covered at liquidation times by the surviving members following (3.8).

**Remark 3.6** The (aggregated) default fund is  $\sum_{j \in N} J^j DF^j$ , a quantity also referred to as the funded default fund, as opposed to the unfunded default fund that refers to the additional amounts a member may have to pay in case of defaults of other members via the default fund replenishment principle. Specifically,

$$u_{lT}^i = \left( \sum_{lT-T < \tau_Z^\delta < lT} \epsilon_{\tau_Z^\delta}^i - DF_{lT-T}^i \right)^+$$

represents the unfunded default fund contribution of the member  $i$  for the period  $(lT - T, lT)$ . The service closure, i.e. the default of a clearinghouse on a given market or service, is typically specified in terms of events related to the funded and the unfunded default funds, such as the unfunded default fund  $u_{lT}^i$  reaching a cap given as, e.g.,  $2DF_{lT-T}^i$  (unless some members are willing to pay more to avoid the service closure). Given the very high levels of margins that are used in practice (initial margins in particular), this is a very extreme tail event. We neglect it in this paper, where we don't model any notion of default of the clearinghouse. See Duffie (2014) about alternative approaches to the design of insolvency and failure resolution regimes for CCPs.

**Remark 3.7** Since the default fund is meant to be depleted by realized breaches, but no more in principle, the unfunded default fund contributions of a member can be interpreted as its unexpected costs. Accordingly, Ghamami (2014) argues that the CCP regulatory capital of the member  $i$ , i.e. the capital guaranteeing its involvement in the clearinghouse, should be based on its expected future unfunded default fund contribution, e.g.  $U_t^i = \widehat{\mathbb{E}}_t \sum_{t < lT} u_{lT}^i$ , where  $\widehat{\mathbb{E}}$  is the conditional expectation under the historical probability measure (rather than risk-neutral, for regulatory capital computations). Instead, the current regulation uses stylized formulas based on the funded default margin, i.e.  $DF_t^i$  instead of  $U_t^i$ . See Sect. A.1 and Basel Committee on Banking Supervision (2012, 2013, 2014).

## 4 Central Clearing Valuation Adjustment

In this main theoretical section of the paper, we study the cost of the clearance framework for a member of the clearinghouse. Following the now classical bilateral counterparty risk methodology, we represent the value of the position of the member, from its own perspective, as the difference between the mark-to-market of its portfolio (cf. (2.1)) and a correction interpreted as the cost of the clearance structure to the member (cost of the hedge of the related exposure). In reference to the bilateral XVA taxonomy, we call this cost CCVA, for central clearing valuation adjustment.

We refer to the (generic) member 0 as “the member” henceforth, the other members being collectively referred to as “the clearinghouse”. For deriving the equation satisfied by the member’s CCVA process  $\Theta$ , we use a risk-neutral analysis that corresponds to a situation where the member would perfectly hedge its CCVA  $\Theta$ , at an initial cost given by  $\Theta_0$ . In practice, a bank neither really wants nor can achieve such a perfect hedge. In particular, the other members positions and margins, which affect the realized breaches and the CCP regulatory capital of the member, cannot be communicated to the member by the clearinghouse, which, conversely, doesn’t have access to the funding data of the member. Hence, a perfect hedge would require some cooperation between the member and the clearinghouse, whereby the hedge, like the margins, would be computed by the clearinghouse (using in particular the funding data communicated by the member) and implemented by the member. Equivalently, the member could pay the CCVA  $\Theta_0$  to the clearinghouse that would implement the hedge on its behalf. In view of an obvious market incompleteness here, one may claim that the risk-neutral analysis of this paper is not enough conservative. But the main motivation for this work is to provide a metric for comparing the performance of different margin schemes and/or centrally cleared versus bilateral trading. For such relative pricing, the first order provided by risk-neutral pricing should be acceptable. Moreover, any incomplete market approach to that matter, adding in some way one layer of optimization to the analysis, would be hardly feasible on real-life portfolios. It would also be more subjective due to the lack of reliable estimation procedures for utility functions or other theoretical optimization tools.

### 4.1 DVA and FVA Issues

For notational simplicity, we remove any index 0 referring to the reference member. From the perspective of the member, the effective time horizon of the problem is  $\bar{\tau}^\delta$ , with an effective incoming dividend stream  $(-J_t dD_t)$  on  $[0, \bar{\tau}]$  (recall that  $D = D^0$  denotes the outgoing promised dividend stream of the member, ignoring its default). The position of the member is closed at  $\tau^\delta$  (if  $\tau < \bar{T}$ ) with a terminal cash-flow from the member’s perspective given, in view of (3.5) and of the analysis developed in the two bullet points of the proof of Lemma 3.1 (for  $i = 0$  here), by

$$\mathfrak{R} = -\mathbf{1}_{\chi > 0}(\mathcal{C}_{\bar{\tau}} + R\chi) - \mathbf{1}_{\chi = 0}Q_{\tau^\delta}. \quad (4.1)$$

In particular, if  $\chi > 0$ , i.e.  $Q_{\tau^\delta} > \mathcal{C}_{\bar{\tau}}$ , then the member gets

$$-\mathcal{C}_{\bar{\tau}} - R\chi = -\mathcal{C}_{\bar{\tau}} - R(Q_{\tau^\delta} - \mathcal{C}_{\bar{\tau}}) = (-Q_{\tau^\delta}) + (1 - R)(Q_{\tau^\delta} - \mathcal{C}_{\bar{\tau}}). \quad (4.2)$$

Assuming the member already has in place a hedge for the market risk of its position, hedge that generates (after it has been taken over by the clearinghouse as liquidator of

the member on  $[\bar{\tau}, \bar{\tau}^\delta]$ ) the amount  $Q_{\tau^\delta}$  at  $\tau^\delta$ , compensating the  $(-Q_{\tau^\delta})$  cash flow in (4.2), then the member still needs a hedge for the nonnegative debit benefit at own default  $(1 - R)(Q_{\tau^\delta} - \mathcal{C}_{\bar{\tau}})^+$ . Otherwise, a risk-neutral valuation of this amount is not legitimate. But, in order to hedge this amount, the member basically needs to sell credit protection on itself, which is barely possible in practice. Consequently, one may find safer to ignore debit benefit at own default and the ensuing DVA (debit valuation adjustment), formally setting  $R = 1$ , which results in  $\mathfrak{R} = -Q_{\tau^\delta}$  in (4.1) and in turn  $\xi = 0$  later in (4.12). Then, of course,  $R$  becomes disconnected from what the clearinghouse would actually recover (if anything) from the member in case it defaults, but this is immaterial for analyzing the costs of this particular member (it only matters for other members). As a consequence, it is possible and convenient to analyze the no DVA case for the member just by setting  $R = 1$  everywhere. In the end, even if one considers that the effective recovery rate of the member is simply zero, playing with a nominal recovery coefficient  $R$  somewhere between 0 and 1 allows reaching any desired level of interpolation between the bullish full DVA ( $R = 0$ ) and the bearish no DVA ( $R = 1$ ) points of view.

If some DVA is accounted for (i.e. if  $R < 1$ ), then one may also want to include, at least for the sake of the analysis, an additional funding benefit at own default and the ensuing DVA2 (in the terminology of Hull and White (2012a, 2012b, 2014)), corresponding to an additional cash-flow to the member of the form

$$(1 - \bar{R})(-\mathcal{W}_{\tau^-} + \mathcal{C}_{\bar{\tau}})^+ \quad (4.3)$$

at time  $\tau$  (if  $< \bar{T}$ ), where  $\mathcal{W}_{\tau^-}$  represents the wealth of the member right before  $\tau$  (see Sect. 4.2) and  $\bar{R}$  is a recovery rate of the member to its funder. Here we call “funder” a third party, possibly composed in practice of several entities or devices (assumed default-free for simplicity), playing the role of lender/borrower of last resort after exhaustion of the internal sources of funding provided to the member through its collateral and its hedge. The amount  $(-\mathcal{W}_{\tau^-} + \mathcal{C}_{\bar{\tau}})^+$  in (4.3) represents the funding debt of the member at its default. As for the DVA, any desired level of interpolation between the bullish full FVA ( $\bar{R} = 0$ ) and the bearish no FVA ( $\bar{R} = 1$ ) points of view can be obtained by playing with a nominal funding recovery coefficient  $\bar{R}$  anywhere between 0 and 1.

**Example 4.1** If the wealth of the member is 10 and that the member must post 4 (resp. 17) as collateral, then the member uses 4 out of its wealth of 10 as collateral (resp. borrows 7 on top of its wealth of 10 and posts the totality as collateral), hence its funding debt equals  $(-10 + 4)^+ = 0$  (resp.  $(-10 + 17)^+ = 7$ ). Since posted collateral is essentially a loan (see the remark 3.5), the wealth of the member is still 10 after the collateral has been posted in both cases.

On the DVA and FVA/DVA2 debates, see Hull and White (2012a, 2012b, 2014) and Burgard and Kjaer (2012). As a personal note, we recommend to ignore windfall benefits at own default as fake benefits. The reason why we introduce DVA is because the regulation allows for DVA recognition in the case of bilateral transactions. Including DVA when assessing the cost of counterparty risk on a bilateral basis, whilst ignoring it when assessing the cost of counterparty risk on a centrally cleared basis, might arguably bias the comparison. The results of Sect. 8, where numbers are computed both with DVA included and without it, demonstrate that this is not the case. The DVA2 cash-flow (4.3) is even more controversial. In fact, we view this cash flow as mainly pedagogical, as it provides a clear mathematical formulation of the FVA debate. Again, for ignoring DVA (respectively DVA2), just use  $R = 1$  (respectively  $\bar{R} = 1$ ).

## 4.2 Wealth Process

The member can hedge its collateralized portfolio and needs to fund its whole position (portfolio, collateral and hedge). We restrict ourselves to the situation of a fully secured funded hedge, entirely implemented by means of swaps, short sales and repurchase agreements (all traded outside the clearinghouse, given our assumption of a constant CCP portfolio of the member), at no upfront payment. As explained in Crépey et al. (2014, Section 4.2.1 page 87)<sup>1</sup>, this assumption encompasses the vast majority of hedges that are used in practice. In particular, it includes CDS contracts that may be used for hedging the jump-to-default exposures. Consistent with arbitrage requirements and our terminology of a risk-neutral measure  $\mathbb{Q}$ , we assume that the vector valued gain process  $\mathcal{M}$  of unit positions in the hedging assets is a  $\mathbb{Q}$  martingale (see Crépey et al. (2014, Remark 4.4.2 pages 96-97)<sup>2</sup> or Bielecki and Rutkowski (2014, Proposition 3.3)). In addition, consistent with the remark 3.5), we assume that the variation margins  $VM_t = P_{t-}$  (resp. the initial margins and default fund contributions  $IM_t + DF_t = \mathcal{C}_t - P_{t-}$ ) are remunerated at a flat OENIA rate  $r_t$  (resp. at a rate  $(r_t + c_t)$  with  $c_t < 0$ , e.g.  $c_t = -20$  bp). We postulate that the member can invest (respectively get unsecured funding) at a rate  $(r_t + \lambda_t)$  (resp.  $(r_t + \bar{\lambda}_t)$ ). Moreover, we model the cost of the regulatory capital required for being part of the clearinghouse as  $k_t K_t dt$ , where  $K_t$  is the CCP regulatory capital of the member, which depends in a formulaic way on its default margin  $DF_t$  (see Sect. A.1), and where  $k_t$  is a proportional cost. The CCP regulatory capital accounts for the implicit senior CDO tranche position each member holds on others through the default fund replenishment principle. From an economical point of view,  $k_t$  represents a cost of opportunity, which may be viewed as a Lagrange multiplier related to the global capital constraint of the member. Since explicitly stating it as a constraint would complicate too much the analysis, we adopt the relaxed formulation of a cost proportional to the CCP regulatory capital of the member, following in this Green et al. (2014), where  $k$  is taken as 10%, consistent with reference orders of magnitude for return on capital or dividend yields to shareholders.

We assume that the member enters its contracts at time 0 against an upfront payment of a certain amount  $\Pi_0$  and that the member sets up a related hedge  $(-\zeta)$ , where by a hedge we mean a left-continuous row-vector process with components yielding the (negative of the) positions in the hedging assets. The “short” negative notation in front of  $\zeta$  is used for consistency with the idea, just to fix the mindset, that the portfolio is “bought” by the member, which therefore “sells” the corresponding hedge. Let  $\mathcal{W}$  denote the value of the corresponding collateralization, hedging and funding portfolio, held by the member itself until  $\bar{\tau}$  and (if  $\tau < \bar{T}$ ) by the clearinghouse, as liquidator of the member’s position, on  $(\bar{\tau}, \bar{\tau}^\delta]$ .

**Lemma 4.1** *Ignoring the close-out cashflow (4.1) at  $\bar{\tau}^\delta$  if  $\tau < \bar{T}$ , which will be added separately later (and is treated accordingly as a boundary condition in the pricing equation (4.7), see Lemma 4.2 and its proof), we have  $\mathcal{W}_0 = -\Pi_0$  and, for  $0 < t \leq \bar{\tau}^\delta$ ,*

$$d\mathcal{W}_t = r_t \mathcal{W}_t dt - J_t \left( dD_t + \sum_{Z \subseteq N} \epsilon_{\tau_Z^\delta} \delta_{\tau_Z^\delta}(dt) + g_t(-\mathcal{W}_t) dt + k_t K_t dt \right) - \mathbf{1}_{\{\tau < \bar{T}\}} (1 - \bar{R})(-\mathcal{W}_{t-} + \mathcal{C}_t)^+ dJ_t - \zeta_t d\mathcal{M}_t, \quad (4.4)$$

<sup>1</sup>Or Crépey (2015, Part I, Section 2.1) in the journal version.

