

Introduction

The extension of mathematical morphology to multivalued images is an important issue. In this context, connected morphological operators based on hierarchical image models have been increasingly considered to provide efficient image segmentation and filtering tools in various application fields, e.g. (bio)medical imaging, astronomy or remote sensing.

We propose a preliminary study that describes how *component-graphs* (that extend the component-tree from a spectral point of view) and *shaping* (that extends the component-tree from a conceptual point of view) can be associated for the effective processing of multivalued images.

Previous works

Component-graphs [1]: Component-trees and multivalued images

A (discrete) image $\mathcal{I} : \Omega \rightarrow \mathbb{V}$ (canonically equipped with an order relation \leq) can be modeled as a **valued graph** $(\mathcal{G}, \mathbb{V}, \mathcal{I})$. If (\mathbb{V}, \leq) is no longer a total order, the Hasse diagram associated to \mathcal{I} is no longer a tree:

- We extend the notion of connected component to **valued connected component**: $\Theta = \bigcup_{v \in \mathbb{V}} \mathcal{C}[\mathcal{G}_v] \times \{v\}$

- We define an **order relation** \preceq on Θ as

$$(X_1, v_1) \preceq (X_2, v_2) \Leftrightarrow \begin{cases} (X_1 \subset X_2) \vee \\ (X_1 = X_2 \wedge v_2 \leq v_1) \end{cases}$$

- The **component-graph** $\mathcal{C}\mathcal{G}$ of the valued graph $(\mathcal{G}, \mathbb{V}, \mathcal{I})$ is the Hasse diagram of the partially ordered set (Θ, \preceq) .

- Each node of $\mathcal{C}\mathcal{G}$ can contain an **attribute value**, interpreted as a function $\mathcal{A} : \Theta \rightarrow \mathbb{R}$. Such enriched component-graph is also interpreted as a valued graph $(\mathcal{C}\mathcal{G}, \mathbb{R}, \mathcal{A})$.

