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Time-dependant cosmological interpretation of quantum mechanics

Emmanuel Moulay*

Abstract

The aim of this article is to define a time-dependant cosmological interpretation of quantum mechanics in the context of a multiverse coming from eternal inflation. A common notion of time is defined for observers in similar observable universes by using the holographic principle. It is the time elapsed since the post-inflationary epoch. With this improvement, the cosmological interpretation of quantum mechanics becomes a full interpretation of quantum mechanics where the unitary evolution of quantum states is preserved. Moreover, it is well suited for eternal inflation.

1 Introduction

A new interpretation of quantum mechanics, called the cosmological interpretation of quantum mechanics, has been developed in order to take into account the new paradigm of eternal inflation [1, 2, 3]. Eternal inflation can lead to an infinite collection of open bubble universes belonging to a multiverse [4, 5, 6, 7]. Such a multiverse implies that there exist an infinite number of observers belonging to similar observable universes which are indistinguishable [1, 8]. A quantum state $|\Psi_i\rangle \in \mathcal{H}$ is associated with each observer i belonging to these similar observable universes and it is possible to define a quantum state gathering all these observers

$$|\Psi_\infty\rangle = \bigotimes_{i=1}^{\infty} |\Psi_i\rangle \quad (1)$$

which belongs to the Hilbert space

$$\mathcal{H}^{\otimes\infty} := \mathcal{H} \otimes \mathcal{H} \otimes \mathcal{H} \otimes \dots \quad (2)$$

The reader may refer to [9, 10, 11, 12] for more details about the notion of infinite quantum states. Such a modelling is compatible with the Born rule [1, 13, 14, 15, 3] and can avoid the problem of the wave function collapse [2].

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However, the notion of time is not well defined because similar observable universes are not causally related. The main objective of this article is to define a notion of time for the quantum state (1). We will see that this problem can be solved by using the Fischler-Susskind cosmological holographic principle [16, 17]. The holographic principle has also been used to render the many-worlds interpretation of quantum mechanics compatible with eternal inflation [18, 19]. The unitary evolution of the time-dependent quantum state of the cosmological interpretation of quantum mechanics is preserved.

The organization of this paper is the following. In Section 2, a common notion of time is defined for observers belonging to similar observable universes by using the Fischler-Susskind cosmological holographic principle. We prove in Section 3 that the time-dependant quantum state associated with these observers does not collapse. Finally, a conclusion is addressed in Section 4.

2 Time-dependant quantum states

Let us consider two similar observers belonging to two similar observable universes which are indistinguishable. Let us denote by $|\Psi_1\rangle$ the quantum state of the observer 1 and by $|\Psi_2\rangle$ the quantum state of the observer 2. We may wonder if it is possible to define a common notion of time for these two observers. We want to use the time elapsed since the post-inflationary epoch as the same reference time. The problem comes from the fact that it is possible for two different initial quantum states to evolve toward two indistinguishable final quantum states. In other words, we may wonder if the two observers, belonging to two similar observable universes at a given time, have a similar past since the post-inflationary epoch.

If there exist two similar observers in two similar observable universes who have different pasts, then it is not possible to define a common notion of time based on the time elapsed since the post-inflationary epoch. One of the two observers may refer to a past which is different from the past of the other observer. Then the unitary evolution and also the notion of time elapsed since the post-inflationary epoch fall down. To the best of our knowledge, it seems impossible under these conditions to define a common notion of time.

The cosmological horizon is defined as the boundary of the observable universe. It is possible to assert that two similar observers in two similar observable universes at a given time have a similar past since the post-inflationary epoch because all the information is stored in the cosmological horizon of each observable universe [20]. This is a consequence of the cosmological holographic principle.

