



**HAL**  
open science

# A MULTI-ITEM BI-LEVEL PRODUCTION PLANNING PROBLEM WITH REUSABLE BY-PRODUCTS

Mehdi Rowshannahad, Nabil Absi, Stéphane Dauzère-Pérès, Bernard Cassini

► **To cite this version:**

Mehdi Rowshannahad, Nabil Absi, Stéphane Dauzère-Pérès, Bernard Cassini. A MULTI-ITEM BI-LEVEL PRODUCTION PLANNING PROBLEM WITH REUSABLE BY-PRODUCTS. MOSIM 2014, 10ème Conférence Francophone de Modélisation, Optimisation et Simulation, Nov 2014, Nancy, France. hal-01166628

**HAL Id: hal-01166628**

**<https://hal.science/hal-01166628v1>**

Submitted on 23 Jun 2015

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# A MULTI-ITEM BI-LEVEL PRODUCTION PLANNING PROBLEM WITH REUSABLE BY-PRODUCTS

Mehdi Rowshannahad<sup>1,2</sup>, Nabil Absi<sup>1</sup>, Stéphane Dauzère-Pérès<sup>1</sup> and Bernard Cassini<sup>2</sup>

<sup>1</sup>Department of Manufacturing Sciences and Logistics  
Ecole des Mines de Saint-Etienne – CMP  
13541 Gardanne - France  
{rowshannahad,absi,dauzere-peres}@emse.fr

<sup>2</sup>Soitec  
Parc Technologique des Fontaines  
38190 Bernin - France  
{mehdi.rowshannahad,bernard.cassini}@soitec.com

**ABSTRACT:** *In this paper, we investigate a multi-item production planning problem in which reusable by-products are generated during production. After further processing, the generated by-products can be reused as raw materials. However, by-products can be “recycled” only a given number of times. The production and recycling processes are performed in internal and external sites with limited capacity. Each product may be produced using specific raw material – newly purchased or recycled – references. The proposed model represents a part of the supply chain of “SOI” (Silicon-On-Insulator) fabrication units. Using numerical examples based on industrial data, the model is validated and some of its characteristics are discussed. Finally, interesting perspectives of this work are proposed.*

**KEYWORDS:** *Bi-level capacitated lot-sizing, by-production, remanufacturing, semiconductor manufacturing.*

## 1 INTRODUCTION

Silicon wafers are extensively used in semiconductor manufacturing to produce microelectronic components such as chips and integrated circuits. However, some devices require higher performance which cannot be delivered by traditional silicon-only wafers. Components built on Silicon-On-Insulator (SOI) wafers offer much more performance while consuming less energy compared to components on silicon-only wafers. SOI wafers may be produced using different technologies: *SIMOX<sup>TM</sup>* (Separation by IMplantation of OXYgen), *wafer bonding* or *Seed methods*. The supply chain studied in this paper considers a type of wafer bonding technology called the *Smart-Cut<sup>TM</sup> Technology*.

The Smart-Cut<sup>TM</sup> Technology transfers a thin layer of crystalline material from a donor substrate to another substrate using bonding and layer splitting processes (Bruel 1995). The used donor substrate can be processed later to be reused again as the donor substrate to produce another SOI wafer. Here, the used donor wafer is considered as a “by-product”, i.e. it has been generated during production. In this study, we consider the supply chain of a SOI production unit in which the used raw material can be recycled and reused several times. The recycling process, of limited capacity, can be internal or external. The demand is assumed to be known over a discrete time horizon. It is possible to stock products to satisfy future demand but no backlogging is allowed.

The paper is organized as follows. In Section 2, we present a brief literature review on the related planning

domains. Then the main aspects of the Smart-Cut<sup>TM</sup> Technology and the studied supply chain are described concisely in Section 3. The mathematical model corresponding to the supply chain under study is proposed in Section 4. Using numerical experiments on test instances generated based on industrial data, the model is validated and some managerial insights are discussed in Section 5. Finally, conclusions are drawn and some perspectives of the study are presented in Section 6.

## 2 LITERATURE REVIEW

The problem studied in this paper is related to different domains of lot-sizing problems. Some related studies are recalled to distinguish the purpose of our study with previous investigations.

For interested readers, we suggest the paper of (Brahimi, Dauzère-Pérès, Najid & Nordli 2006) which provides a survey on both uncapacitated and capacitated single item lot sizing problems. A literature review of the models and the algorithms for uncapacitated and capacitated single level lot sizing problems can be found in (Karimi, Fatemi Ghomi & Wilson 2003).

This study concerns a multi-item supply chain. The classical capacitated multi-item lot-sizing problem with non-stationary costs, demands and setup times is considered in (Trigeiro, Thomas & McClain 1989). The problem is decomposed into a set of uncapacitated single product lot-sizing problems using a Lagrangian relaxation. The single-item problems are solved using a dynamic programming algorithm. A smoothing heuristic is used to

make the dual solution feasible.

If there is a parent-component relationship in the item structure, the problem is classified as a multi-level lot-sizing problem. In single-level problems, only independent demands (from external customers) are considered while, in multi-level problems, the production of each final item generates a dependent demand for its components. The problem studied in this paper is a bi-level lot-sizing problem. The problem of minimizing the setup and inventory costs in a capacitated multi-level lot-sizing problem is discussed in (Billington, McClain & Thomas 1983). An interesting literature review on multi-level capacitated lot-sizing problems together with a solution approach for the dynamic multi-level capacitated lot-sizing problem are discussed in (Helber & Sahling 2010).

