Identification of Robot Dynamic Parameters Using Jacobi Differentiator

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Qi GUO[1], Maxime GAUTIER[2], Da-Yan LIU[3] and Wilfrid PERRUQUETTI[4]

Abstract— **This paper investigates the behavior of central Jacobi differentiator in robot identification applications. Jacobi differentiator is a Jacobi orthogonal based algebraic differentiator. It is applied to compute acceleration from noisy position measurements. Moreover, its frequency domain property is analyzed via a finite impulse response (FIR) filter point of view, indicating clearly the differentiators performance. In the end, a two revolute joints planar robot identification application is presented and comparisons between the Jacobi differentiator and the Euler differentiation combined with Butterworth filter are drawn.**

I. INTRODUCTION

The topic on identification of robot dynamic parameters has been widely studied in the past decades, but there still exist several open questions. One of them is numerical differentiation, which concerns with estimating the derivatives of an unknown signal using its noisy measurement. This is an ill-posed problem in the sense that a small error in the measurement can produce a large error in the estimated derivatives, specially in the case of high order derivatives. Therefore, various numerical methods have been developed to obtain stable algorithms which are robust against corrupting noises. They mainly fall into the following categories:

- finite difference methods [1],
- Savitzky Golay methods [2],
- wavelet differentiation methods [3],
- Fourier transform methods [4],
- mollification methods [5],
- Tikhonov regularization methods [6],
- algebraic methods [7], [8]
- differentiator observer [9], [10]

The recent algebraic differentiators are rooted in a recent algebraic parametric method introduced by Fliess and Sira-Ramírez [8], [11]. These algebraic differentiators are divided into two classes: model-based differentiators and model-free differentiators. The formers were obtained by applying the algebraic method to a differential equation which defines a class of linear systems [12], [13], [14]. Hence, they are mainly used for linear systems. However, the model-free differentiators can be used for nonlinear systems. The first model-free differentiator was introduced in [15] by applying the algebraic method to the truncated Taylor series expansion of the signal to be differentiated. Then, two model-free differentiators were proposed in [16], where the causal Jacobi differentiator is presented. Moreover, it was shown that the causal Jacobi differentiator can also be obtained by taking the truncated Jacobi orthogonal series expansion of the signal to be differentiated. In [17], central Jacobi differentiator was proposed, which is devoted to off-line applications.

The algebraic differentiators have the following advantages. Firstly, they are given by exact integral formula. Thus, estimations at different instants can be obtained using a sliding integration window of finite length. Secondly, the integral formula can be considered as low-pass filters, which show robust properties with respect to corrupting noises [18]. The Jacobi differentiators contain some design parameters. Some error analysis has been done to study the influence of the design parameters [19], [20], [17], [21], where the study was based on some proposed error bounds. In this paper, the influence of the design parameters will be studied in a FIR filter point of view. For this purpose, their frequency domain properties are studied.

In order to identify robot dynamic parameters, a huge variety of methods have been proposed mainly using least-square techniques. The most widely applied approach is based on the robot explicit dynamic model, requiring joint acceleration data which are usually estimated from noisy measurement [22]. The other approaches are based on the robot energy model [23] or the robot power model [24], which require only joint velocity data, but instead they need a derivation operation on an implicit part of velocity. Besides, some authors utilize a parallel scheme to identify robot dynamic parameters by minimizing the output error from a closed loop simulation [25]. In this paper, the robot explicit dynamic model is considered under the condition that velocity and acceleration are well estimated using position data.

The paper is organized as follows: Section II introduces the robot dynamic model and discusses the identification process. Section III presents the central Jacobi differentiator and analyzes its frequency domain properties. Section IV obtains the identification results on a 2 joints planar direct drive prototype robot, respectively by means of the central Jacobi differentiator and the central Euler differentiation combined with the Butterworth filter. Finally, conclusions are given in Section V.