<sup>2</sup>Or Crépey (2015, Part I, Remark 4.1) in the journal version.

where, for any  $\pi \in \mathbb{R}$ ,

$$g_t(\pi) = -c_t(\mathcal{C}_t - P_{\hat{t}-}) + \bar{\lambda}_t(\pi + \mathcal{C}_t)^+ - \lambda_t(\pi + \mathcal{C}_t)^-. \quad (4.5)$$

**Proof.** Formulating in mathematical terms the above described collateralization, hedging and funding policy, we have  $\mathcal{W}_0 = -\Pi_0$  and, for  $0 < t \leq \bar{\tau}^\delta$ :

$$\begin{aligned} d\mathcal{W}_t = & - \underbrace{J_t dD_t}_{\text{member pays dividends}} - \underbrace{J_{t-} \zeta_t d\mathcal{M}_t}_{\text{member pays on its hedge}} \\ & - \underbrace{J_t \sum_{Z \subseteq N} \epsilon_{\tau_Z^\delta} \delta_{\tau_Z^\delta}(dt)}_{\text{member contributes to realized breaches}} + \underbrace{J_t(r_t \mathcal{C}_t + c_t(\mathcal{C}_t - P_{\hat{t}-}))dt}_{\text{margin fees}} \\ & - \underbrace{J_t((r_t + \bar{\lambda}_t)(-\mathcal{W}_t + \mathcal{C}_t)^+ - (r_t + \lambda_t)(-\mathcal{W}_t + \mathcal{C}_t)^-)dt}_{\text{funding costs / benefits to member}} - \underbrace{J_t k_t K_t dt}_{\text{cost in capital to member}} \\ & \quad - \underbrace{\mathbb{1}_{\{\tau < \bar{T}\}}(-\mathcal{W}_{t-} + \mathcal{C}_{\hat{t}})^+ dJ_t}_{\text{windfall funding benefit to member at own default (DVA2 cash-flow)}} \\ & - \underbrace{(1 - J_{t-})\zeta_t d\mathcal{M}_t}_{\text{clearinghouse (liquidator) pays on the hedge of the member during its liquidation period}} \\ & + \underbrace{(1 - J_t)r_t \mathcal{W}_t dt}_{\text{risk-free funding benefits / costs of the clearinghouse (liquidator) during its liquidation period}} \end{aligned}$$

Collecting terms,

$$\begin{aligned} d\mathcal{W}_t = & r_t \mathcal{W}_t dt - \zeta_t d\mathcal{M}_t - \mathbb{1}_{\{\tau < \bar{T}\}}(1 - \bar{R})(-\mathcal{W}_{t-} + \mathcal{C}_{\hat{t}})^+ dJ_t \\ & - J_t \left( dD_t + \sum_{Z \subseteq N} \epsilon_{\tau_Z^\delta} \delta_{\tau_Z^\delta}(dt) + (-c_t(\mathcal{C}_t - P_{\hat{t}-}) + \bar{\lambda}_t(-\mathcal{W}_t + \mathcal{C}_t)^+ - \lambda_t(-\mathcal{W}_t + \mathcal{C}_t)^-) dt + k_t K_t dt \right), \end{aligned} \quad (4.6)$$

which is (4.4), by definition (4.5) of  $g$ . ■

**Remark 4.1** As assumed in Sect. 2.2, during the liquidation period of the member, the clearinghouse, funded at the risk-free rate  $r_t$ , takes over its hedge. Consistent with this, the funding cost coefficient  $g$  of the member sits in the parenthesis in (4.4).

**Remark 4.2** The validity of (4.4) is not restricted to the above described collateralization, hedging and funding policy. It is valid for any funding coefficient  $g_t = g_t(\pi)$  in (4.4) such that  $(-r_t \mathcal{W}_t + g_t(-\mathcal{W}_t))dt$  represents the  $dt$ -funding cost of the member (whilst the member is alive and net of the funding cost of its hedge that is already comprised in the local martingale  $\zeta_t d\mathcal{M}_t$ ).

### 4.3 All-Inclusive Price

The hedging justification for the following definition, which can be compared with Crépey et al. (2014, Definition 4.4.5 page 98)<sup>3</sup> in the case of bilateral trading, is provided by Lemma

<sup>3</sup>Or Definition 4.1 in the journal version Crépey (2015, Part I).

4.2 below. More broadly, see Crépey et al. (2014, Remark 4.4.6 page 99)<sup>4</sup> regarding the use of backward stochastic differential equations (BSDEs) for the analysis of funding costs.

**Definition 4.1** An all-inclusive price of the member's portfolio (price from the member's perspective) is a semimartingale  $\Pi$  that satisfies the following price BSDE on  $[0, \bar{\tau}^\delta]$ :

$$\begin{aligned} \Pi_{\bar{\tau}^\delta} &= \mathbf{1}_{\{\tau < \bar{T}\}} \mathfrak{R} \text{ and, for } t \leq \bar{\tau}^\delta, \\ d\Pi_t &= r_t \Pi_t dt + \mathbf{1}_{\{\tau < \bar{T}\}} (1 - \bar{R})(\Pi_{t-} + \mathcal{C}_{\hat{\tau}})^+ dJ_t \\ &\quad + J_t \left( dD_t + \sum_{Z \subseteq N} \epsilon_{\tau_Z^\delta} \delta_{\tau_Z^\delta}(dt) + g_t(\Pi_t) dt + k_t K_t dt \right) + d\nu_t, \end{aligned} \quad (4.7)$$

for some initially null martingale  $\nu$ .

Equivalent to the above differential formulation (assuming true martingality of  $\nu$ ), we can write: for  $t \in [0, \bar{\tau}^\delta]$ ,

$$\begin{aligned} \beta_t \Pi_t &= \mathbb{E}_t \left[ \mathbf{1}_{\{\tau < \bar{T}\}} (\beta_{\tau^\delta} \mathfrak{R} + \beta_\tau (1 - \bar{R})(\Pi_{\tau-} + \mathcal{C}_{\hat{\tau}})^+ J_t) \right. \\ &\quad \left. - \sum_{t < \tau_Z^\delta < \bar{\tau}} \beta_{\tau_Z^\delta} \epsilon_{\tau_Z^\delta} - \int_t^{\bar{\tau}} \beta_s J_s (dD_s + g_s(\Pi_s) ds + k_s K_s ds) \right]. \end{aligned} \quad (4.8)$$

**Lemma 4.2** *If an all-inclusive price  $\Pi$  can be found with  $\nu_t = \zeta_t d\mathcal{M}_t$  for some hedge  $\zeta$ , then  $(\Pi, \zeta)$  yields a perfect (replicating) hedge to the position of the member in the clearinghouse, i.e.  $\mathcal{W} = -\Pi$  on  $[0, \bar{\tau}^\delta]$ . In particular,*

$$\mathcal{W}_{\bar{\tau}^\delta} = -\Pi_{\bar{\tau}^\delta} = -\mathbf{1}_{\{\tau < \bar{T}\}} \mathfrak{R},$$

so that, after the close-out delivery cash flow  $\mathbf{1}_{\{\tau < \bar{T}\}} \mathfrak{R}$  at  $\bar{\tau}^\delta$ , the position of the member is closed break-even.

**Proof.** If a price  $\Pi$  can be found with  $d\nu_t = \zeta_t d\mathcal{M}_t$  for some hedge  $\zeta$ , then  $Z = \beta\Pi + \beta\mathcal{W}$  satisfies  $Z_0 = 0$  and  $dZ_t = \alpha_t Z_t dt$  on  $[0, \bar{\tau})$ , where  $\alpha_t := \mathbf{1}_{\{\Pi_t \neq \mathcal{W}_t\}} \frac{g_t(\Pi_t) - g_t(-\mathcal{W}_t)}{\Pi_t + \mathcal{W}_t}$  is Lebesgue integrable over  $[0, T]$  (for  $g_t(\pi)$  given by (4.5)). Hence,

$$d(e^{-\int_0^t \alpha_s ds} Z_t) = e^{-\int_0^t \alpha_s ds} (dZ_t - \alpha_t Z_t dt) = 0,$$

i.e.  $e^{-\int_0^t \alpha_s ds} Z_t$  is constant on  $[0, \bar{\tau})$ , equal to 0 in view of the initial condition for  $Z$ , i.e.  $\mathcal{W} = -\Pi$  on  $[0, \bar{\tau})$ . This is followed by a jump of the two processes  $\mathcal{W}$  and  $(-\Pi)$  by the same amount

$$(1 - \bar{R})(-\mathcal{W}_{\tau-} + \mathcal{C}_{\hat{\tau}})^+ = (1 - \bar{R})(\Pi_{\tau-} + \mathcal{C}_{\hat{\tau}})^+$$

at  $\bar{\tau}$  (if  $< \bar{T}$ ), after which  $\mathcal{W}$  and  $(-\Pi)$  coincide again on  $[\bar{\tau}, \bar{\tau}^\delta]$  by the same argument as above. Hence,  $\mathcal{W} = -\Pi$  on  $[0, \bar{\tau}^\delta]$ . ■

More broadly, if an all-inclusive price can be found with  $\nu = \zeta_t d\mathcal{M}_t + d\varepsilon_t$ , for some hedge  $\zeta$  and a “small” cost martingale  $\varepsilon$  (which depends on the depth of the hedging market), then the hedging error  $\rho = \mathcal{W} + \Pi$ , which starts from 0 at time 0, remains “small” all the way through. In particular,

$$\mathcal{W}_{\bar{\tau}^\delta} \approx -\Pi_{\bar{\tau}^\delta} = -\mathbf{1}_{\{\tau < \bar{T}\}} \mathfrak{R}$$

at  $\bar{\tau}^\delta$ , so that after the close-out cash flow  $\mathbf{1}_{\{\tau < \bar{T}\}} \mathfrak{R}$ , the member's position is closed with a “small” hedging error.

<sup>4</sup>Or Remark 4.3 in the journal version Crépey (2015, Part I).



#### 4.4 CCVA Representation

In this section we define the central counterparty valuation adjustment (CCVA) and derive the corresponding BSDE.

**Definition 4.2** *Given an all-inclusive price  $\Pi$  for the member, the corresponding CCVA is the process defined on  $[0, \bar{\tau}^\delta]$  as  $\Theta = -(Q + \Pi)$ .*

**Remark 4.3** Recall from (3.5) that  $Q = P + \Delta$ , with all values viewed from the perspective of the clearinghouse. Consistent with the usual definition of a valuation adjustment (see Brigo et al. (2013) or Crépey et al. (2014)), we have  $\Theta = (-Q) - \Pi$ , where  $(-Q)$  corresponds to valuation from the perspective of the member.

Let

$$\bar{\xi}_t = \mathbb{E}(\beta_t^{-1} \beta_{\tau+\delta} \xi \mid \mathcal{G}_t), \quad (4.9)$$

where  $\xi = (1 - R)(Q_{\tau^\delta} - C_{\bar{\tau}})^+$  as before (cf. (3.5)). Let  $\hat{\xi}$  be a  $\mathbb{G}$  predictable process, which exists by Corollary 3.23 2) in He, Wang, and Yan (1992), such that

$$\hat{\xi}_\tau = \mathbb{E}(\beta_\tau^{-1} \beta_{\tau^\delta} \xi \mid \mathcal{G}_{\tau-}) = \mathbb{E}(\bar{\xi}_\tau \mid \mathcal{G}_{\tau-}). \quad (4.10)$$

**Remark 4.4** In particular, in the special case (most relevant in our view) where the DVA is simply ignored, then  $\xi = \bar{\xi} = \hat{\xi} = 0$ , which will imply that  $\Theta = 0$  on  $[\bar{\tau}, \bar{\tau}^\delta]$  (see the beginning of the proof 4.2), as could be expected.

Let, for  $\vartheta \in \mathbb{R}$ ,

$$\begin{aligned} \hat{f}_t(\vartheta) &= g_t(-P_t - \vartheta) + k_t K_t - \gamma_t \hat{\xi}_t - (1 - \bar{R}) \gamma_t (P_t - C_t + \vartheta)^- \\ &= \underbrace{-\gamma_t \hat{\xi}_t}_{dva_t} + \underbrace{(-c_t(C_t - P_{t-}) + \tilde{\lambda}_t (P_t - C_t + \vartheta)^- - \lambda_t (P_t - C_t + \vartheta)^+)}_{fva_t(\vartheta)} + \underbrace{k_t K_t}_{kva_t}, \end{aligned} \quad (4.11)$$

by definition (4.5) of  $g$ , where  $\tilde{\lambda} = \bar{\lambda} - (1 - \bar{R})\gamma$  can be interpreted as a liquidity borrowing spread of the member, net of its credit spread toward its external funder (recall  $\gamma = \gamma^0$  is the assumed intensity of  $\tau$ ). From the perspective of the member, the three terms in the decomposition (4.11) of the coefficient  $\hat{f}_t(\vartheta)$  can respectively be interpreted as a beneficial debit valuation adjustment coefficient ( $dva_t$  that can be ignored by setting  $R = 1$ ), a funding liquidity valuation adjustment coefficient ( $fva_t(\vartheta)$  in which the DVA2 component can be ignored by setting  $\bar{R} = 1$ ) and a capital valuation adjustment coefficient ( $kva_t$ ) in the sense of Green et al. (2014).

**Proposition 4.1** *Let there be given semimartingales  $\Pi$  and  $\Theta$  such that  $\Theta = -(Q + \Pi)$  on  $[0, \bar{\tau}^\delta]$ . The process  $\Pi$  is an all-inclusive price for the member's portfolio if and only if the process  $\Theta$  satisfies the following BSDE:*

$$\begin{aligned} \beta_t \Theta_t &= \mathbb{E}_t \left[ \sum_{t < \tau_2^\delta < \bar{\tau}} \beta_{\tau_2^\delta} \epsilon_{\tau_2^\delta} - \mathbf{1}_{\{\tau < \bar{T}\}} (\beta_{\tau^\delta} \xi + \beta_\tau (1 - \bar{R})(P_{\tau-} - C_{\bar{\tau}} + \Theta_{\tau-})^- J_t) \right. \\ &\quad \left. + \int_t^{\bar{\tau}} \beta_s (g_s(-P_s - \Theta_s) + k_s K_s) ds \right], \quad t \in [0, \bar{\tau}^\delta]. \end{aligned} \quad (4.12)$$

**Proof.** Assuming  $\Theta$  defined as  $-(Q + \Pi)$  for some all-inclusive price  $\Pi$  on  $[0, \bar{\tau}^\delta]$ , the terminal condition  $\Theta_{\bar{\tau}^\delta} = -\mathbf{1}_{\{\tau < \bar{T}\}}\xi$  that is implicit in (4.12) results from (3.5) and the terminal condition for  $\Pi$  in (4.7). Moreover, for  $t \in [0, \bar{\tau}^\delta]$ ,

$$-\beta_t \Theta_t = \beta_t Q_t + \beta_t \Pi_t = \beta_t P_t + \int_0^t \beta_s dD_s + (\beta_t \Pi_t - \int_0^t \beta_s J_s dD_s), \quad (4.13)$$

hence

$$\begin{aligned} & -\beta_t \Theta_t - \int_0^t \beta_s J_s \left( \sum_{Z \subseteq N} \epsilon_{\tau_Z^\delta} \delta_{\tau_Z^\delta}(ds) + g_s(-P_s - \Theta_s)ds + k_s K_s ds \right) \\ & - \mathbf{1}_{\{\tau < \bar{T}\}} \int_0^t (1 - \bar{R})(-P_{s-} - \Theta_{s-} + \mathcal{C}_{\bar{s}})^+ dJ_s = \left( \beta_t P_t + \int_0^t \beta_s dD_s \right) + \int_0^t \beta_s d\nu_s, \end{aligned}$$

by the price BSDE (4.7) satisfied by  $\Pi$ . In view also of (2.1) (used for  $i = 0$ ), this is a (local) martingale, hence it coincides with the conditional expectation of its terminal condition (assuming true martingality), which establishes (4.12). The converse implication is proven similarly. ■

**Remark 4.5** As an alternative argument equivalent to the above, one can substitute the right-hand side in (4.8) for  $\beta_t \Pi_t$  in (4.13), which, after an application of the tower rule, yields (4.12). One can proceed similarly to show (4.8) if (4.12) is assumed.

