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# SCHEDULING MODEL FOR A BUILDING RENOVATION PROJECT WITH RE-ADD AND STORAGE OF RESOURCE TYPES UNDER SUPPLIER LIMITED PRODUCTION CAPACITY

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**ABSTRACT:** Resource constrained project scheduling problem, RCPSP, in its basic form consists in minimizing total makespan of effectuating a group of activities subject to precedence constraints and resource availability limitations. In this paper, we deal with a new extension of RCPSP's mathematical model. Our work which proposes an optimisation model for a French project of buildings' thermal renovation overcomes existing gap of RCPSPs by either considering the recharge for renewable resources from an external subcontractor or heeding capacity limitations for production and storage of non-renewable resources in problem modelling. The mathematical model aims to minimize total cost of the system implementation that includes recharge cost of non-renewable resources, execution cost of the activities, stock cost of non-renewable resources and rent cost of expensive required equipment through the execution period of the project. In order to obtain the outputs of the model, a solution methodology based on linear programming relaxation has been provided and applied on the proposed mathematical model. The suggested procedure reduces number of integer variables by defining the bounds for earliest and latest start time of the project.

**KEYWORDS:** Resource Constrained Project Scheduling Problem, Stock Capacity Constraint, Resource Recharge, Mixed Integer Linear Programming, Linear Programming Relaxation.

## 1 INTRODUCTION

During the last decades, Resource Constrained Project Scheduling Problem (RCPSP) has attracted attention of many researchers and it has appropriated a main position in Project Management (PM) and Operation Research (OR) studies. The RCPSP implies minimization of processing time of a predefined activities group while trying to satisfy a set of constraints. In the standard form of RCPSP, two types of constraints have been considered to present the mathematical model. According to (Pritsker A.A.B. *et al.*, 1969) and (Badiru, A.B. and Pulat P.S., 1995), the first set of constraints, (i), considers the precedence relations among the activities and the second set of constraints, (ii), satisfies resource availability over the execution time of the project.

By precedence relationships (i), RCPSP articulates that an activity cannot start performing when its predecessors are not totally completed. Usually, three different types of precedence relations have been distinguished in pro-

ject scheduling models (Klein, R., 1999): *Imposed* precedence relationship happens in the time that one activity has to be performed after some others. It happens because they use the same resource and there are some restrictions in presence of the required resources. *Technological* precedence relationships occur by technological interdependences. For example in our project, in order to install thermal insulation panels, it is already necessary to implement panels' fasteners on the external facades of the buildings. Finally *Procedural* relationships impose the precedence regarding some guidelines that define how certain processes should be executed. In this work, the precedence of the activities has been defined by imposed and technological precedencies.

Furthermore, by resource satisfaction constraints (ii), RCPSP articulates that each activity requires certain units of the resources to be carried out in its processing time. The required resources can only be accessible in a restricted quantity over the time horizon of the project. In this context, total demand of the activities must not exceed the available quantities. Different types of the

resources have been introduced for RCPSP models. According to (Shirzadeh Chaleshtari, A. *et al*, 2014) renewable and non-renewable ones are the most common resource types have been used in practice and theory.

In this study, we evenly deal with both the renewable and the non-renewable resources. The main contributions of the study have been considered in planning of these resources where they are supposed in two points:

- First, we consider the possibility of additive capacities for renewable resources. As a matter of fact, in the basic forms of RCPSP and in most of the models defined in literature, the whole amount of the resources especially the renewable ones are supposed to be available for applying from the beginning time of the project whereas this is not always the case of real world problems. In practice, it is so common that some resources are being added or are being procured during the project periods. Here in our work, we deal with the mentioned situation wherein some types of the renewable resources, i.e. the projects' labor works, could be added by an external enterprise during the project time and at different periods of time.