Several terms such as “by-product”, “co-product”, “remanufacturing” and “recycling” are used to designate different concepts in the literature that are close to our research. In this study, the raw material once used for production is considered as “by-product”. This by-product cannot fulfill any demand and must be reworked before coming back to the manufacturing cycle. The process of restoring the generated by-products makes them reusable again as raw materials. Therefore, this process can be considered as a “remanufacturing process”.

A multi-item uncapacitated lot-sizing problem in which co-products are produced at each production run is treated in (Agrali 2011). In this paper, it is considered that the co-products have their own demand and cannot fulfill the demand of the main product. Several MIP formulations are presented for the problem. Using a variant of the zero-inventory property, a dynamic program is used to solve the problem in the single-item case.

Other studies consider the co-production of a range of products with different performances in a single production run. The co-products are then sorted according to their key performance to satisfy demands of each co-product (Tomlin & Wang 2008).

Remanufacturing in reverse logistics is considered in (Helmrich, Jans, van den Heuvel & Wagelmans 2014). By remanufacturing in reverse logistics, it is meant that there is not only a one-way flow of the products to the customers but materials and products may be returned to the manufacturer for remanufacturing and reuse. This is what is called a “closed-loop supply chain”. In the proposed model, known quantities of used products are returned from customers in each period. Once reworked, the returned products are used to satisfy the customer demand as new products. Therefore, in each period, it is determined whether to produce new products or to remanufacture returned products to satisfy the demand. The supply chain is modeled using Mixed-Integer Programming (MIP). The proposed model is shown to NP-hard.

In a more recent study, a closed-loop supply chain with

setup costs, product returns and remanufacturing is considered in (Zhang, Jiang & Pan 2012). The study is inspired from the paper manufacturing industry in which both virgin and deinked pulps are used to make papers. A MIP model and a Lagrangian relaxation based solution approach are further proposed. A manufacturing - remanufacturing closed-loop supply chain in a dynamic continuous time stochastic context is studied in (Kenné, Dejax & Gharbi 2012).

Spengler et al. (Spengler, Puechert, Penkuhn & Rentz 1997) propose mathematical planning models for recycling generated by-products during production, and dismantling and recycling products at the end of their lifetime. Several other studies use the term remanufacturing to denote the restoring or recycling of products (Ferrer & Whybark 2001).

In the mentioned studies, the process of refurbishing the returned products is referred to as remanufacturing. However, in our study no products are returned and only the used raw materials can be freshened (remanufactured) a limited number of times. The proposed problem modeling is original and different from the literature review. In the following section, the problem is unfolded in more detail and the notations are introduced.

### 3 PROBLEM DEFINITION

In this study, we consider the supply chain of a Silicon-on-Insulator (SOI) wafer production unit using the Smart-Cut™ Technology.

In SOI wafers, a thin layer of silicon is laid on a silicon wafer which serves only as a physical support (or handle). These two silicon layers are separated by an insulator: The oxide. Figure 1 illustrates the main steps of the Smart-Cut™ Technology.

Once wafer **A** is oxidized and implanted, it is ready to be bonded with wafer **B**. After the wafers are bonded, they are split to form the SOI wafer. Wafer **A** is the “donor” wafer in the sense that a thin silicon layer of this substrate is deposited on Wafer **B**. In the industrial jargon, Wafer **A** is called “Top” while Wafer **B** is called “Base”.

As only a thin layer of the Top wafer is deposited on the Base Wafer, it is possible to reuse the Top wafer several times to produce other SOI wafers. This is one of the main advantages of the Smart-Cut™ Technology which makes the process cost competitive.

A “used Top”, called “Negative”, must be reworked before returning to the SOI fabrication process. This remanufacturing process is called the *refresh process* or shortly *refresh*. In industrial terminology, a new Top wafer used for the SOI fabrication is called a “Fresh Wafer”. A Fresh Wafer is purchased from silicon wafer suppliers. After the first utilization, the generated Negative of the Fresh Wafer is called “Negative 0” or shortly “Neg 0”. Refresh-

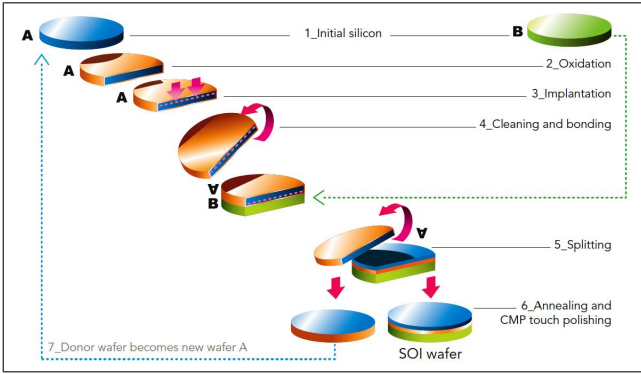


Figure 1: Unibond SOI wafer fabrication steps using the Smart-Cut™ Technology - (Soitec 2014) accessed May 2014

ing “Negative 0” gives a newly usable Top wafer called “Refresh Wafer 1”. A wafer may be refreshed only a maximum number of times (called the “maximum refresh level”). It is economically interesting to refresh a Top wafer as many times as possible. However there is an end to the refresh process because of quality and yield constraints. Mostly, mature products have the highest refresh level because of a better understanding of the characteristics of the product and a higher expertise of the refresh process. Using this logic of numbering, a Fresh Wafer can also be called “Refresh Wafer 0”.