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II. IDENTIFICATION

A. Explicit Dynamic Model

The explicit dynamic model of a rigid robot composed of n moving links calculates the motor torque vector $\mathbf{\Gamma_m}$ as a function of the state variables and their derivatives. It can be deduced from the following Lagrangian formulation:

$$\mathbf{\Gamma_m} = \frac{\mathrm{d}}{\mathrm{dt}}(\frac{\partial \mathrm{L}}{\partial \dot{\mathbf{q}}}) - \frac{\partial \mathrm{L}}{\partial \mathbf{q}} + \mathbf{\Gamma_f}, \tag{1}$$

where $\mathbf{q}$, $\dot{\mathbf{q}}$ are the $n \times 1$ vectors of generalized joint position and velocity, L is the Lagrangian of the system defined as the difference between the kinetic energy $\mathrm{E}(\mathbf{q}, \dot{\mathbf{q}})$ and the potential energy $\mathrm{U}(\mathbf{q})$, with $\mathrm{E} = \frac{1}{2}\dot{\mathbf{q}}^{\mathbf{T}}\mathbf{M}(\mathbf{q})\dot{\mathbf{q}}$, where $\mathbf{M}(\mathbf{q})$ is the $(n \times n)$ robot inertia matrix. Γ_f is the friction torque which is usually modelled at non zero velocity as: $\Gamma_{\mathrm{fj}} = \mathrm{F_{sj}}\mathrm{sign}(\dot{\mathrm{q}}_\mathrm{j}) + \mathrm{F_{vj}}\dot{\mathrm{q}}_\mathrm{j} + \Gamma_{\mathrm{offj}}$, where $\dot{\mathrm{q}}_\mathrm{j}$ is the velocity of joint j, $\mathrm{sign}(\mathrm{x})$ denotes the sign function. $\mathrm{F_{vj}}$, $\mathrm{F_{sj}}$ are the viscous and Coulomb friction coefficients of joint j, Γ_{offj} is an offset parameter which is the dis-symmetry of the Coulomb friction with respect to the sign of the velocity and is due to the current amplifier offset which supplies the motor [26] . Notice that the Coulomb friction contains the non-linear term $\mathrm{sign}(\dot{\mathrm{q}}_\mathrm{j})$ which causes discontinuities on $\mathbf{\Gamma_f}$ during the crossing of 0 velocity $\dot{q}_j$. To avoid this, it is replaced by a continuous $\frac{2}{\pi}\arctan(\alpha_j \dot{q}_j)$ function, where α_j is a ratio of maximum joint velocity $\dot{q}_{jmax}$
Develop Eq. (1) by replacing L with $\mathrm{E} - \mathrm{U}$, then it becomes the explicit inverse dynamic model which depends on the joint acceleration:

$$\mathbf{\Gamma_m} = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{Q}(\mathbf{q}) + \mathbf{\Gamma_f}, \tag{2}$$

where $\ddot{\mathbf{q}}$ is the $n \times 1$ vector of joint acceleration, $\mathbf{M}(\mathbf{q})$ is the $n \times n$ symmetric and positive definite inertia matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$ is the $n \times 1$ vector of Coriolis and centrifugal torques, $\mathbf{Q}(\mathbf{q})$ is the $n \times 1$ vector of gravity torques. The dynamic model is linear with respect to a set of standard dynamic parameters $\mathbf{X}_{dyn}$. From (2) it can be rewritten as:

$$\mathbf{\Gamma_m} = \mathbf{D_s}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\mathbf{X}_{dyn}. \tag{3}$$

The vector $\mathbf{X}_{dyn}$ is of dimension $14n \times 1$, and for each link there are 6 components of the inertia tensor, 3 components of the first moment, 1 mass parameter, 1 total inertia moment for rotor actuator and gears, 2 viscous and Coulomb friction parameters. According to [27], the set of standard dynamic parameters can be simplified into a set of base inertial parameters containing the minimum parameters that can describe the robot dynamics. They are obtained from the standard inertial dynamic parameters by eliminating those that have no effect on the dynamic model and by regrouping those in linear relations. In [28], symbolic and numerical solutions are presented for any open or closed chain robot manipulator to get a minimal dynamic model:

$$\mathbf{\Gamma_m} = \mathbf{D}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\mathbf{X}, \tag{4}$$

where $\mathbf{X}$ is the vector of base parameters.