- Our second point considers capacity limitations that are included in both limited production capacity of the suppliers and limited storage capacity in the project worksites. In our study, the limited production capacity of the manufacturing units makes them to function like the bottlenecks in the supply chain. Thus, for smoothing the flow in the network and supplying the demands completely at the expected times, stocking a certain level of goods is supposed to be done in the worksites. Nevertheless, because of the limited storage capacity, stocking of the goods is only permitted in a limited quantity. Figure 1 depicts a general presentation of the framework.

According to the best of our knowledge, integration of these two mentioned points provides a new framework in RCPSP mathematical models that is not considered in literature by now.

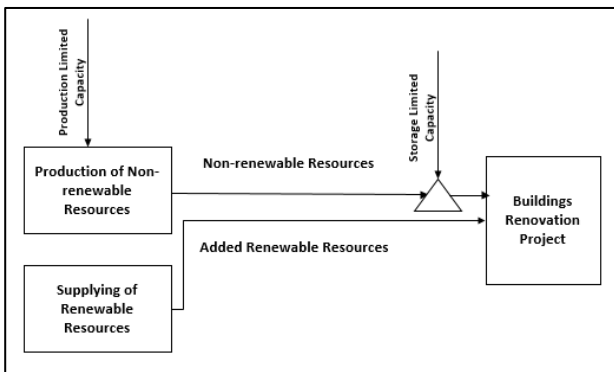


Figure 1: Constrained resources of the project

In order to report the optimum outputs of the model, we apply a solution methodology based on linear program-

ming relaxation method. The proposed procedure reduces the number of binary variables by bounding two related parameters and by that declines the computation time of the model. The outputs of the problem have got by using branch and bound solver in LINGO software (Due to the researchers' preference, the other commercial operation research soft-wares, i.e. CPLEX and GAMS, could be applicable for solving OR's models).

In the rest of the paper, in section 2, we review the literature and provide a study of related works in this research avenue. Then in section 3, we define our problem in building thermal renovation project and subsequently we represent our mathematical formulation for that project. We define the solution approach and numerical results in section 4 and finally we provide conclusion and future research guidelines in section 5.

## 2 LITERATURE REVIEW

The literature on resource constrained project scheduling problems returns to the 1960's. As the first formulation of these problems, we can mention to (Pritsker, A.A.B. *et al.*, 1969). The other types of the mathematical formulation for these problems have been defined latterly in literature. In (McKnew, M.A. *et al.*, 1991) the authors present a new formulation of RCPSPs that is applied on a lot-sizing problem. (Scholl, A., 1999) presents application of RCPSP's concepts on an assembly line balancing problem where the authors use branch and bound procedures for reporting their results. Besides the problem formulation, variants and different extensions of RCPSP have been introduced in literature as well. In this context, (Kaplan, L.A, 1988) represents the initial extension of resource constrained project scheduling problems considering preemption of the activities. That is to say, once an activity is started, it may be interrupted during its processing time. For the other extension of RCPSP, we mention to multimode problems wherein different possible ways of doing one supposed activity is considered. Some of the related works to multi-mode RCPSP are reviewed in following: (Ghoddoussi, P. *et al.*, 2013), (Kyriakidis, T. S. *et al.*, 2012), (Mika, M. *et al.*, 2008), (Van Pedegem, V. *et al.*, 2014) and (Tavan, M. *et al.*, 2014). In order to outstand the other extensions of these models and their vast application due to project activities' carrying out modes, project resources types, mathematical modeling classification and solution methodologies, we refer the readers to following surveys (Brucker, P., *et al*, 1999), (Hartmann, S. Briskorn, D. 2010), (Kolisch, R and Hartmann, S., 2006), (Kolisch, R. and Padman, R., 2001) and (Weglarz, J. *et al* 2011).