Special yield and quality constraints or specific customer specifications may cause the SOI-Refresh planning to be more complicated. Some products may only use a Fresh Wafer as a Top Wafer and not its Refresh Wafers. Or some customers may require to use only up to a certain refresh level for their products. A rare situation which may also occur is that some products can only be produced from the Refresh Wafers of a Fresh Wafer. In this case, in order to obtain the required Refresh Wafers, Fresh Wafers are first used to produce another product. Then the generated Negatives are refreshed.

The refresh process can be done internally or externally. Internal refresh may also be performed in a different site than the one where the SOI wafers are produced. Therefore, the shipping, planning, extra packaging, possible deterioration and increased and less certain cycle time must be considered when the refresh process is not done at the same site where SOI is produced. A yield factor is associated with the refresh process because of the manufacturing line scrap.

In Figures 2 and 3, the landscape of the supply chain under study is depicted. Figure 2 illustrates different levels of the refresh process until the Top Wafer can no longer be refreshed. At this time, it is used as a test wafer for qualification or test purposes or simply as a filler wafer. Figure 3 depicts a simple SOI-Refresh supply chain. Top (Fresh) and Base Wafers are purchased from a bulk supplier. The SOI production site includes also a refresh line. An exter-

nal refresh supplier and a customer are also present.

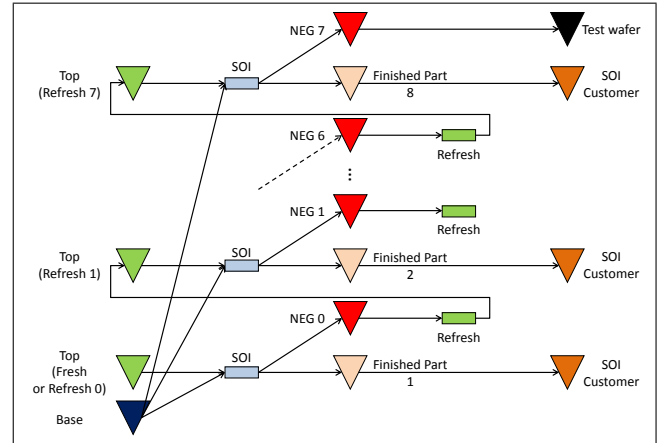


Figure 2: SOI Fabrication and Demonstrative Refresh Process up to 7 Levels ( $I^{max} = 7$ )

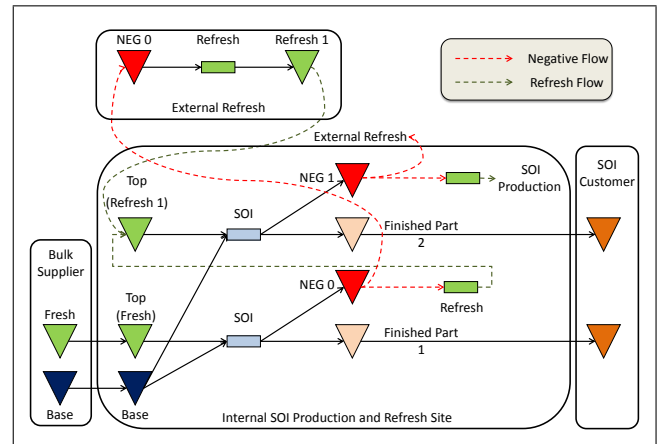


Figure 3: A Simple SOI Production-Refresh Supply Chain

#### 4 MATHEMATICAL MODEL

The objective of our model is to decide when and how much to produce final products (SOI), when and how much to purchase raw materials (Base Wafer and Fresh Wafer), when and how much to refresh used Top Wafers in order to satisfy demand. The demand satisfaction is done over a discrete time horizon while minimizing the total cost (production, purchase, refresh and inventory costs). The plan must satisfy inventory, bill of materials as well as capacity constraints. The parameters, the decisions variables and the mathematical model are presented below.