B. Identification Process

Least squares (LS) technique is commonly used in robot dynamic parameters identification process by solving an overdetermined linear system from n_s sampling points of the dynamic models (4) along a trajectory:

$$\mathbf{Y} = \mathbf{W}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\mathbf{X} + \rho, \tag{5}$$

where ρ is a noise, $\mathbf{Y}$ and $\mathbf{W}$ are the vector of torques and the observation matrix, respectively, which are defined as follows:

$$\mathbf{Y} = \begin{bmatrix} \mathbf{\Gamma_m(1)} \\ \vdots \\ \mathbf{\Gamma_m(n_s)} \end{bmatrix}, \mathbf{W} = \begin{bmatrix} \mathbf{D(1)} \\ \vdots \\ \mathbf{D(n_s)} \end{bmatrix}. \tag{6}$$

The traditional method to estimate joint velocity $\dot{\mathbf{q}}$ and acceleration $\ddot{\mathbf{q}}$ is to apply central Euler difference of joint position $\mathbf{q}$. However, this method can amplify the noise effect in the estimations of $\dot{\mathbf{q}}$, $\ddot{\mathbf{q}}$. To avoid the noise distribution, $(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$ must be filtered by a low-pass filter $\mathrm{F_q(s)}$, with derivative operator s. The filter $\mathrm{F_q(s)}$ should have a flat amplitude without phase shift in the range $[0\ \omega_{\mathrm{c_q}}]$, with the rule of thumb $\omega_{\mathrm{c_q}} > (10 \times \omega_{dyn})$, and ω_{dyn} is the bandwidth of the joint position closed loop [24]. Meanwhile, the torque $\mathbf{\Gamma_m}$ is perturbed by high frequency torque ripple from joint drive chain in the closed loop control. Hence, it has to be filtered. Then, $\mathbf{\Gamma_m}$ and $\mathbf{D}(\mathbf{q}_{\mathrm{f_q}}, \dot{\mathbf{q}}_{\mathrm{f_q}}, \ddot{\mathbf{q}}_{\mathrm{f_q}})$ are both filtered and downsampled through a decimate filter composed of a low-pass filter $\mathrm{F_p(s)}$, where its cutoff frequency $\omega_{\mathrm{f_p}}$ is approximated by $5 \times \omega_{dyn}$, in order to get a new filtered linear system:

$$\mathbf{Y_{fp}} = \mathbf{W_{fp}}(\mathbf{q_{fq}}, \dot{\mathbf{q}}_{\mathbf{fq}}, \ddot{\mathbf{q}}_{\mathbf{fq}})\mathbf{X} + \rho_{\mathbf{fp}}. \tag{7}$$

Finally, we solve the LS problem via:

$$\hat{\mathbf{X}} = \mathbf{W}^{+}_{\mathbf{fp}}(\mathbf{q_{fq}}, \dot{\mathbf{q}}_{\mathbf{fq}}, \ddot{\mathbf{q}}_{\mathbf{fq}})\mathbf{Y_{fp}}, \tag{8}$$

where $\mathbf{W}^{+}_{\mathbf{fp}}$ is the pseudo-inverse matrix of $\mathbf{W_{fp}}$ and $\hat{\mathbf{X}}$ is the unique LS solution using the QR factorization or SVD decomposition. $\hat{\mathbf{X}}$ minimizes the Euclidean norm $\|\rho\|^2$ of the vector of errors ρ. The unicity of $\hat{\mathbf{X}}$ depends on the rank of the observation matrix $\mathbf{W_{fp}}$. The numerical rank deficiency can come from $\mathbf{W_{fp}}$ structural rank deficiency or dissatisfaction of excitation condition due to a bad choice of samples. In order to decrease the sensitivity of LS solution with respect to errors, the condition number of the observation matrix must be close to 1. Some approaches used to calculate the exciting trajectory for identification are proposed in [29].

While in some cases the contribution of certain base parameters is too small that they cannot be experimentally identified even with optimized trajectories. Then, the base parameters should be replaced by a set of essential parameters by eliminating certain insignificant parameters. There are two approaches: the QR factorization of $\mathbf{W_{fp}}diag(\hat{\mathbf{X}})$ with column pivoting [30] or assuming that parameters with relative standard deviation $\sigma_{\hat{X}_j}$ larger than 10% will be eliminated from the model (see section V). This new set of essential parameters allows to improve the noise immunity of the robot identification process and reduce the computation burden for the model based control application.

III. Central Jacobi Differentiator

Consider a noisy measurement $x^{\varpi}: I \to \mathbb{R}$, $x^{\varpi}(t) = x(t) + \varpi(t)$, where I is a finite time open interval of $\mathbb{R}^{+}$, $x \in \mathcal{C}^{n}(I)$ with $n \in \mathbb{N}$, and ϖ is an additive corrupting noise. The objective is to estimate the n^{th} order derivative of x using x^{ϖ}. First, for any $t_0 \in I$, the set $D_{t_0} = \left\{t \in \mathbb{R}_{+}^{*}; [t_0 - t, t_0 + t] \in I\right\}$ is introduced.