Besides the extensions, from view point of objective functions, resource constrained project scheduling problems are grouped according to different objectives: *Time-based objectives* where targets like completion time, earliness, tardiness and lateness are supposed to be minimized (Baker, K., 2014) and (Arabameri, S. and Salmasi, N., 2013). *Quality-based objectives* where the prob-

lem goal is to find the results that maximize the quality of the project. (Icmeli, O., and Rom, W., 1997) was the first that have presented this concept in project scheduling problems. *Cost-based objectives* in which the objective function minimizes the total cost of projects like execution costs, material costs, inventory holding costs or costs related to tardiness or earliness of the project. Some of the recent studies in this area are reviewed in following: (Ranjbar, M., *et al.*, 2012) represents an optimal solution procedure for minimizing total weighted resource tardiness penalty costs in the resource constrained project scheduling problem. In (Rodrigues, B., and Yamashita, D., 2010), the authors consider resource availability cost in their problem and they propose an exact algorithm to report the results. Their algorithm includes a hybrid method where an initial feasible solution is found heuristically and then the branching concepts have been applied to find the results. (Dodin, B., and Elimam, A. A., 2001) pursues a RCPSP problem with minimization of material costs, inventory costs and activity crashing costs. Activity crashing costs implies the costs of shortening the duration of an activity by increasing the required materials. Besides, the authors consider a bonus for early project completion time or a penalty coefficient for delay in project completion. *Net present value objectives* are used when certain predefined cash flows happen according to a predefined time periods. Net present value maximization of the projects reflexes the time value of money in project scheduling problems. Some models and solution methods for these problems are reviewed by (Demeulemeester, E.L., Herroelen, W.S., 2002), (Artigues, C., 2008) and (Wiesemann, W. *et al.*, 2010). Finally *multiple objectives* models in which more than one objective have been considered in problem formulation (Abbasi, B., *et al.* 2006) and (Al-Fawzan, M.A. and Houari, M., 2005). For our model, we consider cost-based objective functions where the target is to minimize total cost of the system implementation that includes recharge cost of non-renewable resources, execution cost of the activities, stock cost of non-renewable resources and rent cost of expensive required equipment through the execution period of the project.

As reviewed related works to our model, we mention to (Shirzadeh Chaleshtarti, A., *et al.*, 2014) where the authors propose a mathematical model for a resource constrained project scheduling problem with defined and pre-scheduled availability of non-renewable resources. They customize four branch and bound algorithms for solving the model. In (Oumar, K., *et al.*, 2013) the authors take into account storage resources which may be produced or consumed by activities. (Ng, S., *et al.*, 2009) presents a construction project where it is considered the stock of materials on site. The authors articulate that in projects like construction projects when the consequences of the activities are not specifically clear, there is always a risk of missing the productivity due to the shortage of material by idleness of manpower. So holding some levels of stocks will be an effective solution.

The authors aim to identify the possible saving in time and cost according to different amounts of stocks on project site. For that, they use an activity-based simulation model. As the other similar works in this area, we mention to partially renewable resources, cumulative resources and spatial resource. The partially renewable resources problems initially were introduced by (Böttcher, J., *et al.*, 1999) to define resource with partial availability in periods of time. The paper represents the application of this concept in shift scheduling of resources. (Alvarez-valdes, R., *et al.*, 2008) develops some preprocessing techniques and several heuristic algorithms for the problem. (Schirmer, A., 2000) and (Schirmer, A. and Drexler, A., 2001) present the other application of these problems. In terms of cumulative resources and spatial resource, the model of cumulative resources implies to availability of resources in a given time period based on its consumption in the past periods and consequently spatial resource supposes the use of resources by a group of activities that these resources totally belong to activities of the group till all of them will be done completely (Boer, R., 1998) and (Artigues, C., 2005).

For the above mentioned categories of RCPSP, different solving algorithms including exact algorithms, heuristics and meta-heuristics are developed in literature. Comprehensive reviews related to this subject provided in literature by (Brucker, P., *et al.*, 1999), (Kolisch, R and Hartmann, S., 2006) and (Węglarz, J. *et al.*, 2011) where the authors represent different algorithms to solve the problems. For our model, because of the NP-hard nature of resource constrained problems and the presence of integer variables in the formulation, we apply linear programming relaxation concept to decrease solution time length of the model. The proposed procedure reduces the numbers of integer variables by defining bounding values for earliest and latest start time of the project.