**Parameters**

$I$	Total number of products,
$F$	Total number of Top (Fresh and Refresh) wafers,
$B$	Total number of Base Wafers,
$T$	Total number of periods,
$M$	Total number of production or supplier sites,
$d_{i,t}$	Demand of product $i$ in period $t$ ,
$n_f^{max}$	Maximum refresh level of Top wafer $f$ ,
$n_f$	Number of times that Top wafer $f$ has been refreshed,
$l$	Lead time of the refresh process,
$a_{i,b}$	$\begin{cases} 1 & \text{if Base Wafer } b \text{ can be used in product } i, \\ 0 & \text{otherwise.} \end{cases}$
$a_{i,f}$	$\begin{cases} 1 & \text{if Top } f \text{ (Fresh or Refresh) wafer can be used in product } i, \\ 0 & \text{otherwise.} \end{cases}$
$b_{f',f}$	$\begin{cases} 1 & \text{if Top } f' \text{ (Refresh) wafer can be obtained via the refresh process from } f \text{ (Negative of either a Fresh or Refresh Wafer),} \\ 0 & \text{otherwise.} \end{cases}$
$\alpha_{f,m}$	Yield of the refresh process for Top wafer $f$ at site $m$ ,
$h_{b,t}$	Inventory cost of Base Wafer $b$ in period $t$ ,
$h_{f,t}^+$	Inventory cost of Top wafer $f$ in period $t$ ,
$h_{f,t}^-$	Inventory cost of Negative of Top wafer $f$ in period $t$ ,
$h_{i,t}$	Inventory cost of product $i$ in period $t$ ,
$S_{b,0}$	Initial inventory level of Base Wafer $b$ ,
$S_{f,0}^+$	Initial inventory level of Top wafer $f$ ,
$S_{f,0}^-$	Initial inventory level of Negative of Top wafer $f$ (a used Fresh or Refresh Wafer),
$S_{i,0}$	Initial inventory level of product $i$ ,
$\hat{B}_{b,t}$	Quantities of Base Wafer $b$ to be received (in transit) in period $t$ ,
$\hat{F}_{f,t}$	Fresh quantities of $f$ to be received (in transit) in period $t$ ,
$\hat{R}_{f,t}$	Refresh quantities of $f$ to be received (in transit) in period $t$ ,
$rc_{m,t}$	Refresh capacity at site $m$ in period $t$ ,
$\beta_{f,m}$	Process time of refreshing a unit of Top wafer $f$ at site $m$ ,
$pc_t$	Production capacity in period $t$ ,
$\eta_i$	Process time of producing a unit of product $i$ ,
$cg_{i,t}$	Production cost of product $i$ in period $t$ ,
$sg_t$	Production setup cost in period $t$ ,
$cr_{f,m,t}$	Refresh cost ( $\forall f n_f \in \{1, \dots, n_f^{max}\}$ ) of Top wafer $f$ at site $m$ in period $t$ ,
$sr_{m,t}$	Refresh setup cost at site $m$ in period $t$ ,
$cp_{b,t}$	Purchase cost of Base Wafer $b$ in period $t$ ,
$cp_{f,t}$	Purchase cost of Top wafer $f$ ( $\forall f n_f = 0$ ) (Fresh Wafer) in period $t$ ,
$sp_t^B$	Base Wafer purchase order cost in period $t$ ,
$sp_t^F$	Fresh Wafer purchase order cost in period $t$ .

**Decision variables**

$G_{i,t}$	Produced quantity of product (SOI) $i$ in period $t$ ,
$B_{b,t}$	Ordered quantity of Base Wafer $b$ in period $t$ ,
$F_{f,t}$	Ordered quantity of Fresh Wafer $f$ ( $f n_f = 0$ ) in period $t$ ,
$R_{f,f',m,t}$	Refreshed quantity $f'$ (Refresh Wafer) obtained from Negative $f$ at site $m$ in period $t$ ,
$X_{i,b,t}^B$	Used quantity of Base Wafer $b$ in period $t$ to produce product $i$ ,
$X_{i,f,t}^F$	Used quantity of Top wafer $f$ in period $t$ to produce product $i$ ,
$S_{i,t}$	Inventory level of product (SOI) $i$ at the end of period $t$ ,
$S_{b,t}$	Inventory level of Base Wafer $b$ at the end of period $t$ ,
$S_{f,t}^+$	Inventory level of Top wafer $f$ at the end of period $t$ ,
$S_{f,t}^-$	Inventory level of Negative of Top wafer $f$ (a used Fresh or Refresh Wafer) at the end of period $t$ ,
$Y_t$	$\begin{cases} 1 & \text{if production occurs in period } t, \\ 0 & \text{otherwise.} \end{cases}$
$V_t^B$	$\begin{cases} 1 & \text{if Base Wafer procurement occurs in period } t, \\ 0 & \text{otherwise.} \end{cases}$
$V_t^F$	$\begin{cases} 1 & \text{if Fresh Wafer procurement occurs in period } t, \\ 0 & \text{otherwise.} \end{cases}$
$W_{m,t}$	$\begin{cases} 1 & \text{if refresh process is done in site } m \text{ in period } t, \\ 0 & \text{otherwise.} \end{cases}$

The objective function (1) minimizes the total cost which is the sum of the purchase cost of new Top wafers (Fresh Wafers) as well as Base Wafers, refresh cost at all sites, finished goods (SOI wafers) production cost, raw material (Bulk, i.e. Fresh and Base Wafers) procurement cost, refresh setup cost, production setup cost, and the inventory cost of Base Wafers, Top wafers (either Fresh or Refresh Wafers), generated Negatives and finished goods (SOI wafers).

Constraint 2 models the flow conservation of finished goods. Constraints 3 and 4 respectively determine the amount of Base Wafers and Top wafers (either Fresh or Refresh Wafers) which are used to satisfy the production plan of product  $i$  in period  $t$  ( $G_{i,t}$ ).

Constraints 5 and 6 respectively model the flow conservation for Base Wafers and Fresh Wafers. Constraint 7 refers to Refresh Wafer inventory balance. It indicates that the inventory of the Refresh Wafers  $f$  in period  $t$  ( $S_{f,t}^+$ ) is equal to the Refresh Wafers to be received in this period (in transit Refresh Wafers) ( $\hat{R}_{f,t}$ ) plus the Refresh Wafer inventory in the previous period ( $S_{f,t-1}^+$ ) plus the Refresh Wafers of that period in all sites (refreshed Negatives obtained in period  $t$ ) ( $\sum_{\forall m} \sum_{\forall f'|b_{f,f'}=1} \alpha_{f,m} R_{f,f',m,t-l}$ ) minus the used Top wafers of  $f$  in that period ( $\sum_{\forall i} X_{i,f,t}^F$ ).