**Proposition 4.2** *The “full CCVA BSDE” for a semimartingale  $\Theta$  satisfying (4.12) on  $[0, \bar{\tau}^\delta]$  is equivalent to the following “reduced CCVA BSDE” for a semimartingale  $\hat{\Theta}$  on  $[0, \bar{\tau}]$ :*

$$\beta_t \hat{\Theta}_t = \mathbb{E}_t \left[ \sum_{t < \tau_Z^\delta < \bar{\tau}} \beta_{\tau_Z^\delta} \epsilon_{\tau_Z^\delta} + \int_t^{\bar{\tau}} \beta_s \hat{f}_s(\hat{\Theta}_s) ds \right], \quad t \in [0, \bar{\tau}], \quad (4.14)$$

equivalent in the sense that if  $\Theta$  solves (4.12), then  $\hat{\Theta} = J\Theta$  solves (4.14), whilst if  $\hat{\Theta}$  solves (4.14), then  $\Theta = J\hat{\Theta} - (1 - J)\mathbf{1}_{\{\tau < \bar{T}\}}\bar{\xi}$  solves (4.12).

**Proof.** The full CCVA BSDE (4.12) is obviously equivalent to  $\Theta = -\mathbf{1}_{\{\tau < \bar{T}\}}\bar{\xi}$  on  $[\bar{\tau}, \bar{\tau}^\delta]$  and

$$\begin{aligned} \beta_t \Theta_t = \mathbb{E}_t \left[ \sum_{t < \tau_Z^\delta < \bar{\tau}} \beta_{\tau_Z^\delta} \epsilon_{\tau_Z^\delta} - \mathbf{1}_{\{\tau < \bar{T}\}} \beta_{\bar{\tau}} (\bar{\xi}_{\bar{\tau}} + (1 - \bar{R})(P_{\bar{\tau}-} - \mathcal{C}_{\bar{\tau}} + \Theta_{\bar{\tau}-})) \right. \\ \left. + \int_t^{\bar{\tau}} \beta_s g_s(-P_s - \Theta_s) ds + \int_t^{\bar{\tau}} \beta_s k_s K_s ds \right] \end{aligned} \quad (4.15)$$

on  $[0, \bar{\tau})$ , which is in turn equivalent to

$$\begin{aligned} \Theta &= -\mathbf{1}_{\{\tau < \bar{T}\}}\bar{\xi} \text{ on } [\bar{\tau}, \bar{\tau}^\delta] \text{ and, on } [0, \bar{\tau}), \\ \beta_t \Theta_t &= \mathbb{E}_t \left[ \sum_{t < \tau_Z^\delta \leq \bar{\tau}} \beta_{\tau_Z^\delta} \epsilon_{\tau_Z^\delta} + \int_t^{\bar{\tau}} \beta_s \hat{f}_s(\Theta_s) ds \right], \end{aligned} \quad (4.16)$$

for on  $[0, \bar{\tau})$  :

$$\begin{aligned}
& \mathbb{E}_t \left[ \mathbb{1}_{\{\tau < \bar{T}\}} \beta_\tau \left( \bar{\xi}_\tau + (1 - \bar{R})(P_{\tau-} - \mathcal{C}_{\bar{\tau}} + \Theta_{\tau-})^- \right) \right] \\
&= \mathbb{E}_t \left[ \mathbb{1}_{\{t < \tau < \bar{T}\}} \beta_\tau \left( \widehat{\xi}_\tau + (1 - \bar{R})(P_{\tau-} - \mathcal{C}_{\bar{\tau}} + \Theta_{\tau-})^- \right) \right] \\
&= -\mathbb{E}_t \left[ \int_t^{\bar{T}} \beta_s \left( \widehat{\xi}_s + (1 - \bar{R})(P_{s-} - \mathcal{C}_{\bar{s}} + \Theta_{s-})^- \right) dJs \right] \\
&= \mathbb{E}_t \left[ \int_t^{\bar{T}} \beta_s \gamma_s \left( \widehat{\xi}_s + (1 - \bar{R})(P_{s-} - \mathcal{C}_{\bar{s}} + \Theta_{s-})^- \right) ds \right],
\end{aligned}$$

where the last identity holds by consideration of the (local, assumed true) martingale

$$\beta_t (\widehat{\xi}_t + (1 - \bar{R})(P_{\tau-} - \mathcal{C}_{\tau-} + \Theta_{\tau-})^-) dJ_t + \beta_t \gamma_t (\widehat{\xi}_t + (1 - \bar{R})(P_t - \mathcal{C}_t + \Theta_t)^-) dt.$$

One readily checks that if  $\Theta$  solves (4.16), then  $\widehat{\Theta} = J\Theta$  solves (4.14), whilst if  $\widehat{\Theta}$  solves (4.14), then  $\Theta = J\widehat{\Theta} - (1 - J)\mathbb{1}_{\{\tau < \bar{T}\}}\bar{\xi}$  solves (4.16). ■

## 5 Common Shock Model of Default Times

In the sequel, we specialize the above to a dynamic Marshall-Olkin (DMO) copula model of the default times  $\tau_i$  (see Crépey et al. (2014, Chapt. 8–10)<sup>5</sup> and Crépey and Song (2015, Sect. 7)). As demonstrated in Crépey et al. (2014, Sect. 8.4)<sup>6</sup>, this model can be efficiently calibrated to marginal and portfolio credit data, e.g. CDS and CDO data (or proxies) on the members. The joint defaults feature of the DMO model is also interesting in regard of the “cover two” EMIR rule (see the remark 3.3)

We define a family  $\mathcal{Y}$  of “shocks”, i.e. subsets  $Y$  of members, typically the singletons  $\{0\}, \{1\}, \dots, \{n\}$  and a small number of “common shocks” representing simultaneous defaults. For  $Y \in \mathcal{Y}$ , we define

$$\eta_Y = \inf\{t > 0; \int_0^t \gamma_Y(s) ds > E_Y\}, \quad J^Y = \mathbb{1}_{[0, \eta_Y)}, \quad (5.1)$$

for a shock intensity function  $\gamma_Y(t)$  and an independent standard exponential random variable  $E_Y$ , and we set

$$\tau_i = \min_{\{Y \in \mathcal{Y}; i \in Y\}} \eta_Y, \quad i \in N.$$

**Example 5.1** Fig. 3 shows one possible default path in a common shock model with  $n = 5$  and  $\mathcal{Y} = \{\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{3, 4\}, \{1, 2, 3\}, \{0, 1\}\}$ . The inner ovals show which shocks happen and cause the observed defaults at successive default times. First, the default of name 1 occurs as the consequence of the shock  $\{1\}$ . Second, names 3 and 4 default simultaneously as a consequence of the shock  $\{3, 4\}$ . Third, the shock  $\{1, 2, 3\}$  triggers the default of name 2 alone (as name 1 and 3 have already defaulted). Fourth, the default of name 0 alone occurs as the consequence of shock  $\{0, 1\}$ .

<sup>5</sup>Or Bielecki et al. (2014b, 2014a) for the journal versions.

<sup>6</sup>Or Bielecki et al. (2014a, Part II) in the journal version.

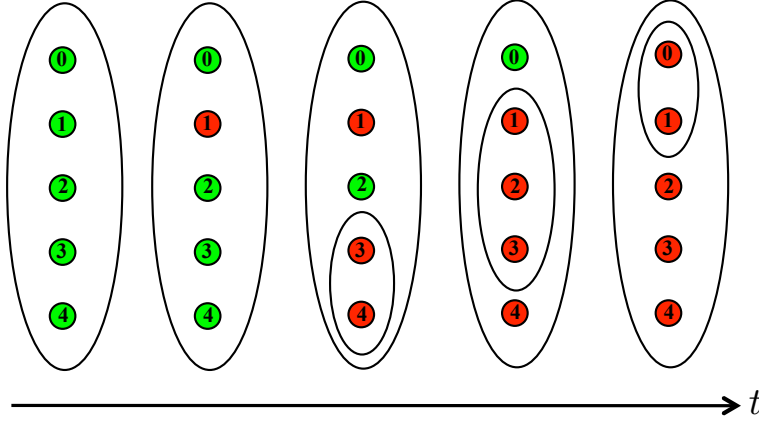


Figure 3: One possible default path in a model with  $n = 4$  and  $\mathcal{Y} = \{\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{3, 4\}, \{1, 2, 3\}, \{0, 1\}\}$ .

Again, in the case of the member (labeled 0), we omit the superscript 0 in the notation. In particular,  $J = \mathbb{1}_{[0, \tau)} = \prod_{Y \in \mathcal{Y}_\bullet} J^Y$ , where  $\mathcal{Y}_\bullet = \{Y \in \mathcal{Y}; 0 \in Y\}$ , hence the intensity  $\gamma$  of  $\tau$  is given as

$$\gamma = J_- \gamma_\bullet, \text{ where } \gamma_\bullet = \sum_{Y \in \mathcal{Y}_\bullet} \gamma_Y. \quad (5.2)$$

We assume that all the market risk factors are gathered in a vector process  $\mathbf{X}$  without jump at  $\tau$  and that the processes  $\mathbf{X}$  and  $X = (\mathbf{X}, \mathbf{J})$ , where  $\mathbf{J} = (J^Y)_{Y \in \mathcal{Y}}$ , are Markov in the full model filtration  $\mathbb{G}$  given as the filtration of  $\mathbf{X}$  progressively enlarged by the random times  $\eta_Y, Y \in \mathcal{Y}$  (see Sect. 7-8 for a numerical illustration where  $\mathbf{X}$  is simply a Black-Scholes stock  $S$ , augmented as mentioned below to cope with the path dependence of dividends and collateral). Setting  $\widehat{\Delta}_t = \int_0^t e^{\int_s^t r_u du} dD_s$ , so that  $\beta_t \Delta_t = \beta_t \widehat{\Delta}_t - \beta_\tau \widehat{\Delta}_{\tau-}$  (for  $t \geq \tau$ ), we assume, consistent with the interpretation of each respective quantity, that

$$\begin{aligned} \epsilon_t &= \epsilon(t, X_t) \text{ for } t = \tau_Z^\delta, \quad Z \subseteq N \\ P_t &= P(t, \mathbf{X}_t), \quad \widehat{\Delta}_t = \widehat{\Delta}(t, \mathbf{X}_t), \quad C_t = C(t, X_t), \quad t \in [0, \bar{\tau}] \end{aligned} \quad (5.3)$$

(augmenting  $\mathbf{X}$  by  $\widehat{\Delta}$  and/or  $C$  if need be), for continuous functions  $\epsilon(t, x)$ ,  $P(t, \mathbf{x})$ ,  $\widehat{\Delta}(t, \mathbf{x})$  and  $C(t, x)$ . In particular,

$$\Delta_\tau = \widehat{\Delta}_\tau - \widehat{\Delta}_{\tau-} = \widehat{\Delta}(\tau, \mathbf{X}_\tau) - \widehat{\Delta}(\tau, \mathbf{X}_{\tau-}) = 0,$$

by continuity of  $\mathbf{X}$  at  $\tau$ . We refer to Crépey and Song (2015) for a possible extension of the following analysis to the “wrong-way risk” case where  $P_t$  and  $\widehat{\Delta}_t$  might also depend on  $\mathbf{J}$  (as, in particular, with credit derivatives), so that  $\Delta_\tau$  might be nonzero.

**Lemma 5.1** *We have*

$$dva_t = dva(t, X_t) = -J_t \bar{\xi}(t, X_t) \gamma_\bullet, \quad \mathbb{Q} \times \lambda \text{ a.e.},$$

for a function  $\bar{\xi}(t, x)$  such that  $\bar{\xi}_\tau = \bar{\xi}(\tau, X_{\tau-})$ .

**Proof.** Since  $\xi = (1 - R)(Q_{\tau^\delta} - \mathcal{C}_{\bar{\tau}})^+$  (cf. (3.5)), where

$$\begin{aligned} \mathcal{C}_{\bar{\tau}} &= \mathcal{C}_{\tau^-} = \mathcal{C}(\tau, X_{\tau^-}) \text{ and} \\ Q_{\tau^\delta} &= P_{\tau^\delta} + \Delta_{\tau^\delta} = P(\tau^\delta, \mathbf{X}_{\tau^\delta}) + \widehat{\Delta}(\tau^\delta, \mathbf{X}_{\tau^\delta}) - e^{\int_\tau^{\tau^\delta} r(u, \mathbf{X}_u) du} \widehat{\Delta}(\tau, \mathbf{X}_\tau), \end{aligned} \quad (5.4)$$

we have by definition (4.9) of  $\bar{\xi}$ :

$$\begin{aligned} \bar{\xi}_\tau &= (1 - R) \times \\ \mathbb{E} &\left[ e^{-\int_\tau^{\tau^\delta} r(u, \mathbf{X}_u) du} \left( P(\tau^\delta, \mathbf{X}_{\tau^\delta}) + \widehat{\Delta}(\tau^\delta, \mathbf{X}_{\tau^\delta}) - e^{\int_\tau^{\tau^\delta} r(u, \mathbf{X}_u) du} \widehat{\Delta}(\tau, \mathbf{X}_\tau) - \mathcal{C}(\tau, X_{\tau^-}) \right)^+ \middle| \mathcal{G}_\tau \right]. \end{aligned} \quad (5.5)$$

Therefore, the Markov property of  $\mathbf{X}$  and the continuity of  $\mathbf{X}$  at time  $\tau$  imply that  $\bar{\xi}_\tau$  can be represented in functional form as  $\bar{\xi}(t, X_{t-})$ . Hence (cf. Crépey and Song (2015, Lemma 5.1)),

$$\gamma_t \widehat{\xi}_t = \gamma_t \bar{\xi}(t, X_t), \quad \mathbb{Q} \times \boldsymbol{\lambda} - a.e.,$$

where (5.2) yields  $\gamma = J_- \gamma_\bullet$ . This gives the result since  $dva = -\gamma \widehat{\xi}$ . ■

## 6 TVA Engines

In this section, we summarize in algorithmic terms the central clearing XVA methodology of this paper, as well as a bilateral CSA XVA analysis adapted, for comparison purposes, from Crépey and Song (2015). In both cases we use the common shock model of Sect. 5 for modeling the default times involved. TVA stands for total valuation adjustment as a unified acronym for CCVA (central clearing valuation adjustment) in the CCP setup and BVA (bilateral valuation adjustment) in the bilateral CSA setup.

