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COMBINATORIAL OPTIMIZATION WITH MULTIPLE RESOURCES AND ENERGY CONSTRAINTS

LAAS-CNRS

Optimisation sous contraintes de ressources énergétiques multiples (OREM) - PGMO project (2013-2015) - 1st year

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OREM project : context and objectives

Context

The integration of energy constraints in deterministic scheduling models, such as job-shop scheduling or resource-constrained project scheduling, yields a combinatorial optimization challenge. It follows that the literature on this subject is sparse. Pre-existing studies involve multiple energy sources and general non-linear efficiency functions, but generally no scheduling. All our previous work on scheduling under energy constraints considered linear (and even identical) energy efficiency functions, which oversimplifies the problem.

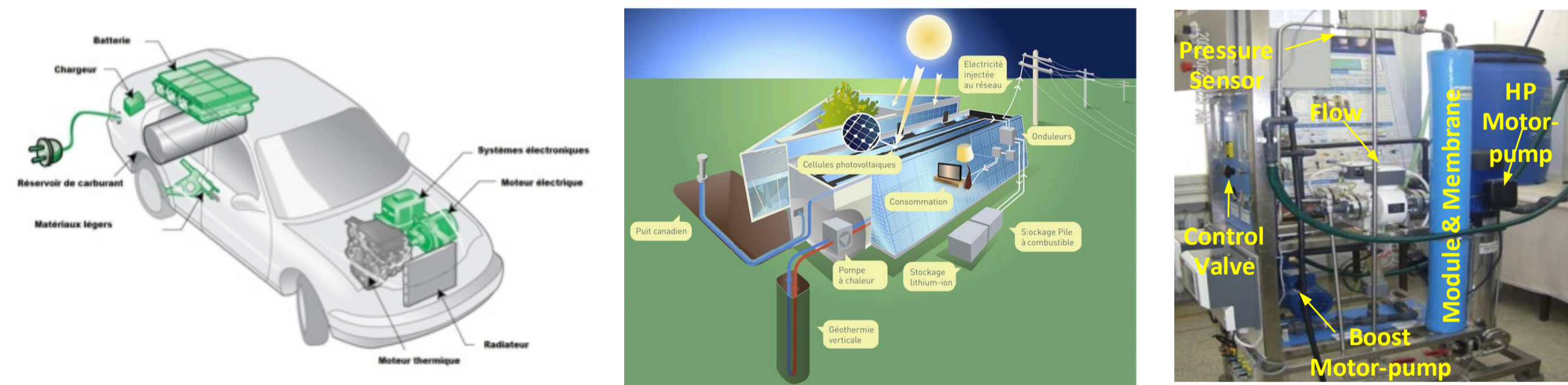
Objectives

- Address the (combinatorial) optimization challenge of integrating energy sources constraints (physical, technological and performance characteristics) in deterministic (scheduling) models.
- Solve explicitly and in an integrated fashion the resulting energy resource allocation problems and energy-consuming activity scheduling problems with non linear energy efficiency functions.