The properties of the cosmological horizon were first studied by Gibbons and Hawking in [21]. Let us recall how it is possible to define a cosmological horizon for a flat de Sitter space. The Friedmann equation is given by

$$3 \left(\frac{1}{c^2} \left(\frac{\dot{a}(t)}{a(t)} \right)^2 + \frac{k}{a(t)^2} \right) - \Lambda = \frac{8\pi G}{c^4} \rho(t) \quad (3)$$

where $a(t)$ is the scale factor, $\rho(t)$ the mass density, $\Lambda > 0$ the cosmological constant homogeneous to the inverse square of a length, G the gravitational constant, $k = 0, 1, +1$ for a flat, open or closed universe respectively. Let

$$H(t) = \frac{\dot{a}(t)}{a(t)} \quad (4)$$

be the Hubble parameter. The Friedmann equation reduces to

$$\frac{H(t)^2}{c^2} + \frac{k}{a(t)^2} = \frac{\Lambda}{3} \quad (5)$$

for a vacuum space with $\rho = 0$. Such a universe where only the cosmological constant Λ has an influence on the scale factor a is called a de Sitter universe. It is the maximally symmetric, vacuum solution of Einstein's field equations with a positive cosmological constant $\Lambda > 0$. In a flat de Sitter space where $k = \rho = 0$, the Friedmann equation becomes

$$H_{dS}^2 = \frac{c^2 \Lambda}{3} \quad (6)$$

and in static coordinates, the de Sitter metric is given by

$$ds^2 = - \left(1 - \frac{c^2 \Lambda r^2}{3} \right) dt^2 + \frac{1}{1 - \frac{c^2 \Lambda r^2}{3}} dr^2 + r^2 d\Omega^2 \quad (7)$$

where $d\Omega^2 = d\theta^2 + \sin^2 \theta d\varphi^2$ (see for instance [21, 22, 23]). The de Sitter metric (7) looks like the Schwarzschild metric and there is an analogy between the properties of de Sitter spaces and those of black holes [24]. At

$$R_{dS} = \frac{1}{H_{dS}} = \frac{1}{c} \sqrt{\frac{3}{\Lambda}} \quad (8)$$

there is the cosmological horizon which is also called the cosmological event horizon. It has a similar role to that of the horizon of a black hole.

If the notion of cosmological horizon is time-independent for a de Sitter universe, this is not the case for a Friedmann-Lemaître-Robertson-Walker (FLRW) universe [20]. The FLRW metric is given by

$$ds^2 = c^2 dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right). \quad (9)$$

A natural definition of cosmological horizon for a FLRW universe is the particle horizon whose radius is defined at time t by

$$R_P(t) = a(t) \int_{t_0}^t \frac{c}{a(s)} ds \quad (10)$$

where t_0 denotes the post-inflationary epoch. The particle horizon is the largest comoving spatial distance from which light could have reached an observer if it

was emitted at time t_0 [25]. It represents the boundary between the observable and the unobservable regions of the universe for an observer.

The cosmological holographic principle, given by Fischler and Susskind, says that the entropy of matter inside the post-inflationary particle horizon must be smaller than the area of the cosmological horizon [16, 26]. The Fischler-Susskind cosmological holographic principle is true for open and classical flat FLRW universes [27, 20]. As eternal inflation can lead to open FLRW universes [4, 5, 6], the Fischler-Susskind cosmological holographic principle can be applied in this context. It implies that an observer can theoretically have access to the past of his observable universe since the post-inflationary epoch because all the information is stored in his cosmological horizon [19]. Thus, we conclude that if two similar observers have indistinguishable observable universes at time t_f after the post-inflationary epoch t_0 then they have a similar past since the post-inflationary epoch, i.e. their observable universes are similar since the post-inflationary epoch t_0 . The observers can be two particles created during the post-inflationary epoch. Let us remark that the Fischler-Susskind cosmological holographic principle has been extended in [27] to more general FLRW universes, as closed FLRW universes, by using another cosmological horizon called the apparent horizon and whose radius is given by

$$R_A(t) = \frac{1}{\sqrt{H(t)^2 + \frac{k}{a(t)^2}}}. \quad (11)$$

We are now able to define the quantum state

$$|\Psi_{12}(t)\rangle = |\Psi_1(t)\rangle \otimes |\Psi_2(t)\rangle \quad (12)$$

for all $t_0 \leq t \leq t_f$ because we know that $|\Psi_1(t)\rangle$ and $|\Psi_2(t)\rangle$ are indistinguishable for all $t_0 \leq t \leq t_f$. If we consider all the quantum states of all the observers having an observable universe similar to the observable universes 1 and 2, then the generalization to the quantum state (1) is straightforward and we obtain the time-dependant quantum state

