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# COMBINATORIAL OPTIMIZATION WITH MULTIPLE RESOURCES AND ENERGY CONSTRAINTS

LAAS-CNRS

Optimisation sous contraintes de ressources énergétiques multiples (OREM) - PGMO project (2013-2015) - 1st year

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## OREM project : context and objectives

### Context

The integration of energy constraints in deterministic scheduling models, such as job-shop scheduling or resource-constrained project scheduling, yields a combinatorial optimization challenge. It follows that the literature on this subject is sparse. Pre-existing studies involve multiple energy sources and general non-linear efficiency functions, but generally no scheduling. All our previous work on scheduling under energy constraints considered linear (and even identical) energy efficiency functions, which oversimplifies the problem.

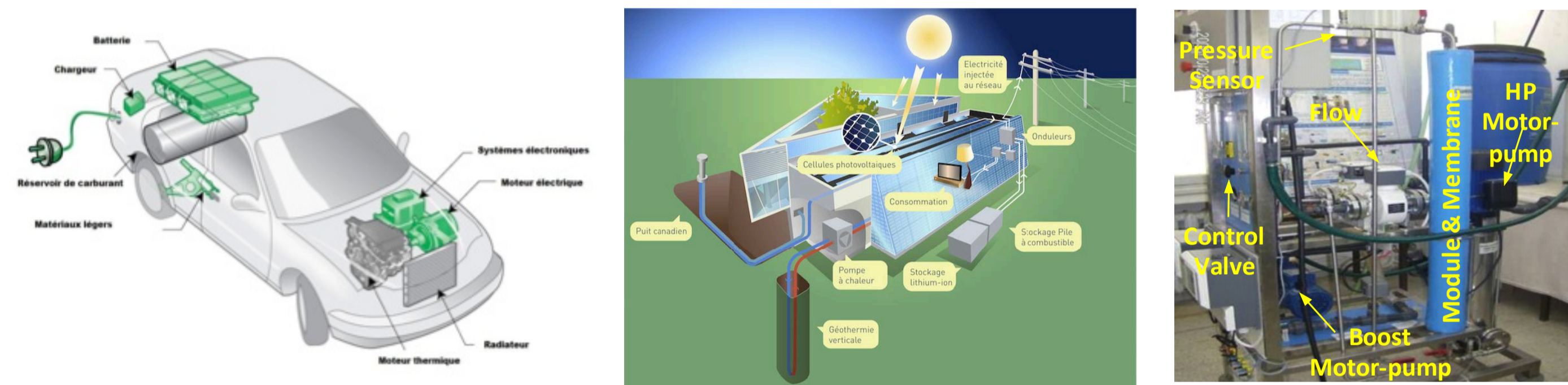
### Objectives

- Address the (combinatorial) optimization challenge of integrating energy sources constraints (physical, technological and performance characteristics) in deterministic (scheduling) models.
- Solve explicitly and in an integrated fashion the resulting energy resource allocation problems and energy-consuming activity scheduling problems with non linear energy efficiency functions.

