



HAL
open science

Combinatorial optimization with multiple resources and energy constraints

Sandra Ulrich Ngueveu, Christian Artigues, Pierre Lopez

► **To cite this version:**

Sandra Ulrich Ngueveu, Christian Artigues, Pierre Lopez. Combinatorial optimization with multiple resources and energy constraints. Gaspard Monge Program for Optimization - Conference on Optimization & Practices in Industry : PGMO-COPT'14, Oct 2014, Paris, France. , 2015. hal-01163069

HAL Id: hal-01163069

<https://hal.science/hal-01163069v1>

Submitted on 12 Jun 2015

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

COMBINATORIAL OPTIMIZATION WITH MULTIPLE RESOURCES AND ENERGY CONSTRAINTS

LAAS-CNRS

Optimisation sous contraintes de ressources énergétiques multiples (OREM) - PGMO project (2013-2015) - 1st year

Sandra Ulrich Ngueveu, Christian Artigues and Pierre Lopez

LAAS-CNRS, Toulouse, France

OREM project : context and objectives

Context

The integration of energy constraints in deterministic scheduling models, such as job-shop scheduling or resource-constrained project scheduling, yields a combinatorial optimization challenge. It follows that the literature on this subject is sparse. Pre-existing studies involve multiple energy sources and general non-linear efficiency functions, but generally no scheduling. All our previous work on scheduling under energy constraints considered linear (and even identical) energy efficiency functions, which oversimplifies the problem.

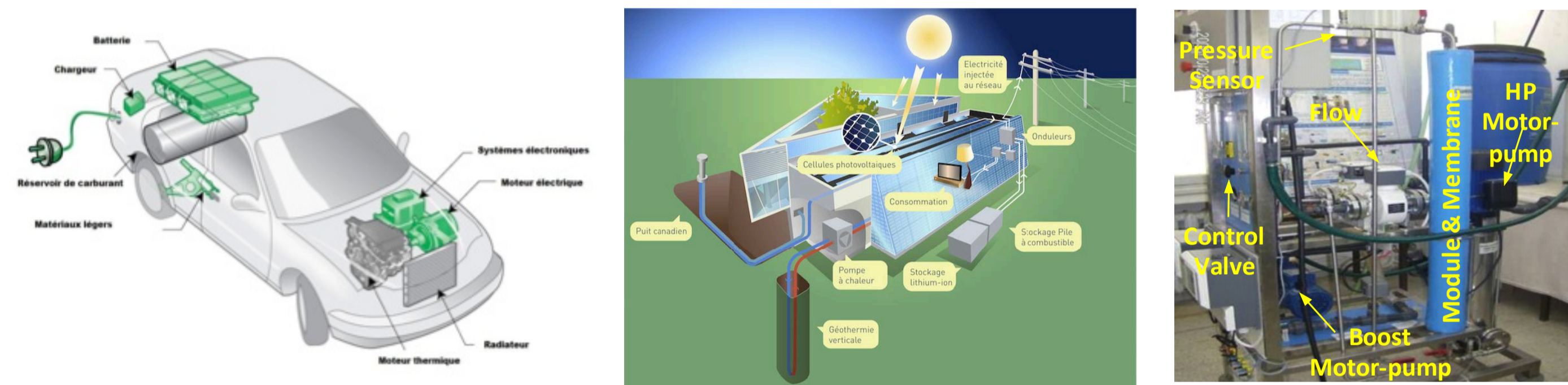
Objectives

- Address the (combinatorial) optimization challenge of integrating energy sources constraints (physical, technological and performance characteristics) in deterministic (scheduling) models.
- Solve explicitly and in an integrated fashion the resulting energy resource allocation problems and energy-consuming activity scheduling problems with non linear energy efficiency functions.