Applications and challenge

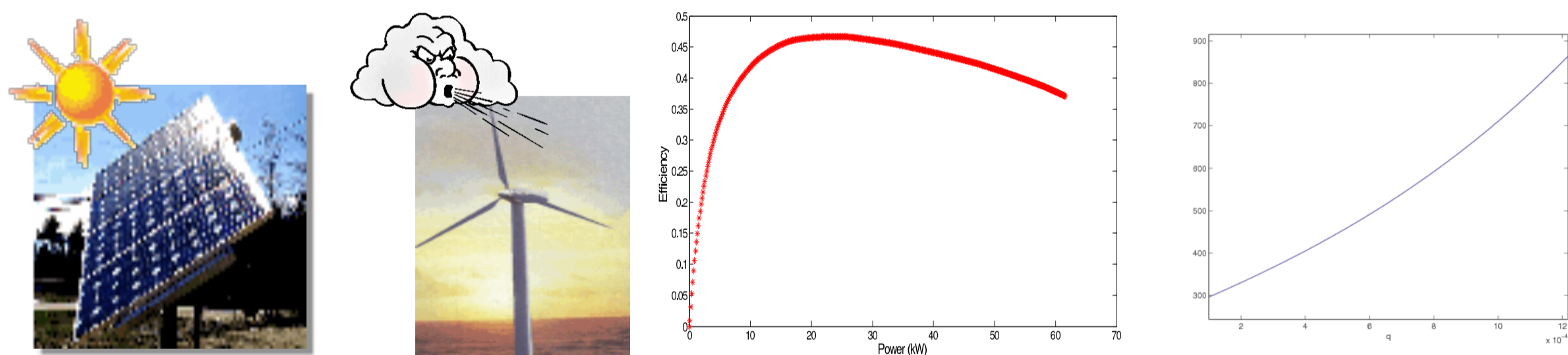
Applications

Scheduling for hybrid electric vehicles, intelligent buildings, processes and manufacturing



