



HAL
open science

STACKELBERG GAME BETWEEN A CUSTOMER AND COOPERATING SUPPLIERS: STABILITY AND EFFICIENCY

Ibtissem Ernez Gahbiche, Khaled Hadj Youssef, Abdelwaheb Dogui, Zied
Jemai

► **To cite this version:**

Ibtissem Ernez Gahbiche, Khaled Hadj Youssef, Abdelwaheb Dogui, Zied Jemai. STACKELBERG GAME BETWEEN A CUSTOMER AND COOPERATING SUPPLIERS: STABILITY AND EFFICIENCY . MOSIM 2014, 10ème Conférence Francophone de Modélisation, Optimisation et Simulation, Nov 2014, Nancy, France. hal-01166623

HAL Id: hal-01166623

<https://hal.science/hal-01166623>

Submitted on 23 Jun 2015

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

STACKELBERG GAME BETWEEN A CUSTOMER AND COOPERATING SUPPLIERS: STABILITY AND EFFICIENCY

Ibtissem ERNEZ GAHBICHE^(a), Khaled HADJ
YOUSSEF^(a), Abdelwaheb DOGUI^(a)

^(a) Mechanical Engineering Laboratory, National
Engineering School of Monastir, Av. Ibn Eljazar, 5019
Monastir, Tunisia
University of Monastir, Tunisia
ibtissem.gahbiche@gmail.com,
khaled.hadjyoussef@gmail.com,
abdel.dogui@enim.rnu.tn

Zied JEMAI^{(b),(c)}

^(b)Ecole Centrale Paris, Grande voies des vignes, 92290
Chatenay-Malabry, France

^(c) OASIS-ENIT , University of Tunis El Manar, B.P. 37
Le Belvédère, 1002 Tunis, Tunisia
zied.jemai@ecp.fr

ABSTRACT: *In this paper, we analyze a Stackelberg game between one customer and multiple suppliers faced with the proposition of a new product manufacturing. Suppliers apply base stock policy to replenish their inventories. The customer, being the leader, anticipates suppliers' optimal base stock levels and allocates demand volume to suppliers in a way to maximize his own profit. In addition, the customer as well as each of the suppliers has the option of accepting or rejecting this new product affair according to its profitability. We let suppliers cooperate by forming coalitions. In order to study coalition structures stability, we endow suppliers with foresight. We show that the grand coalition may be unstable particularly when system load is relatively low. We show that suppliers' profits are better when they cooperate if the system load is high. However, for weakly loaded systems, suppliers may find it better not to cooperate. By comparing to the centralized system performances, we show that decentralization of decisions may lead to the loss of the affair in quite a lot of situations even when suppliers cooperate, and that in the case of acceptance, the system inefficiency is considerable.*

KEYWORDS: *Stackelberg game, Coalition formation, Stochastic Modeling, Queuing Theory.*

1 INTRODUCTION

In the last two decades, the collaboration within a supply chain has been strongly recognized by industrials. Particularly, coalition formation was widely used in several supply chain settings. It enables cooperating players to coordinate their activities and to centralize their decisions which often results in significant additional payoffs.

In this paper, we deal with coalition formation among suppliers that are facing the proposition of a new product manufacturing from a customer. We model a Stackelberg game where the customer is the leader. The customer anticipates suppliers' optimal base stock levels decisions and allocates demand volume between suppliers in a manner to maximize his own profit. In addition, the customer, as well as each supplier has the option of accepting or refusing the new product proposal according to its profitability.

Suppliers decide to cooperate by forming coalitions. We analyze equilibriums resulting from the Stackelberg game between the customer and suppliers' coalitions in all possible coalition partitions settings. The main ques-

tion is then, which coalition structures are stable, that is, which coalition structures are such that, once one among them is formed there is no interest for any set of players to deviate from. We endow suppliers with foresight, in the sense that each player deciding to defect from a coalition to join another, is able to predict the other players possible defections that may occur. We analyze the inefficiency due to decentralization of decisions by studying the system loss in comparison with the centralized version of our model.

The remaining of this paper is organized as follows: section 2 exposes a brief literature review on cooperation in supply chain research works. In section 3 we detail our decentralized model. The centralized system is described in section 4. A numerical example is presented in section 5. We conclude and present some perspectives in section 6.