A. Algebraic differentiator involving Jacobi polynomials

For any $t_0 \in I$, x can be locally expressed on $[t_0 - h, t_0 + h]$ with $h \in D_{t_0}$ by the following Jacobi orthogonal series expansion:

$$\forall \xi \in [-1,1],\ x(t_0 + h\xi) = \sum_{i \geq 0} \frac{\left\langle P_i^{(\mu,\kappa)}(\cdot), x(t_0 + h\cdot)\right\rangle_{\mu,\kappa}}{\|P_i^{(\mu,\kappa)}\|_{\mu,\kappa}^2}\ P_i^{(\mu,\kappa)}(\xi), \quad (9)$$

where $\left\langle P_i^{(\mu,\kappa)}(\cdot), x(t_0 + h\cdot)\right\rangle_{\mu,\kappa} = \int_{-1}^{1} w_{\mu,\kappa}(\tau) P_i^{(\mu,\kappa)}(\tau) x(t_0 + h\tau) d\tau$, $P_i^{(\mu,\kappa)}$ is the i^{th} order Jacobi orthogonal polynomial defined on $[-1, 1]$ as follows (see [31]):

$$P_i^{(\mu,\kappa)}(\tau) = \sum_{j=0}^{i} \binom{i+\mu}{j} \binom{i+\kappa}{i-j} \left(\frac{\tau-1}{2}\right)^{i-j} \left(\frac{\tau+1}{2}\right)^{j},$$

with $\mu, \kappa \in]-1, +\infty[$, $\langle \cdot, \cdot \rangle_{\mu,\kappa}$ is a $\mathcal{L}^2([-1,1])$ scalar product with the associated weight function $w_{\mu,\kappa}(\tau) = (1-\tau)^{\mu}(1+\tau)^{\kappa}$, and the associated norm $\|P_i^{(\mu,\kappa)}\|_{\mu,\kappa}^2 = \frac{2^{\mu+\kappa+1}}{2i+\mu+\kappa+1} \frac{\Gamma(\mu+i+1)\Gamma(\kappa+i+1)}{\Gamma(\mu+\kappa+i+1)\Gamma(i+1)}$, where $\Gamma(\cdot)$ is the classical Gamma function (see [31] p. 255).

In order to approximate x, the $N+1$ first terms in the Jacobi series expansion given in (9) are taken, *i.e.* we locally approximate x by a N^{th} order polynomial on $[t_0 - h, t_0 + h]$. Thus, by denoting the obtained N^{th} order polynomial by $D^{(0)}_{\kappa,\mu,h,N} x(t_0 + h\xi)$, we have:

$$\forall \xi \in [-1,1],\ D^{(0)}_{\kappa,\mu,h,N} x(t_0 + h\xi) := \sum_{i=0}^{N} \frac{\left\langle P_i^{(\mu,\kappa)}(\cdot), x(t_0 + h\cdot)\right\rangle_{\mu,\kappa}}{\|P_i^{(\mu,\kappa)}\|_{\mu,\kappa}^2}\ P_i^{(\mu,\kappa)}(\xi). \quad (10)$$

Hence, the value of x at t_0 can be approximated by $D^{(0)}_{\kappa,\mu,h,N} x(t_0)$ by taking $\xi = 0$. Inspired by this idea, the central Jacobi differentiator is defined by taking the $q+1$ ($q = N - n \in \mathbb{N}$) first terms in the Jacobi series expansion of $x^{(n)}$ with $\xi = 0$:

$$D^{(n)}_{\kappa,\mu,h,q} x(t_0) = \sum_{i=0}^{q} \frac{\left\langle P_i^{(\mu+n,\kappa+n)}(\cdot), x^{(n)}(t_0 + h\cdot)\right\rangle_{\mu+n,\kappa+n}}{\|P_i^{(\mu+n,\kappa+n)}\|_{\mu+n,\kappa+n}^2} P_i^{(\mu+n,\kappa+n)}(0). \quad (11)$$