Applications and challenge

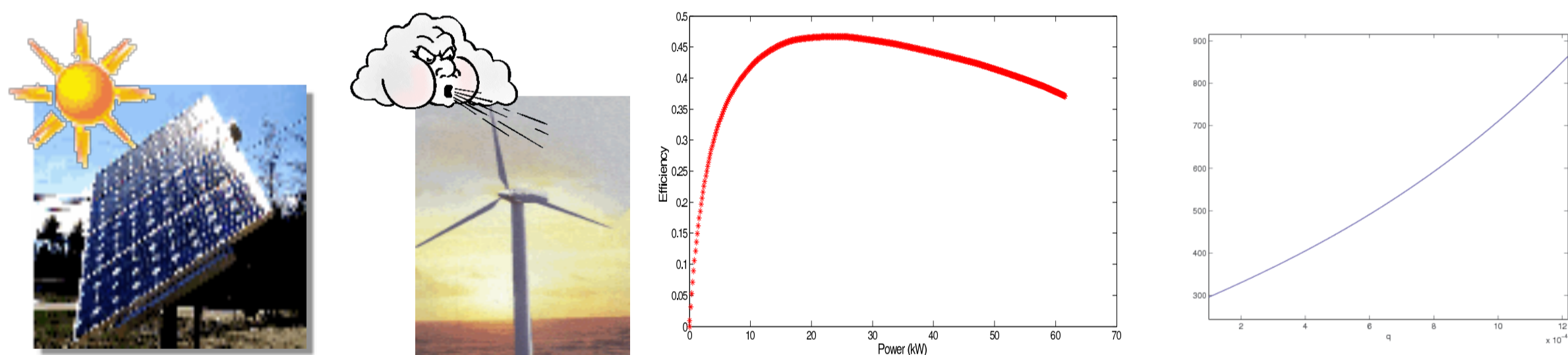
Applications

Scheduling for hybrid electric vehicles, intelligent buildings, processes and manufacturing