2 LITERATURE REVIEW

Stackelberg game theory is detailed by (Webb, 2000). Its application in supply chain management was reviewed by (Cachon & Zipkin, 1999), (Jemai, 2003) and (Caldentey & Wein, 2003) who analyzed a two stage system formed by one supplier and one retailer.

Later, (Arda, 2008) studied a two stage model where the downstream manufacturer pursues fabrication of semi-finished products provided by the supplier. In these papers, the downstream agent decides on either the inventory positioning, or on the sale price and backorder penalty at the upstream stage.

Competition between suppliers was investigated in several papers. Most of them suppose that demand is divided between suppliers according to an allocation policy framework that encourages each supplier to offer the best feasible service level. The most useful allocation rule is the proportional allocation policy, see (Benjaafar, Elahi, & Donohue, 2007), (Ching, Choisy, & Huang, 2010), (Elahi, Benjaafar, & Donohue, 2012), (Elahi, 2013), (Elahi & Blake, 2014).

The benefits of cooperation in supply chains have been investigated in several papers in the recent years.

Some research works study cooperation between retailers selling products with a certain degree of substitutability. Cooperating retailers can interchange products: If a retailer is short of supply, its excess demand is satisfied partly or entirely by retailers within the same coalition which have excess supply, see (Granot & Sošić, 2005), (Sošić, 2006), (Nagarajan & Sošić, 2007) and the references therein.

A second stream of papers considers the joint replenishment models. (Tijs, Meca, & Lopez, 2005) considered a model of retailers who share a storage facility for holding their goods. (Anily & Haviv, 2007) and (Elomri, Ghaffari, Jemai, & Dallery, 2012) consider a system where multiple retailers make orders jointly, so that they reduce the total ordering costs. They analyze different possible allocations of savings and discuss about their stability.

Literature on cooperation among suppliers, however, is still sparse. (Yimin, Benjaafar, & Gerchak, 2009) analyze a resource pooling model where multiple make to order suppliers cooperate by sharing the same facility instead of producing each in his own one. The objective is to reduce the capacity and delay costs of the whole system. (Anily & Haviv, 2010) dealt with a model where a number of servers pool their service capacities to serve the union of customers' orders. Servers look for improving efficiency of the system by reducing the effect of congestion as demand arrival and service time are stochastic processes.

Our paper is in the field of the papers presented above. We study a Stackelberg game between multiple competing suppliers and we include explicitly the customer as a Stackelberg leader in the game. He decides then on demand allocation scheme between suppliers, with foreseeing later suppliers decisions. In addition, we allow each player to accept or to refuse the new product proposal according to its profitability.

Furthermore, we propose that suppliers cooperate by forming coalitions at the aim of getting better profit and we keep competition between coalitions.

3 THE DECENTRALIZED MODEL

We consider a customer who faces the proposition of a new product procurement. He looks for allocating the new product demand amount to n different suppliers in a manner to maximize his own profit. Suppliers use base stock policy to replenish their inventories. Each supplier decides on his base stock level. Furthermore, the customer and suppliers have the option of accepting or refusing the new product proposal. We define a Stackelberg game where the customer is the leader. He foresees suppliers best responses as for the acceptance decisions and base stock levels. Then he determines the demand allocation policy which corresponds to his maximal profit.

Demand occurs according to a Poisson process with rate λ at the customer who doesn't carry inventory. All orders that cannot be satisfied from suppliers' inventories are backordered. The customer incurs a backorder penalty cost B per unit backordered and earns P for each product unit sold. Available unit processing times at each supplier i facility, $i \in \{1, \dots, n\}$, are stochastic, independent and exponentially distributed with mean $1/\mu_i$. Suppliers earn p for each product unit sold to the customer. They are charged a production cost c , a holding cost h per unit of inventory per unit time and a backorder penalty b per unit backordered per unit time. We supposed equal suppliers cost parameters, but our analyses are not limited to this assumption. Suppliers adopt FIFO as service discipline.

Suppliers cooperate by forming coalitions. We denote by $N = \{1, \dots, n\}$ the finite set of suppliers. A coalition S is a subset of N . A coalition structure $Z = \{S_1, \dots, S_m\}$ with $m \leq n$, is a partition of the players' set N , such that

$$S_i \cap S_j = \emptyset \text{ when } i \neq j \text{ and } \bigcup_{i=1}^m S_i = N.$$

We denote by \mathcal{P} the finite set of all possible partitions of N . Each coalition S offers an exponentially distributed service by a combined server whose service rate is $\sum_{i \in S} \mu_i$.