### 3 PROBLEM DEFINITION

In this section, at first we introduce our use case of study and then we present its relevant mathematical model.

#### 3.1 Use Case

This work is a part of a research project in France called CRIBA which aims to industrialize thermal renovation of buildings. In this project, thermal performance of the buildings, either residential or commercial, is improved highly by using industrialized insulation panels that are composed of in particular wooden frames, insulation materials and external coating materials. The panels are produced according to the buildings' geometric characteristics and are installed with fasteners on their external facades. In this project, for utilizing the solar energy, photovoltaic panels are assumed to be installed on the buildings' roof as well. The mentioned resources, i.e. insulation panels, photovoltaic panels and fasteners are the nonrenewable resources of the project which are

outsourced by the projects' suppliers.. For effectuating the installation, three renewable resource types i.e. manpower resources, trucks and cranes are required to be considered. The worksite comprises three buildings including two three floor buildings,  $B_1$  and  $B_2$  and one two floor building,  $B_3$ . For the buildings which have more than one floor, there is a need to rent some cranes. Because of the high rent price of the crane, it is targeted to maintain it on the project site as short as possible.

For resource planning of the project, two major points have been taken into account. The first point considers the possibility of additive capacities for manpower resources of the project within the project time horizon by an external subcontractor and the second integrates the concept of limited capacity for manufacturing units of non-renewable resources and also for stocking places of the required non-renewable resources.

Different activities of the project, their precedence relations and their processing times are depicted in table 1 and figure 2. The first and last activities of the networks are dummy activities.

Code	1	2	3	4
<b>Activity Description</b>	Start	Crane Assembly	Access	Building2 Ground Floor
Code	5	6	7	8
<b>Activity Description</b>	Building1 Ground Floor	Building3 Ground Floor	Building2 First Floor	Building1 First Floor
Code	9	10	11	12
<b>Activity Description</b>	Building3 First Floor	Building 2 Second Floor	Building1 Second Floor	Crane Disassembly
Code	13	14	15	16
<b>Activity Description</b>	Building1 annex	Building2 Edge complement	Building1 Edge complement	End

Table1: Activities description and related codes.

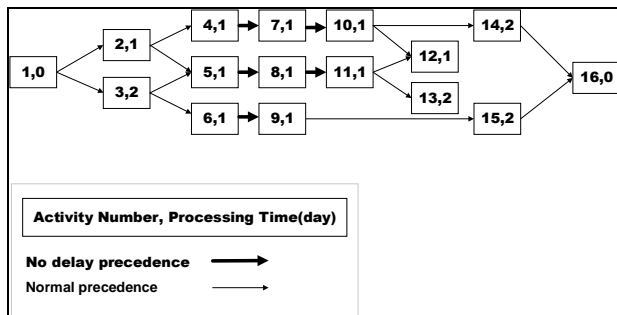


Figure 2: Activity-on-node presentation of activities.

Regarding to figure 2, we represent two types of precedence relationships which are including (1) normal precedence and (2) no delay precedence. The second relationship type encompasses those activities that should proceed immediately after each other because in CRIBA project it is expected to increase the inhabitant satisfaction by shortening the renovation time of the buildings.

### 3.2 Mathematical Formulation

In following, we present our mathematical model. The model considers a multi resource scheduling problems including both renewable and nonrenewable resources. The preemption of the activities is not allowed and there is just one mode for doing the activities. We supposed that all the parameters of the model are deterministic because they are already fully known. That is to say, there is not any uncertainty in model's parameters where it arises by lack of the information about the model attributers. Besides, considering the situation of the problem, we pursue that manpower resources, as the renewable resource of the project, are not available in their full amount at the beginning of the project and they can be charged during the project execution time by an external subcontractor. In order to improve the productivity of the project by eliminating the shortage of the required material in working sites, stock capacities are considered for renewable resources like project materials and insulation panels.