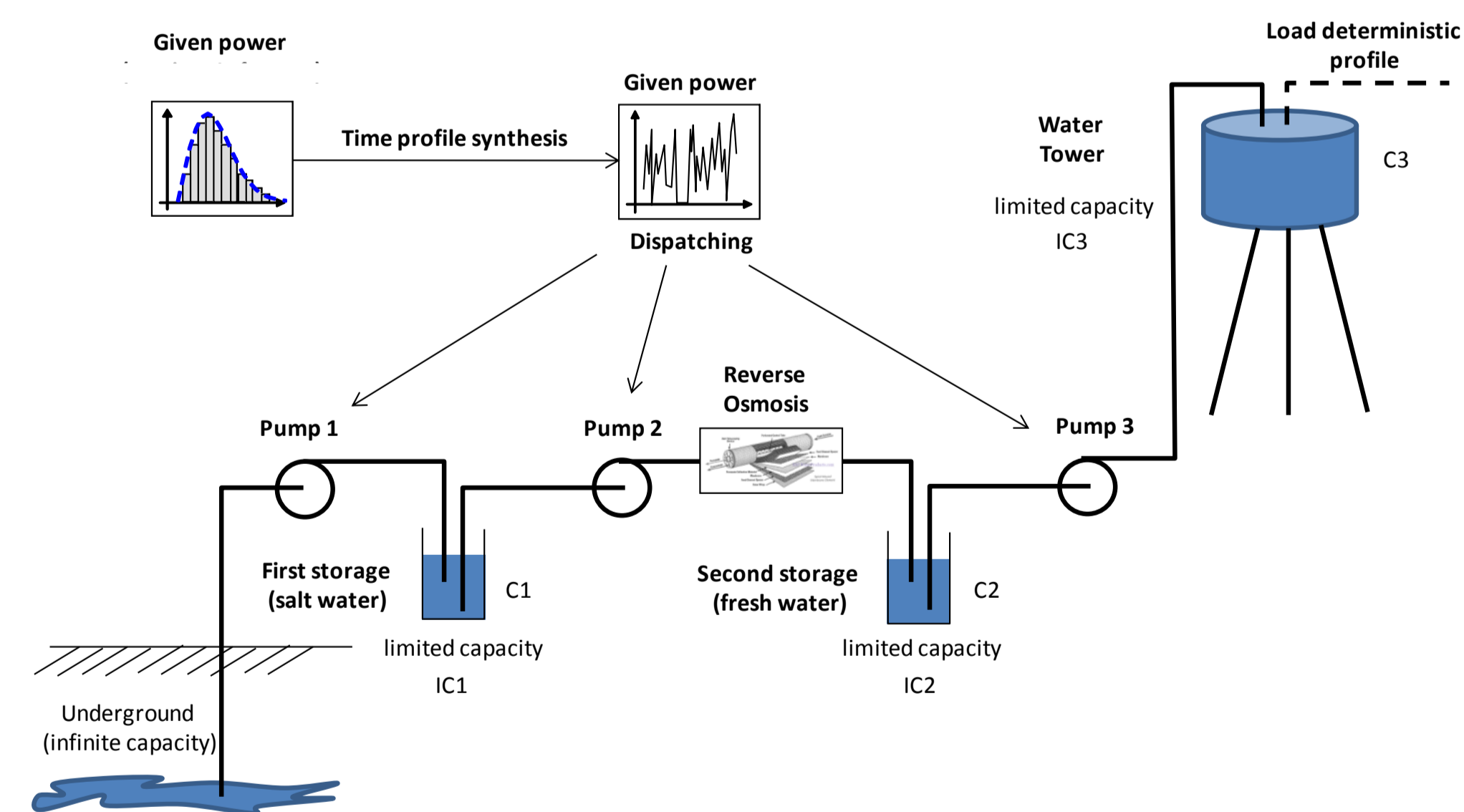
Challenge

Non-linearities come from energy efficiency functions



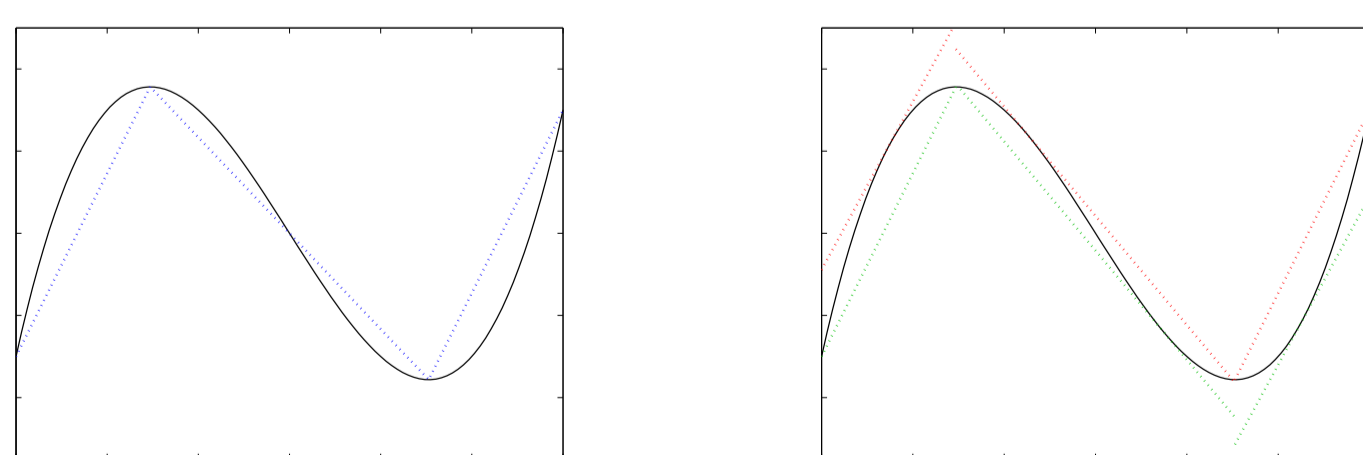
Piecewise linear bounding procedures for the optimal management of water pumping and desalination processes

Proof of concept on a water pumping and desalination system [CLAIO 2014, IFORS 2014]



Solution Method

- Step 1 : Piecewise linear bounding vs classic approx. of the nonlinear energy transfer/efficiency functions



- Step 2 : Reformulation of the problem into two mixed integer problems (MILP and MILP)

- the problem is originally a MINLP
- using the pair of bounding functions previously defined

Number of sectors per tolerance value

ε	Pump 1		(Pump 2+RO)		Pump 3	
	\bar{n}_{p1}	\bar{n}_{p2}	\bar{n}_{p2}	\bar{n}_{p2}	\bar{n}_{p3}	\bar{n}_{p3}
5%	2	2	11	21	3	2
1%	5	5	21	29	8	5
0.5%	8	7	35	62	13	7
0.3%	10	9	43	74	17	9