## Applications and challenge

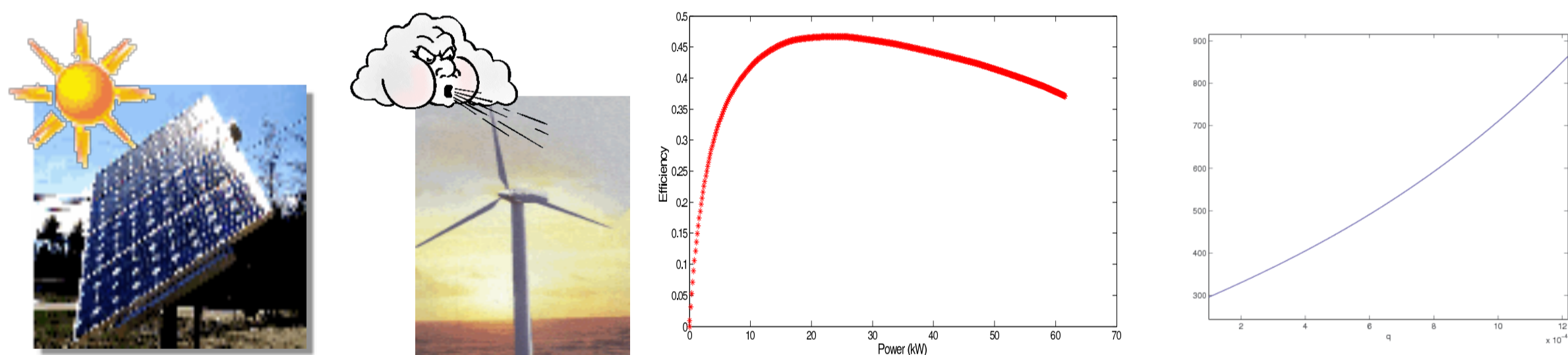
### Applications

Scheduling for hybrid electric vehicles, intelligent buildings, processes and manufacturing