Indices, sets, parameters and decision variables of the model are introduced below:

Indices:

- $i, j$  indices of the activities.
- $r$  indices of renewable resources.
- $s$  indices of nonrenewable resources.
- $t$  indices of time.

Sets:

- $A$  set of activities.
- $P$  set of precedencies  $P \subseteq A \times A$ .
- $H$  set of time periods of the project.
- $R$  renewable resource set.
- $S$  nonrenewable resource set.

Parameters:

- $P_i$  processing time of activity  $i$  measured (day).
- $b_{is}$  demand of activity  $i$  for nonrenewable resource  $s$ .
- $sm_s$  maximum stock capacity of nonrenewable resources.
- $si_s$  initial stock of nonrenewable resource  $s$ .
- $e_i$  earliest start time of activity  $i$ .
- $l_i$  latest start time of activity  $i$ .
- $am_r$  maximum charge of resource  $r$ .
- $vr_r$  daily cost of charged renewable resource  $r$ .
- $c_{ir}$  consumption of renewable resource  $r$  by activity  $i$ .
- $vs_s$  stock holding cost of nonrenewable resource  $s$ .
- $dr_{rt}$  availability quantity of renewable resource  $r$  for time period  $t$ .

$pm_{st}$  delivery capacity of nonrenewable resource  $s$  at time  $t$ .

$S_{ij} = 0$  if activities  $i$  and  $j$  follow no delay precedence, 1 if  $i$  and  $j$  follow normal precedence ( $\forall ij \in P$ ).

$vp$  daily cost of site.

$vg$  rent cost of crane per day.

$vf$  unit cost of available renewable resource  $r$  which has been failed to use in the worksite

Decision Variables:

$A_{rt}$  charge of renewable resource  $r$  at time  $t$ .

$X_{it} = 1$  if activity  $i$  starts at time  $t$ , 0 otherwise.

$Y_{it} = 1$  if activity  $i$  is processing in time  $t$ , 0 otherwise

$PR_{st}$  deliveries of nonrenewable resource  $s$  at time  $t$ .

$ST_{st}$  stock of resource nonrenewable resource  $s$  at time  $t$ .

$SA_i$  start time of activity  $i$ .

$F_{rt}$  quantity of available renewable resource  $r$  which has been failed to use in the worksite.

$CP$  total daily cost of site.

$CG$  total rent cost of crane as high rent cost equipment.

$CS$  total stock cost of nonrenewable resources.

$CA$  total charge cost of the renewable resource.

$CF$  total cost of available renewable resource  $r$  which have been failed to use in the worksite

$Z$  total cost of project execution.

The mathematical optimization model is as follow.

$$\text{Min } Z = CP + CA + CS + CG + CF \quad (1)$$

s.t

$$\sum_{t=e_i}^{l_i} X_{it} = 1 \quad \forall i \in A, \quad \forall t \in H \quad (2)$$

$$SA_1 = 1 \quad (3)$$

$$SA_i = \sum_{t=e_i}^{l_i} t X_{it} \quad \forall i \in A \text{ and } i \geq 2 \quad (4)$$

$$SA_j = SA_i + p_i \quad \forall i, j \in P \mid S_{ij} = 0 \quad (5)$$

$$SA_j \geq SA_i + p_i \quad \forall i, j \in P \mid S_{ij} = 1 \quad (6)$$

$$Y_{it} \leq 1 \quad \forall i \in A, \quad \forall t \in H \quad (7)$$

$$\sum_t Y_{it} = p_i \quad \forall i \in A \quad (8)$$

$$\sum_{u=t-1+p_i}^{t-1+p_i} Y_{iu} \geq p_i X_{it} \quad \forall i \in A, \quad \forall t \in (e_i, l_i) \quad (9)$$

$$\sum_i c_{ir} Y_{it} \leq dr_{rt} + A_{rt} - F_{rt} \quad \forall t \in H, \quad \forall r \in R \quad (10)$$