This differentiator can be given by the following integral formula [17]:

$$D^{(n)}_{\kappa,\mu,h,q} x(t_0) = \frac{1}{h^n} \int_{-1}^{1} Q_{\kappa,\mu,n,q}(\tau)\, x(t_0 + h\tau) d\tau, \quad (12)$$

where $\rho_{n,\kappa,\mu}(\tau) = \frac{2^{-(n+\kappa+\mu+1)} n! \Gamma(2n+\kappa+\mu+2)}{\Gamma(n+\kappa+1)\Gamma(n+\mu+1)}$,

$$Q_{\kappa,\mu,n,q}(\tau) = \sum_{i=0}^{q} P_i^{\mu+n,\kappa+n}(0) \sum_{j=0}^{i} (-1)^{i+j} \binom{i}{j} \times \frac{2i+\kappa+\mu+2n+1}{i+\kappa+\mu+2n+1} \rho_{n,\kappa+i-j,\mu+j}(\tau).$$

Finally, x is substituted in (12) by x^{ϖ} in order to obtain the Jacobi differentiator $D^{(n)}_{\kappa,\mu,h,q} x^{\varpi}(t_0)$ in noisy case.

It is clear that for each $t_0 \in I$, the central Jacobi differentiator $D^{(n)}_{\kappa,\mu,T,q} x^{\varpi}(t_0)$ depends on a set of design parameters, except for the order of the desired derivative n:

- $\kappa, \mu \in]-1, +\infty[$: the parameters of Jacobi polynomials,
- $q \in \mathbb{N}$: the order of truncated Jacobi series expansion,
- $T = 2h$: the length of the sliding integration window.

B. Error Analysis in Time Domain

The estimation error of the central Jacobi differentiator can be decomposed in continuous case as follows:

$$\begin{aligned} D^{(n)}_{\kappa,\mu,h,q} x^{\varpi}(t_0) - x^{(n)}(t_0) &= \left(D^{(n)}_{\kappa,\mu,h,q} x - x^{(n)}(t_0)\right) \\ &+ \left(D^{(n)}_{\kappa,\mu,h,q} x^{\varpi}(t_0) - D^{(n)}_{\kappa,\mu,h,q} x(t_0)\right) \\ &= e_{\varpi}(t_0; n, \kappa, \mu, h, q) + e_{R_n}(t_0; \kappa, \mu, h, q). \end{aligned} \quad (13)$$

where $e_{\varpi}(t_0; n, \kappa, \mu, h, q)$ and $e_{R_n}(t_0; \kappa, \mu, h, q)$ refer to the noise error contribution and the truncated term error, respectively. Corresponding error bounds have been provided in [17]. Finally, by numerically[1] calculating these error bounds, their behaviors with respect to different design parameters can be known. Then, the influence of these design parameters on each source of errors can be deduced. The obtained results are summarized in Table I (see [17], [32] for more details), where the notations $a \uparrow$, $b \nearrow$ and $c \searrow$ mean that if we increase the value for the parameter a, then the error b can be increased and the error c can be decreased. According to Table I, the design parameters' influence on different errors is not the same. Consequently, a compromise among these parameters should be taken.

	Noise error contribution	Truncated term error
$\kappa \uparrow$	$\nearrow$	$\searrow$
$\mu \uparrow$	$\nearrow$	$\searrow$
$q \uparrow$	$\nearrow$	$\searrow$
$T \uparrow$	$\searrow$	$\nearrow$

TABLE I: Influence of design parameters on $D^{(n)}_{\kappa,\mu,h,q} x(t_0)$ in continuous case.

[1] It is very difficult to analytically study the behavior of each error bound due to their complex expressions.

In discrete case, the integral formula should be approximated by applying a numerical integration method, which produces a numerical error.

C. Error Analysis in Frequency Domain

As shown in Eq. (12), the Jacobi differentiator is a combination of integrals with measurements on $[t_0 - \frac{T}{2}, t_0 + \frac{T}{2}]$. In real computation, the numerical integration is actually a discrete operation, with a sampling time T_s in the measurements by calculating the sum of discrete values in the time interval. The discrete version of the Jacobi differentiator is actually a discrete convolution of measurements and a list of weighted coefficients. In this sense, the Jacobi differentiator can be regarded as a FIR filter applied on a discrete system with sampling time T_s. After extracting the weighted coefficients, we can draw the bode magnitude plot of the Jacobi differentiator, as a digital filter with sampling time T_s, in order to express the magnitude of the frequency response. While in the bode phase plot, the phase frequency response is always π or $-\pi$, which means that there is no delay.

Given a signal that is the sum of three sinusoidal waves with amplitude 1 and frequency 4 Hz, 9 Hz, 15 Hz respectively. To get the second order derivative, the central Jacobi differentiator and the Euler central differentiation combined with a central Butterworth filter are applied. The central Butterworth filter is a zero phase forward-backward IIR filter and is referred as a maximally flat magnitude filter. It is widely used in various applications. Hence, this motivates the comparisons values.