**Mathematical model**

$$\begin{aligned}
 & \min \sum_{\forall f|n_f=0} \sum_{\forall t} cp_{f,t} F_{f,t} + \sum_{\forall b} \sum_{\forall t} cp_{b,t} B_{b,t} \\
 & + \sum_{\forall m} \sum_{\forall f|n_f>0} \sum_{\forall f'} cr_{f',m,t} R_{f',f',m,t} + \sum_{\forall i} \sum_{\forall t} cg_{i,t} G_{i,t} \\
 & + \sum_{\forall t} sp_t^B V_t^B + \sum_{\forall t} sp_t^F V_t^F + \sum_{\forall m} \sum_{\forall t} sr_{m,t} W_{m,t} + \sum_{\forall t} sg_t Y_t \\
 & + \sum_{\forall b} \sum_{\forall t} h_{b,t} S_{b,t} + \sum_{\forall f} \sum_{\forall t} h_{f,t}^+ S_{f,t}^+ \\
 & + \sum_{\forall f|n_f \neq n_f^{\max}} \sum_{\forall t} h_{f,t}^- S_{f,t}^- + \sum_{\forall i} \sum_{\forall t} h_{i,t} S_{i,t} \quad (1)
 \end{aligned}$$

$$\begin{aligned}
 & G_{i,t} \geq 0 \\
 & \forall i,t \quad (17) \\
 & S_{b,t}, B_{b,t} \geq 0 \\
 & \forall b,t \quad (18) \\
 & S_{f,t}^+, S_{f,t}^-, S_{i,t}, F_{f,t} \geq 0 \\
 & \forall f,t \quad (19) \\
 & R_{f',f',m,t} \geq 0 \\
 & \forall f',f',m,t \quad (20) \\
 & X_{i,b,t}^B \geq 0 \\
 & \forall i,b,t \quad (21) \\
 & X_{i,f,t}^F \geq 0 \\
 & \forall i,f,t \quad (22)
 \end{aligned}$$

subject to

$$\begin{aligned}
 & S_{i,t-1} + G_{i,t} = d_{i,t} + S_{i,t} \\
 & \forall i,t \quad (2) \\
 & \sum_{\forall b} a_{i,b} X_{i,b,t}^B = G_{i,t} \\
 & \forall i,t \quad (3) \\
 & \sum_{\forall f} a_{i,f} X_{i,f,t}^F = G_{i,t} \\
 & \forall i,t \quad (4) \\
 & \hat{B}_{b,t} + S_{b,t-1} + B_{b,t} - \sum_{\forall i} X_{i,b,t}^B = S_{b,t} \\
 & \forall b,t \quad (5) \\
 & \hat{F}_{f,t} + S_{f,t-1}^+ + F_{f,t} - \sum_{\forall i} X_{i,f,t}^F = S_{f,t}^+ \\
 & \forall f|n_f = 0,t \quad (6) \\
 & \hat{R}_{f,t} + S_{f,t-1}^+ + \sum_{\forall m} \sum_{\forall f'|b_{f',f'}=1} \alpha_{f,m} R_{f',f',m,t-1} - \sum_{\forall i} X_{i,f,t}^F = S_{f,t}^+ \\
 & \forall f|n_f \neq 0,t \quad (7) \\
 & S_{f,t-2}^- + \sum_{\forall i} X_{i,f,t-1}^F - \sum_{\forall m} \sum_{\forall f'} b_{f',f} R_{f',f',m,t} = S_{f,t-1}^- \\
 & \forall f|n_f \neq n_f^{\max},t \quad (8) \\
 & \sum_{\forall i} \eta_i G_{i,t} \leq pc_t \\
 & \forall t \quad (9) \\
 & \sum_{\forall f|n_f \neq 0} \sum_{\forall f'} \beta_{f',m} R_{f',f',m,t} \leq rc_{m,t} \\
 & \forall m,t \quad (10) \\
 & \sum_{\forall i} G_{i,t} \leq M \cdot Y_t \\
 & \forall t \quad (11) \\
 & \sum_{\forall b} B_{b,t} \leq M \cdot V_t^B \\
 & \forall t \quad (12) \\
 & \sum_{\forall f|n_f=0} F_{f,t} \leq M \cdot V_t^F \\
 & \forall t \quad (13) \\
 & \sum_{\forall f'} \sum_{\forall f|n_f>0} R_{f',f',m,t} \leq M \cdot W_{m,t} \\
 & \forall m,t \quad (14) \\
 & Y_t, V_t^B, V_t^F \in \{0,1\} \\
 & \forall t \quad (15) \\
 & W_{m,t} \in \{0,1\} \\
 & \forall m,t \quad (16)
 \end{aligned}$$

Constraint 8 models the inventory balance of Negatives in period  $t$ . It specifies that the inventory of the Negative of the Top wafer  $f$  in period  $t-1$  ( $S_{f,t-1}^-$ ) is equal to the Negative inventory of the Top wafer  $f$  at the previous period ( $S_{f,t-2}^-$ ) plus the Negatives generated at period  $t-1$  ( $\sum_{\forall i} X_{i,f,t-1}^F$ ) minus the Negatives sent to be refreshed in all sites. Note that the Negatives of  $f$  taken in Constraint 8 are returned back refreshed in 7. However, because of the refresh line scraps, not all of the Negatives are transformed into Refresh Wafers. That is why the yield factor  $\alpha_{f,m}$  is used in Constraint 7.