After coalitions are formed, the customer decides how to allocate demand volume between them, foreseeing each coalition optimal base stock decision. Besides, the customer, as well as each coalition has the option to accept or reject the affair according to its profitability.

We suppose that a coalition S earns the same sale price p and is charged the same cost parameters as standalone suppliers: a production cost c , a holding cost h per unit of inventory per unit time and a backorder penalty b per unit backordered per unit time, this penalty is paid to the customer. Our analyses are not limited to similar coalitions' sale prices and cost parameters. We consider that each coalition S adopts FIFO as service discipline.

A coalition S_j , $j \in \{1, \dots, m\}$, $m \leq n$, decides on a pair of strategies (A_j, s_j) where A_j is a binary variable expressing coalition S_j acceptance ($A_j = 1$) or refusal

($A_j = 0$) of the new product proposal, and s_j is the base stock level offered by coalition S_j . The customer has to decide on a vector of strategies $(A_0, \alpha_1, \dots, \alpha_m)$

where A_0 is a binary variable revealing customer's acceptance or refusal of the affair and $\alpha_j \in [0, 1]$, $j \in \{1, \dots, m\}$, is the demand amount allocated to coalition S_j such that $\sum_{j=1}^m \alpha_j = 1$. We denote by α the demand allocation vector $\alpha = (\alpha_1, \dots, \alpha_m)$. Let $\xi_0 = (A_0, \alpha)$ and $\xi_j = (A_j, s_j)$ be respectively the customer and coalition S_j strategies.

We define π_j as the coalition S_j profit function, $j \in \{1, \dots, m\}$, and π_0 the customer's profit function.

$$\pi_0(\xi_0, \xi_1, \dots, \xi_m) = A_0 \sum_{j=1}^m A_j ((P - p)\alpha_j \lambda - (B - b)\bar{Y}_j)$$

$$\pi_j(\xi_0, \xi_j) = A_0 A_j ((p - c)\alpha_j \lambda - b\bar{Y}_j) - h\bar{X}_j$$

Where \bar{X}_j and \bar{Y}_j denote inventory and backorder averages of coalition S_j . As demand arrival is a Poisson Process and service time is an exponential distribution, the production system at each coalition S_j is modeled as an M/M/1 queue. \bar{X}_j and \bar{Y}_j were determined, in this case by (Buzacott & Shanthikumar, 1993) who showed that the two corresponding expressions are of the form:

$$\bar{X}_j = s_j - \frac{\rho_j}{1 - \rho_j} (1 - \rho_j^{s_j}),$$

$$\text{And } \bar{Y}_j = \frac{\rho_j^{s_j+1}}{1 - \rho_j}$$

where ρ_j is such that: $\rho_j = \frac{\alpha_j \lambda}{\mu_j}$ and $\mu_j = \sum_{i \in S_j} \mu_i$.

Each coalition production system stability requires that $\rho_j < 1$. On the other hand, whole system stability re-

quires that system load $\rho = \frac{\lambda}{\sum_{i=1}^n \mu_i}$ verifies $\rho < 1$.

(Shenoy, 1979) highlighted the strong dependency between alliance formation and saving (or payoff) allocation scheme. Obviously, the coalition that a player decides to join depends on the profit that the player would earn. Several research works discussed the influence of the allocation policy on the stability of the coalitions in the context of cost or profit allocation. (Elomri, Ghaffari, Jemai, & Dallery, 2012) used the cost-based

proportional rule which consists of allocating the savings in proportion to the initial cooperating players costs.

Proportional allocation schemes were used in several research works thanks to their attractive properties, such as symmetry, efficiency and additivity. (Nagarajan, Sošić, & Zhang, 2010) used the proportional profit allocation in the context of group purchasing organizations.

In our model, we focus on capacity based allocation rule. According to this rule, each supplier is rewarded proportionally to his capacity.

$$\pi_i = \pi_j \frac{\mu_i}{\mu_j}, \forall i \in S_j, S_j \in \mathcal{Z}.$$

In this case, a supplier who offers the highest capacity earns the best profit.

3.1 Stackelberg Equilibrium

We are in presence of $m+1$ players: the customer and m coalitions, each chooses its strategy with correspondence to its maximal profit value and each player profit depends on the other players strategies.

A coalition decides on accepting or rejecting the new product proposal, and on the base stock to install.