Results

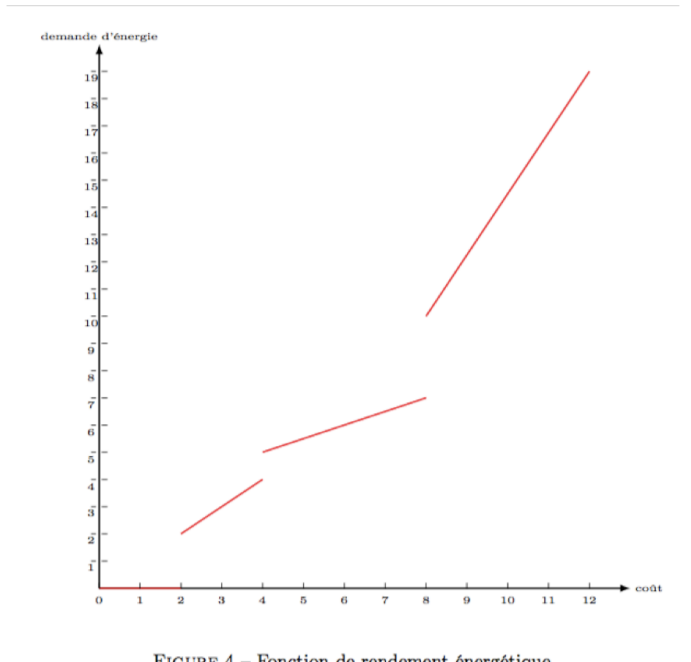
Lower and upper bound obtained

ε	MILP		MILP		UB*	Gap %	opt
	UB	s	LB	s			
5%	20580	4	19740	15	-	4.25	no
1%	20100	15	19920	140	-	0.9	no
0.5%	20040	178	19980	117	-	0.3	no
0.3%	20040	64	19980	321	19980	0.0	yes

Generic problem : Scheduling with non-reversible energy sources

Data

- Set of time periods T
- Set of activities \mathcal{A}
 - r_i, d_i, p_i : release date, due date, duration of activity i
 - b_i : constant instantaneous energy demand of activity i
- Set of non-reversible energy sources \mathcal{S}
 - ρ^s : piecewise-linear efficiency function for source s (x-axis = cost, y-axis = demand and $\rho^s(x) = 0, \forall x < 0, \forall s \in \mathcal{S}$).
- a_{it} : constant term equal to 1 if $t \in [r_i, d_i]$ and 0 otherwise



Decision variables

- x_{it} : binary, = 1 iff activity i is ongoing at time period t

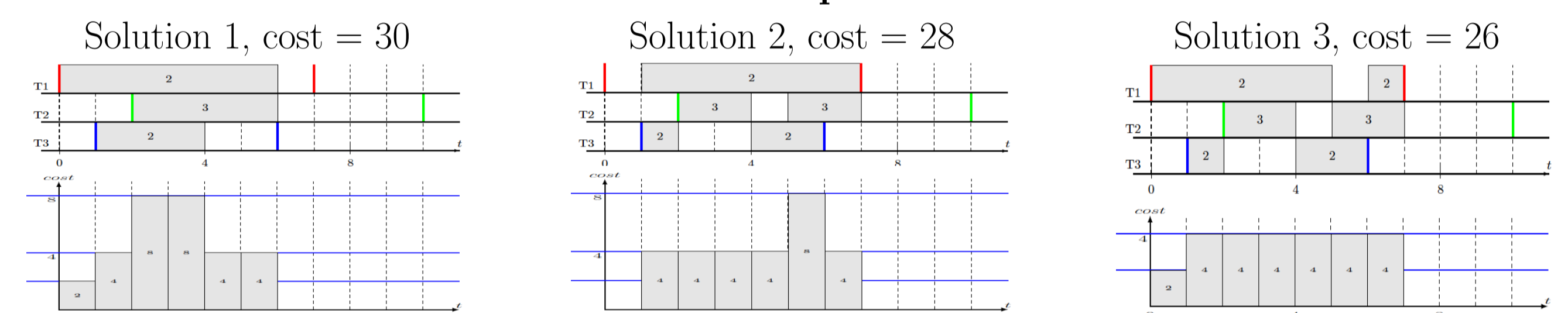
Formulation

$$\begin{aligned} (\text{CF}) \min & \sum_{t \in T} \rho^{-1} \left(\sum_{i \in \mathcal{A}} b_i x_{it} \right) \\ \text{s.t.} & \sum_{t \in T} a_{it} x_{it} \geq p_i, \quad \forall i \in \mathcal{A} \\ & x_{it} \in \{0, 1\}, \quad \forall i \in \mathcal{A}, t \in T \end{aligned}$$