$$|\Psi_\infty(t)\rangle = \bigotimes_{i=1}^{\infty} |\Psi_i(t)\rangle \quad (13)$$

for all $t_0 \leq t \leq t_f$. By using the time-dependant Schrödinger equation associated with each $|\Psi_i(t)\rangle$

$$i\hbar \frac{\partial}{\partial t} |\Psi_i(t)\rangle = \hat{H} |\Psi_i(t)\rangle \quad (14)$$

where \hat{H} is the Hamiltonian associated with each similar observable universe, we obtain the following formal equation associated with $|\Psi_\infty(t)\rangle$

$$i\hbar \frac{\partial}{\partial t} |\Psi_\infty(t)\rangle = \hat{\mathbf{H}}_\infty |\Psi_\infty(t)\rangle \quad (15)$$

where $\frac{\partial}{\partial t} := \frac{\partial}{\partial t} \otimes \frac{\partial}{\partial t} \otimes \dots$ and $\hat{\mathbf{H}}_\infty := \hat{H} \otimes \hat{H} \otimes \dots$.

3 The unitary evolution

We have proved in [2] that the collapse of the time-independent quantum state (1) can be avoided. However, the same reasoning cannot be used for the time-dependent quantum state (13). Indeed, if a measurement is done at time t_m , all quantum states $|\Psi_i(t)\rangle$ collapse at time t_m . So, we may wonder if the unitary evolution of the time-dependent quantum state $|\Psi_\infty(t)\rangle$ can be preserved.

First, let us remark that if we have only a finite fixed number of similar observable universes, the unitary evolution cannot be satisfied. Suppose that we have only a finite fixed number N of similar observers i in similar observable universes having quantum states $|\Psi_i(t)\rangle$. Just after the measurement at time $t_m^+ = t_m + \epsilon$, we may have

$$|\Psi_i(t_m^+)\rangle = |\Psi_j(t_m^+)\rangle \quad (16)$$

for all $i, j \in \{1, \dots, N\}$ where $\epsilon > 0$ is a sufficiently small number. So, the quantum state $\bigotimes_{i=1}^N |\Psi_i(t)\rangle$ collapses at time t_m and all its possible evolutions cannot be explored. Let us remark that the Born rule is also not satisfied in a sufficiently large universe [28, 29].

Then, we prove that the unitary evolution of the time-dependent quantum state $|\Psi_\infty(t)\rangle$ is preserved. Let us denote by t_m^- the time just before the measurement in each similar observable universe and t_m^+ the time just after the measurement. We study the quantum state $|\Psi_\infty(t_m^-)\rangle$ in order to see if its collapse is possible. There exists $1 < K < +\infty$ such that for all $i \in \mathbb{N}^* = \mathbb{N} \setminus \{0\}$

$$|\Psi_i(t_m^-)\rangle = \sum_{k=1}^K \alpha_{ik} |\Psi_k(t_m^+)\rangle \quad (17)$$

where $\alpha_{ik} \in \mathbb{C}$ and $\sum_{k=1}^K |\alpha_{ik}|^2 = 1$. We have

$$|\Psi_\infty(t_m^-)\rangle = \bigotimes_{i=1}^\infty |\Psi_i(t_m^-)\rangle \quad (18)$$

$$= \lim_{N \rightarrow +\infty} \left(\bigotimes_{i=1}^N \sum_{k=1}^K \alpha_{ik} |\Psi_k(t_m^+)\rangle \right) \quad (19)$$

We develop the previous expression in square brackets and we obtain

$$|\Psi_\infty(t_m^-)\rangle = \lim_{N \rightarrow +\infty} \left(\sum_{f_N \in \mathcal{T}_N} \bigotimes_{i=1}^N \alpha_{i f_N(i)} |\Psi_{f_N(i)}(t_m^+)\rangle \right) \quad (20)$$

where \mathcal{T}_N is the set of all the functions between $\{1, \dots, N\}$ and $\{1, \dots, K\}$ and $\text{card}(\mathcal{T}_N) = K^N$. Then, we gather the coefficients α by using the properties of the tensor product and it leads to