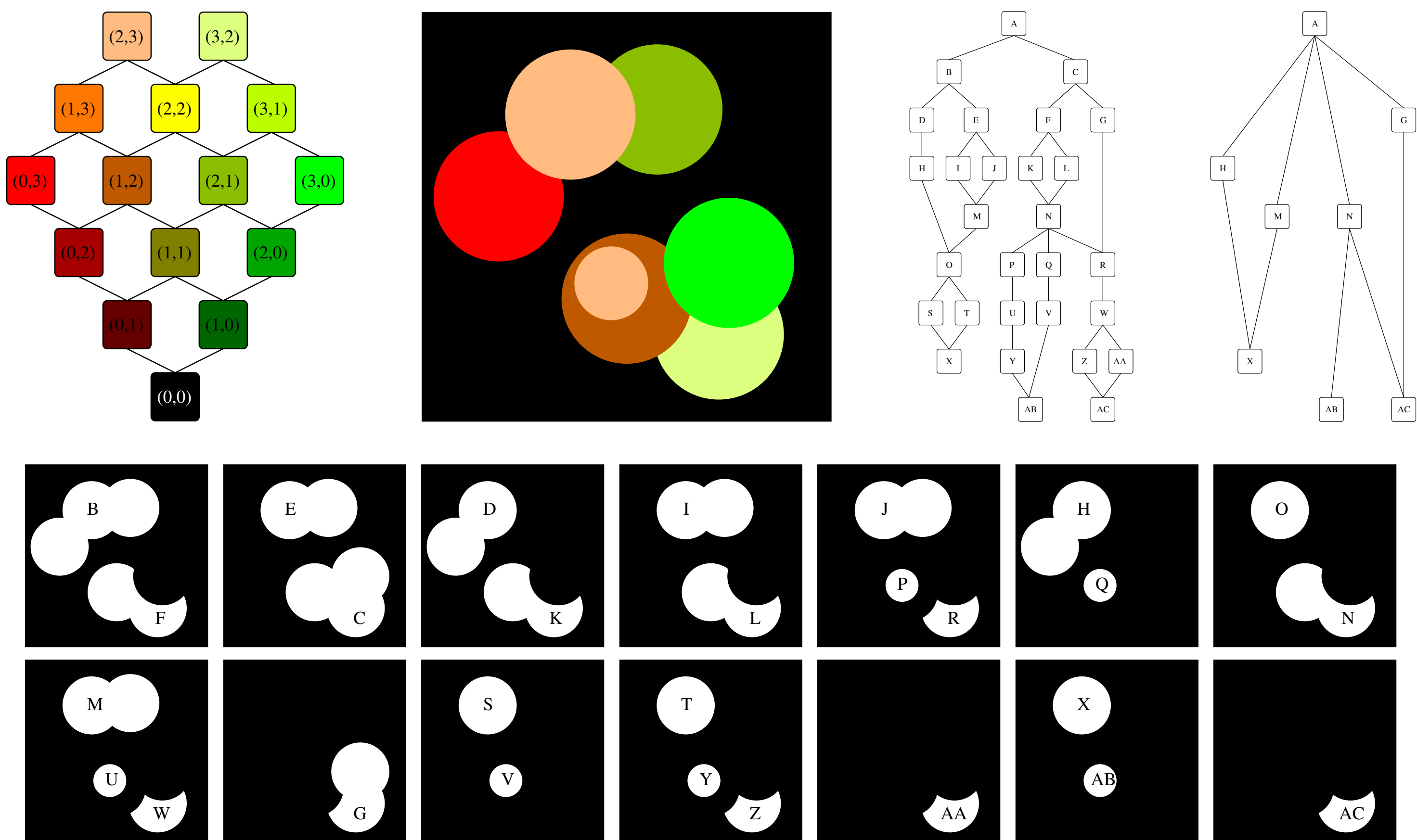


Figure 1: From left to right. First row: the Hasse diagram of the ordered set (\mathbb{V}, \leq) ; a multivalued image, viewed as a valued graph $(\mathcal{G}, \mathbb{V}, \mathcal{I})$ where \mathcal{G} is a part of \mathbb{Z}^2 equipped with the standard 4-adjacency relation; the component-graph $\mathcal{C}\mathcal{G}$ associated to $(\mathcal{G}, \mathbb{V}, \mathcal{I})$; a simplified version of the component-graph. Second and third rows: thresholded images obtained from the initial image. Each (valued) connected component is represented by a letter: A, B, C, etc. Note: A is the support of the figure (the background).