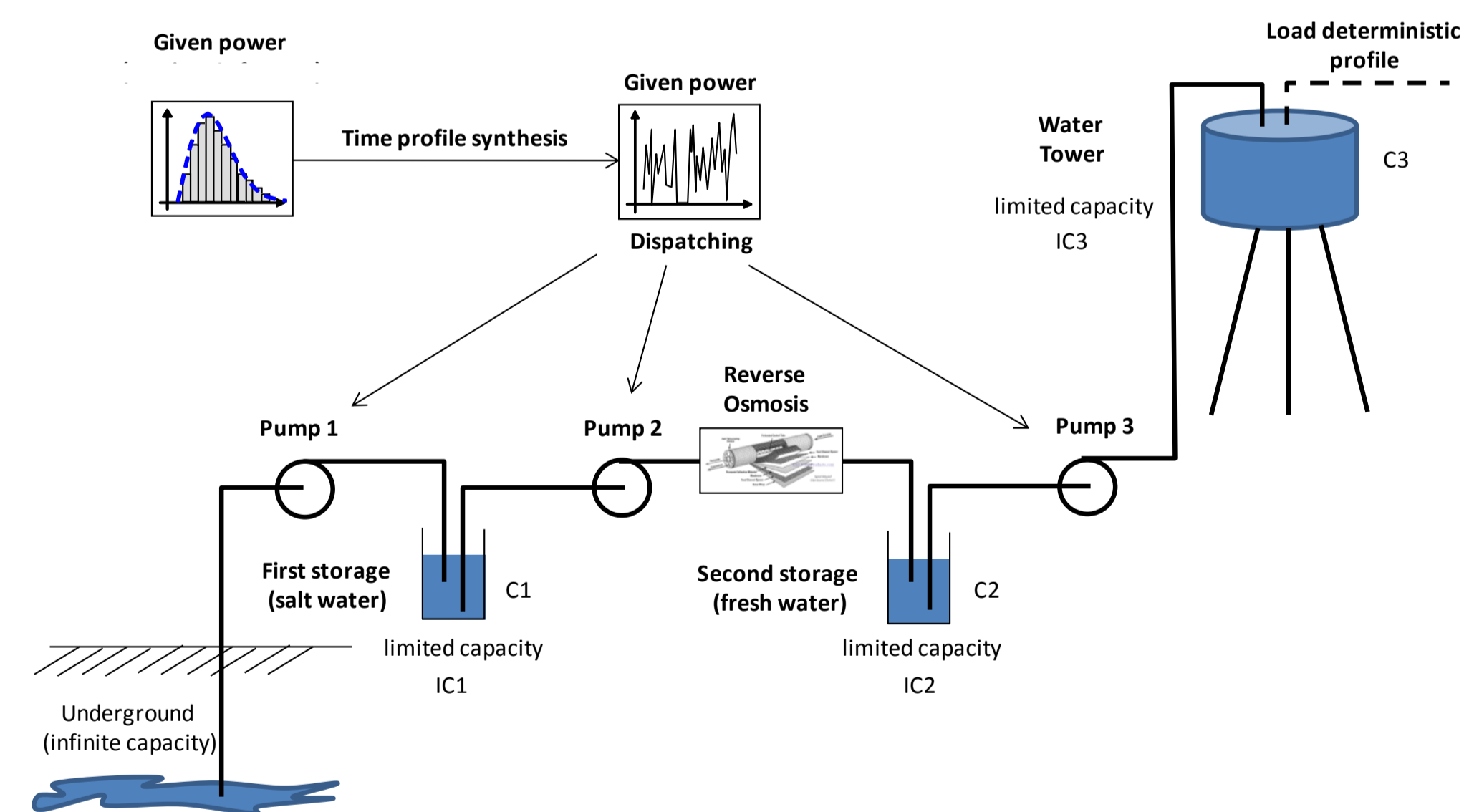
Challenge

Non-linearities come from energy efficiency functions



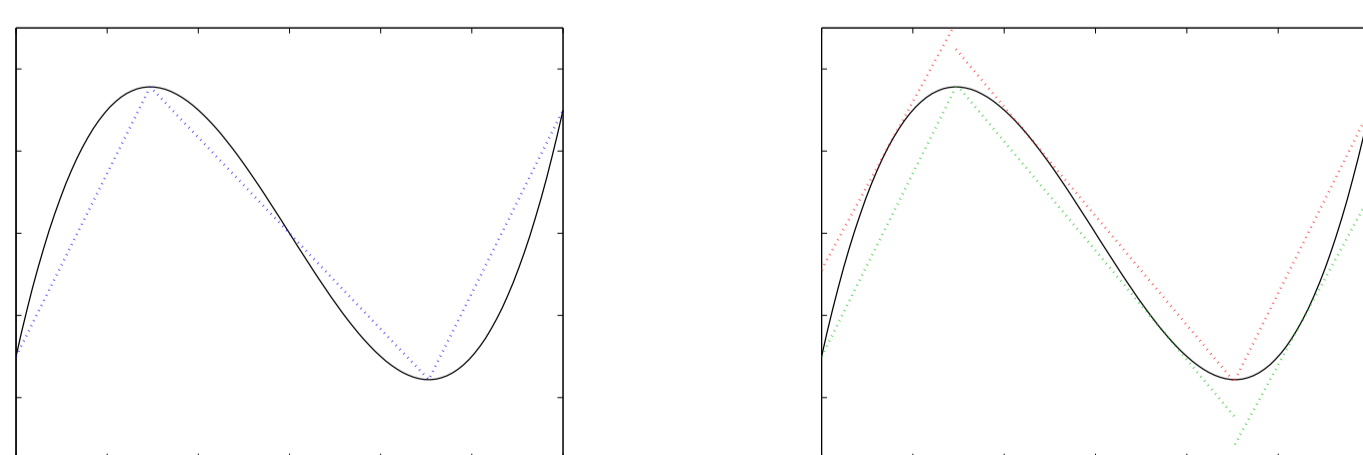
Piecewise linear bounding procedures for the optimal management of water pumping and desalination processes

Proof of concept on a water pumping and desalination system [CLAIO 2014, IFORS 2014]



Solution Method

- Step 1 : Piecewise linear bounding vs classic approx. of the nonlinear energy transfer/efficiency functions



- Step 2 : Reformulation of the problem into two mixed integer problems (MILP and MILP)

- the problem is originally a MINLP
- using the pair of bounding functions previously defined

Number of sectors per tolerance value

ε	Pump 1		(Pump 2+RO)		Pump 3	
	\bar{n}_{p1}	n_{p1}	\bar{n}_{p2}	n_{p2}	\bar{n}_{p3}	n_{p3}
5%	2	2	11	21	3	2
1%	5	5	21	29	8	5
0.5%	8	7	35	62	13	7
0.3%	10	9	43	74	17	9

