Combinatorial optimization with multiple resources and energy constraints
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Underground Control

Combinatorial optimization with multiple resources and energy constraints
Optimisation sous contraintes de ressources énergétiques multiples (OREM) - PGMO project (2013-2015) - 1st year
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OREM project: context and objectives

Context
The integration of energy constraints in deterministic scheduling models, such as job-shop scheduling or resource-constrained project scheduling, yields a combinatorial optimization challenge. It follows that the literature on this subject is sparse. Pre-existing studies involve multiple energy sources and general non-linear efficiency functions, but generally no scheduling. All our previous work on scheduling under energy constraints considered linear (and even identical) energy efficiency functions, which oversimplifies the problem.

Objectives
- Address the (combinatorial) optimization challenge of integrating energy sources constraints (physical, technological and performance characteristics) in deterministic (scheduling) models.
- Solve efficiently and in an integrated fashion the resulting energy resource allocation problems and energy-consuming activity scheduling problems with non linear energy efficiency functions.

Applications and challenges

Applications
Scheduling for hybrid electric vehicles, intelligent buildings, processes and manufacturing

Non-linearities come from energy efficiency functions

Piecewise linear bounding procedures for the optimal management of water pumping and desalination processes

Proof of concept on a water pumping and desalination system [CLAIO 2014, IFORS 2014]

Solution Method
- Step 1: Piecewise linear bounding vs classic approx. of the nonlinear energy transfer/efficiency functions
- Step 2: Reformulation of the problem into two mixed integer problems (MILP and MINLP) - the problem is originally a MINLP

Problem data
- Set of time periods \( T \)
- Set of activities \( A \)
- \( r_i, d_i, p_i \): release date, due date, duration of activity \( i \)
- \( b_i \): constant instantaneous energy demand of activity \( i \)
- Set of non-reversible energy sources \( S \)
- \( \rho^p \): piece-wise linear efficiency function for source \( s \) (\( \rho^p(x) = \begin{cases} \text{cost} & \text{if } x \leq 0, \ x \in S \end{cases} \) otherwise)
- \( a_i \): constant term equal to 1 if \( i \in \{p_i\} \) and 0 otherwise

Decision variables
- \( x_d \): binary, \( x_d = 1 \) if activity \( d \) is ongoing at time \( t \)

Formulation
\[
\begin{align*}
\min \sum_{t \in T} w_{d,t} \sum_{i \in A} x_{d,t} & \quad \forall i \in A, t \in T \\
\sum_{t \in T} x_{d,t} & \geq p_i \quad \forall i \in A \\
\sum_{t \in T} x_{d,t} & \geq -1 \quad \forall i \in T \\
x_{d,t} & \leq 1 \quad \forall i \in A, t \in T \\
x_{d,t} & \geq 0 \quad \forall i \in A, t \in T \\
\end{align*}
\]

Complexity and equivalence between single and multiple sources
- Theorem 1: The problem is NP hard by reduction from discrete bin packing
- Theorem 2: For any problem with multiple sources, there is an equivalent single source problem

The linear relaxation of the master problem
\[
\begin{align*}
\text{min} & \quad \sum_{i \in A} \sum_{t \in T} w_{d,t} x_{d,t} \\
\text{s.t.} & \quad \sum_{i \in A} \sum_{t \in T} w_{d,t} x_{d,t} \geq p_i \quad \forall i \in A \\
\end{align*}
\]

The resulting dual (DLMRP) is:
\[
\begin{align*}
\max & \quad \sum_{i \in A} \sum_{t \in T} c_{d,t} y_{d,t} \\
\text{s.t.} & \quad \sum_{i \in A} \sum_{t \in T} c_{d,t} y_{d,t} \leq w_{d,t} \quad \forall i \in A, t \in T \\
\end{align*}
\]

Therefore, the reduced cost of a column \( y_0 \) is:
\[
\begin{align*}
\epsilon_l - \sum_{i \in A} \sum_{t \in T} c_{d,t} y_{d,t} & \quad \forall i \in A, t \in T \\
\end{align*}
\]

Two column generation schemes:

- Method

Extended formulation based on feasible subsets of activities
- Set of activity sets executable in parallel \( \mathcal{L} \) at any given time period \( t \)
- Set of activities belonging to set \( l \): demand \( b_l \), cost \( c_l \), time window \( [R_l, D_l] \)
- \( x_{d,t} \): binary constant term equal to 1 if \( d \) belongs to set \( l \)
- Variable \( y_{d,t} \): binary, \( x_{d,t} = 1 \) if activity \( d \) is being executed at time \( t \)

The resulting dual (DLMRP) is:
\[
\begin{align*}
\text{min} & \quad \sum_{i \in A} \sum_{t \in T} w_{d,t} x_{d,t} \\
\text{s.t.} & \quad \sum_{i \in A} \sum_{t \in T} w_{d,t} x_{d,t} \geq p_i \quad \forall i \in A \\
\end{align*}
\]

Results

Extended Model | Compact Model
--- | ---
Hypervolume | 261.1 ± 85.8 | 261.1 ± 85.8
Hit ratio | 99.81% (75.9 %, 100%) | 99.81% (75.9 %, 100%)
Number of runs | 10 | 10 | 10 | 10
Number of runs | 500 ± 309.1 | 500 ± 309.1 | 500 ± 309.1 | 500 ± 309.1
Total CPU time | 227 ± 694.1 | 227 ± 694.1 | 227 ± 694.1 | 227 ± 694.1