Shaping: Anti-extensive filtering in the shape-space

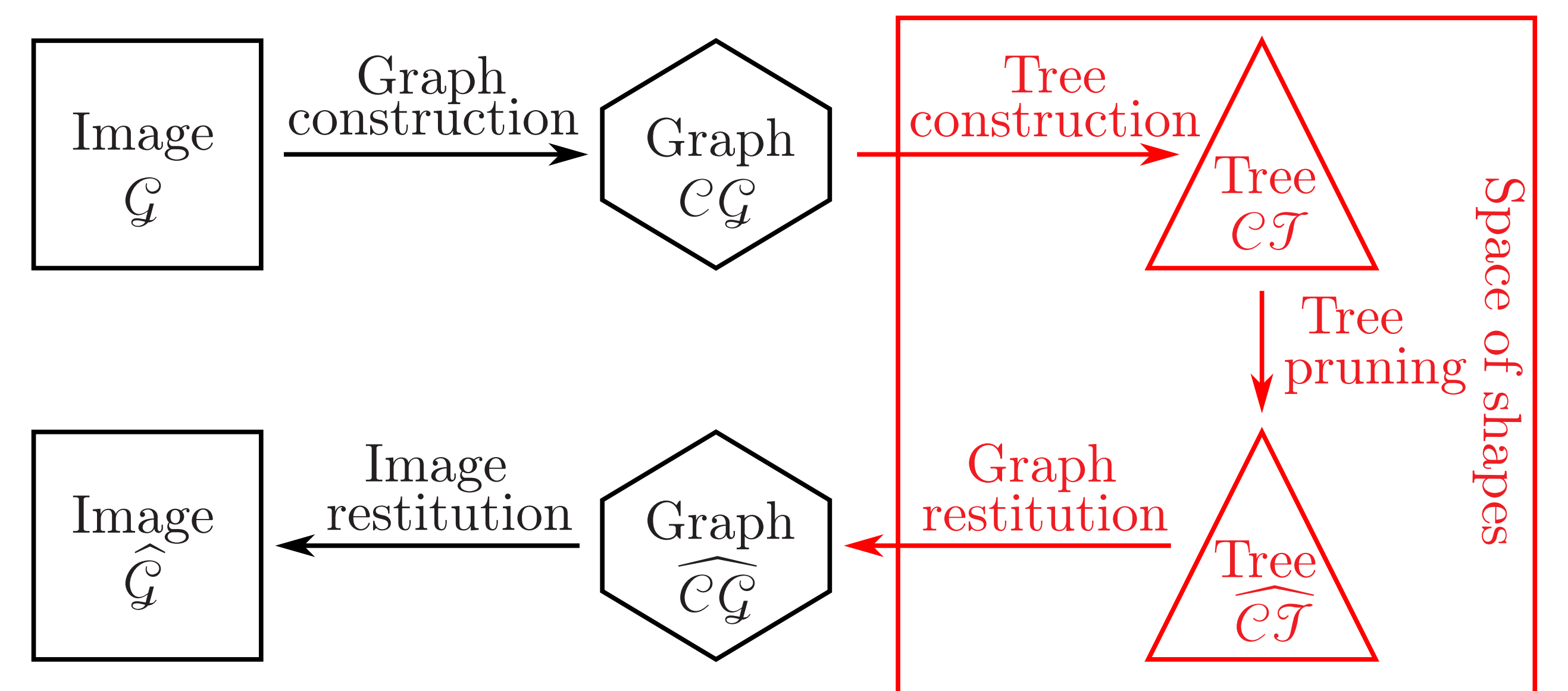
We consider the paradigm of **shapings**, proposed by Xu et al. [2], to extend the filtering capabilities of tree-based representation to **non-monotonic** attributes.

- The **shaping framework** consists of using a two-layer component-tree, i.e. a *tree on a tree*, that transforms any attribute into a monotonic one. This allows us to embed additional information. In particular it facilitates user interaction and real-time threshold-based node selection.
- The first tree is the component-tree of the image: it models its successive binary level-sets. The second is the component-tree of this first tree, considered as an image whose points are the nodes, while intensities correspond to attribute values.
- Limited use to grey-level images considering a tree as intermediate data-structure.

Coupling component-graphs and shaping

We propose to associate the shaping and component-graph in a common framework for developing connected operators on multivalued images.

- We use **vertex-valued graphs** as a **unified formalism** to describe images, component-trees and component-graphs.
- The inner layer of shape-space filtering only **requires a graph**, not necessarily a tree.
- Generalization of the initial shaping paradigm: it can be used not only to build a “**tree on a tree**” but also a “**tree on a graph**”.
- Any images can be processed via shape-based filtering :



Advantages of the framework

- Avoids the complex selection of nodes directly in $\mathcal{C}\mathcal{G}$.
- Extends the initial shaping approach to multivalued images.
- Inherits the good properties of shape-space filtering from increasing attributes: real-time and interactive node selection at higher semantic level.

Application to PET/CT image filtering

Illustrative proof of concept: We illustrate the potential usefulness of this framework by filtering coupled PET and CT images. The criterion considered for filtering is the compactness factor [3], defined as the ratio between the extremal eigenvalues of the matrix of inertia.