$$|\Psi_\infty(t_m^-)\rangle = \lim_{N \rightarrow +\infty} \left(\sum_{f_N \in \mathcal{T}_N} p_N^{f_N} \bigotimes_{i=1}^N |\Psi_{f_N(i)}(t_m^+)\rangle \right) \quad (21)$$

with

$$p_N^{f_N} = \prod_{j=1}^N \alpha_{jf_N(j)}. \quad (22)$$

The term

$$\left| p_N^{f_N} \right|^2 = \left| \prod_{j=1}^N \alpha_{jf_N(j)} \right|^2 = \prod_{j=1}^N |\alpha_{jf_N(j)}|^2 \quad (23)$$

is the probability of having the quantum state $\bigotimes_{i=1}^N |\Psi_{f_N(i)}(t_m^+)\rangle$ for the first N observable universes. Let \mathcal{T} be the set of all the functions between \mathbb{N}^* and $\{1, \dots, K\}$ and

$$p^f = \lim_{N \rightarrow +\infty} p_N^{f_N} = \prod_{j=1}^{\infty} \alpha_{jf(j)} \quad (24)$$

with $f = \lim_{N \rightarrow +\infty} f_N \in \mathcal{T}$ then we have

$$|\Psi_{\infty}(t_m^-)\rangle = \sum_{f \in \mathcal{T}} p^f \bigotimes_{i=1}^{\infty} |\Psi_{f(i)}(t_m^+)\rangle. \quad (25)$$

In [2], we have proved that

$$|\alpha_{ik}| = |\alpha_{jk}| \quad (26)$$

for all $i, j \in \mathbb{N}^*$ by using the Finkelstein-Hartle theorem [14, 15]. As

$$|\alpha_{ik}|^2 < 1 \quad (27)$$

for all $i \in \mathbb{N}^*$, $k \in \{1, \dots, K\}$, we have

$$\lim_{N \rightarrow +\infty} \left| p_N^{f_N} \right|^2 = \prod_{j=1}^{\infty} |\alpha_{jf(j)}|^2 = 0 \quad (28)$$

for all $f \in \mathcal{T}$. Indeed, a necessary condition for the product $\prod_{j=1}^{\infty} |\alpha_{jf(j)}|^2$ to be equal to a finite non zero positive real number is that

$$\lim_{j \rightarrow +\infty} |\alpha_{jf(j)}|^2 = 1. \quad (29)$$

The reader may refer to [30, Chapter 2] for more details on infinite products. It is obvious that Condition (29) cannot be satisfied if we have (26) and (27).

The result given by (28) implies that the probability of having the following quantum state $\bigotimes_{i=1}^{\infty} |\Psi_{f(i)}(t_m^+)\rangle$ is zero for all $f \in \mathcal{T}$. It shows that the collapse of the quantum state $|\Psi_{\infty}(t_m^-)\rangle$ is not possible.

Finally, the wave function collapse is avoided for the quantum state $|\Psi_{\infty}(t)\rangle$. However, we have defined a common notion of time in Section 2 for observers in similar observable universes and we may wonder why the quantum state $|\Psi_{\infty}(t)\rangle$ does not collapse. This is due to the fact that the wave function collapse is

associated with the notion of observer. In an infinite universe, this notion of observer falls down and then also the notion of wave function collapse. An observer i can only see the wave function collapse of his quantum state $|\Psi_i(t)\rangle$ and he also knows that all the other observers in similar observable universes can see the wave function collapse of their quantum state at the same time t_m elapsed since the post-inflationary epoch. However, the global picture is not the wave function collapse of the quantum state $|\Psi_\infty(t)\rangle$ at time t_m .

4 Conclusion

In this article we have defined a notion of time, namely the time elapsed since the post-inflationary epoch, for the quantum states of observers in similar observable universes used in the cosmological interpretation of quantum mechanics. The unitary evolution of these time-dependant quantum states is preserved in the multiverse. This interpretation is now a full interpretation of quantum mechanics requiring the holographic principle and compatible with eternal inflation.