In the next proposition, we determine each coalition optimal decision denoted $\xi_j^* = (A_j^*, s_j^*)$.

Proposition 1

Let $\mathcal{Z} = \{S_1, \dots, S_m\}$ be a partition and \hat{s}_j such that:

$$\bullet \text{ If } \alpha_j = 0, \text{ then } \hat{s}_j = 0 \quad (1)$$

$$\bullet \text{ If } \alpha_j \neq 0 \text{ then } \hat{s}_j = \left\lfloor \text{Log} \left(\frac{h}{h + b} \right) / \text{Log} \rho_j \right\rfloor \quad (2)$$

$j \in \{1, \dots, m\}$.

- If $\pi_j(\xi_0, (1, \hat{s}_j)) > 0$ then $A_j^* = 1, s_j^* = \hat{s}_j, j \in \{1, \dots, m\}$.
- If $\pi_j(\xi_0, (1, \hat{s}_j)) \leq 0$ then $A_j^* = 0$ and $s_j^* = 0, j \in \{1, \dots, m\}$.

$$(3)$$

Proof:

\hat{s}_j is the base stock level that maximizes function π_j when $A_0 = A_j = 1$, in fact:

• If $\alpha_j = 0$ then $\pi_j = -h s_j$. Thus, $\forall s_j > 0, \pi_j < 0$, it implies that the optimal base stock level is $s_j = 0$.

• Given that π_j is concave with respect to s_j and that s_j is an integer value, then the classical method to determine the optimal base stock level is solving the equation $\pi_j(\tilde{s}_j) - \pi_j(\tilde{s}_j - 1) = 0$, according to (Buzacott

& Shanthikumar, 1993). Hence, the optimal base stock level is $\lfloor \tilde{s}_j \rfloor$ where $\lfloor x \rfloor$ is the largest integer that is less than or equal to x .

It turns out that \hat{s}_j is the base stock level that maximizes function π_j when $A_0=A_j=1$.

- If $\pi_j(\xi_0, (1, \hat{s}_j)) > 0$, then coalition S_j accepts the affair as it is profitable. Consequently, $A_j^*=1$ and $s_j^* = \hat{s}_j, j \in \{1, \dots, m\}$.

- If $\pi_j(\xi_0, (1, \hat{s}_j)) \leq 0$, then coalition S_j refuses the affair. Consequently, $A_j^*=0$. $\pi_j(\xi_0, (0, s_j)) = -h\bar{X}_j$. So $\forall s_j > 0, \pi_j(\xi_0, (0, s_j)) < 0$. It turns out that $s_j^* = 0$. \square

In the Stackelberg game, we consider that the customer is the leader. We study Stackelberg equilibrium in each coalition structure setting. The equilibrium solution allows the determination of each coalition profit. Thereby, a supplier knows the profit he would earn in each partition setting.

Let $Z = \{S_1, \dots, S_m\}$ be a coalition structure. The customer moves first by deciding on his pair of strategies $\xi_0 = (A_0, \alpha)$ based on his planning about each coalition later pair of strategies $\xi_i = (A_j, s_j), 1 \leq j \leq m$. He selects ξ_0 that maximizes his profit π_0 with the knowledge that each coalition S_j will choose ξ_i that maximizes his profit.

Let $\xi_Z^* = (\xi_0^*, \xi_1^*, \dots, \xi_m^*)$ be a Stackelberg equilibrium and let Π the following maximization problem:

$$\Pi: \max_{\alpha_1, \dots, \alpha_m, s_1, \dots, s_m, A_1, \dots, A_m} \pi_0((1, \alpha_1, \dots, \alpha_m), \xi_1, \dots, \xi_m)$$

such that

- $0 \leq \alpha_j < \frac{\mu_j}{\lambda}, \forall j \in \{1, \dots, m\}$ (4)

- $\sum_{j=1}^m \alpha_j = 1$ (5)

- $\sum_{j=1}^m A_j \geq 1$ (6)

- ξ_1, \dots, ξ_m are determined according to proposition 1. (7)

Constraint (4) reveals production system stability of coalition S_j ($\rho_j < 1$). Constraint (5) implies that the totality of demand volume must be allocated to coalitions. Con-

straint (6) reveals that at least one coalition will be allocated demand volume, on the other hand, coalitions that are allocated a demand amount must accept the new product affair.

Constraint (7) means that the customer foresees coalitions strategies and use them in his optimization problem.