### 6.1 CCVA Engine

In spite of the nonlinearity inherent to the FVA component of the CCVA, reflected by the  $\cdot^\pm$  in  $g_t(\pi)$  (cf. (4.5)), hence in  $fva_t(\vartheta)$ , standard Monte Carlo loops can be used for estimating a linearized first order CCVA obtained replacing  $g_s(P_s - \Theta_s)$  by  $g_s(P_s)$  and in turn  $fva_s(\Theta_s)$  by  $fva_s(0)$  in (4.11) and  $\widehat{f}_s(\widehat{\Theta}_s)$  by  $\widehat{f}_s(0)$  in (4.14). A nonlinear correction can be estimated based on the Monte Carlo expansion of Fujii and Takahashi (2012a, 2012b) in vanilla cases, with explicit formulas for  $P_t$ , or by the branching particles scheme of Henry-Labordère (2012) in more exotic situations. In a bilateral trading setup, this is studied in Crépey and Song (2015), where, for realistic values of the counterparty risk and funding data, the nonlinear correction is consistently found less than 5% to 10% of the linear part. Hence, in this paper, we just use the linear part. Namely, we obtain by first order linear approximation in the reduced CCVA BSDE (4.14):

$$\begin{aligned} \Theta_0 = \widehat{\Theta}_0 &\approx \mathbb{E} \left[ \sum_{0 < \tau_Z^\delta < \bar{\tau}} \beta_{\tau_Z^\delta} \epsilon_{\tau_Z^\delta} + \int_0^{\bar{\tau}} \beta_s \widehat{f}_s(0) ds \right] = \underbrace{\mathbb{E} \sum_{0 < \tau_Z^\delta < \bar{\tau}} \beta_{\tau_Z^\delta} \epsilon_{\tau_Z^\delta}}_{\text{CVA}} + \underbrace{\mathbb{E} \int_0^{\bar{\tau}} \beta_s dva_s ds}_{\text{DVA}} \\ &+ \underbrace{\mathbb{E} \int_0^{\bar{\tau}} \beta_s \left( -c_s (\mathcal{C}_s - P_{s-}) + \tilde{\lambda}_s (P_s - \mathcal{C}_s)^- - \lambda_s (P_s - \mathcal{C}_s)^+ \right) ds}_{\text{FVA}} + \underbrace{\mathbb{E} \int_0^{\bar{\tau}} \beta_s k_s K_s ds}_{\text{KVA}}, \end{aligned} \quad (6.1)$$

where  $\beta_t = e^{-\int_0^t r_s ds}$ ,  $\tilde{\lambda} = \bar{\lambda} - (1 - \bar{R})\gamma_\bullet$  in which the DVA2 can be ignored by setting  $\bar{R} = 1$ ,  $\mathcal{C} = \mathcal{C}^0$  with, for each member  $i$ ,  $\mathcal{C}^i = VM^i + IM^i + DF^i$ , and where, for each  $t = \tau_Z^\delta < \bar{\tau}$ ,

$$\epsilon_t = (B_t - E_{t-})^+ \frac{DF_t}{\sum_{j \in N} J_t^j DF_t^j}, \quad \text{with } B_t = \sum_{i \in Z} (P_t^i + \Delta_t^i - \mathcal{C}_t^i)^+, \quad \mathcal{C}_t^i = \mathcal{C}_{\bar{\tau}_Z}^i$$

(cf. (3.8) and (3.4)-(3.6)). In addition,  $dva = -\gamma \hat{\xi}$ , where  $\hat{\xi}$  is a predictable process such that  $\hat{\xi}_\tau = \mathbb{E}(\beta_\tau^{-1} \beta_{\tau\delta} \xi | \mathcal{G}_{\tau-})$  (cf. (4.10)), with  $\xi = (1 - R)(P_{\tau\delta} + \Delta_{\tau\delta} - \mathcal{C}_\tau)^+$ , so that the DVA can be ignored by setting  $R = 1$ .

The  $\epsilon$  terms in (6.1) give rise to a costly credit valuation adjustment for the member, in the form of a non standard CVA paid through its contribution to the covering of other members realized breaches whilst it is alive. The three components of  $\hat{f}$  in the second line of (4.11) give rise after integration in (6.1) to a debt valuation adjustment (DVA), a funding valuation adjustment (FVA) and a capital cost (KVA in the sense of Green et al. (2014)). The positive (respectively negative) CCVA terms in (6.1) can be considered as deal adverse (respectively deal friendly) as they increase (respectively decrease) the CCVA  $\Theta$  and therefore increase (respectively decrease) the cost of the hedge  $-\Pi = Q + \Theta$  for the member—with, depending on the sign of  $\Pi$ , a “less positive”  $\Pi$  interpreted as a lower “buyer price” by the member or a “more negative”  $\Pi$  interpreted as a higher “seller price” by the member.

For numerical purposes, we use the following randomized version of (6.1):

$$\mathbb{E} \left[ \sum_{0 < \tau_Z^\delta < \bar{\tau}} \beta_{\tau_Z^\delta} \epsilon_{\tau_Z^\delta} + \mathbb{1}_{\{\zeta < \bar{\tau}\}} \frac{e^{\mu\zeta}}{\mu} \beta_\zeta \hat{f}_\zeta(0) \right], \quad (6.2)$$

where  $\zeta$  denotes an independent exponential time of parameter  $\mu$ . Moreover, to deal with the  $dva_\zeta$  term in  $\hat{f}_\zeta(0)$ , we use the following result with  $h_\zeta = \frac{e^{\mu\zeta}}{\mu}$ .

**Lemma 6.1** *For any predictable process  $h$  and independent diffuse random variable  $\zeta$ , we have:*

$$\mathbb{E}[\mathbb{1}_{\{\zeta < \bar{\tau}\}} h_\zeta \beta_\zeta dva(\zeta, X_\zeta)] = -\mathbb{E} \left[ \mathbb{1}_{\{\zeta < \bar{\tau}\}} h_\zeta \beta_{\zeta+\delta} (1 - R) \gamma_\bullet(\zeta) (Q_{\zeta\delta} - \mathcal{C}_\zeta)^+ \right]. \quad (6.3)$$

**Proof.** We denote by  $\mathcal{T}_\delta$  the transition function of the homogeneous Markov process  $(t, \mathbf{X}_t, \beta_t)$  over the time horizon  $\delta$ , i.e.

$$(\varphi, (t, \mathbf{x}, b)) \rightarrow \mathcal{T}_\delta[\varphi](t, \mathbf{x}, b) = \mathbb{E}[\varphi(t^\delta, \mathbf{X}_{t^\delta}, \beta_{t^\delta}) | \mathbf{X}_t = \mathbf{x}, \beta_t = b] = \mathbb{E}[\varphi(t^\delta, \mathbf{X}_{t^\delta}, \beta_{t^\delta}) | \mathcal{G}_t].$$

Hence, recalling (5.5),

$$\bar{\xi}_\tau = \mathcal{T}_\delta[\xi_\star(\cdot, \cdot, \cdot, \beta_\tau, \mathcal{C}_{\tau-}, \hat{\Delta}_{\tau-})](\tau, \mathbf{X}_\tau, \beta_\tau) = \mathcal{T}_\delta[\xi_\star(\cdot, \cdot, \cdot, \beta_\tau, \mathcal{C}_{\tau-}, \hat{\Delta}_{\tau-})](\tau, \mathbf{X}_{\tau-}, \beta_\tau) \quad (6.4)$$

(since  $\mathbf{X}$  doesn't jump at time  $\tau$ ), where we set

$$\xi_\star(t, \mathbf{x}, b, \beta_\tau, \mathcal{C}_{\tau-}, \hat{\Delta}_{\tau-}) = (1 - R) \beta_\tau^{-1} b \left( P(t, \mathbf{x}) + \hat{\Delta}(t, \mathbf{x}) - \beta_\tau b^{-1} \hat{\Delta}_{\tau-} - \mathcal{C}_{\tau-} \right)^+,$$

in which  $\beta_\tau$ ,  $\mathcal{C}_{\tau-}$  and  $\hat{\Delta}_{\tau-}$  are considered as  $\mathcal{G}_{\tau-}$  measurable parameters. In view of (6.4), we have (cf. Crépey and Song (2015, Lemma 5.1))

$$-dva_t = \gamma_t \hat{\xi}_t = J_{t-} \gamma_t \mathcal{T}_\delta[\xi_\star(\cdot, \cdot, \cdot, \beta_t, \mathcal{C}_t, \hat{\Delta}_{t-})](t, \mathbf{X}_{t-}, \beta_t), \quad \mathbb{Q} \times \lambda \text{ a.e.} \quad (6.5)$$

As a consequence, given an independent random variable  $\zeta$  with density  $p$ , we can write, using (6.5), the definition of  $\mathcal{T}_\delta$  and (5.2) to pass to the second, third and fourth line, respectively:

$$\begin{aligned}
& -\mathbb{E}[h_\zeta \mathbf{1}_{\{\zeta \leq \bar{\tau}\}} \beta_\zeta dva(\zeta, X_\zeta)] = -\int_0^T \mathbb{E}[h_t \beta_t \mathbf{1}_{\{t < \bar{\tau}\}} dva(t, X_t)] p(t) dt \\
& = \int_0^T \mathbb{E}\left[h_t \beta_t \mathbf{1}_{\{t \leq \tau\}} \gamma_t \mathcal{T}_\delta[\xi_\star(\cdot, \cdot, \cdot, \beta_t, \mathcal{C}_t, \widehat{\Delta}_t)](t, \mathbf{X}_t, \beta_t)\right] p(t) dt \\
& = \int_0^T \mathbb{E}\left[h_t \beta_t \mathbf{1}_{\{t \leq \tau\}} \gamma_t \mathbb{E}[\xi_\star(t^\delta, \mathbf{X}_{t^\delta}, \beta_{t^\delta}, \beta_t, \mathcal{C}_t, \widehat{\Delta}_t) | \mathcal{G}_t]\right] p(t) dt \\
& = \int_0^T \mathbb{E}[h_t \beta_t \mathbf{1}_{\{t \leq \tau\}} \gamma_\bullet(t) \xi_\star(t^\delta, \mathbf{X}_{t^\delta}, \beta_{t^\delta}, \beta_t, \mathcal{C}_t, \widehat{\Delta}_t)] p(t) dt \\
& = \mathbb{E}[\mathbf{1}_{\{\zeta \leq T\}} h_\zeta \beta_\zeta \mathbf{1}_{\{\zeta \leq \tau\}} \gamma_\bullet(\zeta) \xi_\star(\zeta^\delta, \mathbf{X}_{\zeta^\delta}, \beta_{\zeta^\delta}, \beta_\zeta, \mathcal{C}_\zeta, \widehat{\Delta}_\zeta)]. \blacksquare
\end{aligned}$$

Plugging  $h_\zeta = \frac{e^{\mu\zeta}}{\mu}$  in (6.3) to deal with the  $dva_\zeta$  term in  $\widehat{f}_\zeta(0)$ , (6.2) is rewritten as

$$\begin{aligned}
\Theta_0 = \widehat{\Theta}_0 \approx & \mathbb{E}\left\{ \sum_{0 < \tau_Z^\delta < \bar{\tau}} \beta_{\tau_Z^\delta} \epsilon_{\tau_Z^\delta} + \mathbf{1}_{\{\zeta < \bar{\tau}\}} \frac{e^{\mu\zeta}}{\mu} \times \right. \\
& \left. \left[ -\beta_{\zeta^\delta} \gamma_\bullet(\zeta) (1-R)(Q_{\zeta^\delta} - \mathcal{C}_\zeta)^+ + \beta_\zeta \left( -c_\zeta (\mathcal{C}_\zeta - P_{\widehat{\zeta}^-}) + \widetilde{\lambda}_\zeta (P_\zeta - \mathcal{C}_\zeta)^- - \lambda_\zeta (P_\zeta - \mathcal{C}_\zeta)^+ + k_\zeta K_\zeta \right) \right] \right\}. \tag{6.6}
\end{aligned}$$

## 6.2 BVA Engine

Here we provide an executive summary of a bilateral CSA setup adapted, for comparison purposes, from Crépey and Song (2015). There is no cost in capital there, but a KVA term can be added to the funding costs as in the present paper.

**Remark 6.1** In Crépey and Song (2015), cash flows are viewed from the perspective of the bank, which will be taken as the member here, whereas we view them in this paper from the perspective of the clearinghouse (opposite to the one of the member). Hence, the sign conventions are opposite, i.e.  $P, \Delta, Q$ , etc... in this paper correspond to their opposites in Crépey and Song (2015), which is why we see  $\cdot^\mp$  here whenever we have  $\cdot^\pm$  there.

In the context of a bilateral CSA (credit support annex), i.e. a legal agreement specifying the margining and default liquidation procedure between a bank, say “the member”, labeled 0, in the above CCP setup, and a counterparty, say another member  $i \neq 0$ , let  $VM$  denote the variation margin, where  $VM \geq 0$  (resp.  $\leq 0$ ) means collateral posted by the bank and received by the counterparty (resp. posted by the counterparty and received by the bank), and let  $IM^b \geq 0$  (resp.  $IM^c \leq 0$ ) represent the initial margin posted by the bank (resp. the negative of the initial margin posted by the counterparty). Hence,

$$\mathfrak{C} = VM + IM^b \quad \text{and} \quad C = VM + IM^c \tag{6.7}$$

represent respectively the collateral guarantee for the counterparty and the negative of the collateral guarantee for the bank. Assuming all the margins re-hypothecable, meaning that received margins can be reused for funding purposes in the bilateral setup, then the collateral funded by the bank is  $\mathcal{C} = VM + IM^b + IM^c$ , where, for consistency with the CCVA setup,  $VM_t$  will be taken as  $P_{\widehat{\zeta}}$ . So, in the spirit of standard CSAs (“sCSA”) that

are becoming the main alternative to CCPs, we are considering full CSA collateralization, and even over-collateralization through the initial margins  $IM^b$  and  $IM^c$ . In the equations below,  $VM$  will be assumed to be remunerated at a flat OENIA rate  $r_t$  and  $IM^b$  and  $IM^c$  at a possibly different rate  $(r_t + c_t)$ , even though, as opposed to the CCP setup where clearinghouses typically charge  $-c_t = 20$  bp for initial margins and default fund contributions, there is no systematic margin fee in bilateral transactions. Accordingly,  $c$  will be taken as 0 in all the CSA numerics in Sect. 8. Following Crépey and Song (2015), at time 0, the difference  $\Theta_0$  between the mark-to-market of the portfolio and an all-inclusive price (both from the perspective of the bank, cf. the remark 4.3), dubbed BVA for bilateral valuation adjustment, is given by the following linear approximation formula (compare (6.1)):

$$\begin{aligned} \Theta_0 = \bar{\Theta}_0 \approx \mathbb{E} \left[ \int_0^{\bar{\tau}} \beta_s \bar{f}_s(0) ds \right] &= \underbrace{\mathbb{E} \int_0^{\bar{\tau}} \beta_s cdva_s ds}_{\text{CDVA}} + \\ &+ \underbrace{\mathbb{E} \int_0^{\bar{\tau}} \beta_s \left( -c_s(C_s - P_{\hat{s}-}) + \tilde{\lambda}_s (P_s - C_s)^- - \lambda_s (P_s - C_s)^+ \right) ds}_{\text{FVA}} + \underbrace{\mathbb{E} \int_0^{\bar{\tau}} \beta_s k_s K_s ds}_{\text{KVA}}, \end{aligned} \quad (6.8)$$

where:

- $P$  means the mark-to-market of the position of the member with the counterparty (viewed from the perspective of the latter),
- the meaning of  $\beta$ ,  $\tilde{\lambda}$ ,  $\lambda$ ,  $k$  and  $K$  is as in the CCP setup, but the formula for the regulatory capital  $K$  is different (compare Sect. A.1 and A.2),
- $\tau = \tau_b \wedge \tau_c$  is the first-to-default time of the bank and its counterparty (as opposed to the default time of the member, i.e. the bank, in the CCP setup),
- $cdva = \gamma \hat{\xi}$ , where  $\hat{\xi}$  is a predictable process such that  $\hat{\xi}_\tau = \mathbb{E}(\beta_\tau^{-1} \beta_{\tau\delta} \xi | \mathcal{G}_{\tau-})$ , with

$$\xi = \mathbb{1}_{\{\tau_c \leq \tau_b^\delta\}} (1 - R_c)(P_{\tau\delta} + \Delta_{\tau\delta} - C_\tau)^- - \mathbb{1}_{\{\tau_b \leq \tau_c^\delta\}} (1 - R_b)(P_{\tau\delta} + \Delta_{\tau\delta} - \mathfrak{C}_\tau)^+,$$

in which the recovery rates  $R_c$  of the counterparty to the bank and  $R_b$  of the bank to the counterparty are usually taken in the bilateral setup as 40%.