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References

- [1] A. Aguirre, M. Tegmark, Born in an infinite universe: A cosmological interpretation of quantum mechanics, *Physical Review D* 84 (10) (2011) 105002.
- [2] E. Moulay, Non-collapsing wave functions in an infinite universe, *Results in Physics* 4 (2014) 164–167.
- [3] A. Vilenkin, A quantum measure of the multiverse, *Journal of Cosmology and Astroparticle Physics* 2014 (05) (2014) 005.
- [4] S. Coleman, F. De Luccia, Gravitational effects on and of vacuum decay, *Physical Review D* 21 (12) (1980) 3305.
- [5] J. Garriga, A. H. Guth, A. Vilenkin, Eternal inflation, bubble collisions, and the persistence of memory, *Physical Review D* 76 (12) (2007) 123512.
- [6] A. H. Guth, Eternal inflation and its implications, *Journal of Physics A: Mathematical and Theoretical* 40 (25) (2007) 6811.
- [7] A. Linde, Toy model for open inflation, *Physical Review D* 59 (2) (1998) 023503.

- [8] M. Tegmark, Parallel universes, in: *Science and Ultimate Reality: Quantum Theory, Cosmology, and Complexity*, Cambridge University Press, 2003, pp. 459–491.
- [9] J. Bub, How to solve the measurement problem of quantum mechanics, *Foundations of physics* 18 (7) (1988) 701–722.
- [10] S. Gutmann, Using classical probability to guarantee properties of infinite quantum sequences, *Physical Review A* 52 (5) (1995) 3560–3562.
- [11] J. von Neumann, On infinite direct products, *Compositio Mathematica* 6 (1939) 1–77.
- [12] J. von Neumann, Quantum mechanics of infinite systems, in: *John von Neumann and the Foundations of Quantum Physics*, Vol. 8 of Vienna Circle Institute Yearbook, Springer, 2001, pp. 249–268.
- [13] E. Farhi, J. Goldstone, S. Gutmann, How probability arises in quantum mechanics, *Annals of Physics* 192 (2) (1989) 368–382.
- [14] D. Finkelstein, The logic of quantum physics, *Transactions of the New York Academy of Sciences* 25 (6) (1963) 621–637.
- [15] J. B. Hartle, Quantum mechanics of individual systems, *American Journal of Physics* 36 (8) (1968) 704–712.
- [16] W. Fischler, L. Susskind, Holography and cosmology, arXiv preprint hep-th/9806039.
- [17] L. Susskind, J. Lindesay, *An introduction to black holes, information and the string theory revolution*, World Scientific, 2005.
- [18] R. Bousso, L. Susskind, Multiverse interpretation of quantum mechanics, *Physical Review D* 85 (4) (2012) 045007.
- [19] Y. Nomura, Physical theories, eternal inflation, and the quantum universe, *Journal of High Energy Physics* 2011 (11) (2011) 63.
- [20] R. Bousso, The holographic principle, *Reviews of Modern Physics* 74 (3) (2002) 825.
- [21] G. W. Gibbons, S. W. Hawking, Cosmological event horizons, thermodynamics, and particle creation, *Physical Review D* 15 (10) (1977) 2738.
- [22] Ø. Grøn, S. Hervik, *Einstein’s general theory of relativity: with modern applications in cosmology*, Springer, 2007.
- [23] M. Spradlin, A. Strominger, A. Volovich, De sitter space, in: *Unity from Duality: Gravity, Gauge Theory and Strings*, Les Houches - Ecole d’Ete de Physique Theorique, Springer, 2002, pp. 423–453.

- [24] C. C. Linder, Particle physics and inflationary cosmology, CRC Press, 1990.
- [25] M. Roos, Introduction to cosmology, John Wiley & Sons, 2004.
- [26] N. Kaloper, A. Linde, Cosmology versus holography, Physical Review D 60 (10) (1999) 103509.
- [27] D. Bak, S.-J. Rey, Cosmic holography, Classical and Quantum Gravity 17 (15) (2000) L83.
- [28] D. N. Page, The Born rule fails in cosmology, Journal of Cosmology and Astroparticle Physics 2009 (07) (2009) 008.
- [29] D. N. Page, Born's rule is insufficient in a large universe, arXiv:1003.2419.
- [30] Y. A. Melnikov, Green's functions and infinite products: bridging the divide, Birkhäuser, 2011.