Results

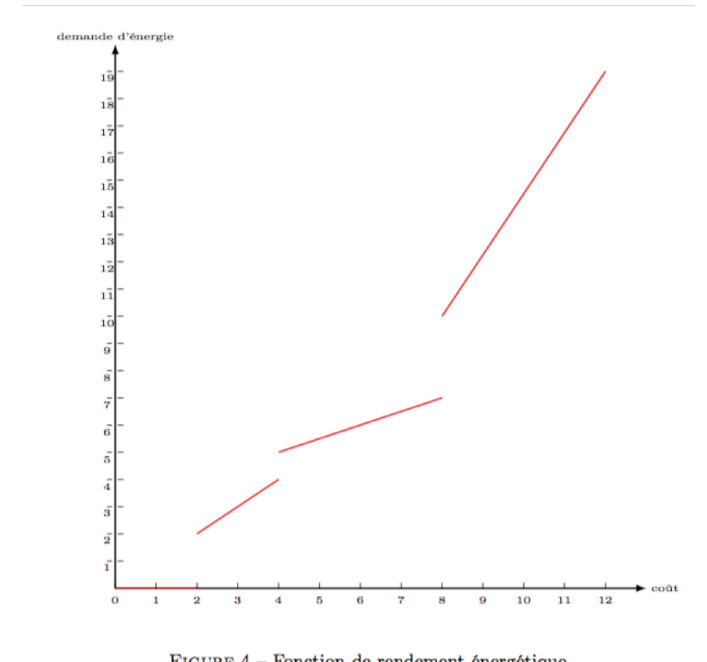
Lower and upper bound obtained

ε	MILP		MILP		UB*	Gap %	opt
	UB	s	LB	s			
5%	20580	4	19740	15	-	4.25	no
1%	20100	15	19920	140	-	0.9	no
0.5%	20040	178	19980	117	-	0.3	no
0.3%	20040	64	19980	321	19980	0.0	yes

Generic problem : Scheduling with non-reversible energy sources

Data

- Set of time periods T
- Set of activities \mathcal{A}
 - r_i, d_i, p_i : release date, due date, duration of activity i
 - b_i : constant instantaneous energy demand of activity i
- Set of non-reversible energy sources \mathcal{S}
 - ρ^s : piecewise-linear efficiency function for source s (x-axis = cost, y-axis = demand and $\rho^s(x) = 0, \forall x < 0, \forall s \in \mathcal{S}$).
- a_{it} : constant term equal to 1 if $t \in [r_i, d_i]$ and 0 otherwise