Problem Π is resolved numerically. After replacing s_j with its expression given in (1) and (2), and replacing

A_j with 1 $\forall j \in \{1, \dots, m\}$, we determine $\alpha = (\alpha_1, \dots, \alpha_m)$ via an iterative algorithm. This algorithm verifies conditions (4) and (5) and saves α which corresponds to the optimal customer profit and which results in a profit to coalitions that are allocated a non null demand amount.

Let $\tilde{\xi}_Z = (\tilde{\xi}_0, \tilde{\xi}_1, \dots, \tilde{\xi}_m)$ be a solution of problem Π .

If the resulting customer profit $\pi_0(\tilde{\xi}_Z) > 0$, then $A_0^*=1$,

$\xi_0^* = \tilde{\xi}_0$ and $\xi_j^* = \tilde{\xi}_j \forall j \in \{1, \dots, m\}$. If however, $\pi_0(\tilde{\xi}_Z) \leq 0$, then $A_0^*=0, \xi_0^* = (0,0)$ and $\xi_j^* = (0,0) \forall j \in \{1, \dots, m\}$.

Analogously, we define for each coalition structure a Stackelberg equilibrium. Consequently, a supplier knows the profit he would earn in all the coalition structures settings.

As each supplier looks for the maximum profit, there will be successive migrations among suppliers before converging to a final coalition structure. This partition is referred to as the stable coalition structure where no player has interest to deviate from. We endow suppliers with foresight i.e they consider the possibility that once they act, another coalition may react, a third coalition might in turn react, and so on, deterring then the first move. We use the Largest Consistent Set to evaluate the stability of coalitions, see (Chwe, 1994), (Diamantoudi & Xue, 2003) for a detailed description of the related theory. According to (Chwe, 1994), as the set \mathcal{P} of all possible partitions of N is finite, and players preferences are irreflexive (a coalition structure Z is not strictly better in terms of profits than itself), then the LCS is non-empty.

4 CENTRALIZED MODEL

In the centralized model, we optimize the entire system formed by the suppliers and the customer. We consider that suppliers cooperate by forming the grand coalition. In fact, thanks to the pooling effect, the grand coalition leads to the best total suppliers' profit.

The whole system receives the entire demand according to a Poisson Process with rate λ , earns P per unit product sold and pays a backorder penalty B per unit backordered per unit time. The grand coalition incurs manufacturing and holding costs c and h . Let G_c refers to the grand coalition, A_c a binary variable which expresses centralized system acceptance ($A_c = 1$) or refusal ($A_c = 0$) of the

new product proposal, and s_{Gc} the base stock level. We denote by π_c the centralized system profit:

$$\pi_c(A_c, s_{Gc}) = A_c \left((P - c)\lambda - B\bar{Y}_{Gc} \right) - h\bar{X}_{Gc}$$

where \bar{X}_{Gc} and \bar{Y}_{Gc} are respectively the inventory and the backorder averages. The system is modeled as an M/M/1 queue as demand inter-arrival and service times are exponential distributions. (Buzacott & Shanthikumar, 1993) showed that \bar{X}_{Gc} and \bar{Y}_{Gc} expressions are as follows:

$$\bar{X}_{Gc} = s_{Gc} - \frac{\rho}{1 - \rho} (1 - \rho^{s_{Gc}})$$

$$\bar{Y}_{Gc} = \frac{\rho^{s_{Gc}+1}}{1 - \rho}$$

and $\rho = \frac{\lambda}{\sum_{i=1}^n \mu_i}$.

We denote by (A_c^*, s_{Gc}^*) the optimal centralized system strategies.

Proposition 2

Let $\tilde{s}_{Gc} = \left\lfloor \text{Log} \left(\frac{h}{B+h} \right) / \text{Log}(\rho) \right\rfloor$.

Then:

- If $\pi_c^*(1, \tilde{s}_{Gc}) > 0$, then $A_c^* = 1$ and $s_{Gc}^* = \tilde{s}_{Gc}$.
- If $\pi_c^*(1, \tilde{s}_{Gc}) \leq 0$, then $A_c^* = 0$ and $s_{Gc}^* = 0$.

Proof

Let $A_c = 1$. By noting that π_c is concave with respect to s_{Gc} , then s_{Gc}^* satisfies: $s_{Gc}^* = \lfloor \tilde{s}_{Gc} \rfloor$ where \tilde{s}_{Gc} is such that $\pi_c(\tilde{s}_{Gc}) - \pi_c(\tilde{s}_{Gc} - 1) = 0$, with reference to (Buzacott & Shanthikumar, 1993).