For numerical purposes, we use the following randomized version of (6.8) (compare (6.2)):

$$\mathbb{E} \left[ \mathbb{1}_{\{\zeta < \bar{\tau}\}} \frac{e^{\mu\zeta}}{\mu} \beta_\zeta \bar{f}_\zeta(0) \right], \quad (6.9)$$

where  $\zeta$  denotes an independent exponential time of parameter  $\mu$ . The  $cdva_\zeta$  term in  $\bar{f}_\zeta(0)$  is treated by the following bilateral analog of Lemma 6.1, which is also a straightforward adaptation of Crépey and Song (2015, Lemma 8.2) and is therefore stated without proof. We write  $\mathcal{Y}_b = \{Y \in \mathcal{Y}; 0 \in Y\}$ ,  $\mathcal{Y}_c = \{Y \in \mathcal{Y}; i \in Y\}$ .



**Lemma 6.2** *For any predictable process  $h$  and independent diffuse random variable  $\zeta$ , we have:*

$$\begin{aligned} \mathbb{E}[\mathbf{1}_{\{\zeta < \bar{\tau}\}} h_\zeta \beta_\zeta cdva(\zeta, X_\zeta)] &= \mathbb{E}\left[\mathbf{1}_{\{\zeta < \bar{\tau}\}} h_\zeta \beta_{\zeta^\delta} \times \right. \\ &\left. \left( \left( \sum_{Y \in \mathcal{Y}_c} \gamma_Y(\zeta) + \mathbf{1}_{\{\tau_c \leq \zeta^\delta\}} \sum_{Y \in \mathcal{Y}_b \setminus \mathcal{Y}_c} \gamma_Y(\zeta) \right) (1 - R_c) (Q_{\zeta^\delta} - C_\zeta)^- \right. \right. \\ &\quad \left. \left. - \left( \sum_{Y \in \mathcal{Y}_b} \gamma_Y(\zeta) + \mathbf{1}_{\{\tau_b \leq \zeta^\delta\}} \sum_{Y \in \mathcal{Y}_c \setminus \mathcal{Y}_b} \gamma_Y(\zeta) \right) (1 - R_b) (Q_{\zeta^\delta} - \mathfrak{C}_\zeta)^+ \right) \right]. \blacksquare \end{aligned} \quad (6.10)$$

Plugging  $h_\zeta = \frac{e^{\mu\zeta}}{\mu}$  in (6.10) to deal with the  $cdva_\zeta$  term in  $\bar{f}_\zeta(0)$ , (6.9) is rewritten as (compare (6.6)):

$$\begin{aligned} \Theta_0 = \bar{\Theta}_0 &\approx \mathbb{E}\left\{ \mathbf{1}_{\{\zeta < \bar{\tau}\}} \frac{e^{\mu\zeta}}{\mu} \left[ \beta_{\zeta^\delta} \left( \left( \sum_{Y \in \mathcal{Y}_c} \gamma_Y(\zeta) + \mathbf{1}_{\{\tau_c \leq \zeta^\delta\}} \sum_{Y \in \mathcal{Y}_b \setminus \mathcal{Y}_c} \gamma_Y(\zeta) \right) (1 - R_c) (Q_{\zeta^\delta} - C_\zeta)^- \right. \right. \\ &\quad \left. \left. - \left( \sum_{Y \in \mathcal{Y}_b} \gamma_Y(\zeta) + \mathbf{1}_{\{\tau_b \leq \zeta^\delta\}} \sum_{Y \in \mathcal{Y}_c \setminus \mathcal{Y}_b} \gamma_Y(\zeta) \right) (1 - R_b) (Q_{\zeta^\delta} - \mathfrak{C}_\zeta)^+ \right) \right. \\ &\quad \left. + \beta_\zeta \left( -c_\zeta (C_\zeta - P_{\zeta^-}) + \tilde{\lambda}_\zeta (P_\zeta - C_\zeta)^- - \lambda_\zeta (P_\zeta - C_\zeta)^+ + k_\zeta K_\zeta \right) \right] \right\}. \end{aligned} \quad (6.11)$$

## 7 Experimental Framework

In this section we design an experimental framework that is used in Sect. 8 for studying numerically the netting benefit of central clearing. We stress that this should only be viewed as illustrative of one among many possible uses of our ‘‘TVA engines’’. The small experiment elaborated in this section is simplistic in many respects. In particular, we only consider one clearinghouse trading one single asset, whereas it is well known since the paper by Duffie and Zhu (2011) that the netting benefit of central clearing can be balanced by fragmentation, namely dispersion of trades among different CCPs. Also, we compare a situation where all trades are centrally cleared with a situation where all trades are bilateral. In practice, vanilla products are most likely to be cleared and exotics most likely bilaterally traded. Therefore, in a more realistic perspective, central clearing also implies a loss of netting between vanillas and exotics, which in some cases might balance the obvious multilateral netting (see Figure 1) offered by CCPs.

### 7.1 Driving Asset

We consider, on underlying FRA Libor rates  $F_t(T_{l-1}, T_l)$ , an interest rate swap with cash-flows  $h_l(K - F_{T_{l-1}}(T_{l-1}, T_l))$  at increasing times  $T_l$ ,  $l = 1, \dots, d$ , where  $h_l = T_l - T_{l-1}$ , assumed to drive all members P&Ls. Moreover, for simplicity, we assume a constant OIS short term interest rate  $r$  and stylized Black-Scholes dynamics with historical (resp. risk-neutral) drift  $\mu$  (resp.  $\kappa$ ) and volatility  $\sigma$  for the  $F_t(T_{l-1}, T_l) = S_t$ ,  $t \leq T_{l-1}$ , independent of  $l$  (see the remark 7.1). Denoting by  $T_t$  the smallest  $T_l > t$ , the mark-to-market for a counterparty paying the above cash flows is given, for  $T_0 = 0 \leq t \leq T_d = \bar{T}$ , by  $P_t = \beta_t^{-1} \beta_{T_t} h_l (K - S_{T_{l-1}}) + P_t^*$ , where

$$P_t^* = \beta_t^{-1} K \sum_{l=l_t+1}^d \beta_{T_l} h_l - S_t \sum_{l=l_t+1}^d \beta_t^{-1} \beta_{T_l} h_l = P_*(t, S_t), \quad (7.1)$$

with  $K = S_0$  so that the swap has zero value at time 0. Moreover, we assume a notional  $Nom$  for this swap such that the time 0 value of each leg of the swap is one, i.e.  $Nom = (\sum_{l=1}^d \beta_{T_l} h_l K)^{-1}$ .

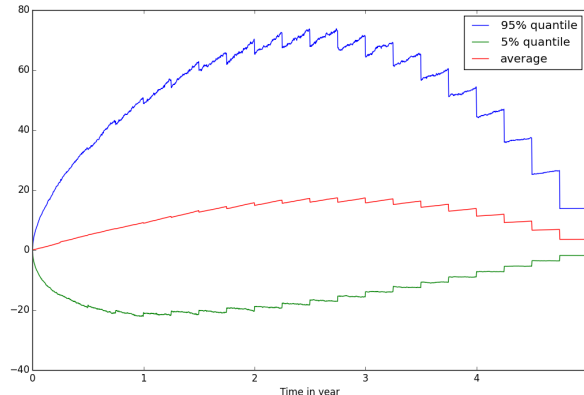


Figure 4: Mark-to-market process of the swap driving all members P&Ls, viewed from the point of view of a party receiving floating and paying fix in the swap, which we call a long unit position in the swap ( $y$  axis in percent). Mean and quantiles as a function of time computed by Monte Carlo simulation of the process  $-P_t = -\beta_t^{-1} \beta_{T_t} h_l (K - S_{T_{t-1}}) - P^*(t, S_t)$  based on the formula (7.1) used along  $m = 10^4$  simulated trajectories of  $S$ .

The following numerical values are used:

$$r = 2\%, \quad S_0 = 100, \quad \mu = \kappa = 12\%, \quad \sigma = 20\%, \quad h_l = 3 \text{ months}, \quad \bar{T} = 5 \text{ years}, \quad (7.2)$$

resulting in the mark-to-market process of the swap, from the point of view of a party receiving floating and paying fix, which we call a long unit position in the swap, displayed in Figure 4. Note that since  $\mu = \kappa$ , this is together the risk-neutral profile (as relevant for the Monte Carlo XVA computations) and the historical profile (as relevant for margin computations).

**Remark 7.1** Figure 4 shows the typical profile of an interest rate swap in an increasing term structure, where expectations of increasing rates make the swap in the money on average (i.e. the red curve is in the positive in Figure 4). This yields to the product some XVA flavor that would be missing in a flat interest rate environment where the red curve would be flat and there would be virtually no CVA/DVA. The present Black–Scholes setup and values of the parameters allow one to obtain this stylized pattern without having to introduce a full flesh interest rate model (one among many possibilities would be the model in Crépey, Gerboud, Grbac, and Ngor (2013)), which would add complexity without real value with respect to the objective pursued in this paper.

## 7.2 Structure of the Clearinghouse

We consider a clearinghouse with  $(n + 1)$  members chosen among the 125 names of the CDX index as of 17 December 2007, a particular day toward the beginning of the global financial crisis. The default times of the 125 names are modeled by a common shock model

with piecewise constant intensities  $\gamma_Y$  constant on the time intervals  $[0, 3]$  and  $[3, 5]$  years, calibrated to the corresponding 3 and 5 year CDS and 5 year CDO data. With five nested common shocks  $Y$  on top of an idiosyncratic shock  $Y = \{i\}$  for each of the 125 names, a nearly perfect calibration is achieved, as developed in Crépey et al. (2014, Sect. 8.4.3)<sup>7</sup>, to which the reader is referred for the details. Specifically, we consider a subset of nine representative members of the index, with increasing CDS spreads  $\Sigma$  (average 3 year and 5 year spread) shown in the first row of Table 1.

$\Sigma$	45	52	56	61	73	108	176	367	1053
$\alpha$	(0.46)	0.09	0.23	(0.05)	0.34	(0.04)	0.69	(0.44)	(0.36)

Table 1: (*Top*) Average 3 and 5 year CDS spreads  $\Sigma$ , in basis points (bp), for a representative subset of nine members of the CDX index as of 17 December 2007. (*Bottom*) Coefficients  $\alpha$  summing up to 0 used for determining the positions in the swap of the nine members.

The coefficients  $\alpha$  in the second row, where parentheses mean negative numbers, will be used as explained below for determining the positions in the swap of the nine members in the simulations. These coefficients were obtained as a vector of nine uniform numbers minus its cyclic shift, so that  $\sum_{i \in N} \alpha_i = 0$ . As we shall see below, the so called compression factor  $\nu_0 = \frac{\sum_{i \in N} |\alpha_i|}{|\alpha_0|} - 1$  will correspond to the gross (as opposed to net) position of the reference member, i.e. the size of its position before netting through the CCP, when trading bilaterally, whereas its net, centrally cleared position will be equal to one.

### 7.3 Member Portfolios

We represent in an antisymmetric matrix form

$$\varpi = \begin{matrix} & \begin{matrix} 0 & 1 & 2 & 3 & \cdots & n \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ \vdots \\ n \end{matrix} & \begin{pmatrix} 0 & \varpi_{0,1} & \varpi_{0,2} & \varpi_{0,3} & \cdots & \varpi_{0,n} \\ \cdot & 0 & \varpi_{1,2} & \varpi_{1,3} & \cdots & \varpi_{1,n} \\ \cdot & \cdot & 0 & \varpi_{2,3} & \cdots & \varpi_{2,n} \\ \cdot & \cdot & \cdot & 0 & \cdots & \varpi_{3,n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \cdot & \cdot & \cdot & \cdot & \cdots & 0 \end{pmatrix} \end{matrix}, \quad (7.3)$$

where each “ $\cdot$ ” represents the negative of the symmetric entry in the matrix, the long positions of each member  $i$  with respect to each member  $j$  in the swap with price process depicted in Figure 4 ( $\varpi_{i,j} \leq 0$  effectively means a short position of  $i$  with respect to  $j$ ). We remark that the data of the CCVA BSDE related to the member 0, or the linearized and randomized form (6.6) of this BSDE, only depend on the matrix  $\varpi$  through the sums of each of its rows (or, equivalently up to sign change, the sums of its columns), corresponding to the vector of the long (short) positions of the different clearing members against the CCP. By contrast, the data of the BVA BSDE related to the member 0, or the linearized and randomized form (6.11) of this BSDE, only depend on the matrix  $\varpi$  through its first row

<sup>7</sup>Or Bielecki, Cousin, Crépey, and Herbertsson (2014a, Part II) in the journal version.

(vector of the long positions of the reference member 0 against its different counterparties  $i = 1, \dots, n$ ). Hence, we can forget about the detail of the above matrix, focusing on the  $\omega_i^{csa} := \varpi_{0,i}$  and  $\omega_i^{ccp} := \sum_{l \neq i} \varpi_{l,i}$ ,  $i \neq 0$ , for comparing two trading setups:

- A bilateral CSA setup as of Sect. 6.2, where the member 0 trades a long  $\omega_i^{csa} \in \mathbb{R}$  position in the swap separately with each member  $i \neq 0$  ( $\omega_i^{csa} \leq 0$  effectively means a short position of member 0), whichever other trades members  $i \neq 0$  may have between each others.
  - For instance, but non necessarily, the member 0 has a long  $\omega_i^{csa} \in \mathbb{R}$  position with each member  $i \neq 0$  and there are no other trades between members (at least after netting at the level of each pair of members), which corresponds to the situation where only the first row and column are nonzero in the matrix  $\varpi$ .
  - In any case, the netted long position of the member 0 is  $\sum_{i \neq 0} \omega_i^{csa}$ . However, netting does not apply across different counterparties in the CSA setup. We call compression factor  $\nu_0$  the gross position of the reference member 0, i.e. the number  $\nu_0 = \sum_{i \neq 0} |\omega_i^{csa}|$  of trades the member 0 is engaged into in absolute terms.
- A CCP setup as of Sect. 6.1, where each member  $i \neq 0$  trades a short  $\omega_i^{ccp} \in \mathbb{R}$  position in the swap through the CCP ( $\omega_i^{ccp} \leq 0$  effectively means a long position of member  $i$ ), whichever way this position may be distributed among other members.
  - For instance, but non necessarily, each member  $i \neq 0$  has a short  $\omega_i^{ccp} \in \mathbb{R}$  position with the member 0 and there are no other trades between members, which again corresponds to the situation where only the first row and column are nonzero in  $\varpi$ .
  - In any case, since members trade between themselves, the member 0 trades a long  $\sum_{i \neq 0} \omega_i^{ccp}$  position in the driving asset after netting through the CCP, instead of a non netted position of size  $\nu_0$  before clearing through the CCP.