Complexity and equivalence between single and multiple sources

- **Theorem 1** The problem is NP hard by reduction from discrete bin packing
- **Theorem 2** For any problem with multiple sources, there is an equivalent single source problem

Example



Dantzig-Wolfe decomposition and Branch&Price Method

Extended formulation based on feasible subsets of activities

- Set of activity sets executable in parallel \mathcal{L} (at any given time period \bar{L}_t)
- Set of activities belonging to set l : demand b_l , cost c_l , time window $[R_l, D_l]$
- a_{il} : binary constant term equal to 1 iff i belongs to set l
- Variable y_{lt} : binary, = 1 iff activity set l is being executed at time t

The linear relaxation of the master problem

$$\begin{aligned} (\text{LRMP}) \min & \sum_{t \in T} \sum_{l \in \bar{L}_t} c_l y_{lt} \\ x_{it} - \sum_{l \in \bar{L}_t} a_{il} y_{lt} &= 0, \quad \forall i \in \mathcal{A}, t \in T \\ \sum_{l \in \bar{L}_t} \sum_{i \in \mathcal{A}} a_{il} y_{lt} &\geq p_i, \quad \forall i \in \mathcal{A} \\ - \sum_{l \in \bar{L}_t} y_{lt} &\geq -1, \quad \forall t \in T \\ x_{it} &\leq 1 \quad \forall i \in \mathcal{A}, t \in T \\ y_{lt} &\geq 0 \quad \forall t \in T, l \in \bar{L}_t \\ x_{it} &\geq 0 \quad \forall i \in \mathcal{A}, t \in T \end{aligned}$$

The resulting dual (DLMRP) is :

$$\begin{aligned} \max & \sum_{i \in \mathcal{A}} p_i u_i - \sum_{t \in T} v_t + \sum_{i \in \mathcal{A}} \sum_{t \in T} z_{it} \\ & \sum_{i \in \mathcal{A}} a_{il} (u_i - w_{it}) - v_t \leq c_l, \quad \forall t \in T, l \in \bar{L}_t \\ & w_{it} + z_{it} \leq 0, \quad \forall i \in \mathcal{A}, t \in T \\ & w_{it} \in \mathbb{R}, \quad \forall i \in \mathcal{A}, t \in T \\ & u_i \geq 0, \quad \forall i \in \mathcal{A} \\ & v_t \geq 0, \quad \forall t \in T \end{aligned}$$

Therefore, the reduced cost of a column y_{lt} is :

$$c_l - \sum_{i \in \mathcal{A}} a_{il} (u_i) - v_t$$

Two column generation schemes

SP1 : all dual values u_i, v_t, w_{it} problem data a_{il}, b_i, ρ^{-1} } \rightarrow { best time t' best task set l' } $\left\| \begin{array}{l} \text{a predefined time } t' \\ \text{SP2 : related dual values } u_i, w_{it}' \\ \text{problem data } a_{il}, b_i, \rho^{-1} \end{array} \right\} \rightarrow$ { best task set l' }

Results

	Extended Model Branch & Price	Compact Model Black Box Solver
#Opt/#Instances	261/288	5/288
Ratio avg (min, max)	99.81% (77.47 %, 100%)	84.21 % (57.85 %, 99.99 %)
Time avg (min, max)	540 s (< 1 s, 3603 s)	1034 s (7 s, 1927 s)
Nbnodes avg (min, max)	171 (1, 9755)	2174044 (57157, 13470373)