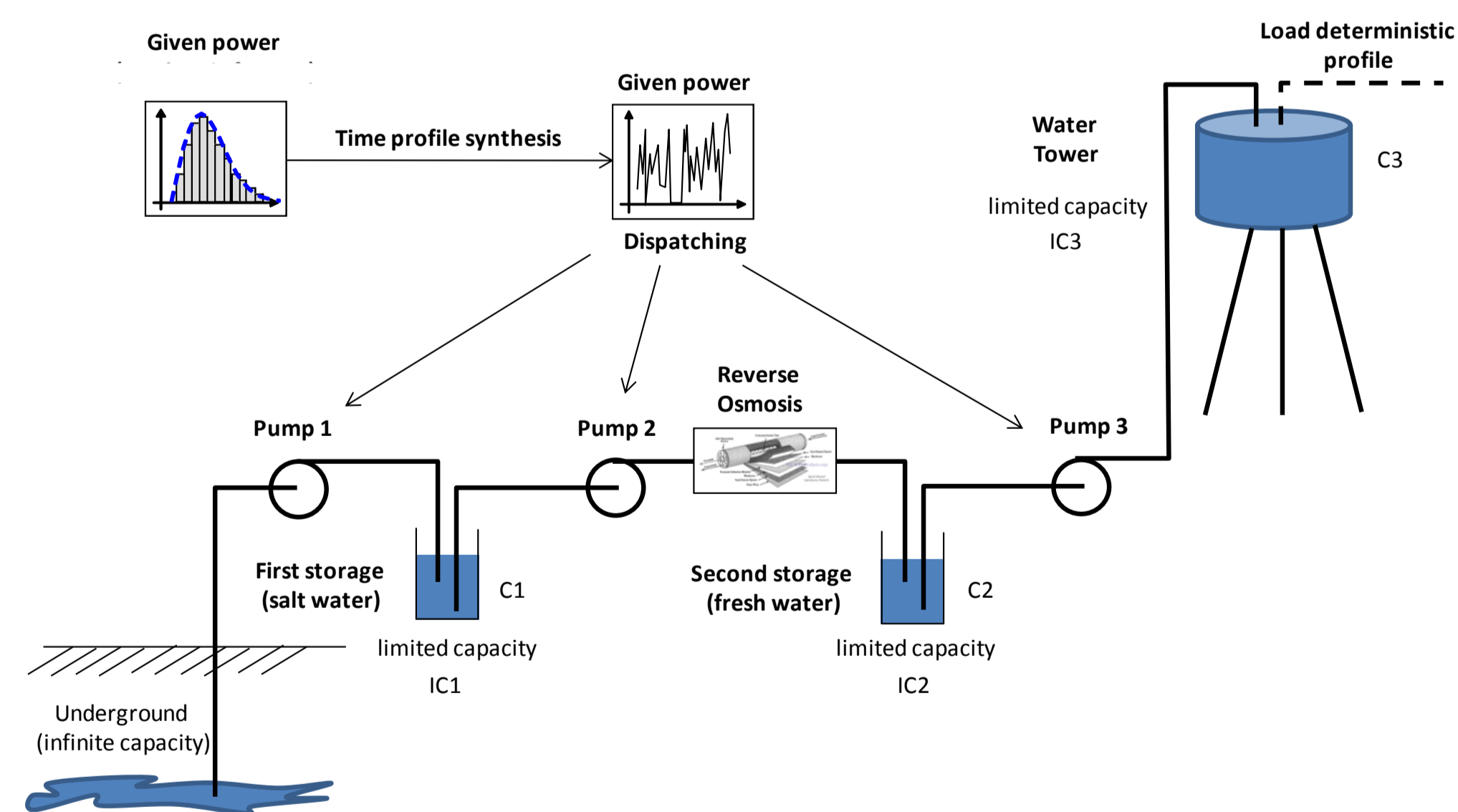
### Challenge

Non-linearities come from energy efficiency functions



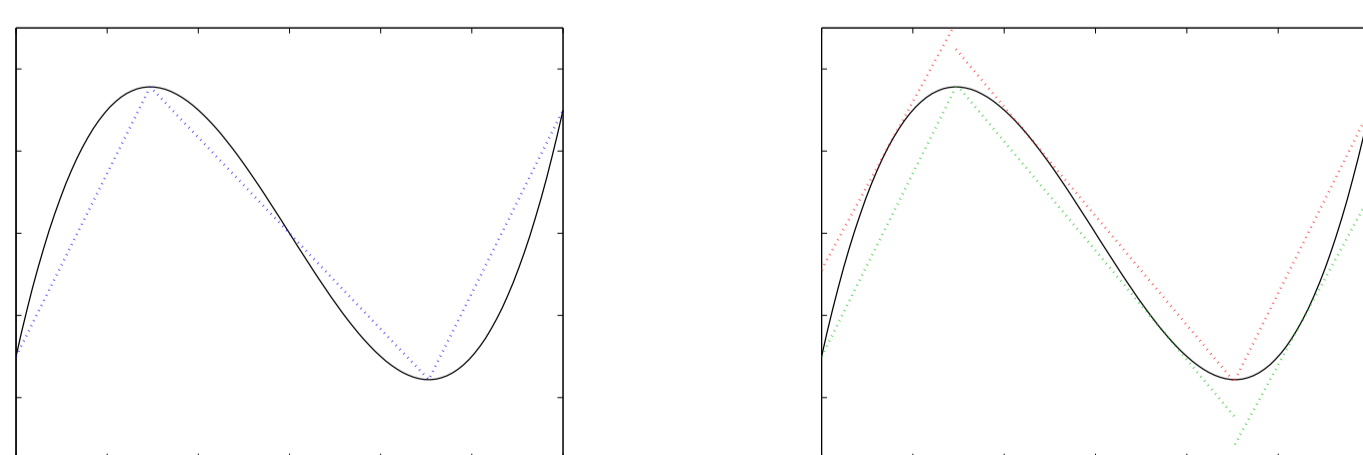
## Piecewise linear bounding procedures for the optimal management of water pumping and desalination processes

Proof of concept on a water pumping and desalination system [CLAIO 2014, IFORS 2014]



### Solution Method

- Step 1 : Piecewise linear bounding vs classic approx. of the nonlinear energy transfer/efficiency functions



- Step 2 : Reformulation of the problem into two mixed integer problems (MILP and MILP)
- the problem is originally a MINLP
- using the pair of bounding functions previously defined

Number of sectors per tolerance value

ε	Pump 1		(Pump 2+RO)		Pump 3	
	$\bar{n}_{p1}$	$\bar{n}_{p2}$	$\bar{n}_{p2}$	$\bar{n}_{p2}$	$\bar{n}_{p3}$	$\bar{n}_{p3}$
5%	2	2	11	21	3	2
1%	5	5	21	29	8	5
0.5%	8	7	35	62	13	7
0.3%	10	9	43	74	17	9