Decision variables

- x_{it} : binary, = 1 iff activity i is ongoing at time period t

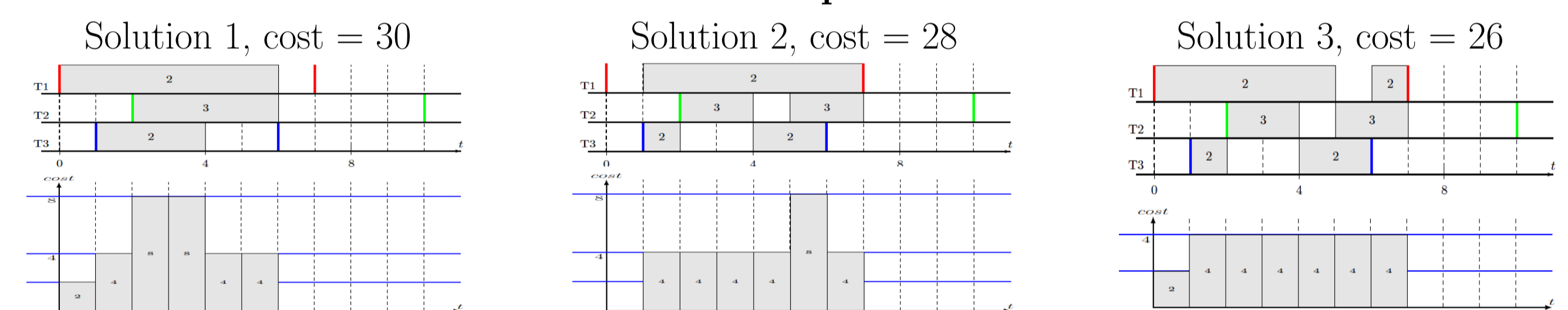
Formulation

$$\begin{aligned} (\text{CF}) \min & \sum_{t \in T} \rho^{-1} \left(\sum_{i \in \mathcal{A}} b_i x_{it} \right) \\ \text{s.t.} & \sum_{t \in T} a_{it} x_{it} \geq p_i, \quad \forall i \in \mathcal{A} \\ & x_{it} \in \{0, 1\}, \quad \forall i \in \mathcal{A}, t \in T \end{aligned}$$

Complexity and equivalence between single and multiple sources

- **Theorem 1** The problem is NP hard by reduction from discrete bin packing
- **Theorem 2** For any problem with multiple sources, there is an equivalent single source problem

Example



Dantzig-Wolfe decomposition and Branch&Price Method

Extended formulation based on feasible subsets of activities

- Set of activity sets executable in parallel \mathcal{L} (at any given time period \bar{L}_t)
- Set of activities belonging to set l : demand b_l , cost c_l , time window $[R_l, D_l]$
- a_{il} : binary constant term equal to 1 iff i belongs to set l
- Variable y_{lt} : binary, = 1 iff activity set l is being executed at time t

The linear relaxation of the master problem

$$\begin{aligned} (\text{LRMP}) \min & \sum_{t \in T} \sum_{l \in \bar{L}_t} c_l y_{lt} \\ x_{it} - \sum_{l \in \bar{L}_t} a_{il} y_{lt} &= 0, \quad \forall i \in \mathcal{A}, t \in T \\ \sum_{l \in \bar{L}_t} \sum_{i \in \mathcal{A}} a_{il} y_{lt} &\geq p_i, \quad \forall i \in \mathcal{A} \\ - \sum_{l \in \bar{L}_t} y_{lt} &\geq -1, \quad \forall t \in T \\ x_{it} &\leq 1 \quad \forall i \in \mathcal{A}, t \in T \\ y_{lt} &\geq 0 \quad \forall t \in T, l \in \bar{L}_t \\ x_{it} &\geq 0 \quad \forall i \in \mathcal{A}, t \in T \end{aligned}$$

The resulting dual (DLMRP) is :

$$\begin{aligned} \max & \sum_{i \in \mathcal{A}} p_i u_i - \sum_{t \in T} v_t + \sum_{i \in \mathcal{A}} \sum_{t \in T} z_{it} \\ \sum_{i \in \mathcal{A}} a_{il} (u_i - w_{it}) - v_t &\leq c_l, \quad \forall t \in T, l \in \bar{L}_t \\ w_{it} + z_{it} &\leq 0, \quad \forall i \in \mathcal{A}, t \in T \\ w_{it} &\in \mathbb{R}, \quad \forall i \in \mathcal{A}, t \in T \\ u_i &\geq 0, \quad \forall i \in \mathcal{A} \\ v_t &\geq 0, \quad \forall t \in T \end{aligned}$$

Therefore, the reduced cost of a column y_{lt} is :

$$c_l - \sum_{i \in \mathcal{A}} a_{il} (u_i) - v_t$$

Two column generation schemes

SP1 : all dual values u_i, v_t, w_{it} problem data a_{il}, b_l, ρ^{-1} } \rightarrow { best time t' best task set l' } \parallel { a predefined time t' SP2 : related dual values u_i, w_{it} problem data a_{il}, b_l, ρ^{-1} } \rightarrow { best task set l' }

Results

	Extended Model Branch & Price	Compact Model Black Box Solver
#Opt/#Instances	261/288	5/288
Ratio avg (min, max)	99.81% (77.47 %, 100%)	84.21 % (57.85 %, 99.99 %)
Time avg (min, max)	540 s (< 1 s, 3603 s)	1034 s (7 s, 1927 s)
Nbnodes avg (min, max)	171 (1, 9755)	2174044 (57157, 13470373)