The study is done in noise case, where the measurement of the signal is simulated by superimposing together on the signal a normally distributed random noise of amplitude 0.2, a 200 Hz high frequency sinusoidal wave of amplitude 0.2 and a Poisson distributed random noise with mean parameter $\lambda = 0.1$ of amplitude 0.2. The sampling frequency is 1 millisecond. In order to estimate the derivatives of the original signal , the central Jacobi differentiator is applied by taking $\kappa = \mu = 12$, $q = 6$ and the sliding integration window $T = 0.21$ second. A well tuned forward Butterworth filter configuration is of order 6 with cutoff frequency at 25 Hz. The forward-backward process is done by adding poles in the denominator of transfer function with negative values.

The estimation errors in velocity and acceleration are shown in Fig. 1. The result shows that central Jacobi differentiator can be accurate and robust as Euler central differentiation with a well tuned Butterworth filter. It can be seen that the estimation errors for the central Jacobi differentiator is larger at the beginning and the end. This is because there is not enough data for the estimation.

In frequency domain, the magnitude bode plots of second order derivative are shown in Fig. 2. The red line presents the ideal case with transfer function $H(s) = s^2$ in continuous time. The Jacobi differentiator and Euler differentiation with Butterworth filter are in discrete case. Under 15 Hz, the magnitude frequency response follows quite well the ideal curve for both Jacobi differentiator and Butterworth method. Above 15 Hz, the magnitude frequency response cuts off

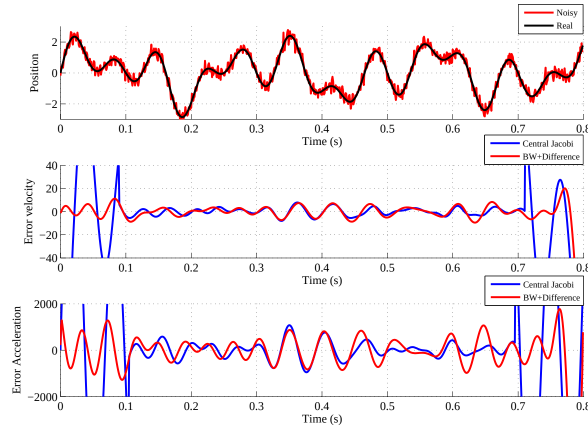

Fig. 1: Derivative errors in velocity and acceleration

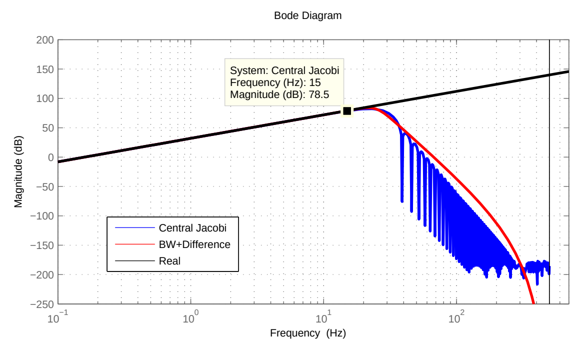

Fig. 2: Bode magnitude plot of second order differentiators

rapidly especially for the Jacobi differentiator. This means that the Jacobi differentiator has a better cutoff property and the unexpected frequency is attenuated quickly to 0 magnitude response.

By varying the differentiator configuration, we can analyze the parameters' influence on the central Jacobi differentiator. The obtained results are shown in Fig. 3, where the following conclusions are given:

1) $\kappa = \mu$, these parameters are chosen to be identical because this configuration reduces the truncated term error [17]. As their value increases, the descending point moves to higher frequency. It means that the unwanted frequency is not filtered and the noise error contribution grows. But the descending period drops rapidly which offers better cutoff property.

2) When q increases, we utilize more terms in the Jacobi orthogonal series expansion. Hence, the truncated term error can be reduced. Similarly, as q increases, the descending point moves to higher frequency and the noise error contribution grows.

3) T is the sliding integration window. When the sampling time T_s is fixed, it represents the points taken for the Jacobi differentiator. As T increases, the descending point moves to lower frequency which cut off more noise components. Thus, the noise error contribution decreases.