$$A_{rt} \leq am_r \quad \forall r \in R, \quad \forall t \in H \quad (11)$$

$$ST_{s1} = si_s \quad \forall s \in S \quad (12)$$

$$ST_{s,t-1} + PR_{st} = ST_{st} + \sum_i b_{is} Y_{it} \quad \forall s \in S, \quad \forall t \in H \quad (13)$$

$$ST_{st} \leq sm_s \quad \forall s \in S \quad (14)$$

$$PR_{st} \leq pm_{st} \quad \forall s \in S, \quad \forall t \in H \quad (15)$$

$$CP = vp S_n \quad (16)$$

$$CA = \sum_r \sum_t vr_r A_{rt} \quad (17)$$

$$CS = \sum_s \sum_t vs_s ST_{st} \quad (18)$$

$$CG = vg (S_{12} + P_{12} - S_2) \quad (19)$$

$$CF = \sum_r \sum_t vf_r F_{rt} \quad (20)$$

$$X_{it} \in \{0, 1\} \quad \forall i \in A, \quad \forall t \in H \quad (21)$$

First equation as an objective function minimizes total cost of the project including (i) total daily cost of the site, (ii) total charge cost of the renewable resources, (iii) total stock cost of the nonrenewable resources, (iv) total rent cost of the equipment with high rent fees, i.e. the crane and (v) total cost of failed renewable resources in the worksite. In the objective function the term  $CA$  (total cost of additive resources) +  $CF$  (total cost of failed

renewable resources) implicitly guaranties to smooth the renewable resources' workload. Constraints (2)-(10) relate to project's activities execution. In this context, constraint (2) guaranties that all of the activities will be done at a time between their earliest start time and latest start time. Equation (3) fixes starting time for the first activity of the network that is defined as a dummy activity. Constraint (4) defines the start time of the activities. Constraints (5) and (6) relate to precedence relationship. According to (5), the start time of the jobs that begin immediately after their predecessors must be equal to the completion time of its predecessor and according to (6), the start time of the activities that may not be started immediately after their predecessors can be greater or equal than their predecessors' completion time. Equations (7) and (8) define respectively the possibility of the activity execution at the time and their processing time length. By constraint (7), an activity can be started executing at a certain period of time and by constraint (8) the processing time of the activities has been met totally in project execution period. Constraint (9) shows that the processing of each activity is not interrupted. In other words, it guaranties the non-preemption of the activities in project network. Constraints (10)-(15) articulate the additive capacities of renewable resources and capacity limitations for non-renewable resources. Equation (10) represents the resource availability limitation. It imposes that the demand for the renewable resources at each time period must not exceed the available quantity plus the quantity which is being charged at each time period and subtracted from the resources that are failed for application in the worksite. Constraint (11) imposes charge capacity limitation. Equation (12) implies the initial stock of nonrenewable resources in the site. Equation (13) represents the stock from the previous period and the new deliveries of the resources must be equal with the sum of the consumed quantity of resources and stocks of the resources at the end of the time period. Constraints (14) and (15) represent properly the stock capacity and the delivery capacity limitations. Equations (16), (17), (18) and (19) calculate respectively the total daily cost of the site, the total charge cost of the renewable resource, the total stock cost of nonrenewable resources and the total rent cost of crane as high rent cost equipment. Constraint (20) calculates total cost of the resources that are employed in the project but are failed to be applied. Finally, constraint (21) defines the variable types of the model. It is worthwhile noting that constraints (7), (8), (9) and (10) imply that the decision variables  $Y_{it}$  will be equal with 0 or 1.

#### 4 SOLUTION METHODOLOGY

In integer programming (IP) problems solution time of the models directly depends on the number of integer and binary variables. Then, one efficient way for decreasing the computation time of an IP problem can be to decline the number of its integer or binary variables. To pursue that purpose, the idea is to limit the possible range

of the variables to take place by defining relevant lower bounds and upper bounds.