Constraints 9 and 10 respectively restrict the production and refresh line capacities in each period  $t$ .

Constraints 11 through 14 respectively model the production setup cost, raw material (Base Wafers and Fresh Wafers) procurement cost and refresh setup cost. Binary and non-negativity sign restrictions are ensured using Constraints 15 through 22.

Note that, if wafer  $f'$  can be obtained from several wafers  $f$  ( $\sum_{\forall f'} b_{f',f} > 1$ ), the condition  $b_{f',f} = 1$  is added to avoid removing several times  $f$  to obtain Refresh Wafer  $f'$ . However if  $\sum_{\forall f'} b_{f',f} = 1$ , there exists only a  $f$  for  $f'$  and, therefore, only one unit of  $f'$  is removed by removing one unit of  $f$ . In this case, the refresh terms in 7, 8, 10 and the objective function could simply be written as  $R_{f,m,t}$  instead of  $R_{f',f',m,t}$ .

**5 NUMERICAL EXPERIMENTS**
**5.1 Data sets**

Based on industrial data, small, medium and large test instances are constructed to run experiments on the model and study its behavior. The parameters for generating data sets are listed in Table 1. In order to avoid infeasibility, coefficients called *capacity tightness factors* (CTF),  $q^p$  and  $q^r$  are used for adjusting production and refresh capacities respectively. Both of the capacity tightness factors are fixed to 1.0 (tight), 1.2 (normal), 1.6 (large), and 2.0 (very large). Initial inventories and in transit wafers are

set to zero. The refresh process can be performed in different sites. Therefore, refresh cost and refresh setup cost ( $cr_{f,m,t}$  and  $sr_{m,t}$ ) are defined based on the refresh site  $m$ . The refresh site can be internal, external, near or remote.

Parameter	Value
$I$	10, 20, 50, 100
$F$	6, 12, 18
$B$	4, 5, 6, 7
$T$	6, 12, 24, 48
$M$	2, 3, 4
$d_{i,t}$	Uniformly drawn from [1000, 3000]
$n_f^{max}$	5
$l$	1
$a_{i,b}$	$a_{i,b} \in [0, 1]   P(a_{i,b} = 1) = 0.90$
$a_{i,f}$	$a_{i,f} \in [0, 1]   P(a_{i,f} = 1) = 0.90$
$b_{f',f}$	$b_{f',f} \in [0, 1]   P(b_{f',f} = 1) = 0.70$
$\alpha_{f,m}$	0.98
$h_{b,t}$	1
$h_{f,t}^+$	2
$h_{f,t}^-$	2
$h_{i,t}$	4
$rc_{m,t}$	$q^r(\sum_{\forall i} d_{i,t} / (\alpha_{f,m} n_f^{max}))$
$\beta_{f,m}$	1
$pc_t$	$q^p(\sum_{\forall i} d_{i,t})$
$\eta_i$	1
$cg_{i,t}$	150
$sg_t$	150000
$cr_{f,m,t}$	20, 30, 40, 50
$sr_{m,t}$	40000, 80000, 120000, 160000
$cp_{b,t}$	50
$cp_{f,t}$	150
$sp_t^B$	30000
$sp_t^F$	30000

Table 1: Parameters for generating data sets

The instances are generated by fixing one of the parameters and by considering all the combinations of the other parameters. In total, 2,304 instances are generated. The reduced MIP of the smallest instance has 2383 constraints and 9822 variables from which 280 are binaries where the reduced MIP of the biggest instance has 16452 constraints and 149356 variables from which 526 are binaries.

## 5.2 Experimental Results

The test instances are solved using IBM ILOG CPLEX™ Optimization Studio 12.5.1. All computational experiments have been run on an AMD Phenom II X2 B57 3.20 GHz with 3.24GB of RAM. The relative MIP gap tolerance is set to 0.5%. A summary of the results can be found in Tables 2 and 3, where the average results of all instances are provided for a given value of each of the parameters  $CTF$ ,  $T$ ,  $I$ ,  $F$ ,  $B$  and  $M$ .

Table 2 shows the average solution gap and the average resolution time. The average solution gap is the percent-

age of  $(UB - LB)/LB$ . Large CPU Times are observed and rather independently of the variations of most parameters. A significant increase of the CPU time is observed when the length of the planning period increases. The average CPU Time is multiplied by more than 17 when the number of periods increases from 6 to 48. The CPU Time also significantly increases when the number of Top wafer references increases.