Moreover, for comparability purposes, we restrict ourselves to a netted long position of the member 0 equal to 1, i.e.  $\sum_{i \neq 0} \omega_i^{ccp} = 1$  (respectively  $\sum_{i \neq 0} \omega_i^{csa} = 1$ , but, again, netting is only effective in the CCP setup). In most of our numerics we will alternately consider as reference member 0 each of the nine members in Table 1, for positions in the driving asset determined by the coefficients  $\alpha$  summing up to zero in the second row of Table 1 using the following rule: “ $\omega_i = -\frac{\alpha_i}{\alpha_0}$ ,  $i \neq 1$ ”, where  $\omega = \omega^{csa}$  or  $\omega^{ccp}$ , as suitable. In particular, since the coefficients  $\alpha$  add up to 0, this specification ensures that  $\sum_{i \neq 0} \omega_i = 1$ , as required. In addition, we have

$$\nu_0 = \sum_{i \neq 0} |\omega_i| = \sum_{i \neq 0} \frac{|\alpha_i|}{|\alpha_0|} = \frac{\sum_{i \in N} |\alpha_i|}{|\alpha_0|} - 1, \quad (7.4)$$

so the smaller  $|\alpha_0|$ , the larger the compression factor  $\nu_0$ . We also define  $\omega_0 = -\frac{\alpha_0}{\alpha_0} = -1$ , consistent with the member 0 being long a +1 (i.e. short a -1) net position in the swap, where we emphasize once more that netting across different counterparties is only effective in the CCP setup.

**Example 7.1** Table 2 shows the resulting values of the  $\omega_i$  of the different members  $i \neq 0$  when the name with CDS spread 61 bp (name with the second smallest  $|\alpha|$  in Table 1, with

corresponding entries emphasized in bold in Table 2), referred to as “Name I” henceforth, is taken as reference member 0 (prototype of a name with a large gross position). Hence, the  $\omega_i$  in Table 2 are proportional to the  $\alpha_i$  in Table 1, modulo a scaling factor tuned so that  $\omega_0$  (corresponding to the member with spread 61 bp) be  $-1$ .

$\Sigma$	45	52	56	<b>61</b>	73	108	176	367	1053
$\omega$	(9.20)	1.80	4.60	<b>(1.00)</b>	6.80	(0.80)	13.80	(8.80)	(7.20)

Table 2: Positions  $\omega$  in the swap of the nine members with CDS spreads  $\Sigma$ , in the respective  $\omega = \omega^{csa}$  or  $\omega^{ccp}$  meaning, when the reference member 0 is Name I with CDS spread 61 bp (name with the second smallest  $|\alpha|$  in Table 1).

The interpretation of the positions  $\omega_i$  is as follows:

- In the bilateral CSA setup, the  $\omega_i$  are understood as  $\omega_i^{csa}$  in the sense explained above, i.e. Name I trades a long  $\omega_i \equiv \omega_i^{csa} \in \mathbb{R}$  position in the driving asset separately with each member  $i$  other than itself. In this case, as we can see from Table 2, the reference member Name I trades  $-9.20$  (i.e. is short 9.20) contracts in the swap with the member with spread 45 bp, 1.80 contracts with the member with spread 52 bp, etc..., which is all we need to know for computing the BVA of Name I based on the BVA formula (6.11), run separately for each counterparty  $i \neq 0$  of Name I. The total number of contracts Name I is engaged into (size of its gross position) amounts to  $\nu_0 = \sum_{i \neq 0} |\omega_i| = 53.00$ , without netting between these (other than the netting implicit to each  $\omega_i$  relative to each counterparty) in the CSA setup.
- In the CCP setup, the  $\omega_i$  are understood as  $\omega_i^{ccp}$  in the sense explained above, i.e. each member  $i$  other than the reference member Name I trades a short  $\omega_i \equiv \omega_i^{ccp} \in \mathbb{R}$  position in the driving asset through the CCP. In this case, the member with spread 45 bp trades a short  $-9.20$  (i.e. long 9.20) contracts position through the CCP, the member with spread 52 bp trades a short 1.80 contracts position through the CCP, etc..., which is all we need to know for computing the CCVA of Name I. Since members trade between themselves, Name I trades a netted long  $\sum_{i \neq 0} \omega_i^{ccp} = +1$  swap units position through the CCP.
- A definite setup consistent with both interpretations, corresponding to a matrix of positions  $\varpi$  where only the first row and column are nonzero, is when Name I trades 9.20 contracts with the member with spread 45 bp,  $-1.80$  contracts with the member with spread 52 bp, etc..., and there are no other trades between members.

**Example 7.2** Table 3 is the analog of Table 2 when the member with spread 367 bp (name with the second largest credit spread in Table 1, with corresponding entries emphasized in bold in Table 2), referred to as “Name II” henceforth, is taken as reference member 0 (prototype of a very risky name). In this case  $\nu_0 = \sum_{i \neq 0} |\omega_i| = 5.14$ .

$\Sigma$	45	52	56	61	73	108	176	<b>367</b>	1053
$\omega$	(1.05)	0.20	0.52	(0.11)	0.77	(0.09)	1.57	<b>(1.00)</b>	(0.82)

Table 3: Analog of Table 2 when the reference member 0 is Name II with CDS spread 367 bp (name with the second largest credit spread  $\Sigma$  in Table 1).

## 7.4 Margins

In the CCP setup the initial margin  $IM^i$  posted by each member  $i \in N$  is set through (3.2) using the value at risk of level  $a_{im}$  for the risk measure  $\rho$ . Since the pricing function  $P_\star$  in (7.1) is decreasing in  $S$ , therefore  $IM^i$  can be proxied, at each simulated time  $\zeta$  in (6.6) or (6.11), by

$$IM_\zeta^i = Nom \times \omega_i \times \begin{cases} P_\star(\zeta, S_\zeta) - P_\star(\zeta, S_\zeta e^{\sigma\sqrt{\delta+h}q(a_{im})+(\mu-\frac{\sigma^2}{2})(\delta+h)}), & \omega_i \geq 0 \\ P_\star(\zeta, S_\zeta) - P_\star(\zeta, S_\zeta e^{\sigma\sqrt{\delta+h}q(1-a_{im})+(\mu-\frac{\sigma^2}{2})(\delta+h)}), & \omega_i \leq 0, \end{cases} \quad (7.5)$$

where  $q(\cdot)$  is the standard normal quantile function (and we recall from (3.2) that  $(\delta + h)$  is the so called margin period of risk).

For instance, the reference member 0, with  $\omega_0^{ccp} = -1$ , is long one unit of the swap with mark-to-market profile shown in Figure 4, hence the exposure of the CCP to member 0 is the opposite profile. Accordingly, the CCP asks initial margins to the member 0 based on  $P_\star(\zeta, S_\zeta e^{\sigma\sqrt{\delta+h}q(1-a_{im})+(\mu-\frac{\sigma^2}{2})(\delta+h)}) - P_\star(\zeta, S_\zeta)$  (recalling Figure 4 shows  $(-P_t)$ ), consistent with the use of the second branch in (7.5) in case  $\omega_i \leq 0$  (here for  $i = 0$ ).

The default fund contribution of each member  $i \in N$  is simply set so that its overall margin level  $IM^i + DF^i$  reaches the value at risk of some level  $a_m > a_{im}$  for the corresponding variation-margined loss-and-profit of the clearinghouse.

In the CSA setup the initial margin  $-IM^c \geq 0$  required by the member 0 from the member  $i \neq 0$  (cf. (6.7)) is given by the right-hand side formula in (7.5) valued at a quantile level  $a'_{im}$ .

For instance, if  $\omega_i^{csa} = +2$ , meaning that the member 0 has a “double Figure 4 exposure” with regard to counterparty  $i$ , then the the member 0 asks the counterparty  $i$  to post initial margins based on  $P_\star(\zeta, S_\zeta) - P_\star(\zeta, S_\zeta e^{\sigma\sqrt{\delta+h}q(a'_{im})+(\mu-\frac{\sigma^2}{2})(\delta+h)})$  (recalling again that Figure 4 shows  $(-P_t)$ ), consistent with the use of the first branch in (7.5) in the case where  $\omega_i^{csa} \geq 0$  (for  $i \neq 0$ ).

Symmetrically, the formula for the initial margin  $IM^b \geq 0$  required by the member  $i$  from the member 0 reads

$$IM_\zeta^b = -\omega_i \times Nom \times \begin{cases} P_\star(\zeta, S_\zeta) - P_\star(\zeta, S_\zeta e^{\sigma\sqrt{\delta+h}q(a'_{im})+(\mu-\frac{\sigma^2}{2})(\delta+h)}), & \omega_i \leq 0 \\ P_\star(\zeta, S_\zeta) - P_\star(\zeta, S_\zeta e^{\sigma\sqrt{\delta+h}q(1-a'_{im})+(\mu-\frac{\sigma^2}{2})(\delta+h)}), & \omega_i \geq 0. \end{cases}$$

Note that for simplicity we assume initial margin quantile levels independent of the reference member 0 and the counterparty  $i$ .

**Remark 7.2** We use quantiles based on the historical distribution of  $S$ , i.e. with drift  $\mu$ , in margin computations. Since all our CCVA and BVA analysis is performed under the pricing measure  $\mathbb{Q}$  (see the first paragraph of Sect. 4), this is the only place where the historical drift  $\mu$  of  $S$  is used (this said, it makes no difference for our numerical data with  $\kappa = \mu$ , i.e. the risk-neutral and historical drifts of  $S$  are the same).

The above margin schemes are of course very elementary, as is also the Black–Scholes model that is used for  $S$ . In particular, in the centrally cleared case, it would be interesting to assess the impact of multivariate risk measures that could be used for fixing the default fund contributions (cf. (3.3)). We leave for further work the study of more sophisticated models and margining schemes.

## 7.5 TVA Data

The following numerical values are used in the sequel:

$$\bar{R} = 1, \quad \bar{\lambda} = \frac{1}{2}\Sigma_0, \quad \lambda = 0, \quad h = 1 \text{ day}, \quad \mu = \frac{2}{T}, \quad m = 10^4, \quad a_{ead} = 85\%, \quad (7.6)$$

where  $m$  is the number of simulations used for estimating each of the expectations in (6.6) or (6.11) and where  $a_{ead}$  is the level of the quantile that is used for computing the exposure at defaults in the regulatory capital formulas as explained in the introductory paragraph to Sect. A.

Moreover, in a CCP setup, unless otherwise stated, we use

$$\begin{aligned} R &= 0, \quad \delta = 5 \text{ days}, \quad a_{im} = 70\%, \quad T = 1 \text{ month}, \quad a_m = 80\%, \quad Y = 1 \text{ year}, \\ E^* &= 25\%K^{ccp}, \quad c = -20 \text{ bp}, \end{aligned} \quad (7.7)$$

where  $K^{ccp}$  is defined in (A.1). The relatively low levels of the quantiles that are used to set the initial and default fund contributions are meant to “compensate” for the excessive simplicity of the Black–Scholes setup without wrong-way risk used for  $S$  (they also yield moderate standard errors with a relatively small number  $m = 10^4$  of simulations). Wrong-way risk add-ons to the present stylized setup could be made following the lines of Crépey and Song (2015). The proportion of the default fund contributions as compared with initial margins reflected by the above quantile levels corresponds roughly to what a CCP would be susceptible to use on interest rate swaps. On equities, the level of the default fund can reach half of the initial margins. With CDSs, it can represent up to three times the initial margins, becoming the main resource of the CCP waterfall. The negative value  $c = -20$  bp is consistent with the actual practice of CCPs of charging a premium proportional to the amount of margins and default fund contributions. This premium is distinct from the commission fees, not reflected in our setup, that the CCP is also charging to the members. This means that the difference (positive or negative) between the resulting BVA and CCVA could be interpreted as the break-even value of this commission ensuring equal costs to centrally cleared and bilateral trading. In practice, commission fees are of the order of a few basis points of the size of the positions, i.e. a few basis points for a unit position in our swap with each leg equal to one at time 0.

In a CSA setup, alternatively to (7.7), unless otherwise stated, we use

$$R_b = R_c = 40\%, \quad \delta = 15 \text{ days}, \quad a'_{im} = 80\%, \quad c = 0. \quad (7.8)$$

The value of  $a'_{im} = 80\%$  used in the bilateral case equals the value of  $a_m$  used in the centrally cleared case, so that the total amount of margins is the same in both cases (initial margins only in the CSA setup versus initial plus default fund contributions in the CCP setup). Moreover, in order to be able to assess the impact of the move from Basel II to Basel III in terms of cost of capital, we sometimes present the  $KVA$  in the CSA case in two parts,  $KVA^{ccr}$  and  $KVA^{cva}$ , corresponding to the respective costs of  $K^{ccr}$  and  $K^{cva}$  (see Sect. A.2). Hence,  $KVA$  should be meant as  $K^{ccr}$  only under a Basel II specification and as  $KVA = KVA^{ccr} + KVA^{cva}$  in a more punitive Basel III specification, which is used by default in all the aggregate BVA numbers below.

## 8 Numerical Results and Their Analysis

All our XVA numbers are stated in basis points (recall that both legs of the swap of Figure 4, which is used for driving all our P&Ls, are worth one at time 0). The standard errors

(not shown) corresponding to the XVA Monte Carlo estimates are typically no more than a few percents in relative terms. For comparability purposes, common random inputs are used in all our Monte Carlo estimates, i.e. we use the same sampled trajectories of  $S$  and sampled sets of default times  $\tau_i$  in all cases, it's only the way these  $m = 10^4$  random input sets are used which changes. The computation times are proportional to the number of members  $n$  and model trajectories  $m$  that are used, e.g. about 5 minutes on a standard laptop to compute a full set of XVAs in Table 5 (four or five TVA components and their sum), where  $n = 8$  and  $m = 10^4$ , using pre-simulated values for all the random inputs as mentioned before. Negative (e.g. DVA) numbers are displayed in parentheses in the tables. Regarding the aggregated XVA numbers, i.e. BVA in the CSA setup, CCVA in the centrally cleared setup and TVA that is sometimes used as a common acronym for covering both cases, we use ' for DVA inclusive results, e.g. CCVA' (resp. CCVA) means the CCVA with the DVA included (resp. ignoring the DVA, i.e. for a coefficient  $R$  set equal to 1).

Given that we use a global margin level equal in both setups (with initial margins in the CSA case equal to the sum of the initial margins plus the default fund contributions in the CCP setup), we expect to obtain not so different CVA/DVA in both setups, but to see a shift from KVA in the CSA case, for which regulatory capital formulas are much more punitive than in the CCP case (see Sect. A), to FVA in the CCP case, due in particular to the 20 bp of margin fees charged by CCPs. But of course the structure of the network of the transactions is completely different in both setups, with full netting in the CCP case as apposed to netting at the level of each counterparty only in the CSA case (see Figure 1 and the matrix  $\varpi$  in (7.3)).