In our problem, the number of binary variables  $X_{it}$  is declined by defining the lower and upper bounds for them. In this context, earliest start times  $e_i$  are considered to define the lower bound of the model and in the same way the latest start times  $l_i$  are assumed to define the upper bounds of  $X_{it}$ . In the model, after defining the values of  $e_i$  and  $l_i$ , by using a procedure which we present below, we limit the binary variables  $X_{it}$  between the calculated bounds of  $e_i$  and  $l_i$  and then we resolve the model to obtain final results. In following, we provide respectively pseudo-code of our applied procedure for calculating the lower bound  $e_i$  and then we extend it to the latest start time  $l_i$ :

1. Relax binary variables  $X_{it}$ .
2. For all  $i$   
 $e_i = 0$  and  $l_i = T$
3. Repeat
  - 3.1 For all  $i$ 
    - 3.1.1  $Z' = \text{Minimum } S_i \text{ subject to 2-19.}$
    - 3.1.2  $Z' \rightarrow e_i$ .
  - 3.2 Until no change between two sequential  $e_i$ .

Figure 3: Pseudo-code of calculating algorithm for earliest start time of activities,  $e_i$ .

In following, we present the related pseudo-code of the procedure for calculation of  $l_i$ :

1. Relax binary variables  $X_{it}$ .
2. For all  $i$   
 $l_i = T$
3. Repeat
  - 3.1 For all  $i$ 
    - 3.1.1  $Z'' = \text{Maximum } S_i \text{ subject to 2-19.}$
    - 3.1.2  $Z'' \rightarrow l_i$ .
  - 3.2 Until no change between two sequential  $l_i$ .

Figure 4: Pseudo-code of calculating algorithm for latest start time of activities,  $l_i$ .

Activity	2	3	4	5	6	7	8
$e_i$	1	2	3	10	9	4	11
$l_i$	15	14	20	17	23	21	18
Activity	9	10	11	12	13	14	15
$e_i$	10	5	12	13	14	7	17
$l_i$	22	22	19	24	21	23	23

Table 2: Earliest and latest start time calculate by the proposed procedure

The results presented in table 2 are obtained from the model solution with  $T=25$ .

In order to verify the utility of the proposed algorithm, we solve a case study of the model applying and not applying the proposed procedure ( $T=25$ ). In both cases we obtain the same optimal solution meanwhile the solution-time is reduced significantly (table 3).

Size of the Model	
<b>Number of Activities= 14</b> <b>Number of Renewable Resources= 3</b> <b>Number of Non-renewable Resources= 3</b> <b>Maximum Numbers of Time Periods= 25</b>	
Computation Time of the Model without Applying the Limiting Algorithm ( $e_i=0, l_i=25$ )	Computation Time of the Model with Applying the Limiting Algorithm ( $e_i$ and $l_i$ based on Table 2)
19'38''	2'59''

Table 3: Comparison of computation time of the model with application and without application of limiting procedure

In Figure 5, we present the final results for sum of the values of three decision variables namely  $A_{rt}$ ,  $PR_{st}$  and  $ST_{st}$  over the different period times of project. They represent the total add of renewable resources at time period  $t$ ,  $rA_{rt}$ , total deliveries of non-renewable resources,  $sPR_{st}$ , and total stock of the non-renewable resources,  $sST_{st}$  are depicted in figure 5. As it is known from table 3, the model is solved for a small size of the problem with 14 activities. Nevertheless, in order to deal with large sizes of the problem, since the exact algorithms consume more time for solving the models, we recommend to researchers to use meta-heuristics or related heuristics to find the results of problem in a reasonable solution time (Klein, R., 1999).

## 5 CONCLUSION AND FUTURE RESEARCHES

In this research, we present a resource constrained project scheduling model for a French project of thermal renovation of the buildings. The model considers the stock capacity for nonrenewable resources and optimal stock capacity for nonrenewable resources and optimal add of the renewable resources by an external enterprise. To the best of our knowledge, these two considerations present a new contribution in resource constrained project scheduling problem and provide a more comprehensive mathematical model in this field.