### Results

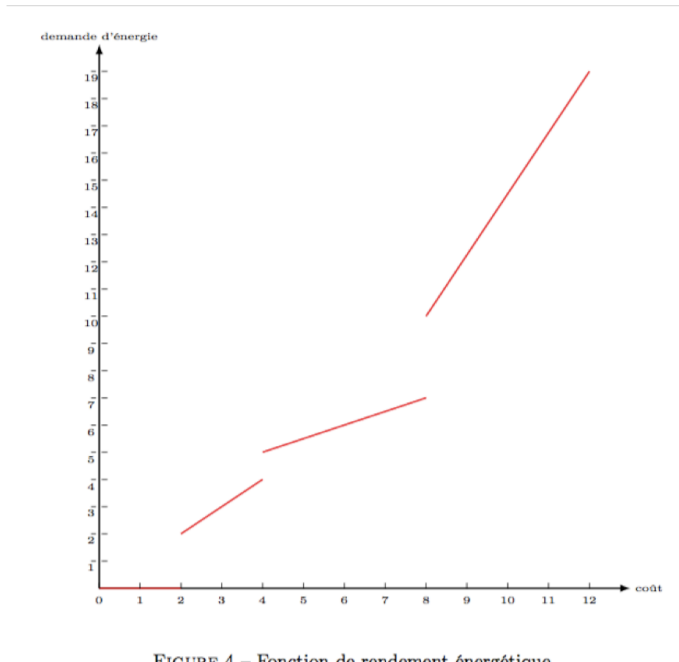
Lower and upper bound obtained

ε	MILP		MILP		Gap %	opt
	UB	s	LB	s		
5%	20580	4	19740	15	-	4.25 no
1%	20100	15	19920	140	-	0.9 no
0.5%	20040	178	19980	117	-	0.3 no
0.3%	20040	64	19980	321	19980	0.0 yes

## Generic problem : Scheduling with non-reversible energy sources

### Data

- Set of time periods  $T$
- Set of activities  $\mathcal{A}$ 
  - $r_i, d_i, p_i$  : release date, due date, duration of activity  $i$
  - $b_i$  : constant instantaneous energy demand of activity  $i$
- Set of non-reversible energy sources  $\mathcal{S}$ 
  - $\rho^s$  : piecewise-linear efficiency function for source  $s$  (x-axis = cost, y-axis = demand and  $\rho^s(x) = 0, \forall x < 0, \forall s \in \mathcal{S}$ ).
- $a_{it}$  : constant term equal to 1 if  $t \in [r_i, d_i]$  and 0 otherwise



### Decision variables

- $x_{it}$  : binary, = 1 iff activity  $i$  is ongoing at time period  $t$

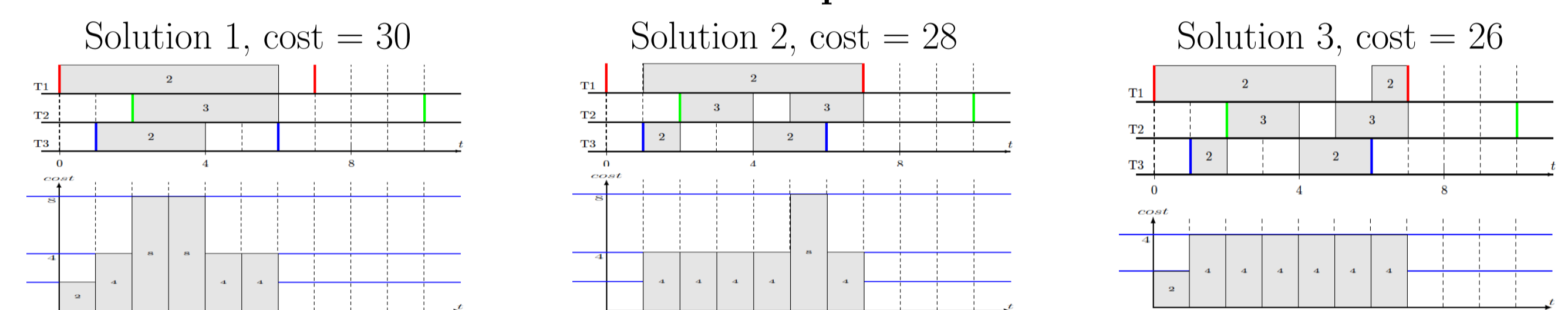
### Formulation

$$\begin{aligned} (\text{CF}) \min & \sum_{t \in T} \rho^{-1} \left( \sum_{i \in \mathcal{A}} b_i x_{it} \right) \\ \text{s.t.} & \sum_{t \in T} a_{it} x_{it} \geq p_i, \quad \forall i \in \mathcal{A} \\ & x_{it} \in \{0, 1\}, \quad \forall i \in \mathcal{A}, t \in T \end{aligned}$$