Parameters	Result (Avg)
<b>CTF</b>	<b>CPU Time (sec.)</b>
1	4402.12
1.2	4304.11
1.6	4417.98
2	4241.19
<b>T</b>	
6	<b>628.81</b>
12	<b>1593.11</b>
24	<b>4127.24</b>
48	<b>10822.78</b>
<b>I</b>	
10	3956.48
20	3265.89
50	3650.13
100	<b>7252.94</b>
<b>F</b>	
6	<b>1746.33</b>
12	<b>2819.48</b>
18	<b>5765.42</b>
<b>B</b>	
4	4317.19
5	4260.83
6	4338.26
7	4447.99
<b>M</b>	
2	<b>3594.66</b>
3	<b>4402.99</b>
4	<b>5011.78</b>

Table 2: Average resolution times

Table 3 shows the percentage of each cost component in the total optimal cost. The percentage of Base Wafer purchase and procurement cost is significantly larger than the percentage of the Fresh Wafer purchase and procurement cost. The reason is that less Fresh Wafer procurement is needed as Refresh Wafers are produced using the refresh process. However, since the refresh process is relatively cheaper than Fresh Wafer procurement, relatively small refresh process and setup costs are incurred. The largest production and setup costs are associated to the SOI fabrication. Inventory costs are negligible for each cost component even if inventory costs are slightly larger for SOI production.

Except for the length of the planning horizon ( $T$ ), the variation of all other parameters do not significantly alter the percentages of the cost components. By increasing the

Parameters	Results											
	Top (Fresh) Wafer Procurement			Base Wafer Procurement			SOI Production			Refresh Process		
CTF	Purchase	Procurement	Inv.	Purchase	Procurement	Inv.	Production	Setup	Inv.	Refresh	Setup	Neg. Inv.
1	9.36%	0.03%	0.01%	20.43%	0.20%	0.00%	61.30%	1.02%	0.34%	7.07%	0.23%	0.00%
1.2	9.33%	0.03%	0.01%	20.43%	0.21%	0.00%	61.29%	1.04%	0.36%	7.07%	0.23%	0.00%
1.6	9.06%	0.03%	0.01%	20.50%	0.20%	0.00%	61.49%	1.01%	0.34%	7.13%	0.23%	0.00%
2	9.17%	0.03%	0.01%	20.46%	0.21%	0.00%	61.39%	1.03%	0.36%	7.11%	0.24%	0.00%
<b>T</b>												
6	<b>18.63%</b>	0.06%	0.02%	<b>18.65%</b>	0.18%	0.00%	<b>55.95%</b>	0.92%	0.34%	<b>5.08%</b>	0.16%	0.00%
12	<b>10.17%</b>	0.03%	0.01%	<b>20.28%</b>	0.20%	0.00%	<b>60.83%</b>	1.01%	0.35%	<b>6.89%</b>	0.22%	0.00%
24	<b>5.32%</b>	0.02%	0.01%	<b>21.20%</b>	0.21%	0.00%	<b>63.60%</b>	1.07%	0.37%	<b>7.93%</b>	0.26%	0.00%
48	<b>2.72%</b>	0.01%	0.00%	<b>21.71%</b>	0.22%	0.00%	<b>65.14%</b>	1.08%	0.34%	<b>8.49%</b>	0.28%	0.00%
<b>I</b>												
10	9.05%	0.05%	0.02%	20.19%	0.31%	0.01%	60.57%	1.54%	0.91%	7.01%	0.35%	0.00%
20	9.33%	0.04%	0.01%	20.37%	0.26%	0.00%	61.11%	1.29%	0.26%	7.05%	0.29%	0.00%
50	9.23%	0.02%	0.00%	20.67%	0.12%	0.00%	62.01%	0.62%	0.00%	7.18%	0.14%	0.00%
100	9.38%	0.01%	0.00%	20.74%	0.06%	0.00%	62.23%	0.31%	0.00%	7.19%	0.07%	0.00%
<b>F</b>												
6	9.32%	0.04%	0.03%	20.22%	0.27%	0.00%	60.66%	1.35%	0.78%	6.98%	0.33%	0.01%
12	9.18%	0.03%	0.01%	20.45%	0.21%	0.00%	61.36%	1.03%	0.39%	7.10%	0.23%	0.00%
18	9.26%	0.03%	0.01%	20.48%	0.20%	0.00%	61.45%	0.98%	0.27%	7.10%	0.22%	0.00%
<b>B</b>												
4	9.20%	0.03%	0.01%	20.47%	0.20%	0.00%	61.40%	1.00%	0.35%	7.10%	0.23%	0.00%
5	9.14%	0.03%	0.01%	20.47%	0.21%	0.00%	61.42%	1.03%	0.34%	7.11%	0.23%	0.00%
6	9.30%	0.03%	0.01%	20.44%	0.21%	0.00%	61.32%	1.03%	0.36%	7.08%	0.23%	0.00%
7	9.27%	0.03%	0.01%	20.45%	0.21%	0.00%	61.34%	1.03%	0.34%	7.08%	0.23%	0.00%
<b>M</b>												
2	9.38%	0.03%	0.01%	20.42%	0.20%	0.00%	61.27%	1.02%	0.37%	7.06%	0.23%	0.00%
3	9.11%	0.03%	0.01%	20.48%	0.20%	0.00%	61.45%	1.02%	0.34%	7.12%	0.23%	0.00%
4	9.20%	0.03%	0.01%	20.46%	0.21%	0.00%	61.39%	1.02%	0.34%	7.10%	0.23%	0.00%

Table 3: Cost components for different data sets

planning horizon, the Fresh Wafer purchase (and also procurement and inventory) cost drastically reduces while the refresh process and setup costs increase. Note that the maximum refresh level  $n_f^{max}$  is 5. This means that a newly bought Fresh Wafer can only be refreshed five times. As the lead time is equal to one period (without any SOI and refresh capacity restrictions), it takes 7 periods to fully use a purchased Fresh Wafer. Therefore, as the planning horizon increases, the refresh process becomes more important. Therefore, when the planning horizon increases, the Fresh Wafer procurement cost decreases and the refresh process cost increases. However, as the Fresh Wafer purchase cost is relatively larger than the refresh process cost, the decrease of the Fresh Wafer procurement cost components is larger than the increase of the refresh process cost components. This illustrates the economical importance of the refresh process and that an efficient production planning contributes to a substantial cost decrease. Note that, as the percentage of the Fresh Wafer procurement cost decreases, the Base Wafer procurement and SOI production cost percentages increase in the total planning cost.