If $\pi_c^* \leq 0$ then the affair is unprofitable $A_c^* = 0$ and $s_{Gc}^* = 0$. □

5 NUMERICAL EXAMPLE

We consider the example of a customer which proposes the production of a new item to three different suppliers. Demand occurs in units of pallets where a pallet contains 1000pcs.

We consider suppliers' production cost: $c=8M\text{€}/\text{pallet}$. A profit margin of 25% leads to suppliers sale price $p=10M\text{€}/\text{Pallet}$. We suppose a same profit margin to the customer which leads to: $P=12.5M\text{€}/\text{Pallet}$. We consider $h=38.4\text{€}/\text{Pallet}/\text{week}$. The backorder penalty: $B= 480\text{€}$

$/\text{Pallet}/\text{week}$. Service rates at suppliers are $\mu_1=4 \text{ pallet}/\text{year}$, $\mu_2=16 \text{ pallet}/\text{year}$ and $\mu_3= 32 \text{ pallet}/\text{year}$.

We propose that suppliers act jointly by forming coalitions and we study Stackelberg equilibrium resulting from each coalition structure. We apply the capacity based allocation policy to determine each supplier profit. Stable coalition structures are those who belong to the largest consistent set, if the LCS contains several coalition structures, we report the one that maximizes the total supply chain profit.

The set of possible partitions is:

$$\mathcal{P} = \{ \{1,2,3\}, \{(1,2),3\}, \{(1,3),2\}, \{(2,3),1\}, \{(1,2,3)\} \}.$$

Figure 1 investigates stable partitions with respect to the whole system load ρ . We varied the arrival rate λ from 13 to 50 which means that ρ varies from 25% to 96%. We notice from the figure that the grand coalition $\{(1,2,3)\}$ is not necessarily stable, particularly, for low system load values. When ρ exceeds nearly 80%, the grand coalition is the only stable coalition structure.

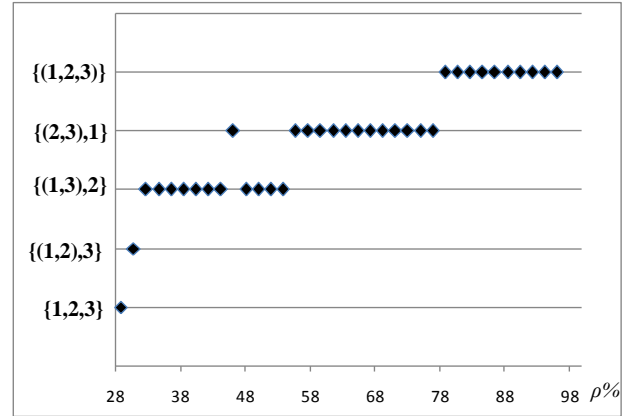


Figure 1: Stable coalition structures with respect to ρ

When system load is under some threshold, some suppliers are better off by deviating from the grand coalition and coupling or choosing not to cooperate. We notice that standalone coalition structure: $\{1,2,3\}$ is stable for low system load.

Table 1 presents suppliers profits in all of the coalition structures settings when $\lambda = 40 \text{ Pallets}$, $\rho = 77\%$.

	α (in the order)	π_1	π_2	π_3	$\sum_{i \in N} \pi_i$
{(1,2,3)}	1	5.24	20.96	41.93	68.13
{(2,3),1}	(0.97,0.03)	0.07	20.99	41.98	63.04
{(1,3),2}	(0.72,0.28)	4.84	13.74	38.78	57.36
{(1,2),3}	(0.34,0.66)	3.83	15.32	36.65	55.8
{1,2,3}	(0.03,0.31, 0.66)	0.07	12.65	36.65	49.37

Table 1: Suppliers profits ($M\text{€}/\text{Pallet}$) in each coalition structure ($\lambda = 40 \text{ Pallet}$, $b=145\text{€}/\text{week}/\text{Pallet}$)

We notice from the table that suppliers' profits are significantly improved when cooperating with comparison to the standalone situation.

In addition, although the grand coalition results in the better suppliers total profit, it is not stable. In fact, suppliers 2 and 3 would migrate from the grand coalition as they earn better profit when they act together.