### Complexity and equivalence between single and multiple sources

- **Theorem 1** The problem is NP hard by reduction from discrete bin packing
- **Theorem 2** For any problem with multiple sources, there is an equivalent single source problem

### Example



## Dantzig-Wolfe decomposition and Branch&Price Method

### Extended formulation based on feasible subsets of activities

- Set of activity sets executable in parallel  $\mathcal{L}$  (at any given time period  $\bar{L}_t$ )
- Set of activities belonging to set  $l$  : demand  $b_l$ , cost  $c_l$ , time window  $[R_l, D_l]$
- $a_{il}$  : binary constant term equal to 1 iff  $i$  belongs to set  $l$
- Variable  $y_{lt}$  : binary, = 1 iff activity set  $l$  is being executed at time  $t$

### The linear relaxation of the master problem

$$\begin{aligned} (\text{LRMP}) \min & \sum_{t \in T} \sum_{l \in \bar{L}_t} c_l y_{lt} \\ x_{it} - \sum_{l \in \bar{L}_t} a_{il} y_{lt} &= 0, \quad \forall i \in \mathcal{A}, t \in T \\ \sum_{l \in \bar{L}_t} \sum_{i \in \mathcal{A}} a_{il} y_{lt} &\geq p_i, \quad \forall i \in \mathcal{A} \\ - \sum_{l \in \bar{L}_t} y_{lt} &\geq -1, \quad \forall t \in T \\ x_{it} &\leq 1 \quad \forall i \in \mathcal{A}, t \in T \\ y_{lt} &\geq 0 \quad \forall t \in T, l \in \bar{L}_t \\ x_{it} &\geq 0 \quad \forall i \in \mathcal{A}, t \in T \end{aligned}$$

The resulting dual (DLMRP) is :

$$\begin{aligned} \max & \sum_{i \in \mathcal{A}} p_i u_i - \sum_{t \in T} v_t + \sum_{i \in \mathcal{A}} \sum_{t \in T} z_{it} \\ & \sum_{i \in \mathcal{A}} a_{il} (u_i - w_{it}) - v_t \leq c_l, \quad \forall t \in T, l \in \bar{L}_t \\ & w_{it} + z_{it} \leq 0, \quad \forall i \in \mathcal{A}, t \in T \\ & w_{it} \in \mathbb{R}, \quad \forall i \in \mathcal{A}, t \in T \\ & u_i \geq 0, \quad \forall i \in \mathcal{A} \\ & v_t \geq 0, \quad \forall t \in T \end{aligned}$$

Therefore, the reduced cost of a column  $y_{lt}$  is :

$$c_l - \sum_{i \in \mathcal{A}} a_{il} (u_i) - v_t$$

### Two column generation schemes

$$\text{SP1 : } \left\{ \begin{array}{l} \text{all dual values } u_i, v_t, w_{it} \\ \text{problem data } a_{il}, b_i, \rho^{-1} \end{array} \right\} \rightarrow \left\{ \begin{array}{l} \text{best time } t' \\ \text{best task set } l' \end{array} \right\} \left\| \begin{array}{l} \text{a predefined time } t' \\ \text{SP2 : related dual values } u_i, w_{it}' \\ \text{problem data } a_{il}, b_i, \rho^{-1} \end{array} \right\} \rightarrow \left\{ \begin{array}{l} \text{best task set } l' \end{array} \right\}$$

## Results

	Extended Model Branch & Price	Compact Model Black Box Solver
#Opt/#Instances	261/288	5/288
Ratio avg (min, max)	99.81% (77.47 %, 100%)	84.21 % (57.85 %, 99.99 %)
Time avg (min, max)	540 s (< 1 s, 3603 s)	1034 s (7 s, 1927 s)
Nbnodes avg (min, max)	171 (1, 9755)	2174044 (57157, 13470373)