## 8.1 Preliminary Computations ( $n = 1$ )

Table 4 displays the XVA estimates when Name I with spread 61 bp is taken as reference member 0, alternately considering each of the other eight members as its unique counterparty in a unit swap position, i.e.  $\omega_i = 1$  for this counterparty and all the other  $\omega_i$  equal to 0. Hence,  $n$  is effectively equal to one and we have  $\nu_0 = 1$  in each of the eight cases. Of course, a CCP setup is rather artificial when  $n = 1$ , it should rather be seen as yielding a "boundary condition" on later results. In a CCP setup with  $n = 1$ , it's only the CVA numbers which really depend on the counterparty, proportionally to its credit spread (as reflected in the table). Consistent with CVA and DVA recovery coefficients  $R_i$  set to 0 in the CCP case and 40% in the CSA case, the CVA and DVA numbers in the CCP setup are found one to two times larger then their analogs in the bilateral CSA setup (note that even in the present case with  $n$  equal to one, the CSA and CCP formulas for the CVA and the DVA differ by other features, such as the time interval on which the related XVAs are computed). The FVA is much greater in the CCP case, due to the 20 bp charged by the CCP to the members proportional to their initial margins and default fund contributions. The capital is found very cheap in the CCP case. The aggregated XVA numbers (e.g. BVA versus CCVA) become in favor of the CSA setup for the two riskiest names due to a CVA that becomes larger in both setups, but is about twice as large in the CCP case. As can be seen from the comparison between the middle and lower panel in Table 4, the impact of a skin in the game  $E^* = 25\% \times K^{ccp}$  (a rather standard specification) is very small, only eroding the CCP CVA by the last digit in the last three columns, consistent with standard expectations of market practitioners in this regard.



$\Sigma_i$	45	52	56	73	108	176	367	1053
CVA	0.58	0.68	0.75	0.91	1.34	2.14	4.50	10.62
DVA	(0.69)	(0.70)	(0.69)	(0.69)	(0.68)	(0.66)	(0.62)	(0.50)
FVA	1.28	1.27	1.27	1.25	1.27	1.21	1.12	0.88
KVA <sup>ccr</sup>	3.22	3.11	2.86	3.84	4.22	4.54	4.20	3.62
KVA <sup>cva</sup>	1.46	0.73	0.72	1.45	1.44	2.12	2.03	1.71
BVA	6.54	5.79	5.60	7.45	8.27	10.01	11.86	16.83
BVA'	5.84	5.09	4.91	6.76	7.59	9.35	11.24	16.33
CVA	0.73	0.85	1.11	1.90	2.31	5.18	8.59	20.45
DVA	(0.74)	(0.74)	(0.74)	(0.74)	(0.74)	(0.74)	(0.74)	(0.74)
FVA	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50
KVA	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19
CCVA	5.43	5.54	5.81	6.59	7.00	9.87	13.28	25.15
CCVA'	4.69	4.80	5.07	5.85	6.27	9.14	12.54	24.41
CVA	0.73	0.85	1.11	1.90	2.31	5.17	8.58	20.44
DVA	(0.74)	(0.74)	(0.74)	(0.74)	(0.74)	(0.74)	(0.74)	(0.74)
FVA	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50
KVA	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19
CCVA	5.43	5.54	5.81	6.59	7.00	9.87	13.27	25.13
CCVA'	4.69	4.80	5.07	5.85	6.26	9.13	12.54	24.39

Table 4: XVA estimates when Name I with spread 61 bp is taken as reference member 0, alternately considering each of the other eight members as its single counterparty in a unit swap position. (*Upper row*) Credit spread of the single counterparty used in each case (ordered by increasing  $\Sigma_i$ ). (*Upper panel*) XVAs in the bilateral CSA setup. (*Middle panel*) XVAs in the CCP setup without skin in the game ( $E^* = 0$ ). (*Lower panel*) XVAs in the CCP setup with skin in the game  $E^* = 25\% \times K^{ccp}$ .

## 8.2 Netting Benefit

Table 5 shows the XVA numbers obtained by considering alternately each of the nine members in Table 1 as reference member, using the  $\alpha$  coefficients for setting the positions of the members in each case as explained in Sect. 7.3 (cf. the examples 7.1 and 7.2). The main conclusion from the table, where the different cases are ordered by increasing values of the compression factor  $\nu_0$ , i.e. by decreasing  $|\alpha_0|$ , is the potentially big netting gain resulting from trading through the CCP, especially for members with a large compression factor  $\nu_0$ . In fact, the CSA XVA numbers are roughly proportional to  $\nu_0$ , whereas the CCP XVA numbers are essentially not impacted by  $\nu_0$ .

Far behind this netting effect that dominates the comparison between the CSA and the CCP XVA numbers, the credit risk structure of the members explains the results within each respective CSA and CCP case. For instance, reference members with similar  $\nu_0$  have DVAs in proportion of their credit spreads, in each respective CSA and CCP setup. We can also see from Table 5 that KVA is often a prominent XVA term in the CSA setup, whereas it is a very small XVA contributor (in absolute as in relative terms) in the CCP setup.

These results ask the question why banks waited Basel III for clearing their trades and even then, often reluctantly so, whereas these results suggest that it was in their interest already with Basel II (i.e. ignoring KVA<sup>cva</sup> in the CSA setup, where KVA<sup>cva</sup> is

typically found of the order of about one-half of  $KVA^{ccr}$ ). Here we emphasize again that our numerical experiments are only an illustration in an extreme case of member P&Ls driven by a single asset and with full netting in the CCP case, as opposed to no netting across the different counterparties of the reference member in the bilateral CSA case. The fragmentation impact of clearing trades with not only one, by many CCPs, and of clearing vanillas versus trading exotics bilaterally, has already been mentioned as a word of warning in the beginning of Sect. 7. In standard cases, compression factors would be much less important than in the case of our “low  $\alpha$ ” names. This will be investigated further in Sect. 8.3. Besides, netting is actually counter-productive in terms of liquidation costs, ignored in our setup, which are proportional to the gross (as opposed to net) size of the positions. In addition, in the CSA setup, it would be more realistic to net at least funding costs across the different counterparties of the reference member (since funding is typically handled at the global level of the treasury of the bank), which we didn’t do for consistency with the stylized setup of Sect. 6.2, postulated separately between the reference member 0 and each counterparty  $i \neq 0$ . Also, not only commission fees charged by the CCP (typically sizing in bp of members positions, where we recall that all our XVA numbers are also expressed in bp), but also the fixed costs that are required for becoming a member of the CCP, are ignored here, as are also ignored the defaultability of the CCP as well as systemic and wrong-way risk. And of course, apart from a comparative costs analysis (even if still driven by the impact of netting after the above reservations have been acknowledged), there are the other aspects of central clearing, i.e. the principles of mutualization and transparency, which could be susceptible to explain why banks may want to keep away from central clearing.

$\nu_0$	2.91	4.87	5.14	6.50	6.94	10.74	29.00	53.00	66.50
$\alpha_0$	0.69	(0.46)	(0.44)	(0.36)	0.34	0.23	0.09	(0.05)	(0.04)
$\Sigma_0$	176	45	367	1053	73	56	52	61	108
CVA	9.41	15.92	12.15	8.66	22.64	34.01	88.36	150.49	187.44
DVA	(5.48)	(2.45)	(20.76)	(63.73)	(5.53)	(7.19)	(17.67)	(36.02)	(79.52)
FVA	9.01	3.99	27.45	74.77	9.61	10.27	27.40	64.35	140.13
$KVA^{ccr}$	10.02	18.92	17.59	18.79	25.87	40.93	108.76	197.83	246.67
$KVA^{cva}$	4.24	8.12	7.50	8.30	11.44	18.25	47.79	85.72	107.13
BVA	32.69	46.95	64.69	110.53	69.56	103.47	272.30	498.40	681.36
BVA'	27.20	44.50	43.93	46.80	64.03	96.28	254.64	462.37	601.84
CVA	5.18	8.67	5.02	2.48	7.38	8.29	8.24	8.84	7.12
DVA	(2.05)	(0.55)	(4.18)	(10.06)	(0.88)	(0.70)	(0.64)	(0.74)	(1.29)
FVA	10.66	3.68	20.83	46.74	5.16	4.40	4.12	4.50	7.04
KVA	0.19	0.19	0.18	0.14	0.19	0.19	0.19	0.19	0.19
CCVA	16.03	12.54	26.02	49.37	12.73	12.88	12.55	13.54	14.34
CCVA'	13.98	11.99	21.84	39.32	11.85	12.18	11.91	12.80	13.05

Table 5: XVA numbers obtained by considering alternately each of the nine members in Table 1 as reference member 0, using the  $\alpha_i$  for setting the positions of the members in each case as explained in Sect. 7.3. (*Up*) Credit spread  $\Sigma_0$ , coefficient  $\alpha_0$  and compression factor  $\nu_0$  of the reference member in each case (ordered by increasing  $\nu_0$ , i.e. decreasing  $|\alpha_0|$ ). (*Middle*) XVAs in the bilateral CSA setup. (*Bottom*) XVAs in the CCP setup.

### 8.3 Impact of the Credit Spread of the Reference Member

In order to confirm the role of netting in explaining the above results and to obtain comparative results free of this first order netting effect (to enhance the effect of other factors), Table 6 shows the same results as Table 5, but with all the CSA XVA numbers scaled by the corresponding compression factor  $\nu_0$ , and ordered by increasing credit spread  $\Sigma_0$  of the reference name, instead of increasing  $\nu_0$  in Table 5. Keeping in mind that the  $KVA^{cva}$  numbers are roughly half of the  $KVA^{ccr}$  numbers, we do not display the two KVA components in the CSA case, only displaying an aggregated KVA that can be directly compared with its CCP analog. From Table 6 we can see that, if we get rid of the netting effect through scaling by  $\nu_0$  (to proxy a situation where there would be netting of the positions of the reference member across its different counterparties also in the CSA setup), then CSA and CCP XVA numbers become of a similar order of magnitude (typically one to two times larger in the CCP case). In both cases, the dominant patterns become a logarithmic decrease of the CVA numbers and a linear increase of the FVA and  $|DVA|$  numbers with respect to the credit spread of the reference name. After scaling of the CSA XVA numbers by  $\nu_0$ , the aggregated TVA numbers become in favor of the CSA setup, especially for reference names with the largest credit spreads. A tentative interpretation of these results is that, if CSA transactions can be compressed in some way (as actually the case in practice, to some smaller or larger extent depending on the considered market, using software such as TriOptima), then a CSA setup can become less costly than a CCP setup, especially for names with a high credit risk, due to a greater impact of funding costs in the CCP setup. That's for the same level of overall margins, but of course even then the two setups have quite different structures and resilience properties. On the other hand, clearing members benefit from a very low KVA. Regarding the differences between the nine different cases within the CCP setup (as also within the CSA setup after scaling by the compression factor), we can see from Table 6 that the main explanatory factor of the XVA numbers is the credit spread of the reference member, risky members being heavily penalized in terms of FVA, especially in the CCP setup.

$\nu_0$	4.87	29.00	10.74	53.00	6.94	66.50	2.91	5.14	6.50
$\alpha_0$	(0.46)	0.09	0.23	(0.05)	0.34	(0.04)	0.69	(0.44)	(0.36)
$\Sigma_0$	45	52	56	61	73	108	176	367	1053
CVA / $\nu_0$	3.27	3.05	3.17	2.84	3.26	2.82	3.23	2.37	1.33
DVA / $\nu_0$	(0.50)	(0.61)	(0.67)	(0.68)	(0.80)	(1.20)	(1.88)	(4.04)	(9.80)
FVA / $\nu_0$	0.82	0.94	0.96	1.21	1.38	2.11	3.09	5.34	11.50
KVA / $\nu_0$	5.55	5.40	5.51	5.35	5.38	5.32	4.90	4.89	4.17
BVA / $\nu_0$	9.64	9.39	9.63	9.40	10.02	10.25	11.22	12.59	17.01
BVA' / $\nu_0$	9.14	8.78	8.97	8.72	9.22	9.05	9.34	8.55	7.20
CVA	8.67	8.24	8.29	8.84	7.38	7.12	5.18	5.02	2.48
DVA	(0.55)	(0.64)	(0.70)	(0.74)	(0.88)	(1.29)	(2.05)	(4.18)	(10.06)
FVA	3.68	4.12	4.40	4.50	5.16	7.04	10.66	20.83	46.74
KVA	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.18	0.14
CCVA	12.54	12.55	12.88	13.54	12.73	14.34	16.03	26.02	49.37
CCVA'	11.99	11.91	12.18	12.80	11.85	13.05	13.98	21.84	39.32

Table 6: XVA numbers obtained by considering alternately each of the nine members in Table 1 as reference member 0, using the  $\alpha_i$  for setting the positions of the members in each case as explained in Sect. 7.3. (*Up*) Credit spread  $\Sigma_0$ , coefficient  $\alpha_0$  and compression factor  $\nu_0$  of the reference member in each case (ordered by increasing  $\Sigma_0$ ). (*Middle*) XVAs in the bilateral CSA setup scaled by the compression factors  $\nu_0$ . (*Bottom*) XVAs in the CCP setup.

## 8.4 Homogeneous Portfolios

To complete the study of the impact of netting, already indirectly addressed through an admittedly artificial scaling of the CSA XVA results by their compression factors  $\nu_0$  in Sect. 8.3, the right panels in Table 7 show what happens when Name I and Name II are successively taken as reference member in the case of homogeneous portfolios with  $\omega_i = \frac{1}{n}$  for  $i \geq 1$  (unrelated to the  $\alpha_i$ ), hence  $\nu_0 = 1$  (as in the  $n = 1$  case of Sect. 8.1). Comparing the left and right panels in Table 7, where the left panels are simply the heterogeneous portfolio XVA results retrieved for comparison from Tables 5 and 6 (genuine XVA numbers and XVA numbers scaled by  $\nu_0$ , in the CSA setup), we see that, in the case of an homogeneous portfolio with only long positions equal to  $\frac{1}{n}$ , the XVA netting benefit of passing through the CCP essentially vanishes. In fact, in the lower panel case of risky Name II and for the homogeneous portfolio in the right (as also for the heterogeneous portfolio in the left if CSA XVAs scaled by the compression factors  $\nu_0$  are considered), the FVA is the dominant CCP XVA number and results in CCP trading being more costly than CSA trading. The right panels corresponding to the homogeneous portfolios for which  $\nu_0 = 1$  (as also the left panels if CSA XVAs scaled by the compression factors  $\nu_0$  are considered) show the expected typical pattern, whereby switching from a CSA to a CCP setup means a transfer of KVA, which is expensive in the bilateral setup, into FVA due to the margin fees in the CCP setup.