Let $Z = \{S_1, \dots, S_m\} \in \mathcal{P}$ and $\pi_d^*(Z)$ be the decentralized system profit at Stackelberg equilibrium. Then,

$$\pi_d^*(Z) = \pi_0^* + \sum_{j=1}^m \pi_j^*$$

where π_0^* and π_j^* are respectively the customer and coalition S_j profits at equilibrium, $j \in \{1, \dots, m\}$.

To investigate the performances of the decentralized system with respect to the corresponding centralized one, let us define the decentralization penalty $DP(Z)$ as follows:

$$DP(Z) = \frac{\pi_c^* - \pi_d^*(Z)}{\pi_c^*}$$

where π_c^* is the optimal centralized system profit.

Figure 2 plots decentralization penalty with respect to b/B for three systems differing in utilization level ρ .

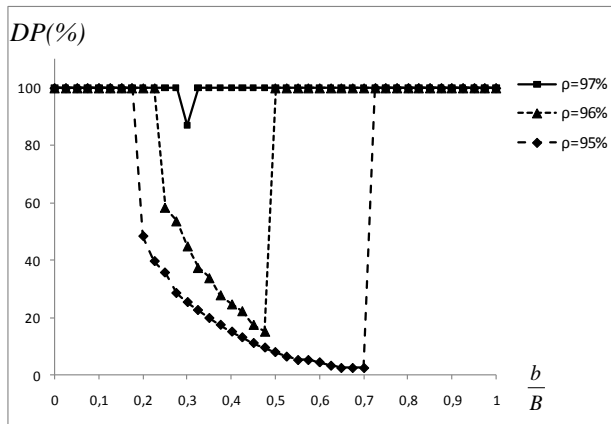


Figure 2: Decentralization penalty with respect to $\frac{b}{B}$ for different values of ρ

As shown in the figure, $DP = 100\%$ in quite a lot of problem settings, which reveals that the new affair is refused by players, while accepted when the supply chain is managed in a centralized manner. Decentralization of decisions leads to the loss of the affair in several common problem settings. We notice that DP achieves 100% when b/B is under some threshold. In fact, the customer refuses the affair as he incurs high backorder penalty share ($B-b$). Analogously, when b/B exceeds a second limit, DP is again 100%. In this range, suppliers refuse the new product proposal as the backorder penalty leads to unprofitability of the new product affair.

Thereby, although suppliers cooperate, the new product is still lost if backorder penalty share is not suitable.

Further, we notice that acceptance range is larger if the whole system is less loaded. In fact, logistic costs decrease which leads to more profits to the players and better acceptance chances of the affair.

Figure 3 displays DP with respect to relative suppliers' profit margin $\frac{P-c}{P-c}$ for three systems differing in utilization rate ρ .

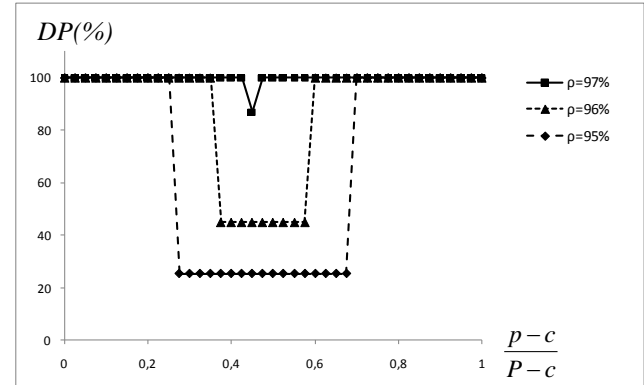


Figure 3: Decentralization penalty percentage with respect to $\frac{p-c}{P-c}$ ($b=145\text{€}/\text{Pallet}/\text{week}$)

When $\frac{p-c}{P-c}$ is under some threshold, $DP=100\%$. The new product is unprofitable to suppliers and then refused.

On the other hand, if $\frac{p-c}{P-c}$ exceeds a second threshold, the affair is again lost as it is no longer profitable to the customer. Analogously, acceptance range is larger when system load is lower. Logistic costs reduction leads to better new product acceptance chances.

Figures 2 and 3 show that the affair may be lost even if suppliers cooperate by forming coalitions and are farsighted.

Even if the new product proposal is accepted, we note that decentralization penalty is important. According to figures 2 and 3, it may exceed 50%.