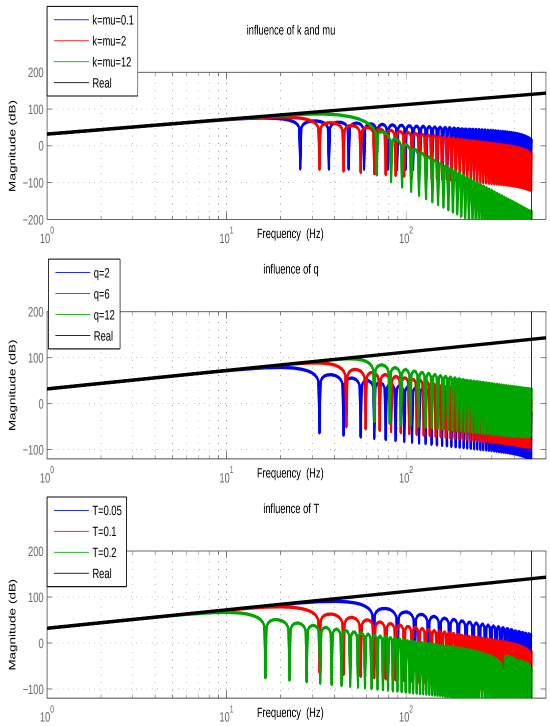

Fig. 3: Parameters influence on bode plot when $T_s = 0.001s$

From the previous analysis, the Jacobi differentiator is regarded as a low-pass differentiator. The low-pass property is inherent because it considers the signal as a certain order polynomial in a small time window and uses the truncations to estimate the derivatives. Compared to the Euler central differentiation combined with Butterworth filter, it can be more robust with respect to noise. However, it is used for off-line application, because the window in central

IV. Experimental Identification with 2R Robot

The experimental work is done on a two revolute joints planar prototype robot which is shown in Fig. 4. It moves in a horizontal plane and has no gravity effect. According to [33], the dynamic model depends on eight minimal base dynamic parameters $\mathbf{X} = [\mathrm{ZZ_{1R}}\ \mathrm{ZZ_2}\ \mathrm{MX_2}\ \mathrm{MY_2}\ \mathrm{F_{V1}}\ \mathrm{F_{C1}}\ \mathrm{F_{V2}}\ \mathrm{F_{C2}}]$, with the regrouped parameter $\mathrm{ZZ_{1R}} = \mathrm{ZZ_1} + \mathrm{M_2L^2}$, where L is the length of first link, $\mathrm{ZZ_1}$ and $\mathrm{ZZ_2}$ are drive side moment of inertial of link 1 and 2 respectively, $\mathrm{MX_2}$, $\mathrm{MY_2}$ are the first moment of link 2, $\mathrm{F_{Vj}}$, $\mathrm{F_{Cj}}$, are the viscous and Coulomb friction coefficients of joint j.

The robot motion is driven by a PD controller with a reference of a successive point to point trajectories which is obtained using a classical 5^{th} order polynomial trajectory generator with a sampling rate $f_s = 100$ Hz. For the central Jacobi differentiator, set $k = \mu = 6$, $q = 2$ and $T = 0.6s$ to estimate the joint velocity, $T = 1.1s$ to estimate the joint acceleration. For filtering method, the joint position and torque are pre-filtered using a fourth order forward-backward Butterworth filter with a cutoff frequency $f_c = 0.1 \times (f_s/2)$.