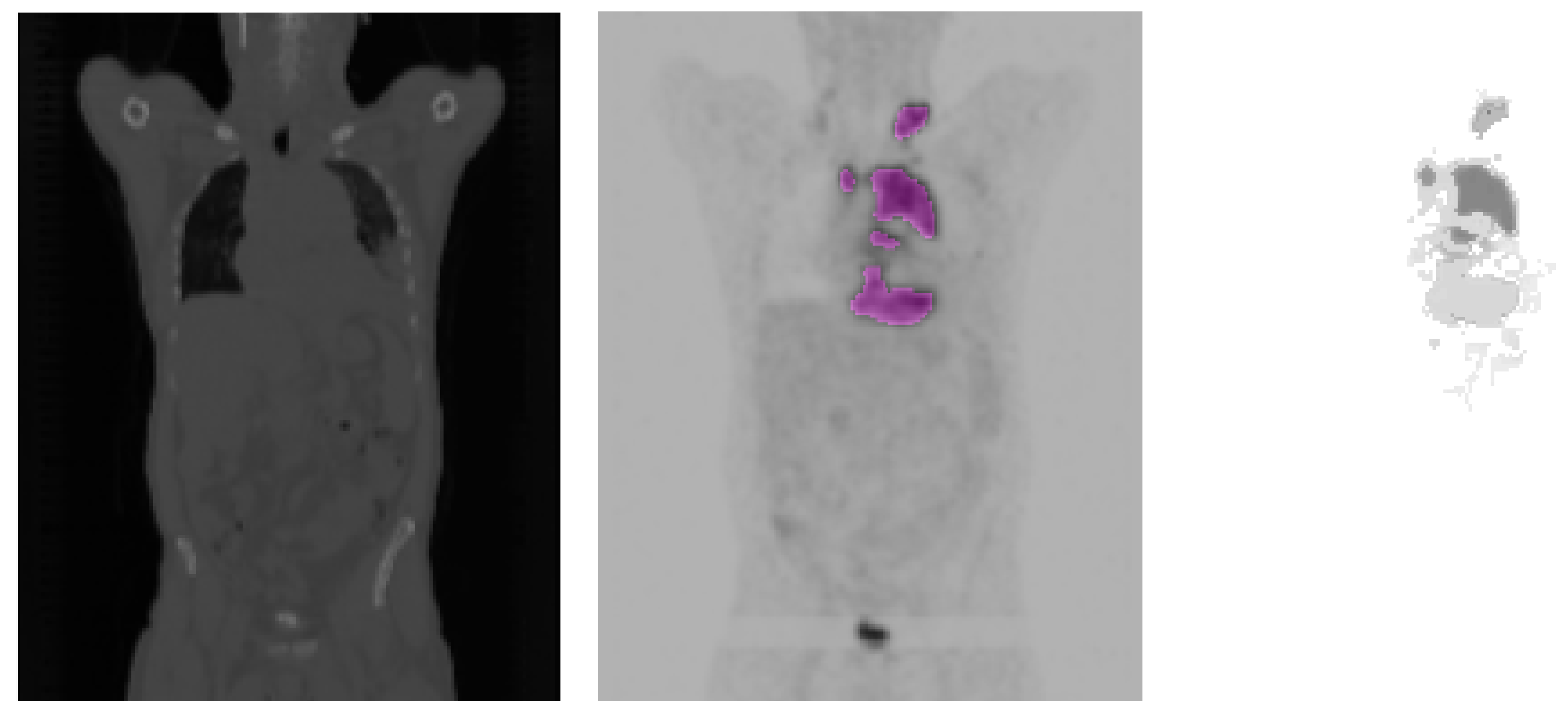


Figure 2: From left to right: CT; PET + ground truth in purple; multivalued shape-based filtering visualized in the PET value space.

References

- [1] Passat, N., Naegel, B., *Component-trees and multivalued images: Structural properties*, in JMIV 49(1) (2014) pp. 37–50.
- [2] Xu, Y., Géraud, Th., Najman, L., *Connected filtering on tree-based shape-spaces*, in PAMI (2015).
- [3] Grossiord, É., Talbot, H., Passat, N., Meignan, M., Tervé, P., Najman, L., *Hierarchies and shape-space for PET image segmentation* in ISBI 2015, pp. 1118–1121.