6 CONCLUDING REMARKS AND PERSPECTIVES

In this paper, we investigated coalition formation among suppliers in a model consisting of n suppliers and a customer facing the proposition of a new product. We characterized Stackelberg equilibrium where the customer is leader. We study coalition structures stability by means of the Largest Consistent Set which allows suppliers to be farsighted. We show that the grand coalition is not stable in some system settings, particularly if system load is lower than some threshold. On the other hand,

we analyze the penalty due to decentralization of decisions and show that the new product affair may be lost for some suppliers' sale prices and backorder penalty share schemes.

It would be interesting to study coalition formation by including side payment between coalitions. On the other hand, coalitions can design coordination arrangement in order to diminish system inefficiency.

The model we presented could be used in the case where players decide on their strategies simultaneously by means of static game theory.

REFERENCES

- Anily, S., & Haviv, M. (2010). Cooperation in service systems. *Operations research* Vol 58 N°3 , 660-673.
- Anily, S., & Haviv, M. (2007). The Cost Allocation Problem for the First Order Interaction Joint Replenishment Model . *Operations Research*, Vol.55, No.2 , 292-302.
- Arda, Y., & Hennet, J.-C. (2008). Inventory control in a decentralized two stage make to stock queueing system. *Int.J. Systems Science* , 741-750.
- Benjaafar, S., Elahi, E., & Donohue, K. L. (2007). Outsourcing via service competition. *Management Science* vol 53 N°2 , 241-259.
- Buzacott, J., & Shanthikumar, J. (1993). *Stochastic models of manufacturing systems*. New Jersey: Prentice Hall.
- Cachon, G. P., & Zipkin, P. H. (1999). Competitive and cooperative inventory policies in a two stage supply chain. *Management Science* 45 , 936-953.
- Caldentey, R., & Wein, L. (2003). Analysis of a decentralized production - inventory system. *Manufacturing & Service Operation Management* vol 5 , 1-17.
- Ching, W.-K., Choiy, S.-M., & Huang, X. (2010). Inducing high service capacities in outsourcing via penalty and competition. *International Journal of Production* , 5169-5182.
- Chwe, M. S.-Y. (1994). Farsighted Coalitional Stability. *Journal Of Economic Theory*,63 , 299-325.
- Diamantoudi, E., & Xue, L. (2003). Farsighted stability in hedonic games. *Social Choice and Welfare* , 39-61.
- Elahi, E. (2013). Outsourcing through competition: What is the best competition parameter? *Int. J. Production Economics* , 370-382.
- Elahi, E., & Blake, R. (2014). An Experimental Investigation of Outsourcing. *Selected works University of Massachusetts Boston* .
- Elahi, E., Benjaafar, S., & Donohue, K. (2012). Optimal Service-Based Competition with Heterogeneous. *Selected works University of Massachusetts Boston* .
- Elomri, A., Ghaffari, A., Jemai, Z., & Dallery, Y. (2012). Coalition Formation and Cost Allocation for Joint Replenishment Systems. *Production and Operations Management Society* , 1015-1027.
- Granot, D., & Sošić, G. (2005). Formation of Alliances in Internet-Based Supply Exchanges. *Management Science*, Vol. 51, No. 1 , 92-105.
- Jemai, Z. (2003). Modèles stochastiques pour l'aide au pilotage des chaînes logistiques: l'impact de la décentralisation. Paris: Ecole Centrale Paris.
- Nagarajan, M., & Sošić, G. (2007). Stable Farsighted Coalitions in Competitive Markets. *Management Science*, Vol. 53, No. 1 , 29-45.
- Nagarajan, M., Sošić, G., & Zhang, H. (2010). Stable Group Purchasing Organizations.
- Shenoy, P. (1979). On coalition formation: a game-theoretical approach. *International Journal of Game Theory*, vol 8, issue 3 , 133-164.
- Sošić, G. (2006). Transshipment of Inventories among Retailers: Myopic vs. Farsighted Stability. *Management Science* Vol. 52, No. 10 , 1493-1508.
- Tijs, S., Meca, A., & Lopez, M. A. (2005). Benefit sharing in holding situations. *European Journal of Operational Research* 162 , 251-269.
- Webb, J. N. (2000). *Game Theory Decision Interaction and Evolution*. UK: Springer Undergraduate Mathematics Series.
- Yimin, Y., Benjaafar, S., & Yigal, G. (2009). Capacity Sharing and Cost Allocation among Independent Firms in the Presence of congestion.