## 6 CONCLUSION AND PERSPECTIVES

In this paper, the supply chain of a SOI fabrication unit using the Smart-Cut™ Technology was modeled. Using this technology, one of the two purchased raw materials can be used several times after reprocessing. The reprocessing which is considered as a kind of “remanufacturing” can be done internally or externally. A mixed-integer linear programming (MILP) model is proposed to model

the supply chain. The model is validated and the optimal solution behavior is studied using generated data sets based on industrial data.

The refresh capacity is constrained by both the refresh line capacity and the SOI production line capacity. In fact, it is the SOI production which determines the rate of the generation of Negatives. Therefore, the refresh process throughput depends on the available refresh capacity and the SOI production (generation of Negatives) rate.

Due to the purchase and refresh cost structure, optimal solutions in our numerical experiments usually include purchase and refresh campaigns, which cause an irregular and fluctuating cost profile along the planning horizon. This may not be desirable both financially and for workforce management, for which a more stable cost expense over the whole planning horizon is preferable. A possible approach is to use a non-linear cost objective function in order to make the sum of all costs closer to their average. However, it makes the model non-linear and its resolution much more difficult.

An interesting challenge is to study the properties and characteristics of the proposed MILP model to develop a more effective resolution approach.

Some bulk (namely Fresh Wafer) suppliers also propose to perform the refresh process. Bulk suppliers, being competitors, do not allow their products to be handled by another supplier. Therefore, Fresh references which are bought from a supplier must be either refreshed internally or externally only if the same supplier proposes the refresh



process. This industrial constraint is also to be considered in future studies.

## ACKNOWLEDGMENTS

This study has been done within the framework of a joint collaboration between SOITEC (Bernin, France), and the Center of Microelectronics in Provence of the École des Mines de St-Étienne (Gardanne, France). The authors are grateful to the Supply Chain Department of SOITEC for their full support and would like to thank the ANRT (*Association Nationale de la Recherche et de la Technologie*) which has partially financed this study.

## References

- Agrali, S. (2011). A Dynamic Uncapacitated Lot-Sizing Problem with Co-Production, *Optimization Letters* **6**(6): 1051–1061.
- Billington, P. J., McClain, J. O. & Thomas, L. J. (1983). Mathematical Programming Approaches to Capacity-Constrained MRP Systems: Review, Formulation and Problem Reduction, *Management Science* **29**(10): 1126–1141.
- Brahimi, N., Dauzère-Pérès, S., Najid, N. M. & Nordli, A. (2006). Single Item Lot Sizing Problems, *European Journal of Operational Research* **168**(1): 1–16.
- Bruel, M. (1995). Silicon on Insulator Material Technology, *Electronics Letters* **31**(14): 1201–1202.
- Ferrer, G. & Whybark, D. C. (2001). Material Planning for a Remanufacturing Facility, *Production and Operations Management* **10**(2): 112–124.
- Helber, S. & Sahling, F. (2010). A Fix-and-Optimize Approach for the Multi-Level Capacitated Lot Sizing Problem, *International Journal of Production Economics* **123**(2): 247–256.
- Helmrich, M. J. R., Jans, R., van den Heuvel, W. & Wagelmans, A. P. (2014). Economic Lot-Sizing with Remanufacturing: Complexity and Efficient Formulations, *IIE Transactions* **46**(1): 67–86.
- Karimi, B., Fatemi Ghomi, S. & Wilson, J. (2003). The Capacitated Lot Sizing Problem: A Review of Models and Algorithms, *Omega* **31**(5): 365–378.
- Kenné, J.-P., Dejax, P. & Gharbi, A. (2012). Production Planning of a Hybrid Manufacturing-Remanufacturing System under Uncertainty within a Closed-Loop Supply Chain, *International Journal of Production Economics* **135**(1): 81–93.
- Soitec, W. (2014). Soitec Smart Cut Technology.
- Spengler, T., Puechert, H., Penkuhn, T. & Rentz, O. (1997). Environmental Integrated Production and Recycling Management, *European Journal of Operational Research* **97**(2): 308–326.
- Tomlin, B. & Wang, Y. (2008). Pricing and Operational Recourse in Coproduction Systems, *Management Science* **54**(3): 522–537.
- Trigeiro, W. W., Thomas, L. J. & McClain, J. O. (1989). Capacitated Lot Sizing with Setup Times, *Management Science* **35**(3): 353–366.
- Zhang, Z.-H., Jiang, H. & Pan, X. (2012). A Lagrangian Relaxation Based Approach for the Capacitated Lot Sizing Problem in Closed-Loop Supply Chain, *International Journal of Production Economics* **140**(1): 249–255.