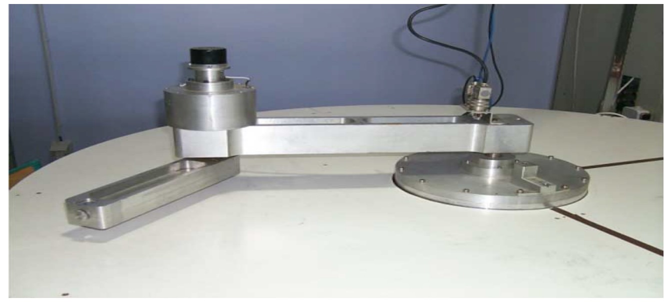

Fig. 4: 2R scara prototype robot

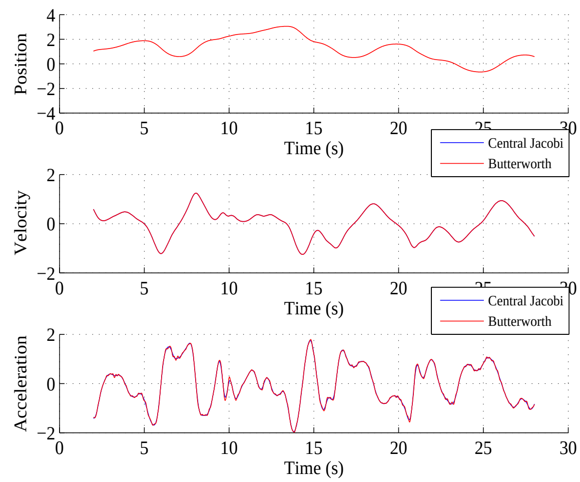

Fig. 5: Real trajectory and estimation of velocities, acceleration of q_1

The estimations of velocity and acceleration are shown in Fig. 5.

The identification method is presented in section II. Then, the estimation of base dynamic parameters $\hat{\mathbf{X}}$ is calculated by LS method. Standard deviations $\sigma_{\hat{\mathbf{X}}_i}$ are estimated using classical and simple results from statistics, assuming the matrix $\mathbf{W}$ to be a deterministic one, and ρ to be a zero mean additive independent noise, with a standard deviation σ_ρ such that $C_{\rho\rho} = E(\rho^T \rho) = \sigma_\rho^2 \mathbf{I}_r$, where E is the expectation operator. The variance-covariance matrix of the estimation error and standard deviations can be calculated by:

$$C_{\hat{\mathbf{X}}\hat{\mathbf{X}}} = E[(\hat{\mathbf{X}} - \hat{\mathbf{X}})(\hat{\mathbf{X}} - \hat{\mathbf{X}})^T] = \sigma_\rho^2 (\mathbf{W}^T \mathbf{W})^{-1},$$

where $\sigma^2_{\hat{\mathbf{X}}_i} = C_{\hat{\mathbf{X}}\hat{\mathbf{X}}ii}$ is the diagonal coefficient of $C_{\hat{\mathbf{X}}\hat{\mathbf{X}}}$.

An unbiased estimation of σ_ρ is used to get the relative standard deviation $\sigma_{\hat{\mathbf{X}}ri}$ by the expression:

$$\hat{\sigma}_\rho^2 = \frac{||Y - W\hat{\mathbf{X}}||^2}{r - c}, \ \%\sigma_{\hat{\mathbf{X}}ri} = 100 \times \frac{\sigma_{\hat{\mathbf{X}}i}}{\hat{\mathbf{X}}i},$$

where r is the total number of equations and c is the number of unknown parameters.

Identification results are given in table II which are quite similar for both methods. Compared to Butterworth filter

	Jacobi			Butterworth		
P	$\hat{\mathbf{X}}$	$2\sigma_{\hat{\mathbf{X}}}$	$\sigma_{\hat{\mathbf{X}}r}$ %	$\hat{\mathbf{X}}$	$2\sigma_{\hat{\mathbf{X}}}$	$\sigma_{\hat{\mathbf{X}}r}$ %
$\mathrm{ZZ}_{1\mathrm{R}}$	3.496	0.032	0.46	3.482	0.033	0.47
F_{V1}	0.248	0.087	17.6	0.247	0.089	17.9
F_{C1}	0.431	0.048	5.58	0.433	0.049	5.61
ZZ_2	0.059	0.004	3.41	0.059	0.004	3.41
MX_2	0.124	0.003	1.07	0.125	0.003	1.08
MY_2	0.0019	0.002	65.99	0.0006	0.003	200
F_{V2}	0.0144	0.015	52.3	0.014	0.015	54.4
F_{C2}	0.1268	0.043	17.0	0.1272	0.043	17.1
number of equations= 522						
$cond(W) = 38$ for both cases						

TABLE II: Comparison of experimental estimation

approach, the Jacobi differentiator method presents a better precision in identification results on error norm and relative error norm. When the trajectory is not of high frequency, the central Jacobi differentiator is a robust differentiator to get high order derivatives.

V. Conclusion

In this paper, the robot identification process has been reviewed by introducing the central Jacobi differentiator. By considering it as a FIR filter, its frequency domain properties have been investigated using bode plot and an insight into its differentiation performance has been given. Comparisons have been drawn between the Jacobi central differentiator and the Euler central differentiation combined with a well tuned forward-backward Butterworth filter. From the results, they all present good attenuation with respect to high frequency components. Especially, the Jacobi differentiator has a fast descending period, which make it resistant to high frequency noises. For future work, the causal Jacobi differentiator used for on-line applications will be studied.

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