$\nu_0 = 53$	CSA	CSA/ $\nu_0$	CCP	$\nu_0 = 1$	CSA	CCP
CVA	150.49	2.84	8.84	CVA	2.69	2.9
DVA	(36.02)	(0.68)	(0.74)	DVA	(0.65)	(0.74)
FVA	64.35	1.21	4.50	FVA	1.19	4.50
KVA	283.55	5.35	0.19	KVA	5.16	0.19
TVA	498.40	9.40	13.54	TVA	9.04	7.61
TVA'	462.37	8.72	12.80	TVA'	8.39	6.87
$\nu_0 = 5.14$	CSA	CSA/ $\nu_0$	CCP	$\nu_0 = 1$	CSA	CCP
CVA	12.15	2.37	5.02	CVA	2.08	1.93
DVA	(20.76)	(4.04)	(4.18)	DVA	(3.91)	(4.18)
FVA	27.45	5.34	20.83	FVA	4.10	20.83
KVA	25.09	4.89	0.18	KVA	4.69	0.18
TVA	64.69	12.59	26.02	TVA	10.87	22.93
TVA'	43.93	8.55	21.84	TVA'	6.96	18.74

Table 7: (*Left*) Heterogeneous portfolios as in Sect. 8.2-8.3. (*Right*) Homogeneous portfolios with  $\omega_i = \frac{1}{n}$ ,  $i \neq 0$ . (*Top*) Reference member Name I with CDS spread 61 bp. (*Bottom*) Reference member Name II with CDS spread 367 bp.

## 8.5 Impact of the Liquidation Period

Back to the portfolio rule determined by the  $\alpha_i$ , as in Sect. 8.2-8.3, and focusing on the reference members Name I and Name II, Table 8 shows the impact of changing the length  $\delta$  of the liquidation period from 5 days to 15 days in the CSA setup and/or vice versa in the CCP setup. In the CSA setup, we show both the genuine XVA numbers, as in Sect. 8.2, and these numbers scaled by the corresponding compression factor  $\nu_0$  as in Sect. 8.3 (fictitious XVA numbers in a tentative CSA setup where netting would be effective across the different counterparties of the reference member). The CSA 15 days and CCP 5 days numbers in Table 8 are simply retrieved from Tables 5 and 6, for comparison purposes with the additional CSA 5 days and CCP 15 days results. The results are in line with the scaling in square root of  $\delta$  inherent to the Gaussian distribution that underlies the Black–Scholes model used for  $S$ . As evident from the comparison between the middle and bottom panel in Table 8, the square root or so impact of  $\delta$  cannot compete with the first order impact of netting that we observe when moving from the CSA to the CCP setup. This is interesting with respect to the ongoing debate regarding whether EMIR ruled CCPs and US regulation (CFTC) ruled CCPs are equivalent (or what should be done for ensuring their convergence), knowing that “1 day gross” and “2 days net” are currently used for fixing initial margins under the CFTC and EMIR rules, respectively. The above results suggest that the question is more one of “gross versus net” rather than one of “1 day versus 2 days”. But one must be careful because, regarding IM requirements for customers trading (between members of the CCP and their clients), which is mostly at stake here (as opposed to proprietary trading between members of the CCP in this paper), clients have no IM netting benefit, so the fair comparison should rather be between “1 day gross” and “2 days gross”. Then, of course, there are more IMs with EMIR than with CFTC.

	5d	15d	5d	15d
CVA / $\nu_0$	1.58	2.84	1.31	2.36
DVA / $\nu_0$	(0.38)	(0.68)	(2.25)	(4.04)
FVA / $\nu_0$	0.41	1.21	1.73	5.34
KVA / $\nu_0$	3.19	5.35	2.90	4.88
BVA / $\nu_0$	5.18	9.40	5.94	12.59
BVA' / $\nu_0$	4.80	8.72	3.69	8.55
CVA	83.86	150.49	6.73	12.15
DVA	(20.18)	(36.02)	(11.56)	(20.76)
FVA	21.96	64.35	8.90	27.45
KVA	168.93	283.55	14.92	25.09
BVA	274.76	498.40	30.55	64.69
BVA'	254.58	462.37	18.99	43.93
CVA	8.84	13.62	5.02	7.60
DVA	(0.74)	(1.28)	(4.18)	(7.58)
FVA	4.50	7.85	20.83	36.35
KVA	0.19	0.32	0.18	0.30
CCVA	13.54	21.80	26.02	44.25
CCVA'	12.80	20.51	21.84	36.67

Table 8: Impact of the liquidation period. (*Left*) Reference member Name I with  $\nu_0 = 53.00$ . (*Right*) Reference member Name II with  $\nu_0 = 5.14$ . (*Top*) CSA setup with all XVA numbers scaled by  $\nu_0$ . (*Middle*) Genuine XVA numbers in the CSA setup. (*Bottom*) CCP setup.

## 8.6 Margin Optimization

Table 9 shows that the above findings are essentially not biased by the low levels of the quantiles that are used to set the initial margins and default fund contributions in our simulations so far, with the double motivation exposed in Sect. 7.5. The left column in each of the two main panels, retrieved from our previous tables, corresponds to our base case where  $a_{im} = 70\%$ ,  $a_m = a'_{im} = 80\%$  and  $a_{ead} = 85\%$ . When higher values are used for  $a_{im}$ ,  $a_m = a'_{im}$  and/or  $a_{ead}$ , i.e. going from the left to the right column in each panel, we observe the same qualitative patterns as before in terms of the comparison between the CSA and the CCP setup, which is mainly driven by the compression factor  $\nu_0$ . Considering now the impact of higher quantiles inside each setup (CSA or CCP), we observe an expected shift from CVA and DVA into KVA (resp. FVA) in the CSA (resp. CCP) setup. Ultimately, for very high quantiles, CVA and DVA would reach zero whereas KVA and FVA would keep increasing, meaning that excessive margins become useless and a pure cost to the system, in the CSA as in the CCP setup.

	$a'_{ead} = 85\%$ $a'_{im} = 80\%$	$a'_{ead} = 95\%$ $a'_{im} = 90\%$	$a'_{ead} = 99.7\%$ $a'_{im} = 99\%$	$a'_{ead} = 85\%$ $a'_{im} = 80\%$	$a'_{ead} = 95\%$ $a'_{im} = 90\%$	$a'_{ead} = 99.7\%$ $a'_{im} = 99\%$
CVA / $\nu_0$	2.84	1.25	0.14	2.36	1.04	0.12
DVA / $\nu_0$	(0.68)	(0.30)	(0.03)	(4.04)	(1.79)	(0.21)
FVA / $\nu_0$	1.21	1.34	1.80	5.34	6.01	8.24
KVA <sup>ccr</sup> / $\nu_0$	3.73	7.00	8.20	3.42	6.45	7.62
KVA <sup>cva</sup> / $\nu_0$	1.62	3.03	3.54	1.46	2.75	3.24
BVA / $\nu_0$	9.40	12.62	13.67	12.59	16.24	19.22
BVA' / $\nu_0$	8.72	12.32	13.64	8.55	14.45	19.00
CVA	150.49	66.24	7.20	12.15	5.33	0.61
DVA	(36.02)	(16.06)	(1.83)	(20.76)	(9.21)	(1.09)
FVA	64.35	71.28	95.31	27.45	30.89	42.34
KVA <sup>ccr</sup>	197.83	371.05	434.59	17.59	33.15	39.17
KVA <sup>cva</sup>	85.72	160.54	187.42	7.50	14.12	16.65
BVA	498.40	669.10	724.52	64.69	83.50	98.77
BVA'	462.37	653.04	722.69	43.93	74.29	97.68
	$a_{im} = 70\%$	$a_{im} = 80\%$	$a_{im} = 95\%$	$a_{im} = 70\%$	$a_{im} = 80\%$	$a_{im} = 95\%$
CVA	8.84	5.52	1.69	5.02	3.11	0.93
DVA	(0.74)	(0.32)	(0.03)	(4.18)	(1.83)	(0.19)
FVA	4.50	6.74	12.15	20.83	31.21	56.34
KVA	0.19	0.36	0.43	0.18	0.33	0.39
CCVA	13.54	12.62	14.27	26.02	34.66	57.66
CCVA'	12.80	12.30	14.24	21.84	32.82	57.47

Table 9: Impact of the level of the quantiles that are used for setting initial margins, default fund contributions and exposures at default. (*Left*) Reference member Name I with  $\nu_0 = 53.00$ . (*Right*) Reference member Name II with  $\nu_0 = 5.14$ . (*Top*) CSA setup with all XVA numbers scaled by  $\nu_0$ . (*Middle*) Genuine XVA numbers in the CSA setup. (*Bottom*) CCP setup with  $a_m = a'_{im}$  everywhere, for comparison purposes.

## 9 Conclusion and Perspectives

In this paper we propose the vision of a clearinghouse effectively eliminating counterparty risk (we don't incorporate the default of the clearinghouse in our setup), but at a certain cost that we analyze. The corresponding costs for a member, dubbed CCVA for central clearing valuation adjustment, are decomposed into CVA, FVA and KVA components, where the CVA is the cost for a member of its losses on the default fund due to other members realized breaches whilst it is alive (and for completeness we also incorporate a DVA term). Beside the theoretical interest, this framework can be used for various purposes by a clearinghouse. Numerical experiments put into evidence the huge netting gains that can result from central clearing. These results are in line with the fact that netting has actually been, together with transparency and mutualization, one of the main motivation for the development of CCPs. The second found more explanatory factor is credit risk, where, passing from a bilateral CSA to a CCP setup, risky members are more penalized in terms of funding costs than rewarded in terms of CVA and KVA. We also find capital much cheaper when trading through a CCP. The conclusion regarding the overall netting benefit of CCPs might need be mitigated as netting can also be implemented at the level of bilateral transactions across different counterparties. Moreover, fragmentation across different CCPs reduces the netting benefit of centrally cleared trading. The default of the clearinghouse as well as systemic risk

at a global markets level and wrong way risk, including procyclical margin effects, could be considered. More underlying assets (as opposed to a single swap in our experiment) could be used. Market incompleteness could be acknowledged by the introduction of suitable risk premia. The simplifying assumption of a risk-free member used as buffer at other members defaults could be relaxed and more realistic liquidation procedures be modeled.

## A Regulatory Capital Formulas

A primitive of all the regulatory capital formulas are the exposures at default  $EAD^i = (EBRM^i - IM^i - DF^i)^+$ ,  $i \in N$  (with  $DF^i = 0$  in the bilateral case), where a stylized  $EBRM^i$  (exposure before risk mitigation) is taken in our numerics, similar to margins in Sect. 7.4, as a Var of level  $a_{ead} > a_m = a'_{im}$  of  $P\&L^i$  over the time horizon  $(\delta + h)$  of the margin period of risk.

### A.1 Centrally Cleared Case

Under centrally cleared trading, the regulatory capital  $K = K^{cm}$  of the reference member is defined, following Basel Committee on Banking Supervision (2014, page 11), as:

$$K^{cm} = \max \left( K^{ccp} \times \frac{DF}{E + \sum_{i \in N} J^i DF^i}, 8\% \times 2\% \times DF \right),$$

where

$$K^{ccp} = RW \times CapRatio \times \sum_{i \in N} J^i EAD^i \quad (\text{A.1})$$

with  $RW = 20\%$  and  $CapRatio = 8\%$ .

### A.2 Bilateral case

In the bilateral setup, the regulatory capital  $K$  is decomposed in three terms, a capital  $K^{ccr}$  for credit counterparty risk, a capital  $K^{cva}$  for CVA risk and a capital  $K^m$  for market risk. As explained in Green et al. (2014),  $K^m$  can be neglected assuming the position of the reference member (bank) perfectly hedged in terms of market risk. Adopting this perspective, we just use

$$K = K^{ccr} + K^{cva}. \quad (\text{A.2})$$

Since we focus on the member 0 with  $n$  counterparties  $i \in N^* = \{1, \dots, n\}$ , all the related formulas below are summed over  $N^*$ .

#### A.2.1 $K^{ccr}$

The Basel II regulatory capital specified for counterparty credit risk is defined as  $K^{ccr} = CapRatio \sum_{i \in N^*} RWA^i$ , where

$$RWA^i = 12.5 \times w_i \times 1.4 \times EAD^i,$$

where  $CapRatio \geq 8\%$  (the value that we use in the numerics) is the chosen capital ratio the bank must hold. The capital weight regulatory value  $w_i$  is given by the IRB formula (see e.g. Basel Committee on Banking Supervision (2005, page 7)):



$$w_i = (1 - R_i) \left( \Phi \left( \frac{\Phi^{-1}(DP_i)}{\sqrt{1 - \rho_i}} + \sqrt{\frac{\rho_i}{1 - \rho_i}} \Phi^{-1}(0.999) \right) - DP_i \right) \frac{1 + (\widehat{T}^i - 2.5)b(DP_i)}{1 - 1.5b(DP_i)},$$

where:

- $R_i$  is the recovery rate of the counterparty  $i$ ,
- $\Phi$  is the standard normal cdf,
- $DP_i$  is the one year default probability of the counterparty  $i$ , historical in principle, proxied in our numerics by the risk-neutral default probability extracted from the corresponding CDS spread,
- $\rho_i$  is the asset-counterparty  $i$  correlation in the sense of

$$\rho_i = 0.12 \frac{1 - e^{-50DP_i}}{1 - e^{-50}} + 0.24 \frac{1 - (1 - e^{-50DP_i})}{1 - e^{-50}}$$

- $\widehat{T}^i$  is the effective time to maturity of the netting set, i.e. simply the time to maturity of the swap in our numerical case where a single derivative is considered,
- $b(p) = (0.11852 - 0.05478 \ln(p))^2$ .

### A.2.2 $K^{cva}$

The standardized CVA risk capital charge in Basel Committee on Banking Supervision (2011, §104) gives the formula to generate CVA capital:

$$K^{cva} = 2.33\sqrt{Y} \left[ \left( 0.5 \sum_{i \in N^*} w_i \widehat{T}^i \widetilde{EAD}^i \right)^2 + 0.75 \sum_{i \in N^*} \left( w_i \widehat{T}^i \widetilde{EAD}^i \right)^2 \right]^{0.5},$$

which we approximate as in Green et al. (2014) by

$$\frac{2.33}{2} \sqrt{Y} \sum_{i \in N^*} w_i \widehat{T}^i \widetilde{EAD}^i,$$

where:

- $Y$  is the one year risk horizon, i.e.  $Y = 1$ ,
- $\widehat{T}^i$  is the effective (time to) maturity of the netting set with counterparty  $i$ , i.e. simply the time to maturity of the swap in our numerical example where a single derivative is considered,
- $\widetilde{EAD}^i = \frac{1 - \exp(0.05\widehat{T}^i)}{0.05\widehat{T}^i} EAD^i$ ,
- $w_i$  is the weight based on the external rating extracted from the one year default probability  $DP_i$  as follows, where the left part comes from Moody's and the right part is taken from Basel Committee on Banking Supervision (2011, §104):

Default Prob	Rating	Weight
0.00%	AAA	0.7%
0.02%	AA	0.7%
0.06%	A	0.8%
0.17%	BBB	1.0%
1.06%	BB	2.0%
3.71%	B	3.0%
12.81%	CCC	10.0%

**Acknowledgements** This paper greatly benefited from regular exchanges with the quant team of LCH.Clearnet in Paris, Quentin Archer and Julien Dosseur in particular. We are grateful to the following people for stimulating discussions: Claudio Albanese (Global Valuation Ltd), Tom Bielecki (IIT Chicago), Igor Cialenco (IIT Chicago), Christophe Hurlin (Orleans university), Samuel Drapeau (Humboldt University Berlin), Nicole El Karoui (university Paris 6-Jussieu), Samim Ghamami (Federal Reserve), Christophe Perignon (HEC Paris) and Marek Rutkowski (university of Sydney).

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