Because of the NP-hard nature of the model, the problem is solvable in an exponential time period. For decreasing

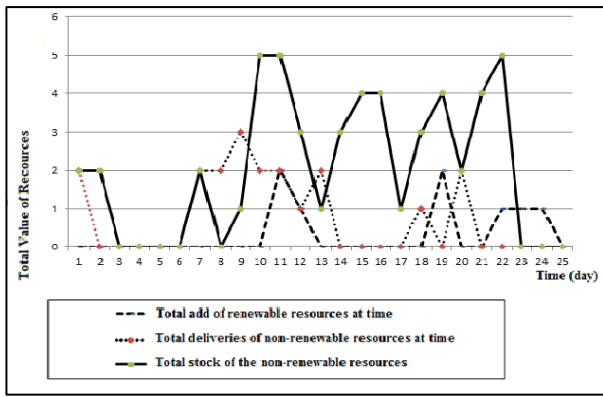


Figure 5: Total add of renewable resources, total deliveries and total stock of the non-renewable resources.

the computation time of the model, we propose and apply a linear programming relaxation based procedure. Our proposed algorithm reduces the number of the binary variables  $X_{it}$  by defining the bounds for earliest and latest start time of the project's activities. So, we reduce the number of binary variables and consequently we decline the computation time of the model. We provide the results and outputs of applying the procedure in section 4.

For future researches, we suppose to consider multi-methods for processing of activities. This develops the problem to multi-mode resource constrained project scheduling models where more attention of the researchers is paid on because of their vast application in real world projects and their more comprehensive framework of planning. Extending the model by considering multiple projects with different priorities of project effectuation can provide a more applicable and more interesting framework of study. Since in real world situation, we face with the uncertainty which is originated by ambiguous information of the model attributers, we propose to consider that approach in the related model parameters and treat with them by relevant methods like stochastic programming, robust optimisation or fuzzy programming. Considering new other objectives such as net present values and project quality could lead the researchers to develop more interesting models. Besides, applying the other solution methodologies based on the efficient heuristics or exact methods are proposed in this field.

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## APPENDIX

Activity Code	Photovoltaic panels	Insulation panels	Fasteners	Manpower	Truck	Crane
2	0	0	0	2	1	1

3	2	2	2	2	0	1
4	2	2	1	2	1	1
5	2	2	2	2	0	1
6	2	3	1	2	1	0
7	1	2	1	2	0	1
8	1	2	2	2	1	1
9	2	2	1	3	1	0
10	1	2	1	2	0	1
11	1	2	1	3	0	1
12	0	0	1	1	1	1
13	2	3	1	2	0	0
14	0	0	1	2	1	0
15	0	0	0	3	1	0

Table4: Demands of activities for renewable and non-renewable resources.

Day	Photovoltaic panels	Insulation panels	Fasteners	Manpower	Truck	Crane
1	2	1	1	2	1	1
2	2	1	1	2	1	1
3	1	1	3	2	1	1
4	1	1	2	2	1	1
5	0	2	2	2	0	1
6	2	0	0	2	0	1
7	2	2	2	0	0	0
8	3	0	0	2	1	1
9	2	1	1	2	1	1
10	2	1	1	2	0	1
11	2	2	2	2	0	1
12	2	1	2	2	0	1
13	2	1	2	2	1	1
14	2	0	3	2	1	1
15	3	2	1	2	1	1
16	1	2	4	2	1	1
17	2	1	3	2	1	1
18	2	1	2	2	1	1
19	1	2	1	2	1	1
20	2	2	3	2	1	1
21	3	1	3	2	1	1
22	2	2	4	2	1	1
23	1	1	2	2	1	1
24	3	1	1	2	1	1
25	1	2	4	2	1	1
MAX STOCK	5	3	0	2	1	0

Table 5: Daily availability of the resource